

## Numerical Modeling for Prediction of the Dissolved Oxygen Depletion Affected by Heated Water Discharged in Rivers

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### Abstract :

*In this research a numerical model is proposed to study the depletion of dissolved oxygen caused by discharge of heated water in the rivers. This problem was formulated by equations of momentum conservation, energy, turbulences and the relation between temperature and dissolved oxygen were used to describe the dissolved oxygen distribution along the study reach. The partial differential equations were simplified to obtain the system of linear equations that were then solved using Gauss-Elimination method .The numerical model was verified by conducting comparison of observed data from South-Baghdad power station with those computed by the model. The comparison results show an agreement with maximum absolute error (11%) and correlation factor of (0.921). The results of the study showed that increasing the bed roughness from 0.04m to 0.11m causes a longitudinal retardation of dissolved oxygen distribution about (26%) and vertical advance about (14%). Increasing the water-surface slope from 6 cm/km to 12 cm/km causes a longitudinal advance of dissolved oxygen depletion about (19%) and vertical retardation of about (16%) and Assumption of constant density of water causes a longitudinal and vertical retardation of dissolved oxygen depletion about 28% and 6% respectively. Neglecting the effect of viscosity cause longitudinal and vertical advancing of dissolved oxygen depletion about 25% and 5% respectively.*

### النمذجة العددية للتنبؤ بنضوب الاوكسجين المذاب نتيجة لاطلاق المياه الحارة في الانهار

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### الخلاصة :

يهدف البحث الى بناء موديل عددي رياضي لدراسة نضوب الاوكسجين المذاب نتيجة المياه الحارة المطروحة في الانهار. استخدمت معادلات حفظ الزخم والطاقة ومعادلات الاضطراب والعلاقات الرياضية بين درجات الحرارة والاكسجين المذاب لمعرفة توزيع وانتشار كمية الاوكسجين المذاب خلال منطقة الدراسة. تم تحويل المعادلات التفاضلية المختلفة الى معادلات جبرية خطية تم حلها بطريقة الحذف المتناوب. اجريت معايرة الموديل العددي من خلال مقارنة النتائج المحسوبة مع النتائج العملية المقاسة من قبل محطة كهرباء جنوب- بغداد على نهر دجلة و اظهرت

نتائج المقارنة ان الموديل مقبول وفق نسبة خطأ مقدارها ( 11% ) ومعامل ارتباط مقداره ( 0.921 ). أظهرت نتائج الدراسة ان زيادة خشونة قاع النهر من ( 0.04 متر الى 0.11 متر ) يسبب إعاقة لنضوب الاوكسجين المذاب بالاتجاه الطولي بمقدار ( 26% ) وتقدم بالاتجاه العمودي مقداره ( 14% ) كما ان زيادة ميل سطح الماء من 6 سم / كم الى 12 سم / كم يسبب تقدم لنضوب الاوكسجين المذاب بالاتجاه الطولي مقداره ( 19% ) واعاقة بالاتجاه العمودي مقدارها ( 16% ). ان افتراض ثبات كثافة الماء يسبب اعاقه لنضوب الاوكسجين المذاب بالاتجاه الطولي والعمودي بنسبة 28% و 6% على التوالي، كذلك ان اهمال تأثير لزوجة الماء يسبب تقدم طولي وعمودي لنضوب الاوكسجين المذاب بنسبة 25% و 5% على التوالي .

## 1. Introduction :

Dissolved oxygen is an indicator of the health of freshwater. Fish and other aquatic life which requires dissolved oxygen to breath. When dissolved oxygen levels are depleted, aquatic animals become stressed and died. Oxygen depletion is commonly caused by organic pollutants breaking down in waterways, elevated water temperatures or night-time respiration by dense algal blooms in nutrient-rich water. [1]

Dissolved oxygen gets into the water by diffusion from the atmosphere, aeration of the water as it tumbles over falls and rapids, and as a waste product of photosynthesis. Warm water may cause reduced dissolved oxygen levels in stream water. The increased molecular activity of the warm water pushes the oxygen molecules out of the spaces between the moving water molecules. [2]

As the temperature increases the saturation of dissolved oxygen in water decreases and vice versa. Temperature changes may be caused by changes in climate or by human activities such as removing stream-bank vegetation, storing water in dams or discharging heated or cooled water after it has been used in industrial processes. [3]

