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Controlling Peak-To-Average Power Ratio In Orthogonal Frequency Division Multiplexing System

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Abstract

A major drawback of Orthogonal Frequency Division Multiplexing at the transmitter side is the high peak-to-average power ratio (PAPR) of the signal. Conventional solutions for this problem are to use a linear amplifier (or to back-off the operating point of a nonlinear amplifier). Recently many techniques are introduced to reduce PAPR , Selected mapping (SLM) is one of the promising PAPR reduction techniques , where some statistically independent data blocks (carrying same information) are generated from an Orthogonal Frequency Division Multiplexing data block using a set of phase sequences and one with the lowest PAPR is chosen and transmitted .The proposed PAPR reduction scheme in this paper used Riemann sequences as a phase vectors for Selected Mapping (in the frequency domain) along with simple mathematic processing in time domain will enhanced PAPR reduction by changing the mean and variance values of the processed Orthogonal Frequency Division Multiplexing signals (Time Domain Statistical Control) .

For system with 128 sub-carriers and QPSK mapping , the proposed method shows almost 2dB and 4dB for conventional SLM and SLM with square rooting SQRT techniques respectively at probability $(PAPR > threashold\ value)$ *of* 10^{-3} *.*

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الخلاصة

التقسيم الترددي المتعامد المتعدد هو الحل الواحد لمشكلة أرسال البيانات بسرعة عالية خلال القنوات ذات أنتقائية ترددية للأضمحلال . مشكلة منظومات التقسيم الترددي المتعامد المتعدد الرئيسية هي نسبة (أعلى-الى-معدل القدرة) PAPR العالية للأشارة المرسلة . من الحلول التقليدية للمشكلة هو أستخدام مضخم قدرة بخواص خطية واسعة أو بأرجاع نقطة العمل للمضخم اللاخطي ليعمل بالمنطقة الخطية . تم مؤخرآ أستحداث عدة تقنيات لتقليل PAPR، تقنية الأنتقاء SELECTIVE MAPPING هي واحدة من التقنيات الواعدة حيث يتم توليد عدة صور لبيانات التقسيم الترددي المتعامد المتعدد المراد أرسالها (تحمل نفس المعلومات الا أنها غير معتمدة أحصائيآ) وذلك

بأستخدام مجموعة من سلاسل طورية يتم أنتقاء سلسلة واحدة فقط بأقل PAPR ليتم أرسالها . تقنية تقليل PAPR المقترحة في هذا البحث تستخدم سلسلة ريمان كمتجه طوري لتقنية SLM (في المجال الترددي للأشارة) مع معالجة رياضية بسيطة للأشارة في المجال الزمني تعزز تقليل PAPR بتغير قيم المتوسط والتباين الأحصائي لأشارة التقسيم الترددي المتعامد المتعدد (سيطرة أحصائية في المجال الزمني) .

بالنسبة لمنظومة ذات 128 حامل للمعلومات التي تستخدم تعديل أزاحة طور رباعية QPSK فأن الطريقة المقترحة تظهر تقليل في قيمة PAPR مامقداره 2dB و4dB بالمقارنة مع أستخدام SLM التقليدية وSLM مع الجذر التربيعي على التوالي وذلك عند أحتمالية (PAPR < قيمة العتبة)مقدارها 10^{-3}.

1. Introduction

Orthogonal Frequency Division Multiplexing systems has been widely used for high data rate transmission applications due to its high spectral efficiency and robustness to the frequency selective fading channels [1]. One major drawback of Orthogonal Frequency Division Multiplexing is the high peak to average power ratio (PAPR) of the output signal. So to overcome nonlinearity effects which cause adjacent channel interference , highly linear power amplifiers with a large back-off required at the transmitter . High values of PAPR result in low efficient usage of the word length used in digital to analogue convertor results in quantization noise . Hence dynamic range reduction plays an important role in power and band-limited communication systems.

