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Predication of A Mathematical Model for A Hardness in Steel-52100

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Abstract

This paper reports a research in which an attempt was made to predict a mathematical model of hardness for steel-52100 . The primary objective was to investigate the effect of heat treatment parameters on the hardness of steel-52100, where the value of hardness is influenced by these parameters (Austenitizing temperature, tempering temperature and tempering time) . To determine the influence of these parameters , experiments were conducted using Taguchi's orthogonal array . The results of these experiments were used to plot response graph , Pareto diagram and to calculate S/N ratios . An empirical model for the hardness in terms of the parameters mentioned was also developed . Coefficient of determination was used to check the adequacy and accuracy of the model . The empirical model so developed was checked by conducting confirmation experiments . Contour plots were plotted to visualize the effect of those heat treatment parameters on the hardness. Those parameters were optimized to maximize the hardness using the larger-the-better concept of robust design and also compared with the optimized parameters obtained using the developed empirical model . Finally, this study show a good agreement between the mathematical model and experimental results .

Keywords: steel52100, heat treatment, robust design, Pareto diagram .

التنبأ بالنموذج (الموديل) الرياضي للصلادة في الفولاذ نوع 52100

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الخلاصة :

يتضمن هذا البحث محاولة للتنبأ بالموديل الرياضي لصلادة الفولاذ 52100 ، أما الهدف الرئيسي فإنه يتضمن دراسة تأثير عوامل المعاملة الحرارية على صلادة الفولاذ 52100 . حيث أن قيمة الصلادة يمكن أن تتأثر بهذه العوامل (درجة حرارة تكون الأوستينايت، درجة حرارة المراجعة ، و زمن المراجعة) . و لغرض تحديد تأثير هذه العوامل ، أجريت مجموعة من التجارب باستخدام مصفوفة تاكوجي . و قد استخدمت نتائج هذه التجارب لرسم مخطط الاستجابة، مخطط باريتو، و حساب نسبة الإشارة/الضبابية S/N ratio . كما تم الحصول على الموديل التجريبي الذي يعبّر عن العلاقة

مابين الصلادة و عوامل المعاملة الحرارية . و تم استخدام معامل التحديد و للتحقق من كفاية و دقة الموديل الرياضي ، ومن ثم التحقق من ذلك الموديل التجريبي من خلال إجراء مجموعة من تجارب الإثبات (التأكيد) . و تم رسم الرسومات الكفافية (الكنتورية) لملاحظة تأثير عوامل المعاملة الحرارية على الصلادة . ومن ثم تم الحصول على العوامل المثلى التي يمكن من خلالها الحصول على القيمة القصوى للصلادة وفقاً لمفهوم جودة التصميم "الأكبر-الأفضل" و تم مقارنتها مع العوامل المثلى التي تم الحصول عليها من الموديل الرياضي . و أخيراً، أظهرت هذه الدراسة التطابق الجيد مابين نتائج الموديل الرياضي و النتائج التجريبية .

Introduction

In general, the heat treatment of critical bearing components fabricated from 52100 steel is applied to achieve high hardness for dimensional stability in service, wear resistance and load bearing strength[1,2] . Rolling bearings of 21st century are expected to deliver superior performance for prolong duration while operating under most hostile (ultrahigh speed and load with insignificant lubrication) conditions . To meet these exponentially increasing service demands, bearing tribologists have constantly been exploring newer avenues to improve the performance. The SAE 52100 steel, in hardened and tempered condition with a predominantly tempered-martensitic microstructure and appropriate amount of retained austenite (RA) is strenuously developed as a promising rolling bearing material for many automotive applications. Owing to this synergic combination of fine martensitic microstructure and RA, sufficient abrasive wear resistance and mechanical (fatigue and tensile) strength at ambient temperature are obtained. Majority of the failures in rolling bearings are due to rolling contact fatigue (RCF) and are defined as the mechanism of crack propagation caused by the near surface alternating load cycle within the rolling-contact bodies, which eventually leads to material removal by cracking or pitting/delamination . Recently, there have been lots of researches in SAE 52100 steel to understand and improve the resistance to RCF . It can be noted that the surface characteristics of the bearings greatly affect the RCF, for all the fatigue failure initiates at the surface. Engineering the surface condition to achieve high hardness is an inevitable solution to avoid/postpone the failure [3,4,5].

Recently, there have been lots of researches in SAE 52100 steel to understand and improve the hardness of surface and core. It can be noted that the surface characteristics of many applications like bearings greatly affected by the hardness, for all the fatigue failure initiates at the surface[6,7,8] .

Taguchi design of experiment (DOE) methods incorporate fractional factorial matrices or orthogonal arrays to minimize the number of experiments required to achieve a given set of performance characteristics. Iterative Taguchi experiments can be designed to systematically approach optimal parameters for a complicated process or as a quality assurance tool to identify the important parameters to monitor for Statistical Process Control (SPC). The Taguchi experimental approach allows a statistically sound experiment to be completed, while investigating a minimum number of possible combinations of parameters or factors. A

Taguchi experiment can beaccomplished in a timely manner and at a reduced costwith results comparable to a full factorial experiment [9,10].