Dissolved oxygen and temperatures are two of the fundamental variables in lake and pond ecology. By measuring dissolved oxygen and temperature, scientists can gauge the overall condition of water bodies. Aquatic organisms need dissolved oxygen for their survival. While water temperature also directly influences aquatic organisms. It regulates dissolved oxygen concentrations within a lake. Dissolved oxygen and temperature are also used to classify lakes. This fact sheet describes why lakes need dissolved oxygen. [4]

Oxygen is a necessary element to all form of life and for good water quality .Natural stream purification processes require adequate oxygen levels in order to provide for aerobic life forms. As dissolved oxygen levels in water drop below 5 mg/l, aquatic life is put under stress. The lower concentration is the greater stress. Depletion in dissolved oxygen can cause major shifts in the kinds of aquatic organisms found in water bodies. Species that cannot tolerate low levels of dissolved oxygen - mayfly nymphs, stonefly nymphs, and beetle larvae - will be replaced by a few kinds of pollution-tolerant organisms, such as worms and fly larvae. Nuisance algae and anaerobic organisms (that live without oxygen) may also become abundant in waters with low levels of dissolved oxygen. [2]

David [5] studied the dissolved oxygen depletion in Willamette River in U.S.A .The results show the concentration of dissolved oxygen affected by carbonaceous decay and the

nitrification of industries discharge ammonia waste. Misra, et al.<sup>[6]</sup> applied nonlinear mathematical model to study the depletion of dissolved oxygen in a water body caused by industrial and household discharge of organic matters. It is found that if the organic pollutants are continuously discharge into water body, the concentration of dissolved oxygen may become negligibly small.

Naik and Manjapp<sup>[7]</sup> had been studied the water quality aspect with regard to dissolved oxygen for a 24 kilometer stretch of Malaprabha River in Karnataka State, India. This study was to simulate the predicted dissolved oxygen depletion due to the waste load allocation in the river with the ambient observed values of dissolved oxygen, and to ascertain the application of mathematical modeling for predicting the dissolved oxygen in a mixing zone.

The main objectives of the present work are to apply a two-dimensional numerical model to simulate the depletion of dissolved oxygen in rivers and to study the effect of hydraulic conditions of Tigris river such as bed roughness and water-surface slope, also the water river properties such as density and viscosity on the depletion of dissolved oxygen.

**2. Governing Equations and Modeling :**

The numerical model of dissolved oxygen depletion is a set of partial differential equations, in which to account velocity, buoyancy and diffusivity. These parameters will be presented in order to obtain an algebraic equation to be solved by a numerical solution. Equations should be set with the proper initial and boundary conditions. These governing equations that formulated the problem of distribution dissolved oxygen in water bodies are:<sup>[8, 9]</sup>

**1) Momentum conservation equations:**

The momentum equation for an incompressible fluid has the following form:

- Horizontal momentum equation:

$$r \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + W \frac{\partial U}{\partial Z} = \frac{\partial}{\partial X} \left( m \frac{\partial U}{\partial X} \right) + \frac{\partial}{\partial Z} \left( m \frac{\partial U}{\partial Z} \right) \dots\dots\dots (1)$$

- Vertical momentum equation:

$$r \frac{\partial W}{\partial t} + U \frac{\partial W}{\partial X} + W \frac{\partial W}{\partial Z} = \frac{\partial}{\partial X} \left( m \frac{\partial W}{\partial X} \right) + \frac{\partial}{\partial Z} \left( m \frac{\partial W}{\partial Z} \right) - \frac{\partial p}{\partial Z} + r g_z \dots\dots\dots (2)$$

where:

X, Z: Horizontal and vertical coordinates respectively (m).

U, W: Velocity components in the X, Z, direction respectively (m/s).

P: Pressure (N/m<sup>2</sup>).

g : Acceleration due to gravity (m/s<sup>2</sup>).

$\rho$  : Density (kg/m<sup>3</sup>).

$\mu$  : Viscosity (N.s/m<sup>2</sup>).

**2) Thermal energy equation:**

The two dimensional of conservation of thermal energy for the river is:

$$r \frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + W \frac{\partial T}{\partial Z} = \frac{\partial}{\partial X} \left( \frac{m}{s} \frac{\partial T}{\partial X} \right) + \frac{\partial}{\partial Z} \left( \frac{m}{s} \frac{\partial T}{\partial Z} \right) - \frac{\alpha (T - T_r)}{C_p \cdot H} \dots\dots (3)$$

where:

$C_p$ : Specific heat ( w.s/(kg. °K).

$H$ : Heat exchange across the water surface ( kg. °K/(m<sup>2</sup>.s).

$T$ : Heated water temperature (°C).