In literature, a variety of different PAPR reduction schemes has been proposed. The most popular schemes are based on multiple signal representation selected mapping (SLM) [2], partial transmit sequences (PTS) [3] , tone reservation [4], PARP reduction based on redundant coding techniques [5] , in [1] traditional SLM method has been combined with square rooting as a new PAPR reduction method for the Orthogonal Frequency Division Multiplexing system. However the literature [6] gives an overview of different PAR reduction schemes . The aim of this work is to study this important issue i.e. PAPR in more details .

2. PAPR of Orthogonal Frequency Division Multiplexing Signal

2.1 Orthogonal Frequency Division Multiplexing Signal

In an Orthogonal Frequency Division Multiplexing system a frequency bandwidth B is divided into N non-overlapping orthogonal subcarriers of bandwidth Δf where $B = N.\Delta f$. For a given symbol, each subcarrier is modulated with a complex value taken from a known constellation (e.g. QAM, PSK, etc.). Any block period (e.g. with index-m) of the signal can be expressed as [1] ;

$$x(t)_m = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_{m,n} w_n (t - mT) \qquad \ldots\ldots\ldots\ldots\ldots (1)$$

Where m ,n are time and subcarrier indexes respectively , Xm,n is the complex modulation symbol that modulate sub-carrier n and $w_n(t)$ is a window (rectangular pulse) applied to each subcarrier n ;

$$w_n(t) = e^{j2\pi n\Delta f} \quad \text{for } 0 \le t \le T \quad \text{and zero elsewhere} \quad \text{...............} \quad (2)$$

For the all data blocks the total continuous time signal $x(t)$ will be as follow ;

$$x(t)_m = \frac{1}{\sqrt{N}} \sum_{m=0}^{\infty} \sum_{n=0}^{N-1} X_{m,n} w_n(t - mT) \quad \text{.........................} \quad (3)$$

Now , since there is no overlapping between adjacent symbols one can consider a single symbol ,so **equation (3)** will be ;

$$x(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi n\Delta f} \quad \text{..............} \quad (4)$$

If the signal ***x(t)*** is sampled with sampling rate of *1/ N.Df* to produce the discrete time signal ;

$$x_k = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_n e^{j2\pi nk\Delta f/N} \quad for\ 0 \le k \le N-1 \text{..............} \quad (5)$$

2.2 The PAPR Problem

From **equation (5)** the average power of the signal is $E[|x_k|^2] = E[|X_k|^2]$. The conventional definition of the PAPR for the Orthogonal Frequency Division Multiplexing symbol in the time domain x_k can be defined as:

$$PAPR(x_k) = \frac{\max\limits_{0 \le k \le N} |x_k|^2}{E[|x_k|^2]} \text{.................} \quad (6)$$

Where E[.] denotes expected value of a random variable. In fact the PAPR is a figure of merit that describes the dynamic range of the time domain signal. Since symbol spaced sampling given in **(5)** some time misses some of the signal peaks so for better calculation of PAPR we have to oversampling time domain signal samples in **(5)** by factor e.g. *L* this is done by taking *Lk* Inverse Fast Fourier Transform (IFFT) for Data block with (L-1) k zero padding;

$$x_{k/L} = IFFT\left(\left[X_0, X_1, \dots \dots X_{N-1}, 0, \dots . 0_{(L-1)k\ zero}\right]^T\right) \dots \dots \dots (7)$$

The PAPR of L times oversampled signal is ;

$$PAPR(x_{k/L}) = \frac{\max_{0 \le k \le NL} |x_{k/L}|^2}{E[|x_{k/L}|^2]} \dots \dots \dots \dots \dots \dots (8)$$