This paper describes an application of a Taguchi approach to predict hardness of steel-52100 under a set of heat treatment parameters that could be used to develop a mathematical model for hardness in steel52100 using Minitab16® statistical package.

Plan of Investigation

The research work was planned to be carried out as follows:

a) Identifying the important heat treatment control variables and finding their upper and lower limits .
b) Selecting an orthogonal array and conducting the experiments as per the array .
c) Recording responses.
d) Calculating S/N ratio ($SN = -10log\left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i^2}\right)$ where y_i is the response for i-th observation, n, the number of observations) and finding the contributions of each factor .
e) Developing mathematical model, then calculating the coefficients of the regression model .
f) Checking the adequacy of the model developed .
g) Conducting the conformity test runs and comparing the results .
h) Presenting the effects of heat treatment parameters .
i) Optimizing the heat treatment parameters to maximize the hardness .

Identifying the Important Heat Treatment Control Variables and Finding Their Upper and Lower Limits

The independently controllable heat treatment process parameters identified were austenitizing temperature (A) , tempering temperature (B) and tempering time (C) . The three parameters or factors identified are primarily affecting the hardness [1,2]

These factors are normally specified in heat treating references as being the most important. The austenitizing temperature is the temperature to whichsteel is heated in order to transform the BCC (body centered cubic) ferrite to homogeneous FCC (face centered cubic) austenite increasing the stability ofcarbon. Austenitizing is performed prior to thequenching operation (using water as quench medium) that hardens the steel trapping thecarbon to form martensitic. The temperature specified

for austenitizing is the maximum temperature to whichthe material is heated during the heat treating process. The 52100 steel bar used during this investigationwas purchased in an annealed condition with an initial hardness less than 30 HRC . Disks that were approximately 2 mm thickwere sectioned from the bar to be used in theanalysis. **Table (1)** shows the average chemical composition steel (from Spectra analysis-University of Technology /Department of materials Engineering) .It can be seen that the compositions are reasonably close to the nominal composition of SAE 52100 steel [1].The factors and levels selected for the DOE (design of experiments) analysis are shown in **Table (2).** The well-established heat treatment of 52100 steel [2] was used to aid theselection of the factors and levels shown.

Table .(1) Chemical composition of steel-52100

Material used	Elements weight (%)									
Steel-52100	Fe	C	Mn	Si	Cr	S	P	Ni	Cu	Mo
	Rem	1.01	0.39	0.37	1.8	0.031	0.021	0.22	0.2	0.05

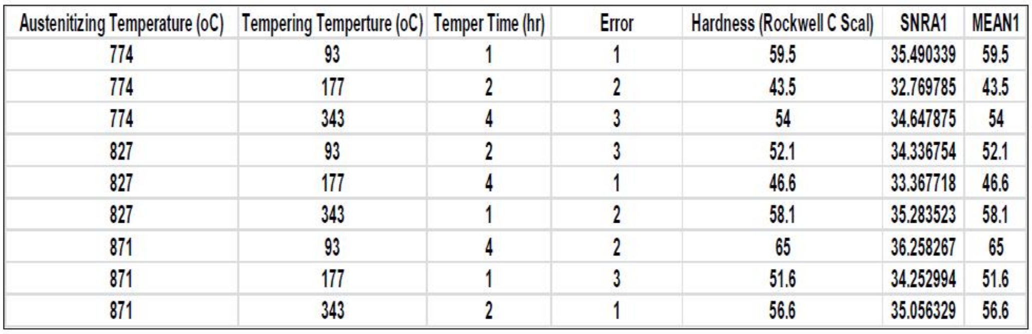
Table .(2) Heat Treatment variables and its bounds

Heat Treatment Variables	Notation	Factor levels		
		1	2	3
Austenitizing temperature (^oC)		774	827	871
Tempering temperature (^oC)		93	177	343
Tempering time (hr)		1	2	4

Selecting an Orthogonal Array and Conducting the Experiments as Per the Array

This investigation involves three factors at three levels . Since , each three-level factor has 2 degrees of freedom (dof) , thus, three , three-level factors will require 6 dof (3x2 = 6) . Hence, it is required to select an orthogonal array with at least 6 dof and L9 orthogonal array which is suitable for three factors , at three levels with 8 dof is selected to conduct the experiments [6] . Column 1 of the array is assigned the factor austenitizing temperature , Column 2 to tempering temperature and Column 3 to tempering time . The remaining fourth Column is assigned for error . The orthogonal array is shown in **Table (3) .**

