

Behavior Of Rectangular Reinforced Concrete Beams Subjected To Bi-Axial Shear Loading

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Abstract :

Shear failure of the concrete beam, is undesirable mode of failure because these kinds of failure gives a few warnings, and have disastrous consequences. So this study comes to investigate the mechanism of failure and the maximum loading capacity of rectangular reinforced concrete beam under biaxial shear load through the test of eight reinforced concrete beam.

The simple experimental set-up for a simply supported beam under one point loading is introduced in this study by applying the load to the tilted beam. This requires only one hydraulic jack to produce the complicated bi-axial shear loading. The measured ultimate bi-axial shear capacities of reinforced concrete beams with shear reinforcement were separately defined for concrete (V_c) and shear reinforcement (V_s) contributions. Their results were discussed individually with respect to calculation using ellipse function (which is currently used in JSCE design specification) with JSCE code requirement and ACI code requirement. Through the test of four reinforced concrete members with changing the shear reinforcement ratio the results show that the ellipse interaction relation seems to underestimate bi-axial shear capacity of concrete about (119 to 188%) and (43 to 50%) for ACI and JSCE design codes respectively and overestimate bi-axial shear capacity of shear reinforcement of reinforced concrete members with rectangular section by the range of (21) to (6) % and (25) to (11) % for ACI and JSCE code, respectively.

Key words: Reinforced concrete beam, Bi-axial shear, Ellipse function, Experimental test.

سلوك العتبات الخرسانية المسلحة المستطيلة المقطع المعرضة لقوى القص ثنائية المحاور

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الخلاصة :

فشل القص في العتبات الخرسانية هو فشل غير مرغوب فيه لأن هذا النوع من الفشل يعطي تحذيرات قليلة قبل الفشل النهائي، وتكون له عواقب كارثية. ولذلك تأتي هذه الدراسة لتحري آلية الفشل و قوة التحمل النهائية للعتبات الخرسانية المسلحة مستطيلة الشكل تحت قوة القص ثنائية المحور من خلال اختبار اربعة عتبات خرسانية مسلحة؛ في هذا البحث تم استخدام الطريقة البسيطة لفحص العتبة بسيطة الأسناد تحت نقطة تحميل واحدة وذلك من خلال تسليط الحمل على العتبة المائلة، وهذا يتطلب مكبس ميكانيكي واحد لتسليط قوة الانضغاط. المتغير هو نسبة حديد تسليح القص. ان قوة الفشل النهائية للعتبة قد قسمت الى قوة تحمل الخرسانة للقص (V_c) و قوة تحمل حديد التسليح العرضي للقص (V_s)، و من ثم نوقشت هذه النتائج مع الأخذ بنظر الاعتبار المعادلة الموضوعية و التي تربط بين القوة المؤثرة على العتبة بالاتجاهين الأفقي و العمودي و المستخدمة حالياً في تحليل العتبات المعرضة لهذا النوع من القوى (و الموجودة في المدونة اليابانية لتصميم الخرسانة JSCE)، و قد تم استخدام هذه المعادلة مع متطلبات تصميم القص في المدونة اليابانية JSCE و كذلك مع متطلبات تصميم القص في المدونة الاميريكية ACI. و قد اظهرت النتائج العملية الى أن تلك المعادلة قد قللت من المقاومة المتوقعة للخرسانة المعرضة للقوة ثنائية المحور، بمقدار (119 الى 188%) و (43 الى 50%) بالنسبة للمقاومة المحسوبة وفقاً لمتطلبات المدونة الاميريكية (ACI) و المدونة اليابانية (JSCE) على التوالي، و قد بلغت تلك المعادلة في تقدير قدرة مقاومة حديد التسليح العرضي للقوة المؤثرة بالاتجاهين العمودي و الأفقي بمقدار (6 الى 21%) و (11 الى 25%) بالنسبة للمقاومة المحسوبة وفقاً لمتطلبات المدونة الاميريكية (ACI) و المدونة اليابانية (JSCE) على التوالي.

الكلمات الدالة : قوة القص ثنائية المحاور؛ عتبة خرسانية مسلحة؛ دالة الشكل البيضي.

Notations :

V : Applied shear force kN

V_c : Shear strength provided by concrete of beams without stirrups, kN

V_s : Shear strength provided by shear reinforcement, kN

V_u : Applied factored shear force on a reinforced concrete beam section, kN

V_x : Shear capacity along x - axis under biaxial shear forces. (kN)

V_y : Shear capacity along y - axis under biaxial shear forces. (kN)

V_{uy} : Uniaxial shear capacity along x -axis. (kN)

V_{ux} : Uniaxial shear capacity along y -axis. (kN)

V_{cx} : Uniaxial concrete capacity along y -axis. (kN)

V_{cy} : Uniaxial concrete capacity along x -axis. (kN)

V_{sx} : Uniaxial shear reinforcement capacity along x -axis. (kN)

V_{sy} : Uniaxial shear reinforcement capacity along y -axis. (kN)

V_R : Resultant shear capacity of reinforced concrete beam under biaxial shear. (kN)

b : Angle between principal axes and line of load P degree.