$T_r$ : River water temperature (°C).

$\alpha$ : Heat transfer coefficient ( W/(m<sup>2</sup>.K).

$\sigma$  : Prandtle Number.

**3) Turbulence kinetic equation:**

The turbulence kinetic energy can be estimated from the following equations:

$$r \frac{\partial K}{\partial t} + U \frac{\partial K}{\partial X} + W \frac{\partial K}{\partial Z} = \frac{\partial}{\partial X} \left( \frac{m}{s} \frac{\partial K}{\partial X} \right) + \frac{\partial}{\partial Z} \left( \frac{m}{s} \frac{\partial K}{\partial Z} \right) + G - r e \dots\dots (4)$$

where:

$K$ : Turbulent Kinetic energy (kg/s).

$G$  : Rate of production of turbulence kinetic energy.

**4) Dissipation rate equation:**

The dissipation rate can be computed from the following equation:

$$r \frac{\partial e}{\partial t} + U \frac{\partial e}{\partial X} + W \frac{\partial e}{\partial Z} = \frac{\partial}{\partial X} \left( \frac{m}{s} \frac{\partial e}{\partial X} \right) + \frac{\partial}{\partial Z} \left( \frac{m}{s} \frac{\partial e}{\partial Z} \right) + C_1 \frac{e}{K} G - C_2 r \frac{e}{K} \dots\dots (5)$$

where:

$\varepsilon$  : Dissipation rate ( m<sup>2</sup>/s<sup>3</sup> ).

$C_1, C_2$  : Constants in kinetic energy turbulence model.

**5) Temperature - dissolved oxygen distribution:**

$$C_s = 14.6 - 0.3943 T + 0.007741 T^2 - 0.0000646 T^3 \dots\dots\dots (6)$$

where  $C_s$  is the concentration of dissolved oxygen ( mg/l).

**6) Turbulent dynamic viscosity distribution:**

$$\mu = C_\mu \rho (K^2 / \varepsilon) \dots\dots\dots (7)$$

where  $C_\mu$  is constant.

**3. Numerical Model Solution :**

**3.1. Solution Technique**

In order to solve the above partial differential equations, finite difference technique is used. Some assumptions are used to transform these equations from non-linear to linear equations <sup>[10 , 11]</sup>. Formulation and simplification of these equations in a two-dimensional numerical model were achieved by using Alternative Direction Implicit-Explicit method (ADI) with Upwinding technique. Then the resulting linear equations were solved using Gauss-Elimination method. A computer program written in Visual Fortran language was used to perform calculations of the simulation model.

**3.2. Input Data**

The input data needed to operate the model and run the computer program are the length reach of the river, depth, hydraulic river properties (such as slope and roughness) , time interval and total time of the study, water river properties (density and viscosity), and the temperature of heated water which were obtained from. <sup>[12]</sup>

**3.3. Initial and Boundary Conditions**

The initial Conditions are:

$U = \max$  (at surface of the river).

$U = 0$  (at the bottom).

$W = 0$  (at every point in the horizontal vertical plane).

$T_r = \text{constant}$ .

The turbulent viscosity ( $\mu$ ) and dissipation rate ( $\varepsilon$ ) in the river are obtained from the following equations: <sup>[11]</sup>

$$m = 0.077 r U_f h \dots\dots\dots (8)$$

$$U_f = (G R S)^{1/2} \dots\dots\dots(9)$$

$$e = S g U \dots\dots\dots (10)$$

where:

*h* : Water depth measured at Z-axis (m).

*S* : Slope of the water surface (m/m).

*U* : Longitudinal velocity components (m/s).

*U<sub>f</sub>* : Friction velocity (m/s).

*R* : Hydraulic radius.

*G* : Acceleration due to gravity (m/s<sup>2</sup>).

Using initial conditions of ( $\mu$ ) and ( $\epsilon$ ) as calculated from equations 8 and 10, the initial conditions for the turbulence kinetic energy (*K*) in river can be obtained from equation 7.

The boundary conditions used in the computer program are:

1. At the upstream boundary:  
At  $X = 0$  and  $0 < Z < D$ .  $W, U, T, K$  and  $\epsilon$  are constants, where *D* is the depth of the river.
2. At the down stream boundary:  
At  $X = L$  and  $0 < Z < D$ , The gradient of  $W, U, T, K, \epsilon$  are assumed equal zero, where *L* is the length of the river.

$$\frac{\partial}{\partial X} (W, U, T, K, e) = 0$$

3. At the surface boundary:  $0 < X < L$  and  $Z = 0$ ,  $W = 0$ , the gradient of  $U, K, \epsilon$  are assumed equal to zero.