Table .(3) The L9 orthogonal array with experimental results



Austenitizing Temperature (oC)	Tempering Temperture (oC)	Temper Time (hr)	Error	Hardness (Rockwell C Scal)	SNRA1	MEAN1
774	93	1	1	59.5	35.490339	59.5
774	177	2	2	43.5	32.769785	43.5
774	343	4	3	54	34.647875	54
827	93	2	3	52.1	34.336754	52.1
827	177	4	1	46.6	33.367718	46.6
827	343	1	2	58.1	35.283523	58.1
871	93	4	2	65	36.258267	65
871	177	1	3	51.6	34.252994	51.6
871	343	2	1	56.6	35.056329	56.6

Recording of response

Nine experimental runs were conducted as per the orthogonal array at random to prevent any systematic error creeping into the system . The Rockwell C hardness measurements were acquired using an Instruments Europe CV-600MA Rockwell Hardness tester – (University of technology – materials engineering department) . A standard sphero-conical diamond penetrator was used with a load of 150 kgf . The hardness readings reported are an average of six measurements . The results of these measurements are given in **Table (3) .**

Computation of S/N Ratio

The S/N ratio (SN) is the ratio of signal-to-noise where the signal represents the desired value (i.e. mean of output characteristics) and noise represents undesirable value (i.e. square deviation for the output characteristics) . The S/N ratio is used to measure the quality characteristics . To achieve desirable mechanical properties in a heat treated steel-52100 , the hardness needs to be maximized hence, larger-the-better characteristics equation is used to calculate S/N ratio **[9,10]** .

$$SN = -10log\left(\frac{1}{n}\sum_{i=1}^{n}\frac{1}{y_i^2}\right) \qquad (1)$$

where y_i is the response for i-th observation, n, the number of of observations) and finding the contributions of each factor . The calculated SN values are presented in **Table (3)** .

Response Table

From the results of experiments , the average response for each level is calculated and entered into the response tables **(Tables 4 and 5)** . From the response tables **(Tables 4 and 5)** , it can be observed that if the austenitizing temperature is increased from 774 to 871^{o}C , hardness increase from 52.33 to 57.73 . Similarly , when the tempering temperature is increased from 93 to 343^{o}C , the hardness decrease from 58.87 to 56.23 . Also, it can be observed , the decrease of hardness from 56.40 to 55.20 when the tempering time increase from 1 to 4 hr.

Table .(4) Response table for signal to noise ratio

Response Table for Signal to Noise Ratios

Larger is better

	Austenitizing	**Tempering**	**Temper**
Level	**Temperature**	**Temperture**	**Time**
1	**34.30**	**35.36**	**35.01**
2	**34.33**	**33.46**	**34.05**
3	**35.19**	**35.00**	**34.76**

Table .(5) Response table for mean hardness

Response Table for Means

	Austenitizing	**Tempering**	**Temper**
Level	**Temperature**	**Temperature**	**Time**
1	**52.33**	**58.87**	**56.40**
2	**52.27**	**47.23**	**50.73**
3	**57.73**	**56.23**	**55.20**

Analysis using Response Curve

Response curves are graphical representation of change in performance characteristics with the variation of heat treatment parameter level [11,12]. This analysis is aimed at determining influential parameters and their optimum levels . **Figure 1(a) and (b**) depict the effects of the parameter levels on SN ratios and response mean (Hardness) respectively .