ρ_l : Ratio of longitudinal reinforcement

ρ_s : Ratio of shear reinforcement

1. Introduction:

Beams generally carry vertical gravitational forces but can also be used to carry horizontal loads (i.e., loads due to an earthquake, wind, unexpected load or accidental load). An extent of deviation of the line of shear application and the major axis exists, especially when it is out of expectation or under earthquake attack, during which arbitrary of shear load direction is probably observed. Moreover, the complex member shape may lead to the action of load transferred in three-dimensional aspect. A typical example is the short RC column under multilateral loading in shear and/or torsion caused by earthquake or accidental impact. With such kind of load and structure, the member is subjected to bi-axial loading and hence, the design of such a reinforced concrete member should be performed in accordance with such kind of loading.

In this case, some codes such as JSCE provide a method to calculate bi-axial shear capacity by using interaction curve, which some codes as the ACI one do not provide. Hence in this paper a comparison between the present experimental results with the capacities calculated by the available interaction formula in JSCE design code, and ACI design code by importing the same interaction formula from JSCE design code, is presented.

One of the direct and typical methods of bi-axial shear test is to apply the shear loads in two directions, as shown in **Figure (1)** taken from the test series conducted by Yoshimura^[1]. In his test series, the reinforced concrete columns fixed at basis and free at top; were subjected not only to horizontal shear loads in two directions (PX and PY in Figure.1) but also vertical axial load (N in Figure.1). This type of test needs at least two hydraulic jacks, and hence the load controls are complicated when the two horizontal shear loads are proportionally increased.

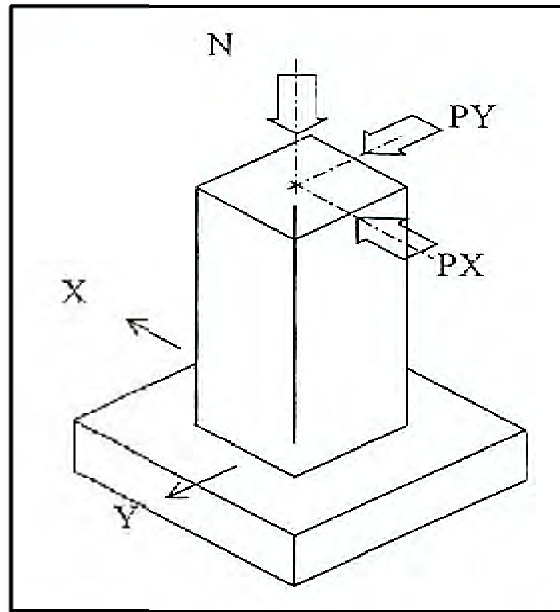


Fig. (1) Bi-axial Shear Test of Short RC Column (Yoshimura 1996) ^[1]

This study aims to clarify shear resisting mechanism of shear reinforcement of RC beams subjected to bi-axial shear by performing the bi-axial shear test of rectangular reinforced concrete beams with changing the shear reinforcement ratio. The test method is quite simple due to the inclination between principal axis of beam section and line of application of load. The simplified bi-axial shear test is presented here by using a simple set-up of loading frame and only one hydraulic jack, as shown in **Figure (2)**. Load stub was provided at the center of span. The vertical force P is resolved into the principal axes of a beam section, and hence shear in two directions are applied simultaneously. To verify the simplified bi-axial shear test, a test was made by Hansapinyo and *et al* ^[2] consisted of three square reinforced concrete beams with different angles of 0, 20, 45 degrees, , the normalized bi-axial shear capacities obtained from the simplified test and Yoshimura's test are plotted in the interaction diagram as shown in **Figure (3)**. The agreement of the two test results with the ellipse function, which is used in the current design practice of JSCE code ^[3], can be seen. By this agreement, the verification of the simplified test of bi-axial shear was made.

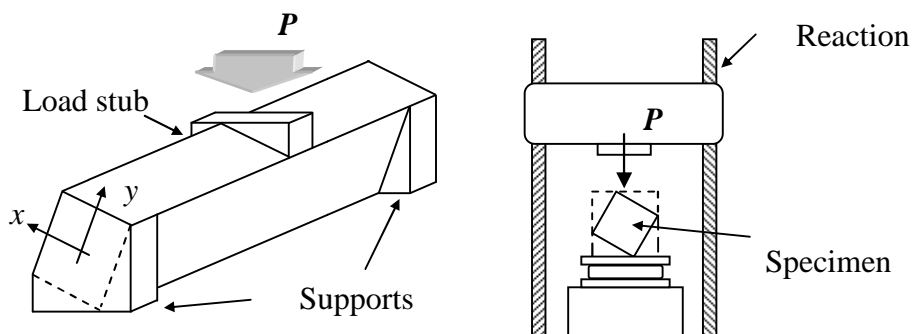


Fig. (2) Loading Scheme of Simplified Bi-axial Shear Test ^[2]