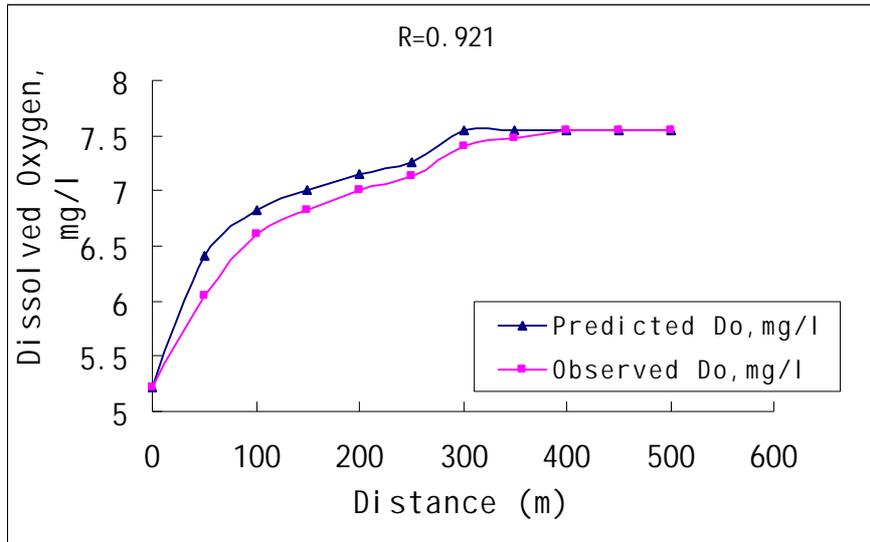
$$\frac{\partial}{\partial Z} (U, K, e) = 0$$

4. At the bottom boundary:  
where  $0 < X < L, Z = D$  and  $W = 0$ , the gradient of  $U, T, K$ , and  $\epsilon$  are equal to zero.

$$\frac{\partial}{\partial Z} (U, T, K, e) = 0$$

#### 4. Field Work and Model Verification :

Observed data of the dissolved oxygen in Tigris River were obtained from South Baghdad Power Station. Other data about Tigris River were obtained from [12]. The model calibration and verification were carried out by comparison of the observed data with that predicted results obtained from the model. The results of this comparison show agreement with maximum absolute error (11%) and correlation factor of ( $R = 0.921$ ) as shown in figure (1).



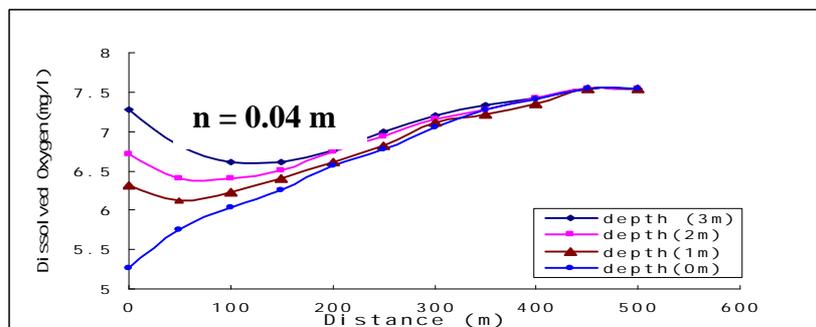
**Fig. (1): The comparison between observed data from Tigris River and predicted data from the numerical model.**

### 5. Results and Discussion :

The effects of hydraulics and water river properties on the longitudinal and vertical dissolved oxygen distribution were studied. These parameters have a direct effect on river velocity. The results were taken at heated water temperature equal to 48°C into 500 m length reach of Tigris River downstream of outfall at water temperature equal to 30°C.

Figures 2 , 3 and 4 show the effect of roughness height on dissolved oxygen depletion . In these figures, increasing the roughness height causes retard in the dissolved oxygen depletion in longitudinal direction due to decrease in river velocity. For example, the longitudinal of oxygen depletion extends to (400 m) downstream of the outfall in case of  $n = 0.04$  m, while it has reached to 300 m, 275 m distance from the outfall in case of  $n = 0.08$  m and 0.11 m, respectively. This is caused by the effect of longitudinal convection which decreases due to the decreases in the longitudinal velocity.