However, a baseband Orthogonal Frequency Division Multiplexing signal with N sub-channels has PAPR *max* = 10 *log10 N* , and from the central limit theorem it follows that for large values of N (more than 64), the real and imaginary values of x_k become Gaussian distributed ,therefore the amplitude of the signal has a Rayleigh distribution, with a cumulative distribution given by ;

$$F(z) = 1 - e^{-z} \dots \dots \dots (9)$$

The probability that the PAPR is below some threshold level z can be written as,

$$P(PAPR \le z) = (1 - e^{-z})^N \dots \dots \dots \dots \dots (10)$$

The complementary cumulative distribution function (CCDF) of PAPR of an Orthogonal Frequency Division Multiplexing signal is usually used, and can be given as (CCDF = 1 – CDF) so ;

$$P(PAPR > z) = 1 - (1 - e^{-z})^N \dots \dots \dots \dots \dots (11)$$

2.3 Selected Mapping Technique

Selected mapping (SLM) scheme for PAPR reduction [2] takes advantage of the fact that the PAPR of an Orthogonal Frequency Division Multiplexing signal is very sensitive to phase shifts in the frequency-domain data. So PAPR reduction is achieved by multiplying independent phase sequences to the original data and calculating the PAPR of each phase sequence data combination then the combination with the lowest PAPR is transmitted. In other words, the data sequence X is element-wise phased by defined number of phase sequences U each with N-length (number of sub-carrier) , **Figure.1** shows the block diagram of the SLM technique. In the ordinary SLM technique U is a set of markedly different, distinct, pseudo-random but fixed sequences ;

$$P^{(u)} = \left[P_0^{(u)}, P_1^{(u)}, \dots \dots \dots \dots P_{N-1}^{(u)}\right]^T \dots \dots \dots (12)$$

Where; $P_n^{(U)} = e^{j\varphi_n^{(u)}}$, $\varphi_n^{(u)} \in [0,2\pi)$, $for\ \ 0 \le n \le N-1\ \ and\ \ 1 \le u \le U$. as depicted in **Figure.1**, the input information sequence is divided into data block *X*, which consists of *N* symbols, by the serial to parallel (S/P) conversion and then data block X is multiplied carrier wise with each one of the *U* different phase sequences P(u), resulting in a set of *U* different Orthogonal Frequency Division Multiplexing data blocks $X^{(u)}$;

$$X^{(u)} = \left[X_0^{(u)}, X_1^{(u)}, \ldots\ldots\ldots\ldots X_{N-1}^{(u)}\right]^T \ldots\ldots\ldots(13)$$

Where; $X_n^{(U)} = X^{(u)}.P_n^{(U)}$ $,for\ \ 0 \le n \le N-1\ \ and\ \ 1 \le u \le U$. Using Inverse Fast Fourier Transform these U alternative data blocks transformed into time domain $x^{(u)}$ i.e, $x^{(u)} = IFFT(X^{(u)})$, then the transmit sequence $\tilde{x}$ will be selected (the one with the lowest PAPR is selected for transmission) ; $\tilde{x} = x^{(\tilde{u})}$, where the index $\tilde{u}$ chosen according to the min-max argument ;

$$\tilde{u} = \arg\{\min_u \max|x^{(u)}|\} \ldots\ldots\ldots\ldots(14)$$

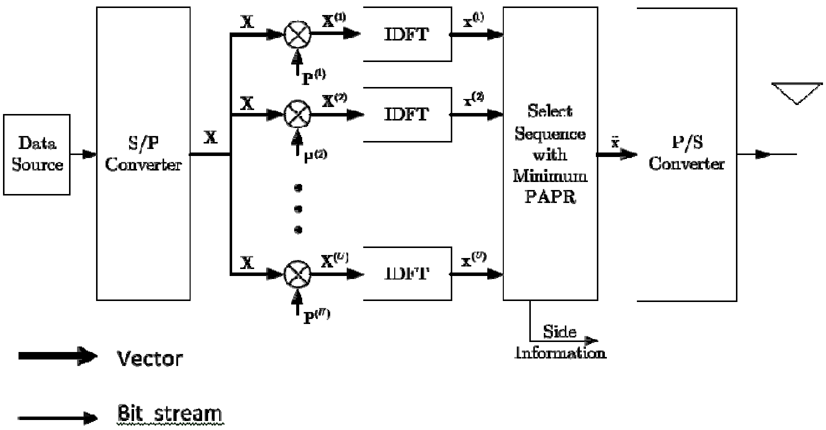


Fig.(1) SLM technique in the transmitter side .