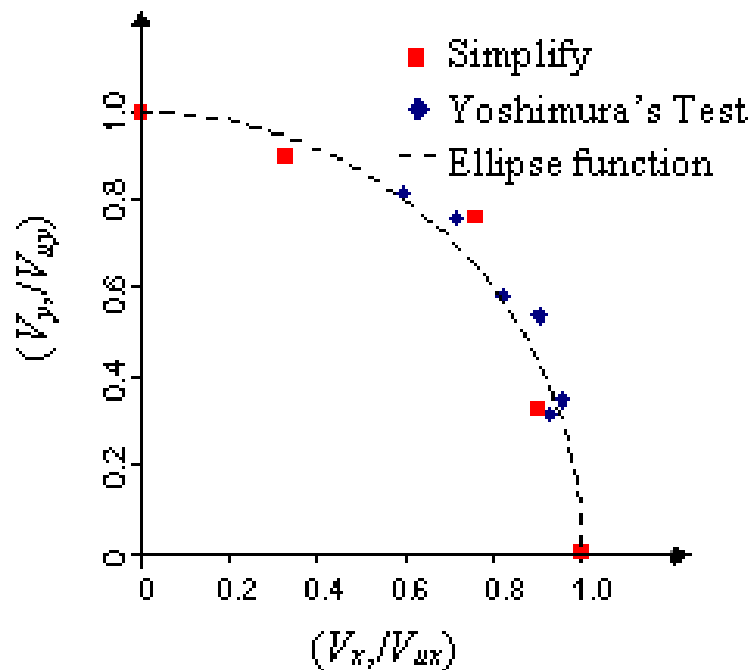


Fig. (3) Verification of the Results of Simplified Test Method with Interaction Diagram in JSCE code ^[2]

JSCE design code mentions the reduction of capacity when the orthogonal shear force is present. It is also considered that the present of two shear reduce each other uni-axial shear capacity. This reduction is sometimes called the “Interaction Formula” for bi-axial shear loading ^[3]. Such interaction can be shown in **Figure (4)**. The process is started by calculating two major uni-axial shear capacities, x and y , correspondingly represented by circles in the figure. Substitution into the interaction formula, shear capacities in x and y directions reduced from the uni-axial shear capacities are obtained. The application for other levels of axial loading becomes possible by expanding or contracting the curve upon direction (compression/tension) of axial loading is applied. The appropriate designed member can be achieved unless the magnitude of applied external shear $|S_R|$ is larger than the absolute resultant shear capacity $|V_R|$.

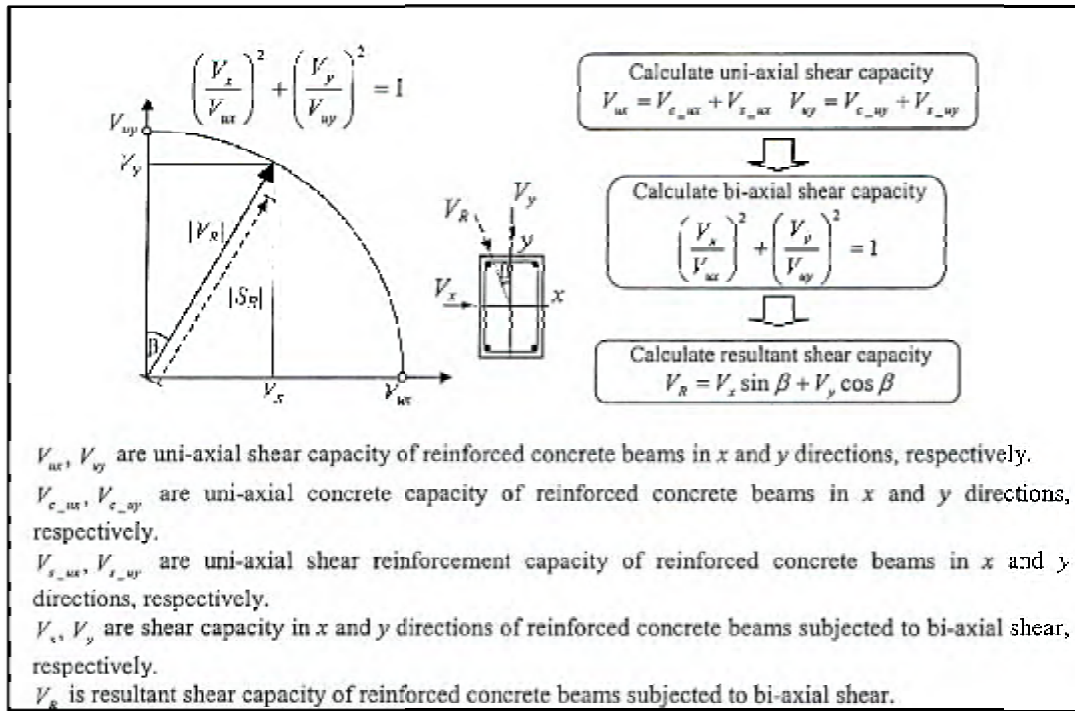


Fig. (4) JSCE Design Recommendation for Reinforced Concrete Member Under Bi-axial Shear^[2]

2. Experimental Work :

The bi-axial shear capacity test was composed of four specimens which were designed primary to investigate the influence of shear reinforcement ratio. The details of these beams were listed in **Table (1)**.

Table (1) Details of Specimen

Beam Designation	β degree	ρ_l	ρ_s	Stirrup Spacing (mm)
B-45-R	45	0.0241	0.00377	100
B-45-WS	45	0.0241	0	-----
B-45-S2	45	0.0241	0.00502	75
B-45-S3	45	0.0241	0.00754	50

One concrete mix was made and used for all tested specimens with nominal cube compressive strength of (27 MPa) after 28 days.

The tested beams (in 45 degree) were placed in the machine on two supports with clear span of (1140mm), the supports were made of iron and designed to fit the spherical seats of the machine, the angle of (45 degree) was obtained through rectangular steel caps having (100 mm) in depth and (4mm) in thickness and tilted by (45 degree), those caps were welded to a steel plate of (4 mm) thick was attached to other three steel plates to get a box fitting spherical seats, as shown in **Plates (1) and (2)**. To achieve the simply supported beam

situation, a thin steel angle was fixed between the beams and cap from one side to prevent vertical movement.