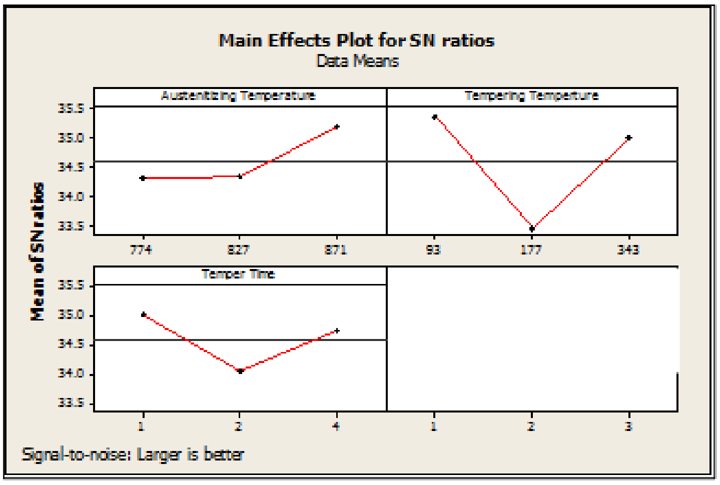


Fig .(1_a) Response curves for SN ratios

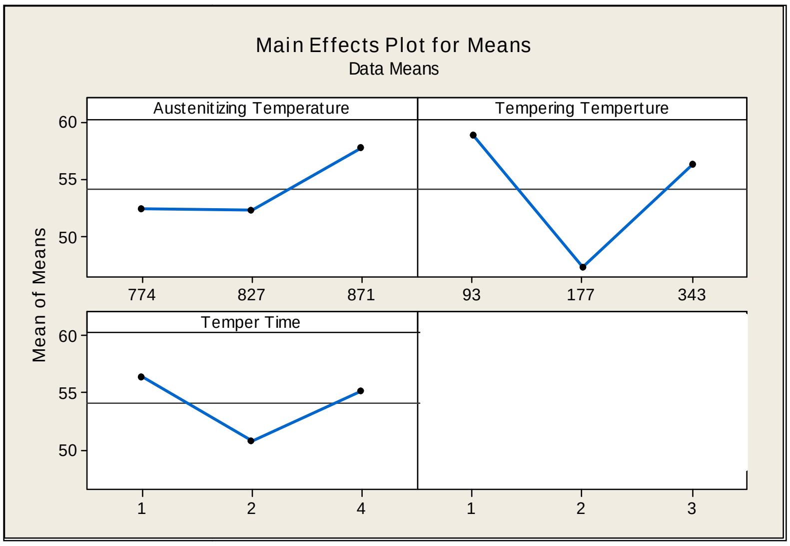
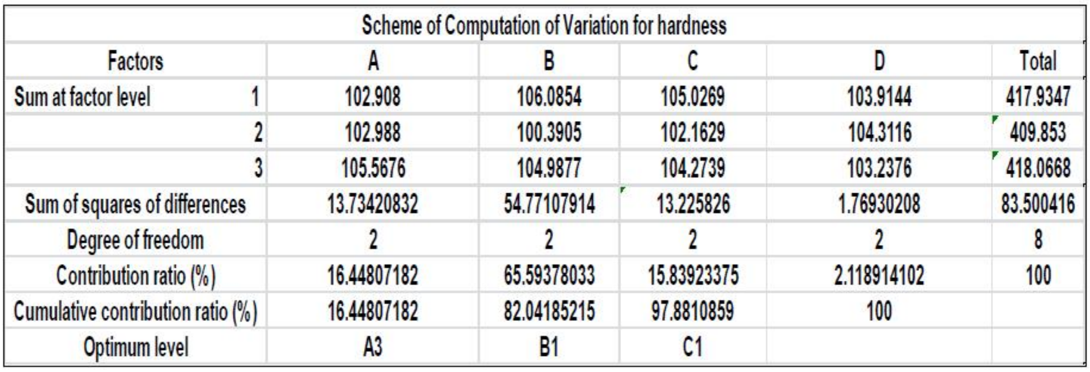


Fig .(1_b) Response curves for hardness

Analysis of Variance for Orthogonal Array Experiments

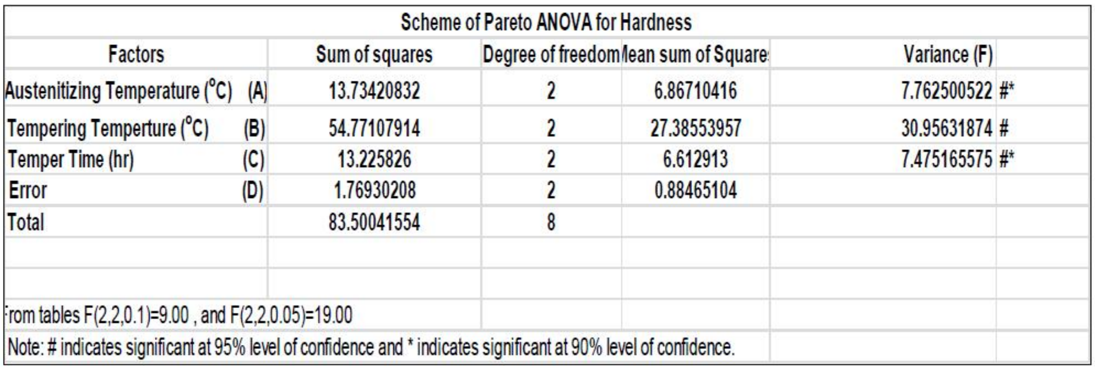
(**Tables 6 and 7),** give the computation of variation of hardness and the scheme of Pareto ANOVA for orthogonal array experiments .

Table .(6) Scheme of computation of variation for hardness



Scheme of Computation of Variation for hardness						
Factors		A	B	C	D	Total
Sum at factor level	1	102.908	106.0854	105.0269	103.9144	417.9347
	2	102.988	100.3905	102.1629	104.3116	409.853
	3	105.5676	104.9877	104.2739	103.2376	418.0668
Sum of squares of differences		13.73420832	54.77107914	13.225826	1.76930208	83.500416
Degree of freedom		2	2	2	2	8
Contribution ratio (%)		16.44807182	65.59378033	15.83923375	2.118914102	100
Cumulative contribution ratio (%)		16.44807182	82.04185215	97.8810859	100	
Optimum level		A3	B1	C1		

Table .(7) Scheme of Pareto ANOVA for hardness



Scheme of Pareto ANOVA for Hardness						
Factors		Sum of squares	Degree of freedom	lean sum of Square	Variance (F)	
Austenitizing Temperature (^{o}C)	(A)	13.73420832	2	6.86710416	7.762500522	#*
Tempering Temperture (^{o}C)	(B)	54.77107914	2	27.38553957	30.95631874	#
Temper Time (hr)	(C)	13.225826	2	6.612913	7.475165575	#*
Error	(D)	1.76930208	2	0.88465104		
Total		83.50041554	8			
rom tables F(2,2,0.1)=9.00 , and F(2,2,0.05)=19.00						
Note: # indicates significant at 95% level of confidence and * indicates significant at 90% level of confidence.						