In order to recover the data sequence X , side information must be transmitted to the receiver which is the information on the selected phase sequence results in some loss of transmission efficiency .

2.4 Phase Sequence vectors

2.4.1 Review of Phase Sequence

Generation of phase sequence is one of the important aspects of SLM technique, phase sequences (conventional SLM) are first generated randomly by Bauml et. al [2] from set {±1, ±j}. SLM technique was described. In [11] Zhou et. al. used cubic phase sequences SLM which offer better PAPR reducing capability. Waung et. al [3] show the using of Pseudo random code sequence in multi-carrier CDMA system . In [8] the rows of the Hadamard matrix are used as phase sequence set in SLM scheme . *Zhou* et. al. [12] prove that the performance of chaotic phase sequence is better than that of Walsh-Hadamard sequence and Shapiro-Rudin sequence sets

2.4.2 Riemann Sequence

If we define a matrix A as [13] ;

$$A(i,j) = \begin{cases} i-1 & \text{when i divides j} \quad \text{else ;} \\ & -1 \end{cases} \dots\dots\dots\dots\dots\dots\dots..(15)$$

then *Riemann matrix* B is obtained by removing the first row and first column of the matrix A, So matrix B of fourth order can be expressed as ;

$$\mathrm{B} = \begin{bmatrix} 1 & -1 & 1 & -1 \\ -1 & 2 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 4 \end{bmatrix} \dots\dots\dots\dots\dots\dots....(16)$$

Now rows of ***normalized Riemann matrix*** **(B)** can be used as phase rotation vectors $P^{(u)}$ in equation (12) for PAPR reduction in SLM technique [8]**.** From (16) it's clear that Riemann matrix has a definite structure where in existing phase sequences sets e.g. chaotic sequence the generation of phase sequence, is very random (initial Sequence is random) and this is one of the important aspects of SLM .

2.5 Controlling Time Domain Statistical

The statistical Distribution of signals can be changed using some mathematical processing [8]. The Rayleigh envelope (for example) of any signal will change into Gaussian distribution if the signal is treated with square root function . This varying in statistical distribution accompanied with varying in the mean and variance values of the signal. Same technique (i.e. square rooting) can be used to change the Chi-Square distribution into

Rayleigh distribution . However, the absolute values of the symbols, $|x(n)|$ is Rayleigh distributed and given by [7]as ;

$$|x(n)| = \left|\sqrt{\text{Re.}\{x[n]\}^2 + \text{Im.}\{x[n]\}^2}\right| \quad \ldots\ldots\ldots\ldots\ldots (17)$$

From the central limit theorem and for large number of input samples, the imaginary and real parts of the IFFT outputs will follow Gaussian distributions. Hence, the amplitude (envelope) of the complex valued symbols will have Rayleigh distribution .

Where, $\text{Re.}\{x[n]\}$, $\text{Im.}\{x[n]\}$ are Gaussian distributed real and imaginary parts respectively , and so Orthogonal Frequency Division Multiplexing signal power $|x(n)|^2$ is Gaussian distributed with mean value of $2\sigma^2$, square rooting i.e. $\sqrt[2]{|\,.\,|}$, is exploited here to achieve reduction of the PAPR value of the discrete symbol (equation (5)) ;

$$\tilde{x}_k = \sqrt{|x_k|}\, e^{j\emptyset k} \quad \ldots\ldots \ldots\ldots \ldots\ldots (18)$$

Given that , $\emptyset k$ is the phase of x_k . At the receive opposite operation is applied on the samples i.e. $|x(n)|^2$. The changes of the mean and variance values of the processed signals are reflected directly on the peak and average power values. However the impact of this operation on the average power value is higher than that on the peak power value, which always leads to reduction in the PAPR .