During the testing, the main characteristics of the structural behavior of the tested beam were detected at every stage of loading. For each test, the first diagonal crack and the ultimate load were recorded. The deflection measurement was taken at three points; two were in vertical direction (under different faces of the beams) at beams center; one in the horizontal direction at the center of the beams, also.

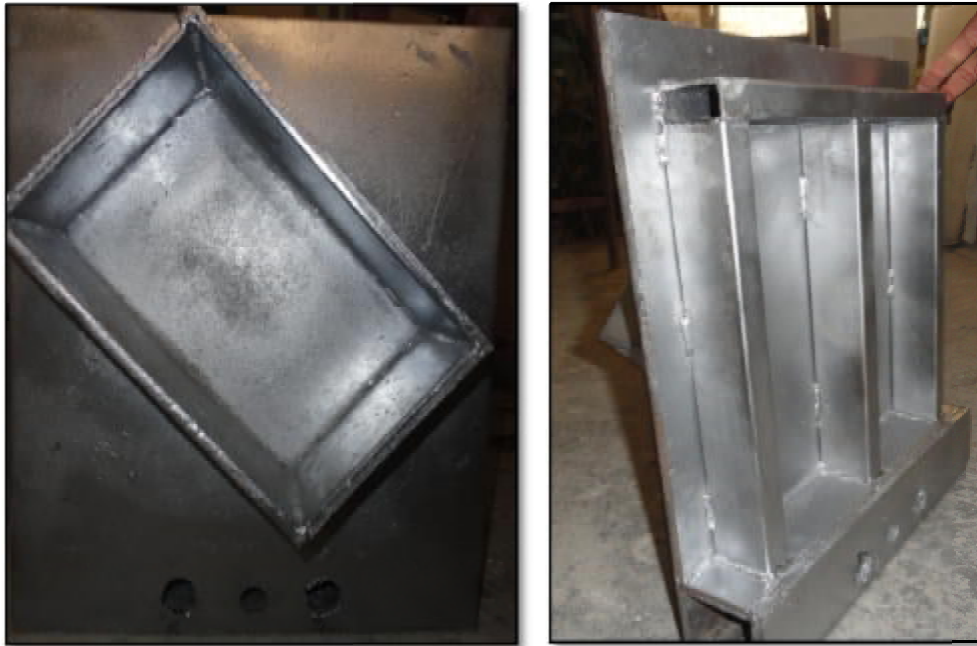


Plate (1) and (2) the Supports of Beams

3. Results and Discussion

3.1 Crack Propagation

At section near midspan, flexural crack were fully opened on the face C and flexural crack tips were on faces B and D, shown in **Plates (3) to (6)**. Hence, it can be said that the neutral axis was parallel to faces A and C. With the continuing increase of load, the crack formed previously was extended and new flexural cracks were observed. First diagonal crack was observed on faces B and D at section near mid span and extended to top fibers of faces B and D at mid span. After that, the diagonal cracks became wider and propagated down to the supports. When the applied load approached the ultimate one, splitting cracks were observed to take a place on faces A and B taken place along the member axis. Due to the small compression area on faces A and B, the cracks observed on these faces were generated by highly applied compressive stress. The propagation of splitting cracks on face A also confirms the direction of neutral surface on cross section. **Plate (3)** shows the crack pattern of specimen B-45-R.