If the calculated value of variance of a factor is greater than the tabulated value , then it is considered as a significant factor [12]. From **(Table 7)** , it can be observed that the austenitizing temperature and tempering time becomes insignificant at 95% and 99% level of confidence , while tempering temperature is significant at both levels .

Contribution Ratio

Using the data provided in **(Table 6**) , a Pareto analysis can be performed . From this, the significant factor is one which cumulatively contribute about 90% and it can be observed that the most significant factor is the tempering temperature , which contributes 65.59% of the total hardness , the next significant factor is austenitizing temperature contributing 16.44% (cumulative contribution ratio 16.44%) and the tempering time is least significant factor contributing only 15.83 of the total hardness. The results of Pareto analysis are given in **Figure (2) .**

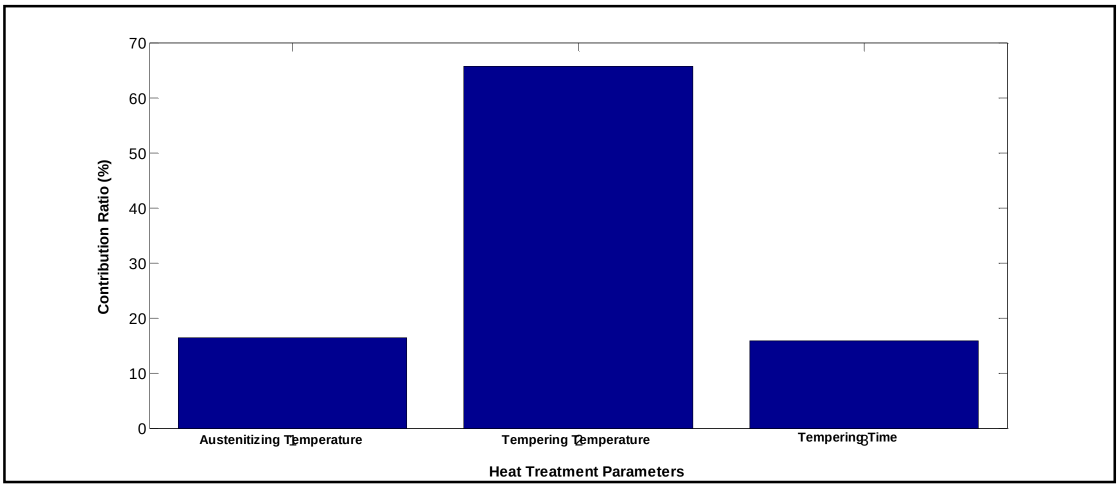


Fig .(2) Results of Pareto analysis

Determination of Optimum Parameters using Taguchi's Technique

From the response tables (Tables 4 and 5) , the optimum set of conditions is selected by choosing all factor levels with high hardness (HRC) since the HRC is the larger-the-better characteristic .

Prediction of the Average Hardness at Optimum Conditions

From Table 5 , the optimum conditions are found to be A3, B1, C1 . But C1 term becomes insignificant as it has lesser contribution **(Table 6)** . Having determined the optimum conditions for orthogonal array experiment , average hardness can be predicted at optimum conditions and given below [9]:

$$\mu_{predicted} = \overline{HRC} + (\overline{A3} - \overline{HRC}) + (\overline{B1} - \overline{HRC}) + (\overline{C1} - \overline{HRC}) \qquad (2)$$

Where , overall of the experiments = $\overline{HRC} = \frac{\sum HRC}{N} = 54.1111$

$$\mu_{predicted} = 54.1111 + (57.73 - 54.1111) + (58.87 - 54.1111) = 62.4889$$

Prediction ofSN Ratios at Optimum Conditions

From **Table 4** , the SN ratio for the optimum condition can be predicted . The optimum condition is found to be A3, B1, C1, but C1 is an insignificant term because it has lesser contribution **(Table 6) .**

Hence , the predicted S/N ratio for this condition is:

$$\mu(A3, B1, C1) = A3 + B1 - \frac{T}{9} \qquad (3)$$

$$\mu(A3, B1, C1) = 35.19 + 35.36 + \frac{311.4636}{9} = 35.94293$$

Confirmation Experiment

A confirmation test is conducted to check whether the obtained optimum condition really produces the desired hardness . The results of the confirmation experiment is given in (**Table 8**) .