On the other hand, increasing the roughness height causes advance of dissolved oxygen depletion toward the bottom bellow the outfall. For example, at depth 3m the dissolved oxygen had reached 7.28 mg/l when  $n = 0.04$  m , while at the same depth, the dissolved oxygen had reached to (7.05 mg/l and 6.81 mg/l) for  $n = 0.08$  m and 0.11 m, respectively .



**Fig.( 2) Effect of bed roughness ( $n = 0.04$  m) on dissolved oxygen depletion at different depths**

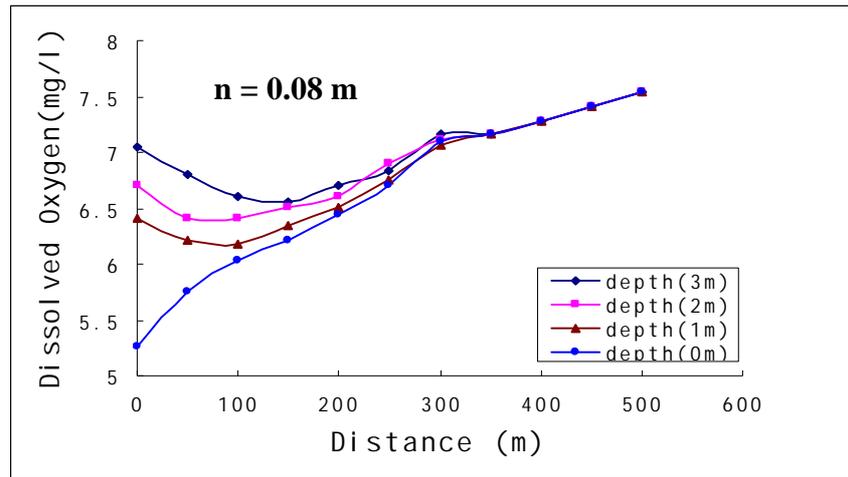


Fig.( 3) Effect of bed roughness ( $n = 0.08$  m) on dissolved oxygen depletion at different depths.

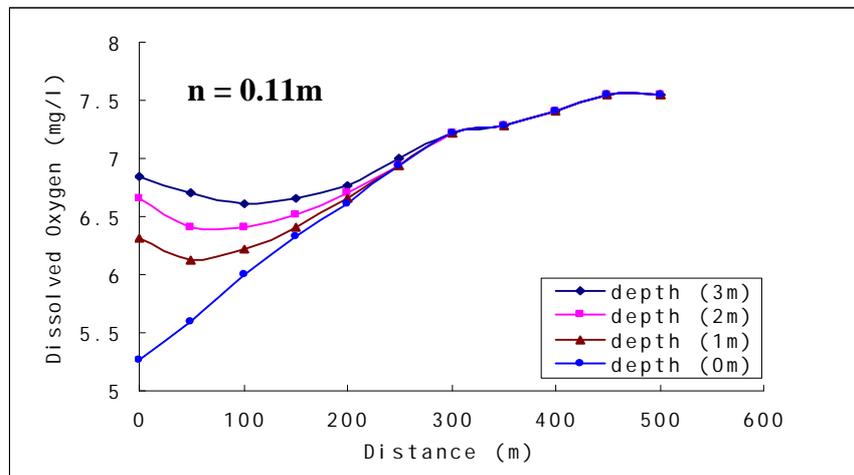


Fig. (4) Effect of bed roughness ( $n = 0.11$  m) on dissolved oxygen depletion at different depths.

This can be attributed to the effect of buoyancy forces which result when polluted water is hotter than ambient river water.

Figure 5 , 6 and 7 show the effect of changing the water surface slope on dissolved oxygen depletion . From these figures it can be seen that increasing the water surface slope will cause a advance in the dissolved oxygen depletion in longitudinal direction due to increase in the river velocity. For example, at depth 3 m and 300 m along reach study the concentration of dissolved oxygen had reached 7.31 mg/l in length in case of  $S = 6$  cm/km , while at the same long it had reached to 7.06 mg/l and 6.72 mg/l in case of  $S = 9$  cm/km and 12 cm/km respectively . This can be attributed to the effect of longitudinal convection which is greater with a higher water-surface slope.