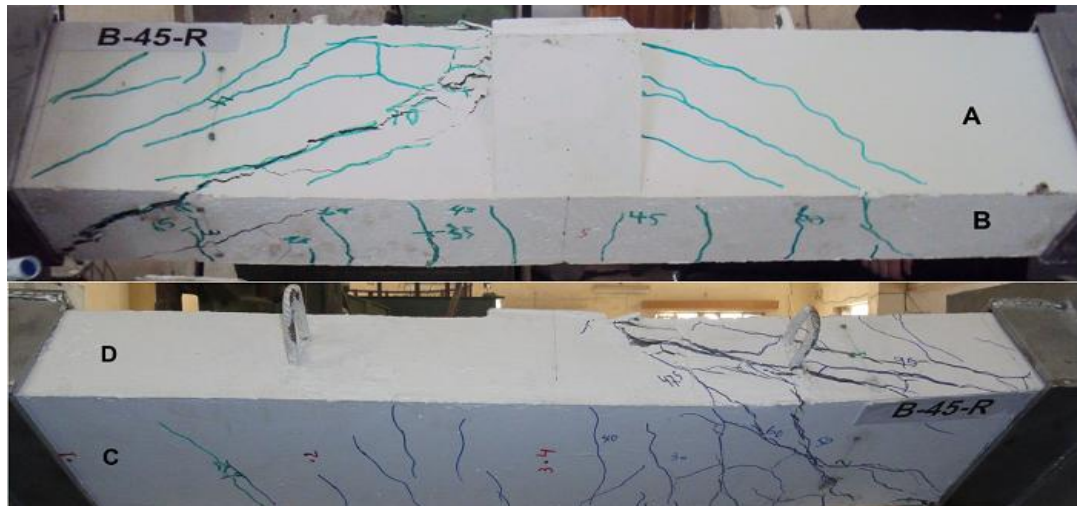


Plate (3) Crack Pattern of Specimen B-45-R

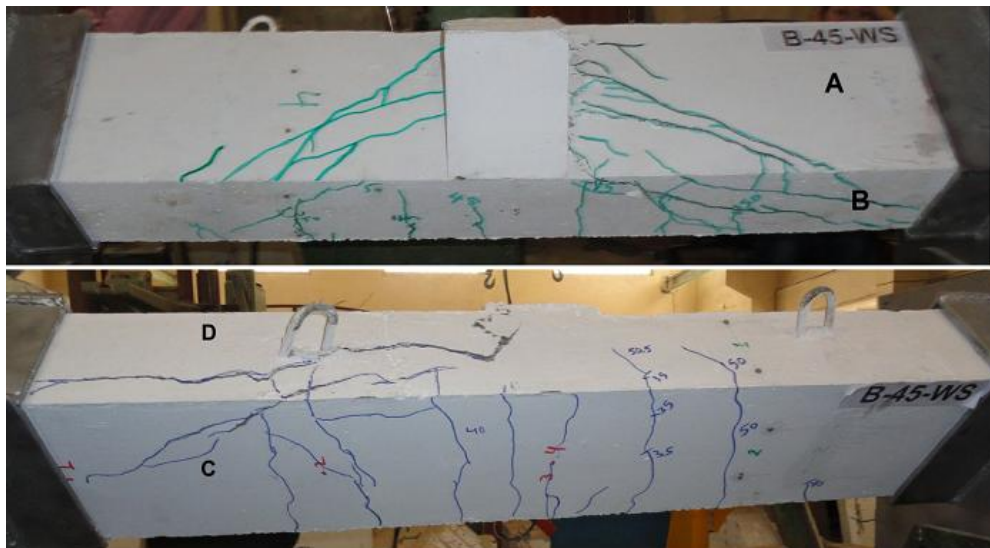


Plate (4) Crack Pattern of Specimen B-45-WS

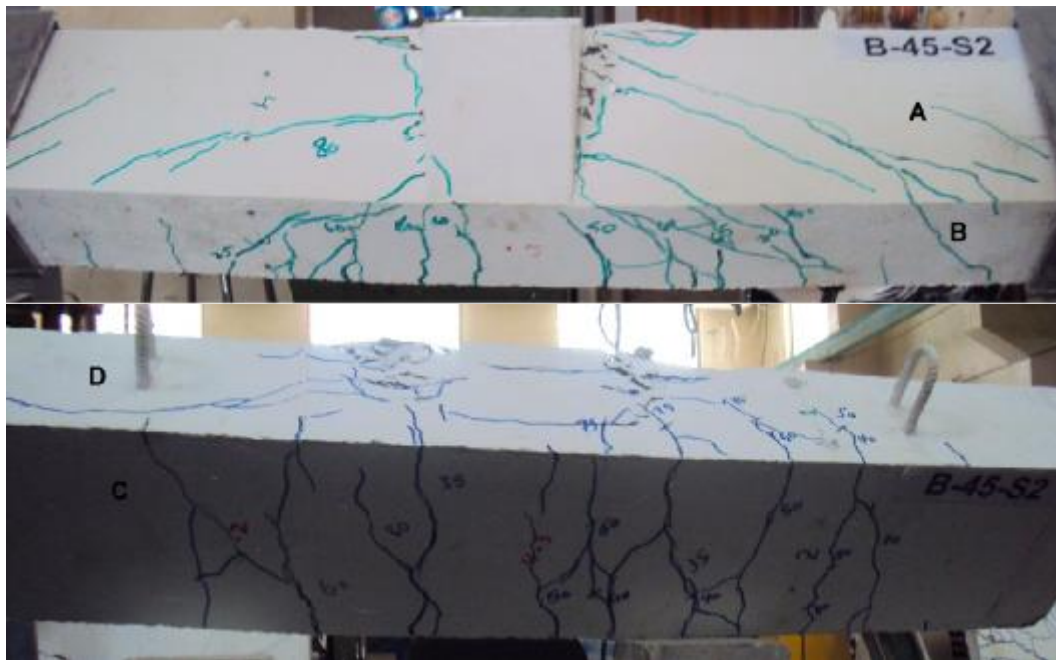


Plate (5) Crack Pattern of Specimen B-45-S2

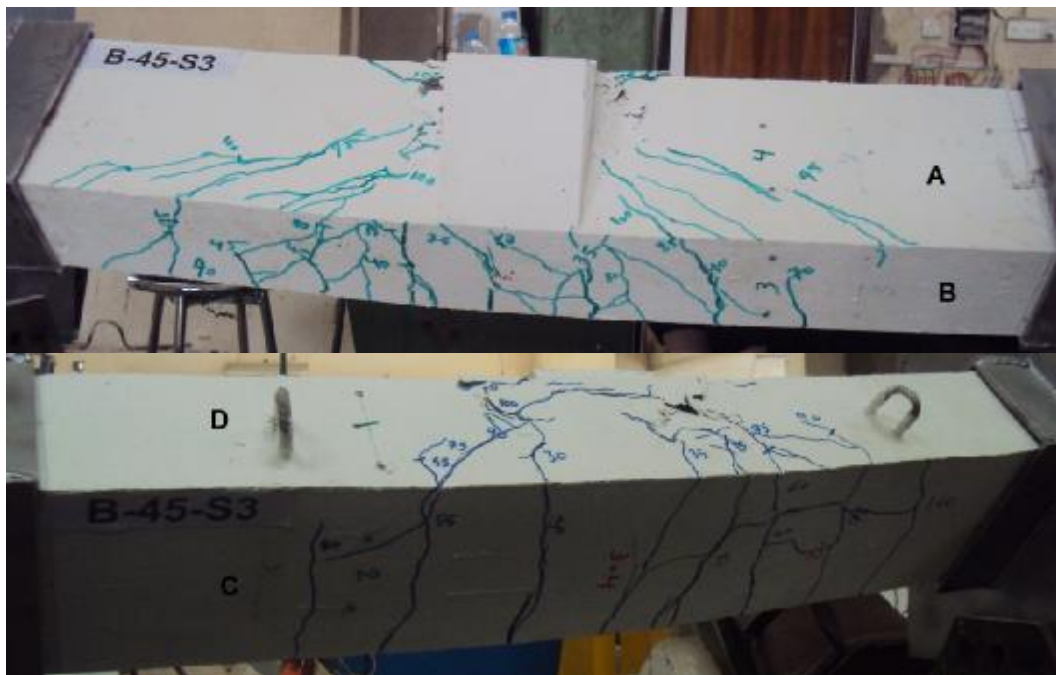


Plate (6) Crack Pattern of Specimen B-45-S3

3.2 Effect of Web Reinforcement on Crack Propagation

The crack propagation from the beginning applied load up to the occurrence of the diagonal cracking of reinforced concrete beams B-45-R, B-42-S2 and B-44-S3 with transverse steel ratio of 0.00377, 0.00502 and 0.00754 respectively appeared in similar pattern to the corresponding beam without shear reinforcement B-45-WS. It is due to that the shear

reinforcement does not pay effect to the generation of cracking at load. In other words, the diagonal cracking load is beam strength regardless of shear reinforcement. The marked differences of providing shear reinforcement are those, for specimen without shear reinforcement, diagonal cracks were generated in smaller number and the generated cracks were opened wider since redistribution of stresses was not accomplished. In addition it can be inferred that the grater stirrups spacing leads to greater diagonal crack spacing, confirming that there is a significant influence of stirrup spacing on the spacing between shear cracks. The reason for this behavior is attributed to the reduction of the effective concrete area, in which shear crack width is controlled by stirrups, and hence the increasing bond effect between the stirrups and surrounding concrete. Increasing the bond effect results in reducing the transfer length (or crack spacing) in which the forces to cause a crack are transferred into the concrete between the cracks by the bond stresses ^[4]. Another possible reason for this behavior is due to the difference in flexural crack. Stirrups often act as crack initiator and thus affect the flexural crack spacing as proved by Rizkalla ^[5]. Hence, smaller flexural crack spacing would be formed in the case of closer stirrup spacing. Plates (4) through (6) show the effect of web reinforcement on crack pattern.

3.3 Bi-axial Shear Capacity

Three types of bi-axial shear capacities are the main interests discussed in this study, i.e. ultimate load capacities (V_u), concrete contribution (V_c) and shear reinforcement contribution (V_s). The capacities obtained from tests are summarized in Table (2). Failure mode of all beams with and without stirrups was the “diagonal tension mode”. In the section that shear reinforcement was provided crossing the diagonal crack plane, it has been found to develop its yield strength at ultimate providing excessive cracking extension. It is often considered that ultimate shear capacity is