

## Design LQR Controller for Higher Order Systems Based on the Reduction Model

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### Abstract

*This paper present a simple method for reducing the linear higher order systems. The coefficient of the reduction model is obtained by using the modified dominant pole technique, while the coefficients of the numerator are obtained by reduced order model. Linear Quadratic Regulator (LQR) is used to design a controller for the higher order system depended on the linear reducing equations. LQR without and with integral effect method are used and compared for controlling the higher order systems. The LQR controller are designed and simulated using MATLAB/Simulink. Simulation results show that both the controllers are capable of controlling the systems successfully. The LQR with integral gives the better performance, compared to when the system without it. the LQR controller scheme with smith predictor connection is suggested in order to maintain the stability of the higher order plant in addition to improve the performance of the controlled system. The simulation results for this scheme are tested with two higher order time delay systems to illustrate the efficient performance for the proposed control scheme.*

### تصميم مسيطر LQR للانظمة ذات الرتبة العالية اعتمادا على نموذج ذات رتبة اقل

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#### الخلاصة

في هذا البحث تم تقديم طريقة بسيطة لتقليل الانظمة ذات الرتب العالية. حيث تم الحصول على معاملات بسط معادلة التقليل باستخدام طريقة (dominant pole) بينما يتم الحصول على معاملات المقام باستخدام طريقة (reduced order). تم استخدام اسلوب السيطرة (LQR) للسيطرة على الانظمة ذات الرتب العالية. حيث تم تطبيق هذا المسيطر مع وبدون استخدام التكامل، ثم اجريت المقارنة بين الاسلوبين وبيان تأثيرهم على الانظمة ذات الرتب العالية. النظام تم تصميمه باستخدام برنامج (MATLAB/Simulink)، وقد اظهرت النتائج ان الاسلوبين قادران على السيطرة على النظام بصورة ناجحة، لكن المسيطر مع استخدام التكامل قد اعطى نتائج افضل مقارنة بدونه. ايضا، في هذا البحث تم طرح مخطط مسيطر (LQR) مع متنبئ (Smith) وذلك للحفاظ على استقرارية المنظومة ذات الدرجة العالية بالإضافة الى تحسين اداء المنظومة المسيطر عليها. وقد تم اختبار مثالين لتوضيح الاداء الكفوء لمخطط السيطرة المقترح.

**keywords:-** higher order, dominant pole method, LQR controller, smith predictor, time delay.

## **1. Introduction**

The reduction of complex high-order models to simple low-order models has been the interest of many researchers [1]. Simple lower order models are very desired in the design of control systems, as the analysis of higher order systems is exhausted and mathematically dense. The reason of obtaining reduced model is to improve a good reduction for the higher order systems, with keeping the characteristics of it as much as possible [1].

Different reduction methods have been proposed. Two approaches have attracted more interested in this paper. These are dominant pole [2] and reduced order model [3]. Several model order reduction techniques have been presented [4-7]. Each one has their own features and applications. In recent times, mixed mathematical techniques [8-9] have taken much significance in model order reduction Single-Input Single-Output (SISO) systems. Shamash [4] has suggested a multivariable system reduction using Padé Approximation and dominant Eigen values. This method proposed that the dominant pole of the higher order system are known, then suffer from the flaw of its conformity to systems with no dominant poles, or where the dominant poles are difficult to determine. Pal [5] has developed a system reduction using the continued partition approach and Routh Hurwitz Array, in which the initial transient response of the reduced order might not applicable with the higher order system, only the first few time moments are assumed depending on the order of the reduced model. Control methods based on traditional unity feedback control structure and (LQR) controller has been developed [6]. A higher order system is reduced to a low order form plus a time delay. It is known the smith predictor (SP) control structure which is more active for processes with large time delay compared with a traditional unity feedback control structure [10]. The common denominators of the transfer function of the reduced order model is determine by using a dominant pole method, while the numerators is obtained using reduced order model.