Increasing the water surface slope causes vertical retardation of dissolved oxygen depletion bellow the outfall. For example, at depth of (3 m) the dissolved oxygen had reached (6.93 mg/l) at  $S = 6$  cm/km, while at the same depth, it had reached (7.16 mg/l) and (7.41

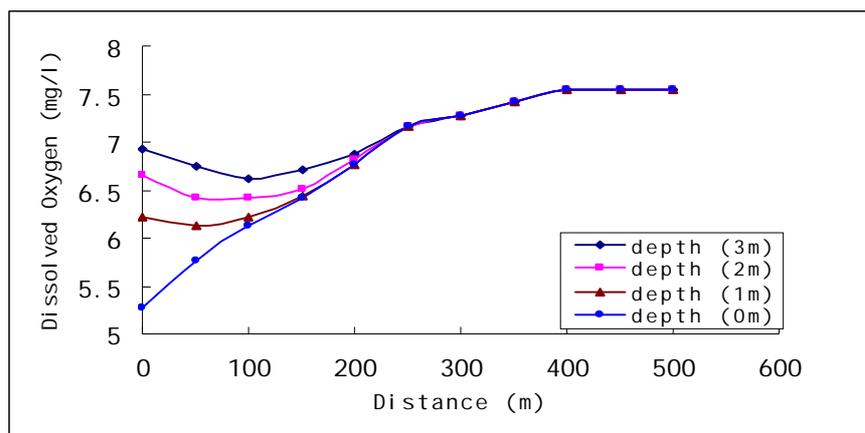
mg/l) at  $S = 9$  cm/km and  $12$  cm/km respectively. This can be attributed to the effect of buoyancy force in which that increase in vertical diffusion with increase in water surface slope.

**Figures 8 and 9** show the effect of density on the dissolved oxygen depletion. It can be seen that assuming constant density in the numerical model (**figure 8**) will cause longitudinal retardation of dissolved oxygen depletion. For example, the depletion of dissolved oxygen had reached to  $250$  m in case of constant density while it had reached to  $350$  m in case of variable density. This can be attributed to the effect of a buoyancy force on the spreading of dissolved oxygen.

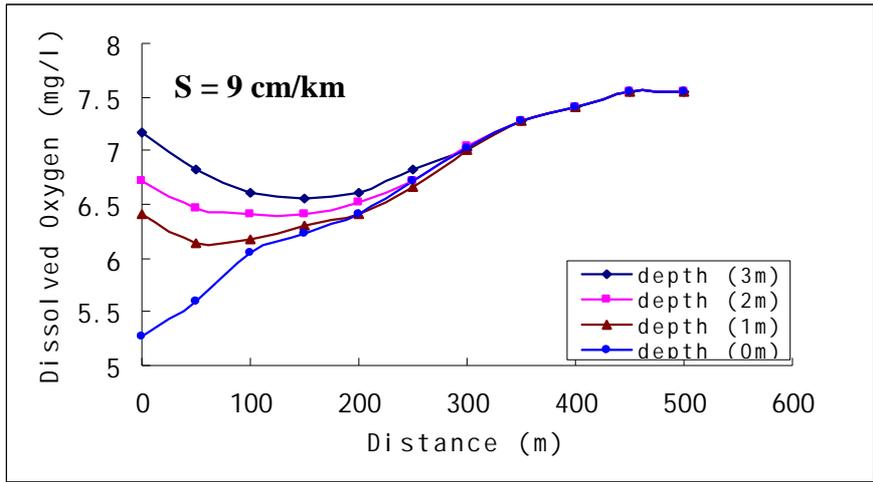
On the other hand, assuming constant density (**Figure 8**) causes vertical retardation of dissolved oxygen depletion toward the surface bellow the outfall. For example at depth of ( $3$  m), the concentration of dissolved oxygen had reached ( $7.18$  mg/l) in case of density is constant , while at the same depth, it had reached ( $6.82$  mg/l) in case of the density is variable (**Figure 9**). This is due to decreasing of the downward vertical motion of hot part of water.

The effect of viscosity on the dissolved oxygen depletion is shown in **Figures 10 and 11**. It can be seen that neglecting the viscosity effect in the numerical model (figure 10) will cause advance vertically of the dissolved oxygen depletion toward the bottom more than that in case of including the viscosity effect (figure 11). For example at depth of  $3$  m, the concentration of dissolved oxygen had reached to  $6.82$  mg/l in case viscosity is variable and it had reached to  $7.16$  mg/l in case viscosity is constant. This can be attributed to the effect of vertical diffusion in heated water toward the bottom due to decrease in shearing stress.