composed of the *concrete contribution V_c and shear reinforcement contribution V_s , and the concrete contribution is considerable taken as diagonal cracking strength. For slender reinforced concrete beams, which have moderate shear span-to-depth ratio without shear reinforcements, this assumption is quite reasonable as shown by experimental results (Cassio, and Siess, (1960) ^[6] and Colunga, Aranda and Cuevas (2008) ^[7]for example). With the known of bi-axial shear capacity of concrete, is then deducted from the ultimate capacity to obtain the bi-axial shear capacity of shear reinforcement.

For calculation of bi-axial shear capacity, as mentioned in JSCE design code (JSCE 2007) ^[3], ellipse function is used to estimate the capacity from uni-axial shear capacities in uni-axial x and y directions. The calculations based on the standard, bi-axial shear capacity of shear reinforcement cannot be obtained directly. The code merely defines bi-axial shear capacity at ultimate stage implying that the bi-axial shear capacity of concrete can be estimated by using ellipse function (for a reinforced concrete member without shear reinforcement). As a result, use of ellipse function is adopted to estimate bi-axial shear

capacity contributed from concrete and at ultimate of reinforced concrete beams with stirrups. Hence, the ultimate bi-axial shear capacity (V_u) is reduced with bi-axial shear capacity contributed by concrete (V_c) to obtain the bi-axial capacity of shear reinforcement.

To compare the bi-axial shear capacity from experimental results with the results of ACI-Committee 318 (2011) [8] equations, the same ellipse function of JSCE (2007) is used for capacity estimation from the uni-axial shear capacity in x and y directions which are calculated by ACI-Committee 318 (2011) equations.

The specimen of reinforced concrete beam without shear reinforcement was tested for concrete contribution for the companion of tilted beams with shear reinforcement. Hence, the concrete contribution in this case is the average value between the test members without shear reinforcement and that obtained the member without shear reinforcement.

The average concrete shear capacities (V_c) of (B-45-R), (B-45-WS) are (140%) and (50%) higher than the calculation for ACI and JSCE codes respectively. They were also higher in range of (119 to 140%) for the calculation of ACI- 318 Committee 2011 and of (43 to 50%) for the calculation of JSCE 2007.

The comparison for the part of shear reinforcement capacity (V_s) indicates that the ellipse formula used for the calculation of ACI or JSCE code, is overestimating the capacity of shear reinforcement of all beams tested in 45° in range of (-21) to (-6%) and (-25) to (-11%) for ACI and JSCE codes, respectively as shown in **Table (2)**.