3. The Proposed Method

In this paper we combine the Riemann Sequence - SLM technique and square rooting method which depicted (transmitter block diagram) in **Figure .(2)**

Firstly , input data bits are mapped to one of constellation points (M-PSK or M-QAM) symbol sequences are produced then divided into blocks X each of length N symbols which is the number of subcarriers , then block sequences is element-wise phased by phase sequence from normalized Riemann matrix at the *default-index* (reducing scheme complexity) which is the index with smallest E_i (the average of the squared sum of error of all data blocks K that utilizing the same i_{th} index of the phase sequence). This is conducted only once for a large data set (data blocks number denoted by K) and can summarized in following pseudo-code;

Given $U, [P^{(1)}, P^{(2)}, \ldots\ldots\ldots\ldots P^{(U)}]$

for $k = 1, \dots, K$, $i = 1, \dots, U$, $n = 1, \dots, N$

$$x_{i,n,k} = IDFT\{X_{i,n,k}\},\ calc.PAR_{i,k}\ matrix$$

$$P_i = \min_{k=0,\dots,K-1} PAR_{i,k}$$

$$E_k = \frac{1}{K}\sum_{i=0}^{K-1}\left[PAR_{i,k} - P_i\right]^2$$

end for

The function min returns the minimum PAR , ($\mathbf{P_i}$) for the $\mathbf{i_{th}}$ data block , now ;

$x_{new} = IDFT\{X_{new} \odot P^{(u)}\}$,calculate $PAPR_{new}$,the symbol $\odot$ denotes element-wise multiplication, $P^{(u)}$is the default-index phase sequence .

If $PAPR_{new} > PAPR_{threshold}$ then second comparison is repeated for the same data block with the phase vector at the decremented index (i.e. index-1), this comparison is repeated for all data blocks, then total number of comparisons = $k_{64} \times 1 + k_{63} \times 2 + k_{62} \times 3 + \dots k_2 \times 63$.

Finally the selected time domain signal (i.e. x with min. PAR) is passed through square rooting process to obtain signal with lower PAPR than in the case of Riemann - SLM method alone.

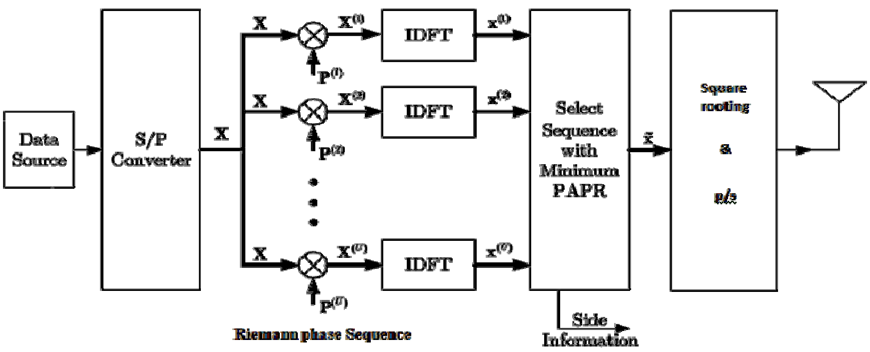


Fig. (2) propose transmitter block diagram .

4. Computer simulations and results

In order to evaluate the performance of the proposed method , system of **Figure.(2)** simulated with MATLAB version R2011a program according to flowchart of **Figure.(3)** and parameters of **Table 1**.