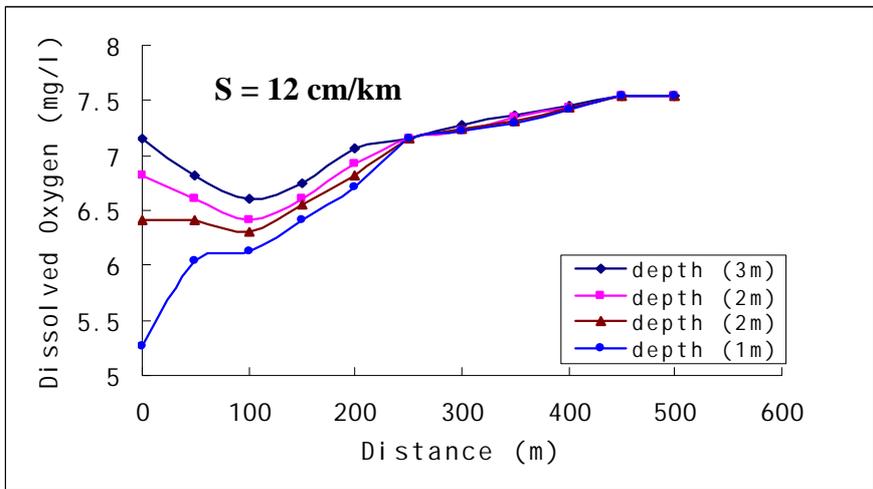
On the other hand, neglecting the viscosity effect the dissolved oxygen depletion was advanced longitudinally near the surface. For example, the depletion of dissolved oxygen has reached to  $400$  m when the viscosity effect was neglected, while it had reached to  $300$  m when the viscosity effect was including. This is because of a longitudinal turbulent mixing which is responsible for the transport of heated water at the longitudinal direction.



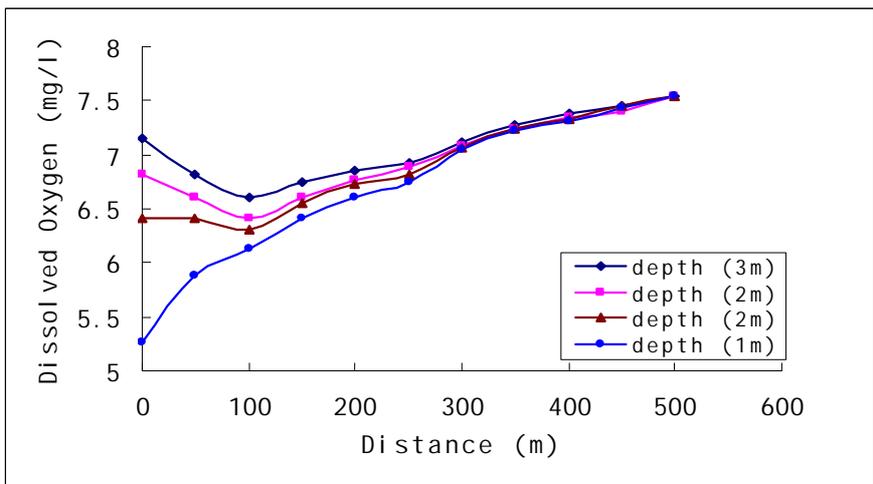
**Fig.(5) Effect of water surface slope ( $S = 6$  cm/km) on dissolved oxygen depletion at different depths.**



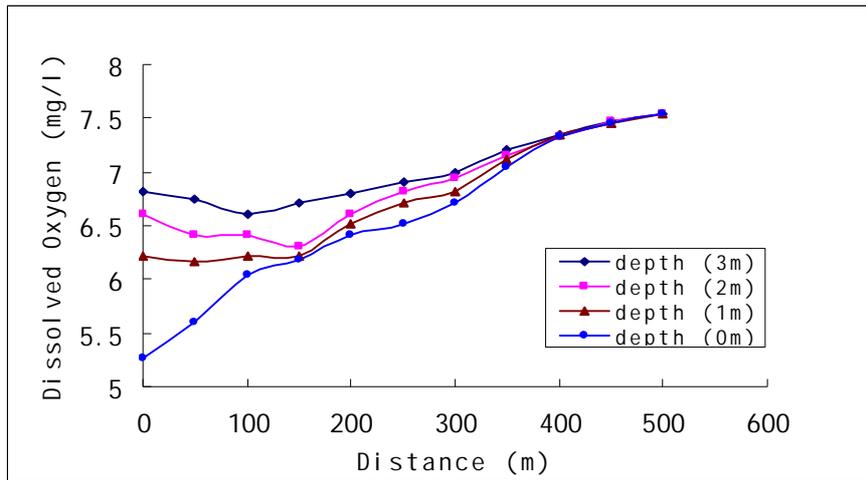
**Fig.( 6) Effect of water surface slope (S = 9 cm/km) on dissolved oxygen depletion at different depths.**