Table .(8) Results of the confirmation experiment for optimum condition

Austenitizing Temperature (^{o}C)	Tempering Temperature(oC)	Tempering Time (hr)	Hardness(HRC)
774	93	1	61.11

The S/N ratio for this observation is:

$$SN\ Ratio = -10 \log\left[\frac{1}{61.11^2}\right] = 35.72225$$

The confidence interval (CI) of the predicted estimation can be calculated using equation [12]:

$$Confidence\ interval(CI) = t\left(\varphi_e, \frac{\alpha}{2}\right)\sqrt{\frac{V_e}{n_{eff}}} \qquad (4)$$

Where $t\left(\varphi, \frac{\alpha}{2}\right)$ is the *t*-distribution required for α = risk , confidence = 1-risk , V_e , pooled error variance , n_{eff} the effective number of replication , φ_e , degree of freedom for insignificant term :

$$V_e = \frac{pooled\ variance\ of\ insignificant\ source}{Pooled\ degree\ of\ \ insignificant\ source} = \frac{\frac{13.225}{4} + \frac{1.769}{9}}{4} = 0.8757$$

$$n_{eff} = \frac{Total\ number\ of\ experiments}{1 + degree\ of\ freedom\ of\ significant\ sourse} = \frac{9}{(1 + 2 + 2)} = 1.8$$

Thus, confidence interval = 0.6974552

A 95% confidence interval for the average S/N ratio at the optimum condition is calculated by using equation(4):

$$35.94293 \pm t\left(\varphi_e, \frac{\alpha}{2}\right)\sqrt{\frac{V_e}{n_{eff}}} = 35.94293 \pm t(4{,}0.025)\sqrt{\frac{V_e}{n_{eff}}}$$

From t-distribution , $t(4{,}0.025) = 2.776$

$$35.94293 \pm t(4{,}0.025)\sqrt{\frac{V_e}{n_{eff}}} = 35.94293 \pm (2.776) \times 0.6974552$$

$= 35.94293 \pm 1.936135635$

I.E

$[-CI] < \mu < +[CI]$

$34.00679433 < 35.7225 < 37.87906564$

As the predicted SN ratio lies within this confidence interval , it is confirmed that the predicted settings produce the desired hardness .

Development of Mathematical Model

Calculation of Regression Coefficients

The response function representing the hardness can be expressed as [10]:

$$Y = f(A, B, C) \tag{5}$$

Where Y is the response , i.e. hardness , A, Austenitizing temperature, B tempering temperature and C , tempering time . The second-order polynomial (regression) equation used to represent the response function for K factors is given by:

$$Y = b_0 + \sum_{i=1}^{k} b_i x_i + \sum_{i,j=1}^{k} b_{ij} x_i x_j + \sum_{i=1}^{k} b_{ii} x_i^2 \qquad (6)$$

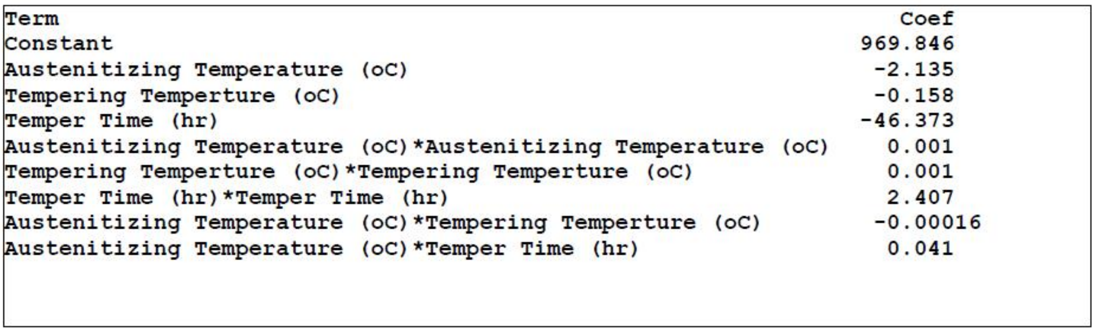
Where b_0 is the free term of the regression equation, the coefficients b_1, b_2 ….b_k are linear terms , the coefficients b_{11}, b_{22}, b_{kk} are the quadratic terms and the coefficients b_{12}, b_{13}…..b_{k-1k} are the interaction terms [13] .

For three factors , the selected polynomial could be expressed as:

$$Y = b_0 + b_1 A + b_2 B + b_3 C + b_{12} Ab + b_{13} Ac + b_{23} BC + b_{11} A^2 + b_{22} B^2 + b_{33} C^2 \qquad (7)$$

The regression coefficients of response function are presented in **(Table 9)** .