The paper is organized as follows. Section 2 summarizes some well known properties of scalar transfer functions and formulates the problem of computing the dominant poles and reduced order model of a scalar transfer function. Numerical tested example is presented in section 3 .Section 4 discusses the LQR controller with and without integral. In section 5, simulation results are described. Section 6 A simple control scheme with smith predictor connection is suggested for time delay higher order systems. Finally conclusion is summarized.

## 2. The Proposed Reduction Method

In this proposed method the features of mixed method of order reduction is used for (SISO), where the numerator is reduced by reduced order model and second order reduced denominator is derived by dominant pole method of higher order system.

The procedure for determining the reduced order model is described as follow:

Consider an  $n^{th}$  order stable linear single-input-single-output (SISO) system described by the transfer function [3].

$$G_n(s) = \frac{N(s)}{D(s)} = \frac{\sum_{i=0}^{n-1} a_i s^i}{\sum_{i=0}^n b_i s^i} \quad \dots (1)$$

where  $a_i (0 \leq i \leq n-1)$  and  $b_i (0 \leq i \leq n)$  are scalar constants. The corresponding stable  $k^{th} (k < n)$  order reduced model is of the form:

$$R_k(s) = \frac{N_k(s)}{D_k(s)} = \frac{\sum_{i=0}^{k-1} d_i s^i}{\sum_{i=0}^k e_i s^i} \quad \dots (2)$$

where  $d_i (0 \leq i \leq k-1)$  and  $e_i (0 \leq i \leq k)$  are scalar constants.

Assuming the original system described by Eq. (1), the required is to find a numerator of the transfer function of the reduced order model in the form of Eq. (2) where the reduced order model maintain the important characteristics of the original system and approximates its response as closely as possible for the same type of inputs.

The  $n^{th}$  order original system given in Eq. (1) is equated to the  $k^{th}$  order reduced model with unknown parameters represented by Eq. (2).

Hence,

$$G_n(s) = R_k(s) \quad \dots (3)$$

$$\frac{a_0 + a_1 s + a_2 s^2 + \dots + a_{n-1} s^{n-1}}{b_0 + b_1 s + b_2 s^2 + \dots + b_n s^n} = \frac{d_0 + d_1 s + d_2 s^2 + \dots + d_{k-1} s^{k-1}}{e_0 + e_1 s + e_2 s^2 + \dots + e_k s^k} \quad \dots (4)$$

Cross multiplying and rearranging the Eq. (4).

$$\begin{aligned} & a_0 e_0 + (a_0 e_1 + a_1 e_0) s + (a_0 e_2 + a_1 e_1 + a_2 e_0) s^2 + \dots + a_{n-1} e_k s^{k-1+k} \\ & = b_0 d_0 + (b_0 d_1 + b_1 d_0) s + (b_0 d_2 + b_1 d_1 + b_2 d_0) s^2 + \dots + b_n d_{k-1} s^{n-1+k} \quad \dots (5) \end{aligned}$$

Equating the coefficients of the corresponding terms in the Eq. (5), the following relations are obtained:

$$\begin{aligned} a_0 e_0 &= b_0 d_0 \\ a_0 e_1 + a_1 e_0 &= b_0 d_1 + b_1 d_0 \\ a_0 e_2 + a_1 e_1 + a_2 e_0 &= b_0 d_2 + b_1 d_1 + b_2 d_0 \\ \dots & \\ \dots & \end{aligned}$$

The unknown parameters are determined by using the above relations and the other unknown parameters are determined by the dominant pole method.

Otherwise, the procedure for determining the dominant pole method to find the denominator of the transfer function is described below [2]:

The transfer function of a Single-Input Single-Output (SISO) system:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = C^T x(t) + Du(t), \end{cases}$$

where

$$\begin{aligned} A \in R^{n \times n}, x(t), B, C \in R^n \text{ and } u(t), y(t), D \in R, \text{ is defined as:} \\ H(s) = C^T (sI - A)^{-1} B + D \end{aligned} \quad \dots (6)$$

where  $I \in R^{n \times n}$  is the identity matrix and  $s \in C$  and  $D = 0$  in the following.