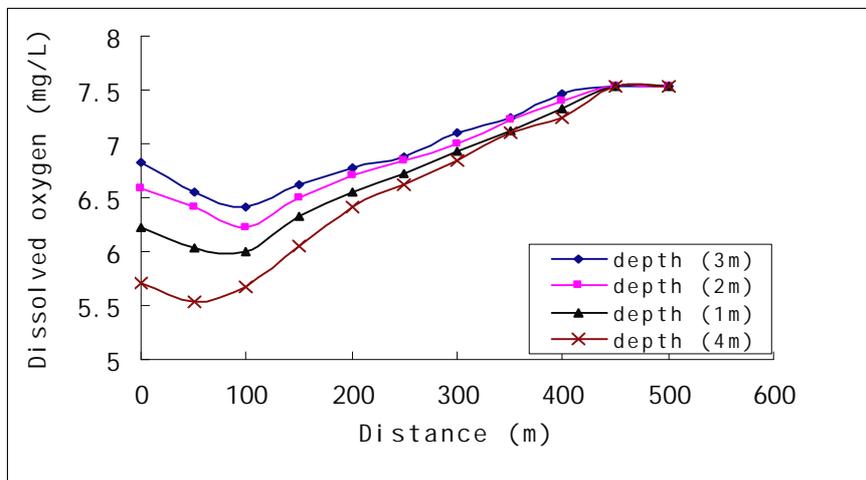
**Fig.(7) Effect of water surface slope (S = 12 cm/km) on dissolved oxygen depletion at different depths.**



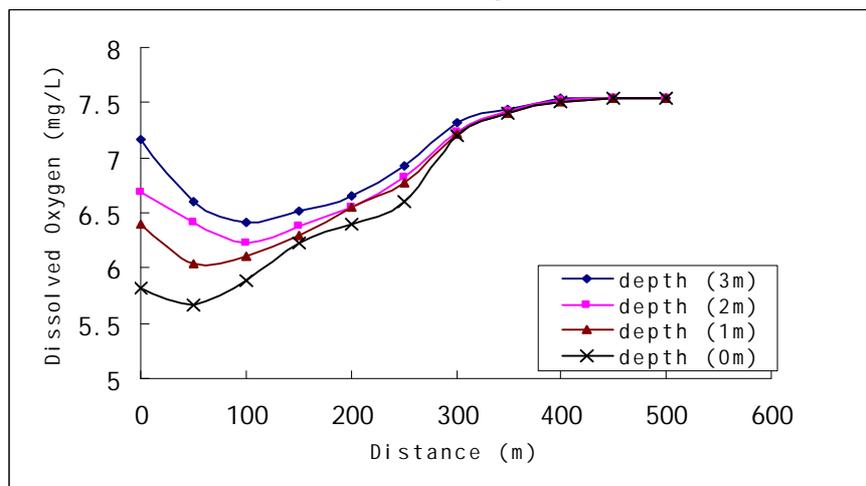
**Fig. (8) Effect of constant density on dissolved oxygen depletion at different depths.**



**Fig.( 9) Effect of variable density on dissolved oxygen depletion at different depths.**



**Fig. (10 ) Effect of neglecting viscosity on dissolved oxygen depletion at different depths.**



**Fig.(11) Effect of including viscosity on dissolved oxygen depletion at different depths**

## 6. Conclusions :

1. The numerical model for prediction the depletion of dissolved oxygen due to thermal pollution in river was applied by considering physical process in water quality.
2. The numerical model developed in this study was verified by the agreement of the model results with those observed from Tigris River. The model can predict the depletion of dissolved oxygen with maximum absolute error of 11% and correlation coefficient of 0.921.
3. The roughness height of the river bed has a considerable effect on the dissolved oxygen depletion in rivers. Increasing the roughness height from 0.04 m to 0.11 m caused a longitudinal retardation of dissolved oxygen depletion about (26% ) and vertical advance of about (14%).
4. The water surface slope has a considerable effect on the dissolved oxygen distribution in the river. Increasing the slope from 6 cm/km to 12 cm/km caused a vertical retardation of dissolved oxygen depletion about (16%) and a longitudinal advance of depletion about (19%).
5. Assuming of constant density causes a longitudinal and vertical retardation of dissolved oxygen depletion about 28% and 6% respectively.
6. Neglecting the viscosity effect causes longitudinal and vertical advancing of dissolved oxygen depletion about 25% and 5% respectively.

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