Table (2) Capacities of tested specimens

Beam Designation	Theoretical (by JSCE Code) kN			Theoretical (by ACI Code) kN			Experimental kN			Exp./Cal. By JSCE code			Exp./Cal. By ACI code		
	V_u	V_c	$*V_s$	V_u	V_c	$*V_s$	V_u	V_c	$*V_s$	V_u	V_c	V_s	V_u	V_c	V_s
B-45-R	63.38	34.24	29.14	49.4	21.74	27.66	74.5	47.5	22	1.17	1.5	0.75	1.5	2.4	0.79
B-45-WS	34.24	34.24	—	21.74	21.74	—	50.5	50.5	—	1.5	1.5	—	2.4	—	—
B-45-S2	73.84	34.98	38.85	59.32	22.45	36.86	82.5	50	32.5	1.11	1.43	0.83	1.39	2.2	0.88
B-45-S3	92.52	34.24	58.28	78.1	22.80	55.3	102	50	52	1.1	1.46	0.89	1.3	2.19	0.94
*is considered by taking difference between ultimate V_u and concrete part V_c							** Range			(10_17) %			(43_50) (30_50) (119_140) % (-25_-11) % (-21_-6) %		
**considered for specimens subjected to bi-axial shear with shear reinforcement only															

Regarding the ultimate load (V_u), i.e. the summation of concrete and reinforcement capacity, the present ACI and JSCE design practice seems to be conservative, for the concrete contribution (V_c). However for the shear reinforcement contribution (V_s) the calculation by using current ellipse formula seems to be not conservative.

3.4 Effect of Transverse Ratio on Bi-axial Shear Capacity

Figure (5) shows the shear forces versus the deflection of beams B-45-WS, B-45-R, B-42-S2 and B-44-S3 with transverse steel ratios of 0, 0.00377, 0.00502 and 0.00754, respectively. In these beams the longitudinal steel ratio was kept constants at $\rho_l=0.0241$. Beyond the formation of major diagonal cracking, the beams with bigger shear reinforcement ratio behave stiffer than those reinforced with less web reinforcement. This means, that before forming of diagonal cracks, shear reinforcement has no considerable efficiency on beam stiffness, but beyond the formation of major diagonal cracks appear the improving effect of shear reinforcement on beam stiffness appears to have a dominant role which increases with increasing the amount of web reinforcement.

Table (3) shows the effect of variation in transverse reinforcement ratio on (V_u) and (V_s) of these beams. It reveals that when transverse steel ratio is increased from zero to 0.00377, 0.00502 and 0.00754, the ultimate load increases by 47.5%, 63% and 101%, respectively. On the contrary when transverse steel ratio is stepwise increased from 0.00377 to 0.00502 then to 0.00754, the shear reinforcement contribution is increased by 47.7% and 136% respectively. Also by increasing the transverse steel ratio from 0.00377 to 0.00502 then to 0.00754 the ratio of (experimental /theoretical values) is increased by 11.4% and 20%, respectively for the ACI calculations, and increased by 10.6% and 18.6% respectively for the JSCE calculation, as shown in **Table (4)**. The reason of this increase may be the small spacing between stirrups which made them work together more effectively than the reference beam B-45-R.

Figure (6) and **Table (3)** show that both the ultimate load and V_s increase as the shear reinforcement ratio (ρ_s) is increased. The results show that the behavior of these beams which were reinforced with stirrups is generally similar to that of beams without stirrups up to the stage of diagonal cracking. However, beyond this stage, as the inclined crack crosses the shear reinforcement, the force is then transmitted from one plane to another by the action of stirrups. Therefore, the presence of stirrups allows the beam to redistribute the internal forces across the inclined cracks and restrain the widening of the inclined crack and tie the longitudinal reinforcement in place so that it increases or at least maintains the forces carried the by aggregate interlock and the dowel action by confining the core of concrete beam

Table (3) Effect of Variation in Transverse Reinforcement Ratio on Ultimate Load and Shear Reinforcement contribution

Beam Designation	Transverse ratio	V_u	Percent of Increase (%)	V_s	Percent of Increase (%)
B-45-WS	$\rho_s = 0$	50.5	---	----	----
B-45-R	=0.00377	74.5	47.5	22	----
B-45-S2	=0.00502	82.5	63	32.5	47.7
B-45-S3	=0.00754	102	101	52	136

Table (4) Effect of Variation in Transverse Reinforcement Ratio on Experimental/ Calculation Ratio

Beam Designation	Transverse Ratio	Exp./ACI Calc. of V_s	Percent of increase (%)	Exp./JSCE Calc. of V_s	Percent of Increase (%)
B-45-R	$\rho_s = 0.00377$	0.79	----	0.75	----
B-45-S2	$\rho_s = 0.00502$	0.88	11.4	0.83	10.6
B-45-S3	$\rho_s = 0.00754$	0.94	20.2	0.89	18.6