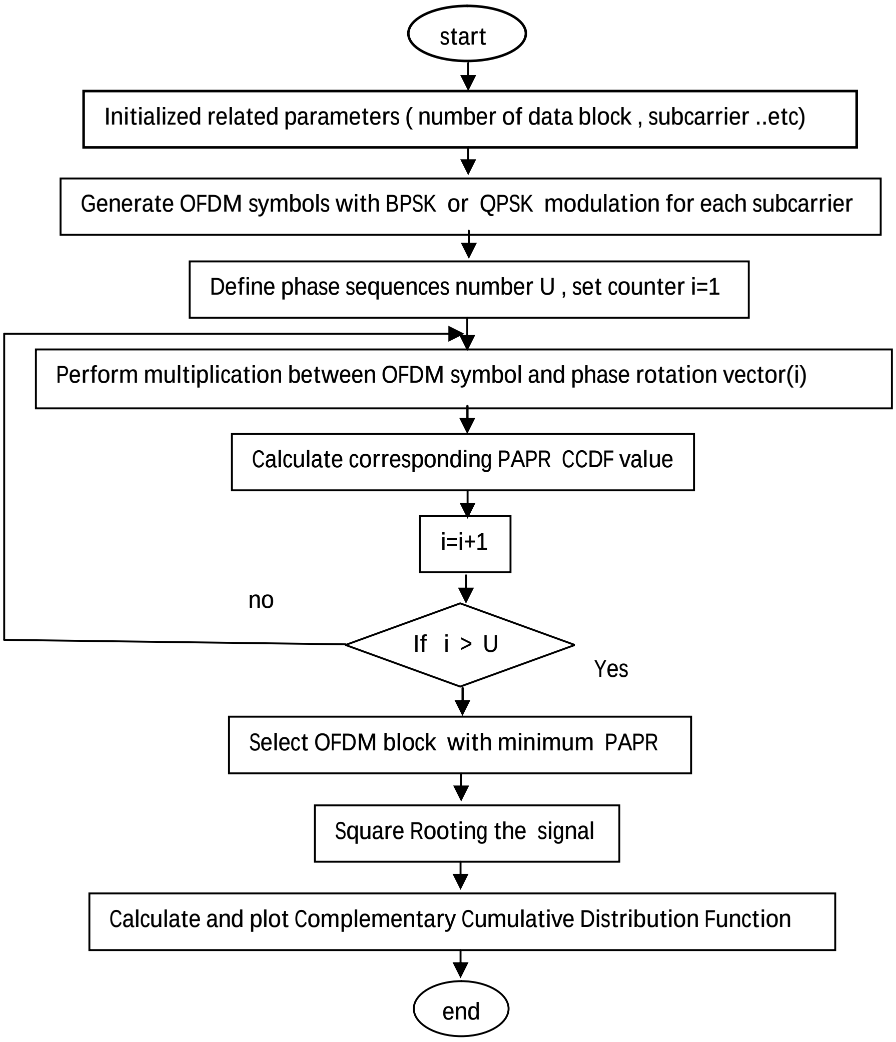
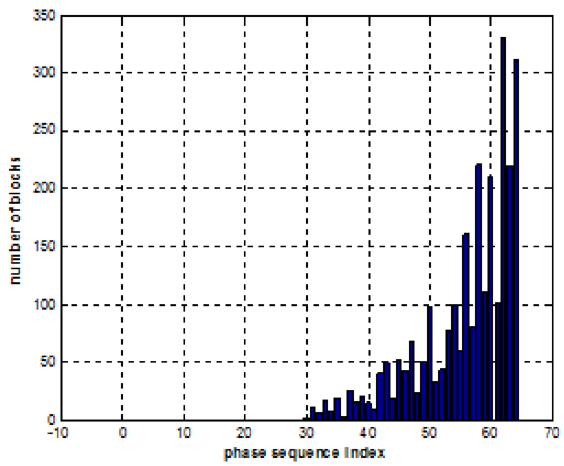
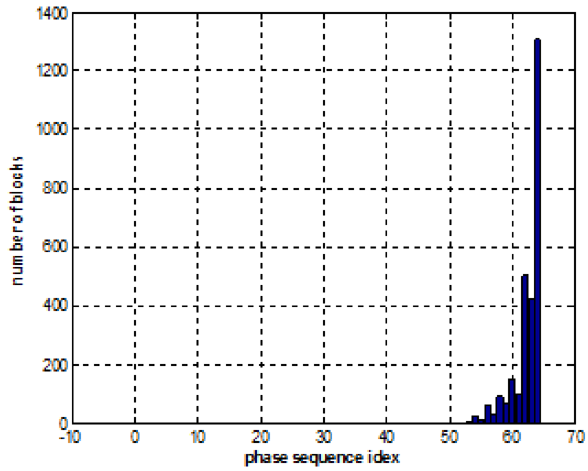


Fig. (3) propose transmitter flowchart .