Let the eigenvalues (poles) of  $A$  and the corresponding right and left eigenvectors be given by the triplets  $(\lambda_j, x_j, v_j)$ . Let the right and left eigenvectors be scaled so that  $v_j x_j = I$ , where  $(v_j x_k = 0 \text{ for } j \neq k)$ . The transfer function  $G(s)$  can be expressed as a sum of residues  $R_j$  over first order poles:

$$G(s) = \sum_{j=1}^n \frac{R_j}{s - \lambda_j}, \quad \dots (7)$$

where the residues  $R_j$  are:

$$R_j = (x_j^T C)(v_j^* B) \quad \dots (8)$$

A pole  $\lambda_j$  that corresponds to a residue  $R_j$  with large magnitude  $|R_j|$  is called a dominant pole, i.e. a pole that is well controllable in the transfer function. An reduction of  $G(s)$  that consists of  $k < n$  terms with  $|R_j|$  above some value, determines the active transfer function:

$$G_k(s) = \sum_{j=1}^k \frac{R_j}{s - \lambda_j}, \quad \dots (9)$$

The problem of concern can now be formulated as: given a *SISO* linear, time invariant, dynamical system  $(A, B, C, D)$ , compute  $k \ll n$  dominant poles  $\lambda_j$  and the corresponding right and left eigenvectors  $x_j$  and  $v_j$ .

### 3. Numerical Tested Example

Consider the fourth order system transfer function given in [11]:

$$G_n(s) = \frac{2400 + 1800s + 496s^2 + 28s^3}{240 + 360s + 204s^2 + 36s^3 + 2s^4} \quad \dots (10)$$

The denominator of the transfer function of the reduced order model is specified by using a dominant pole method. First, we must find the roots of the denominator, then take the roots that have the smallest magnitude, in this example the roots are:

$$s_{1,2} = -7.8033 \pm j 1.3576, s_{3,4} = 1.1967 \pm j 0.6934$$

Here we choose the smallest one  $s_{3,4}$  when the magnitude is 1.38.

The numerator of the transfer function is presented by using a reduced order model method. Consider a second order reduced model represented by:

$$R_k = \frac{d_0 + d_1s}{e_0 + e_1s + e_2s^2} = \frac{d_0 + d_1s}{1.913 + 2.393s + s^2} \quad \dots (11)$$

where  $d_0, d_1$  are unknown parameters. Equating Eq. (10) and Eq. (11) then cross multiplying, we obtain:

$$4591.2 + (3443.4 + 5743.2)s + (2400 + 4307.4 + 948.84)s^2 + (1800 + 1186.928 + 53.56)s^3 + (496 + 67.004)s^4 + 28s^5 = 240d_0 + (360d_0 + 240d_1)s + (204d_0 + 360d_1)s^2 + (36d_0 + 204d_1)s^3 + (2d_0 + 36d_1)s^4 + 2d_1s^5 \quad \dots (12)$$

Comparing the terms in equation (12), we obtain the following:

$$4591.2 = 240 d_0 \quad \dots (13)$$

$$5743.2 + 3443.4 = 360d_0 + 240d_1 \quad \dots (14)$$

$$2400 + 4307.4 + 948.84 = 204d_0 + 360d_1 \quad \dots (15)$$

$$1800 + 1186.928 + 53.564 = 36d_0 + 204d_1 \quad \dots (16)$$

$$496 + 67.004 = 2d_0 + 36d_1 \quad \dots (17)$$

$$28 = 2d_1 \quad \dots (18)$$

By substitute any equations of (13-18), the unknown parameters are solved and the second order reduced model is obtained as:

$$R_2 = \frac{19.13 + 9.5825s}{1.913 + 2.393s + s^2} \quad \dots (19)$$

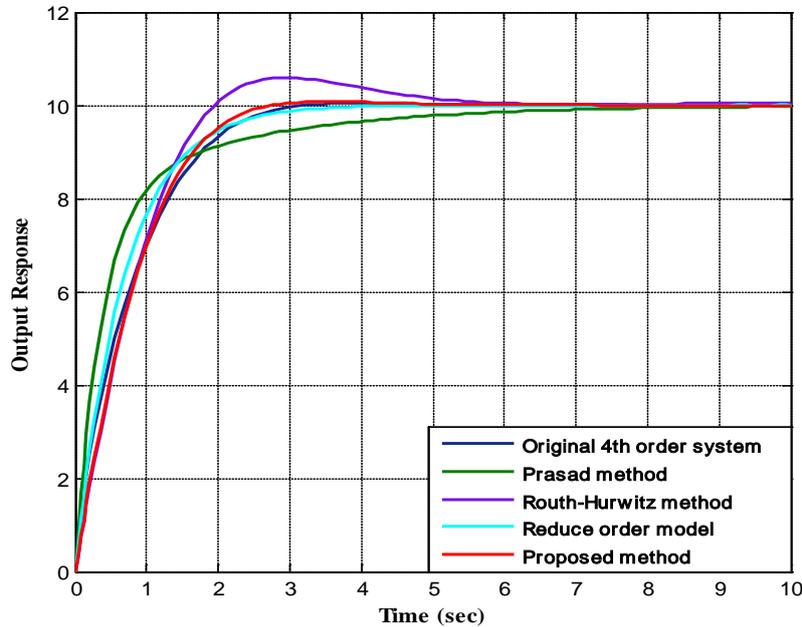
For comparing the proposed method with different reduced order models [3, 12 and 13], an error 'J' is used as bellow [12]:

$$J = \sum_{i=0}^N [Y(t_i) - Y_r(t_i)] \quad \dots (20)$$

Where  $Y(t_i)$  and  $Y_r(t_i)$  are the outputs of the original and the reduce order systems respectively at the  $i^{th}$  sampling instant  $t$  and  $N$  is the number of sampling periods. The unit step responses of the reduced models [3, 13 and 14] with that the original system are shown in figure (1) . Otherwise, the comparison of different order reduction methods is given in table 1:

**Table 1. Comparison of Methods**

Method of Reduction	Reduced Model	Cumulative error index 'J' at 10 sec
1- Prasad method [13]	$R_2(s) = \frac{11.903720 + 22.533255s}{1.190362 + 3.164997s + s^2}$	27.42
2- Routh-Hurwitz array method [14]	$R_2(s) = \frac{13.174378 + 9.046883s}{1.304548 + 1.701321s + s^2}$	12.17
3-Reduced Order Model [3]	$R_2(s) = \frac{1411.256 + 14s}{41.0236 + 29.5887s + s^2}$	4.39
4-Proposed method	$R_2(s) = \frac{19.13 + 9.5825s}{1.913 + 2.393s + s^2}$	1.93



**Fig. (1): Comparison of unit step responses**

#### **4. LQR control design for higher order system**

In this section, LQR controller and LQR with integral effect is proposed and explained in detail, which is suggested from [15] for using it with the reduce higher order system to meet the desired performance specifications.

#### **4.1 LQR (Linear Quadratic Regulator)**

LQR control is a method to find the solution for the problem of reduction that ensures the system stability in closed loop, also its easy in calculation. The most general problem that solved by this method can given by the follow equation of the dynamic system [15]:

$$\dot{x}(t) = Ax(t) + Bu(t); \quad x(t_0) = x_0 \quad \dots (15)$$

with  $t \in [t_0, t_1]$

The Eq. (16) represent the quadratic cost function to minimize:

$$J = \frac{1}{2} \int_{t_0}^{t_1} [x^T(t)Qx + u^T(t)R_u u(t)] dt + \frac{1}{2} x^T(t_1)P_{t_1}x(t_1) \quad \dots (16)$$