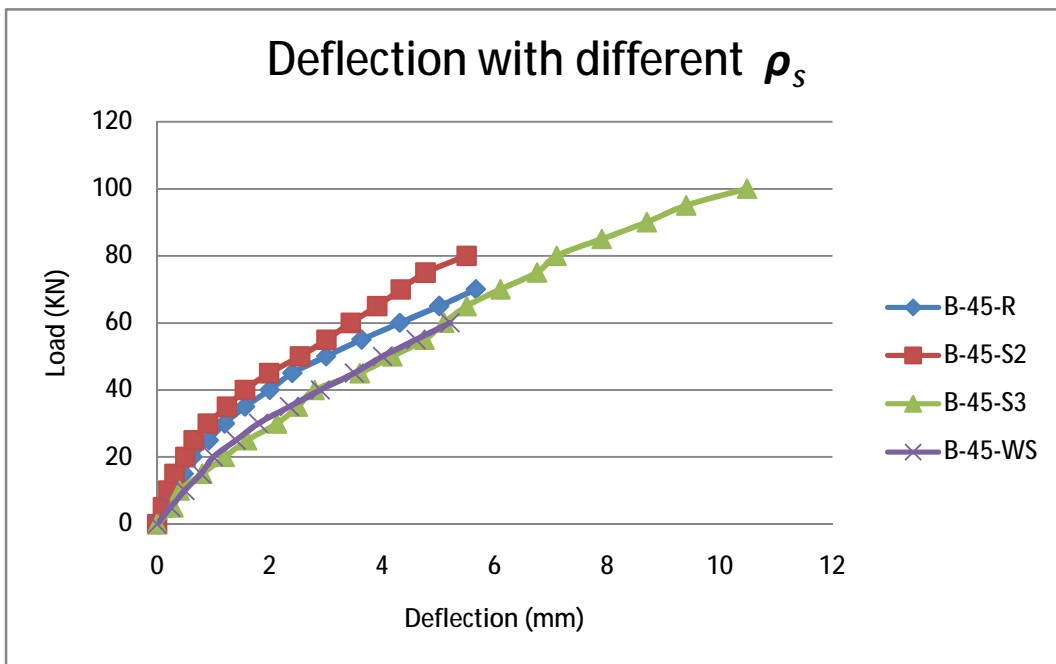


Fig.(5) Load-Vertical Mid-span Deflection for Different Transverse Steel Ratios

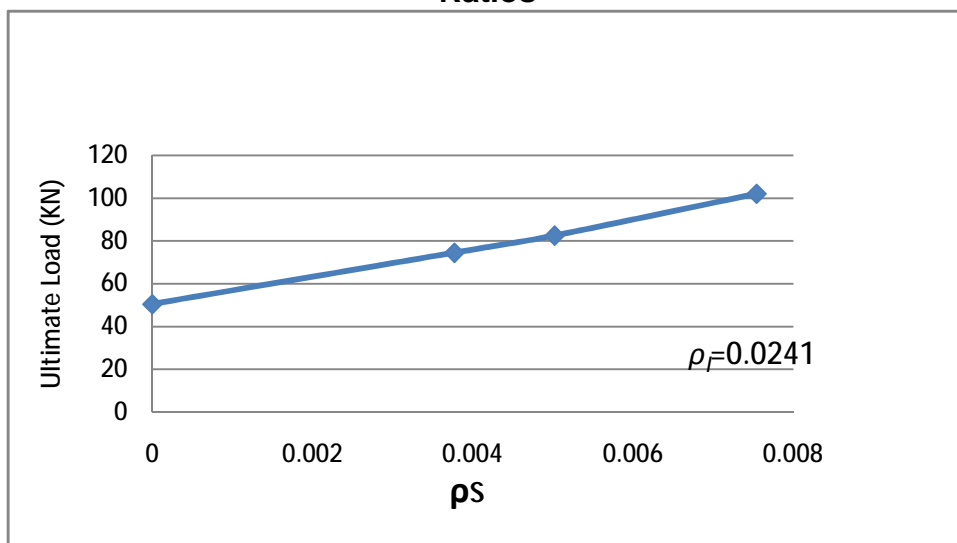


Fig. (6) Experimental Ultimate Load for Different Transverse Reinforcement Ratios

4. Conclusions :

The ultimate capacity in this study is discussed separately in terms of the concrete and the shear reinforcement contributions based on the current design method using ellipse interaction formula (JSCE code 2007).

From experimental results of the tested reinforced concrete beams, the following conclusions can be drawn.

1. The comparison of ultimate capacities from experimental results with the calculated values from the ellipse formula using the ACI and JSCE codes indicates that these codes gives quite conservative values of ultimate capacity where reductions of about (30 to 76)% and (7 to 20)%, respectively, are observed.
2. The ellipse formula underestimates the capacity of concrete part by about (119 to 188%) and about (43 to 50%) for ACI and JSCE design, respectively. This is due to the increase of shearing area along diagonal crack plane. Tilting of the specimen significantly increases the effective depth while shear span remains constant; hence the shear span-to-depth ratio is decreased, especially with the high reinforcement ratio used in the tested specimen.
3. For the contribution of shear reinforcement, the calculations using ellipse formula overestimate the shear reinforcement part by (-21) to (-6) % and by (-25) to (-11) % for the ACI and the JSCE codes, respectively. Therefore the experimental results have exposed weakness in existing ellipse formula.
4. When transverse steel ratio is stepwise increased from zero to 0.00377, 0.00502 and 0.00754, the ultimate load is increased by 47.5%, 63% and 101% respectively. While when transverse steel ratio is increased from 0.00377 to 0.00502 and 0.00754, shear reinforcement contribution is increased by 47.7% and 136% respectively. Also by increasing the transverse steel ratio from 0.00377 to 0.00502 and 0.00754 the ratio of (experimental /calculation) was increased by 10.6% and 18.6% respectively.

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