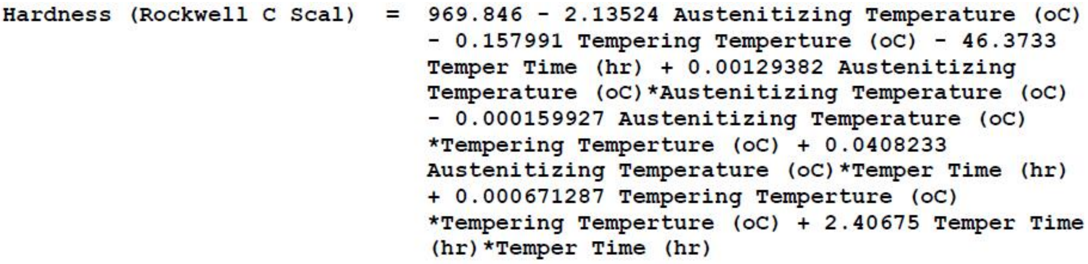
Table .(9) Estimated regression coefficients of the mathematical model for hardness



Term	Coef
Constant	969.846
Austenitizing Temperature (oC)	-2.135
Tempering Temperture (oC)	-0.158
Temper Time (hr)	-46.373
Austenitizing Temperature (oC)*Austenitizing Temperature (oC)	0.001
Tempering Temperture (oC)*Tempering Temperture (oC)	0.001
Temper Time (hr)*Temper Time (hr)	2.407
Austenitizing Temperature (oC)*Tempering Temperture (oC)	-0.00016
Austenitizing Temperature (oC)*Temper Time (hr)	0.041

Mathematical model

The values of the regression coefficients calculated earlier were used to formulate a second order polynomial (regression) , with parameters is:



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Hardness (Rockwell C Scal)  =  969.846 - 2.13524 Austenitizing Temperature (oC)
                               - 0.157991 Tempering Temperture (oC) - 46.3733
                               Temper Time (hr) + 0.00129382 Austenitizing
                               Temperature (oC)*Austenitizing Temperature (oC)
                               - 0.000159927 Austenitizing Temperature (oC)
                               *Tempering Temperture (oC) + 0.0408233
                               Austenitizing Temperature (oC)*Temper Time (hr)
                               + 0.000671287 Tempering Temperture (oC)
                               *Tempering Temperture (oC) + 2.40675 Temper Time
                               (hr)*Temper Time (hr)
```

(8)

Checking the adequacy of the Model developed

The estimated coefficients obtained earlier were used to construct the model for the response parameter .The adequacy of the model so developed was then tested by using the coefficients of determination (R^2) .

The coefficients of determination (R^2) , is given by the following equation [13]:

$$R^2 = \frac{SSR}{SST} = \frac{\sum(\hat{y}-\bar{y})^2}{\sum(y_i-\bar{y})^2} = 1 - \frac{SSE}{SST} \qquad (9)$$

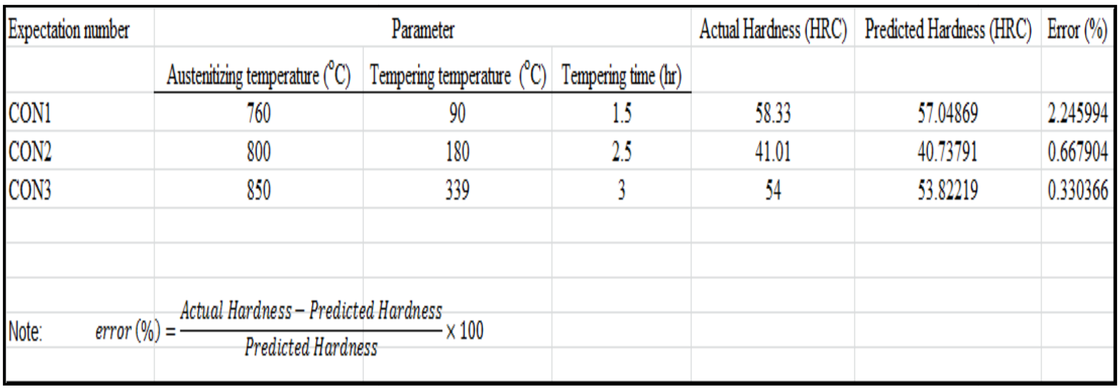
Where , SSR is the regression sum of square , SST , total sum of squares , SSE, error sum of square , y_i , observed value of the response, $\hat{y}$, estimated value of the response using the regression model , and $\bar{y}$, the mean of the observed values of the response .

For the model developed the calculated R^2 value was 99.99% which indicates that the regression model is quite adequate and that 99.99% of the variation in the response has been explained by this regression model .

Confirmation Experiments for the Empirical Model Developed

Experiments were conducted to verify the regression equation obtained (equation 8) . three hardness tests were made using different values of austenitizing temperature, tempering temperature and tempering time other than that used in the design matrix . The results obtained were quite satisfactory and the details are presented in (**Table 10) .**

Table .(10) results of confirmation experiments for empirical model developed



Expectation number	Parameter			Actual Hardness (HRC)	Predicted Hardness (HRC)	Error (%)
	Austenitizing temperature (°C)	Tempering temperature (°C)	Tempering time (hr)			
CON1	760	90	1.5	58.33	57.04869	2.245994
CON2	800	180	2.5	41.01	40.73791	0.667904
CON3	850	339	3	54	53.82219	0.330366

Note: $error\,(\%) = \frac{Actual\ Hardness - Predicted\ Hardness}{Predicted\ Hardness} \times 100$

Interaction Effects of Heat Treatments Parameters on Hardness

(Figures 3 and 4) , depict the distribution of hardness for different levels of austenitizing temperature, tempering temperature and tempering time . From these figures it is evident that the area between the contour lines indicates the combinations between austenitizing temperature, tempering temperature and tempering time for a given heat treatment parameter to achieve desirable range of hardness of steel52100 .

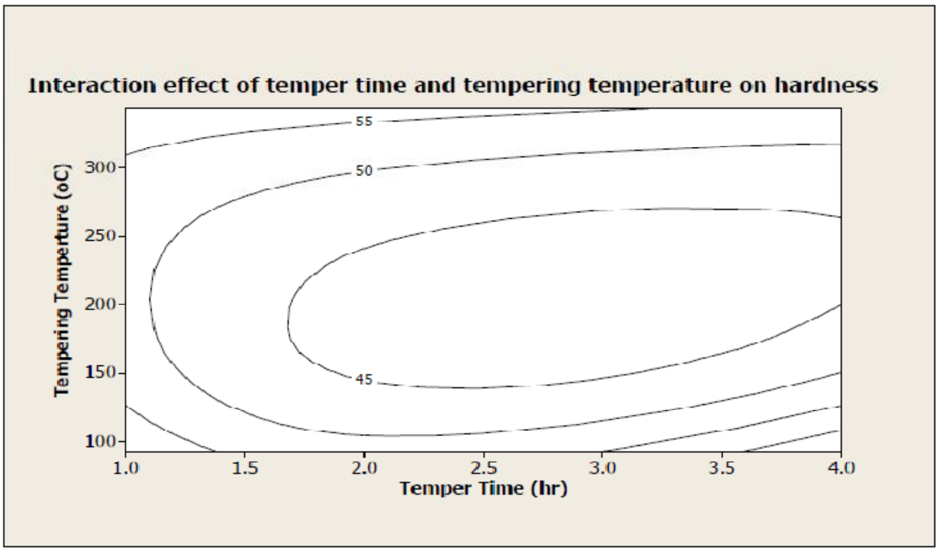


Fig .(3) Interaction effect of tempering time and tempering temperature on hardness (Values shown on contour represent hardness)

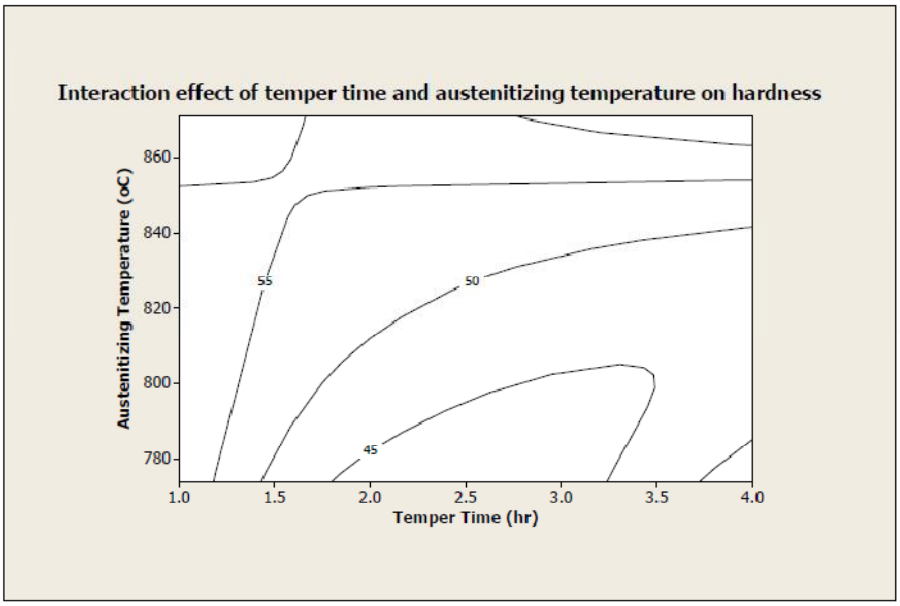


Fig .(4) Interaction effect of tempering time and austenitizing temperature on hardness (Values shown on contour represent hardness)

Results and Discussions

Response tables and response graphs of Taguchi's technique give useful information regarding the significant factors responsible for the hardness , but at the same time they don't give any empirical relation between the hardness and the heat treatment parameters .

Taguchi's concept of optimization gives optimum values only for the level of parameter values for which experiments were conducted , whereas , an empirical model developed by regression analysis can be to give optimized values , in between levels of heat treatment parameters .

From the optimization results using Taguchi's technique , the maximum value of hardness (HRC) of 62.4889 when the austenitizing temperature is set at 871°C , tempering temperature at 93°C and tempering time at 1 hr .

Conclusions

1. Response tables , response graph and Pareto diagram show that the tempering temperature is the most significant parameter influencing the hardness of steel 52100 .
2. The experimental results obtained were used for developing mathematical model for predicting the hardness of steel 52100 .
3. The empirical model developed by regression analysis can be able to give better-optimized solution compared to optimized solution obtained using Taguchi's technique .

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