Table 1:Simulation parameters.

PAP reduction method	OFDM Data symbols	Subcarriers N	Number of phase sequences U	Modulation technique	
SLM	10^{-4}	128	64	QPSK	BPSK
SLM with SQRT					
proposed					



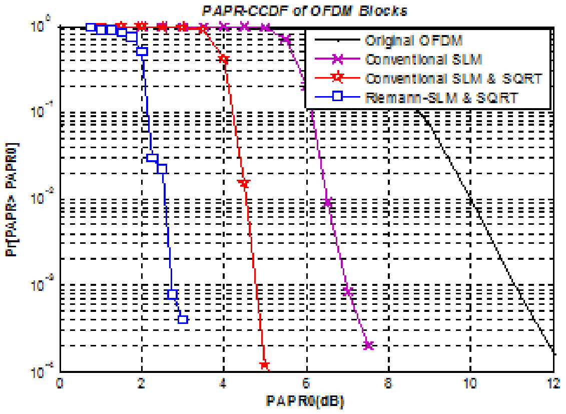


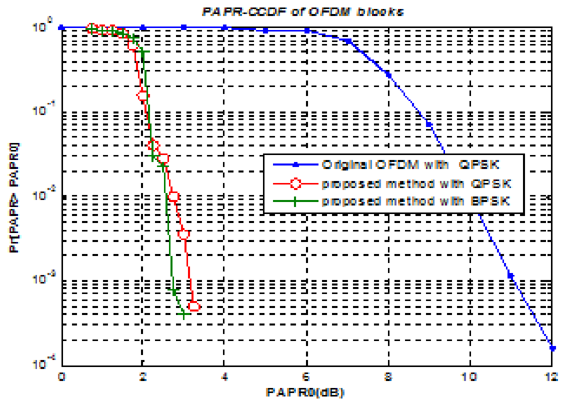
Firstly we will show the results of PAPR distribution for the original Orthogonal Frequency Division Multiplexing signal with N = 128 subcarriers and Quadrature phase shift keying (QPSK) mapping. The results are presented in terms of complementary cumulative distribution function CCDF for an Orthogonal Frequency Division Multiplexing output symbol blocks, then Performance of the proposed method is compared with the conventional technique with same environments **(Figure 4)** , also the influence of modulation technique is investigated for Binary phase shift keying BPSK and Quadrature phase shift keying QPSK mapping , the results is showing in **Figure 5.**

In term of complexity (Number of comparisons) the number of blocks represents the data blocks that satisfy the threshold for different indexes of the phase sequence plotted in figures 6 and 7 for conventional SLM and proposed scheme respectively, where $PAPR_{threshold}$ in the iteration is setting to 2.5dB . Mean and variance of the PAPR observed for all data blocks obtained using different methods is calculated and shown in **Table 2.**

Table 2 . Mean and Standard deviation of PAPR compared in the Confidence interval for whole data.

	N	Mean	Variance
Original OFDM	13	7.8744	11.3243
Conventional SLM plus SQRT	9	3.9433	0.8778
proposed	10	0.1875	0.2104





5. CONCLUSION

In this paper, both time and frequency dimensions of Orthogonal Frequency Division Multiplexing signal are used to obtain a lower PAPR. In term of PAPR performance we can see from PAPR-CCDF plots that using of normalized Riemann matrix as phase sequence vectors with square rooting outperforms traditional SLM technique employing almost the same complexity (number of comparisons that needed to achieve the desired result),while the proposed technique gives improvement in the PAPR performance by 2 dB and 4dB for conventional SLM and SLM with square rooting SQRT respectively for the probability $(PAPR > z)$ of 10^{-3}.

Also ,the mean and variance of PAPR of the whole data block for normalized Riemann sequence set is small compared to ordinary SLM , it allows reduction in PAPR value with no out-of band radiation so the proposed method is efficient in term of spectral distribution and noise immunity, however the threshold value of the power amplifier can be increased for more reduction in the system complexity .

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