$R_u > 0$ ,  $P_{t_1} \geq 0$ ,  $A(t)$  and  $B(t)$  are continuous-time function,  $R_u$  and  $P_{t_1}$  are continuous-time and bounded functions respectively. Thus the general problem in LQR method is to find an input  $u(t)$  in time domain between the initial and final given times. The input is defined by the Eq. (17):

$$u(t) = -R_u^{-1} B^T P(t) x(t) = -K(t)x(t) \quad \dots (17)$$

where  $P(t)$  in Eq. (18) is the solution to the Ricatti differential equation:

$$-\dot{P}(t) = A^T P(t) + P(t)A + Q - P(t)BR_u^{-1} B^T P(t) \quad \dots (18)$$

For linear time invariant systems, Eq. (18) reaches a value in stable state that is reduced to the Eq. (19):

$$A^T P + PA + Q - PBR^{-1} B^T P = 0 \quad \dots (19)$$

named the Control Algebraic Ricatti Equation (CARE) and it is find the optimum value of  $P$ . The optimum input is defined

in Eq. (20):

$$u(t) = -Kx(t) \quad \dots (20)$$

The value  $K$  is found by the Matlab control toolbox using the syntax:  $K = \text{lqr}(A; B; Q; R)$ . The Matlab function `lqr` help to choose two parameters  $R$  and  $Q$  which will balance the relative importance of the control effort ( $u$ ) and the error. In the cost function that we are trying to optimize, the simplest case is to assume  $R = I$  and  $Q = C^T R C$ . The diagram that describe the stabilizing control is shown in the figure 2.  $A$ ,  $B$  and  $C$  are the matrix associated to the linearized system and  $R$  is weight matrix.

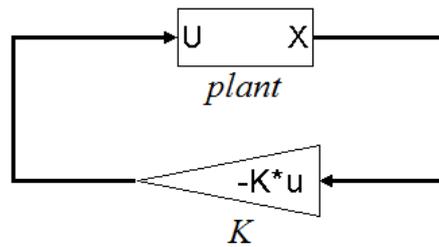


Fig. (2): LQR control to stabilize the system

#### 4.2 LQR with Integral Effect

By the concept of the control from the previous section and adding the integral to the block diagram to solve the oscillation problem of the system with LQR controller without affecting on it, therefore with  $y(t) = C x(t)$  the outputs and  $r(t)$  the reference which is the additional new states  $x_I(t)$ , therefore the equations states  $x_I(t)$  with its respective costs in the cost function.

With  $x(t) = [x(t) \ x_I(t)]^T$  the new cost function is defined in Eq. (21):

$$J = \int_0^{\infty} [x^T(t) Q x(t) + u^T(t) R u(t)] dt \quad \dots (21)$$

therefore:

$$u(t) = - [K \ K_I] \begin{bmatrix} x(t) \\ x_I(t) \end{bmatrix} = -K x(t) \quad \dots (22)$$

When the control has been designed, it must take the block in the closed loop transfer function shown in the diagram in figure 2:

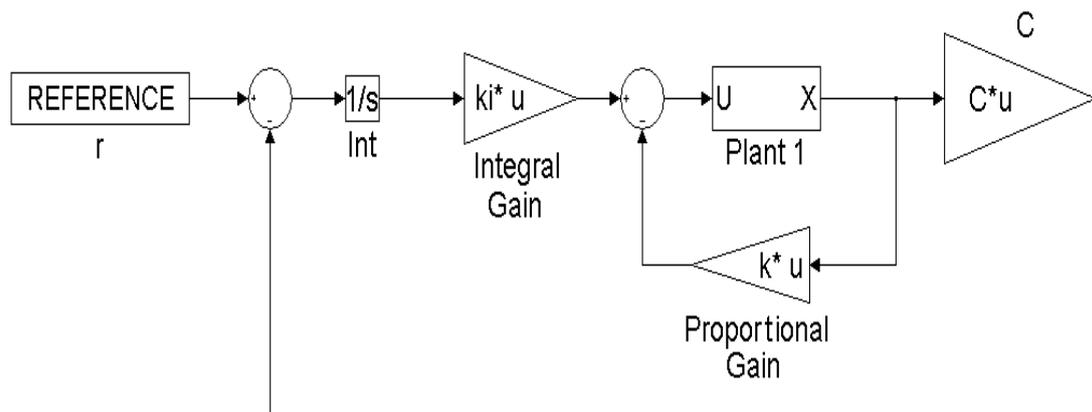


Fig. (3): LQR control with integral effect following the reference

## 5. Simulation results

In this section, the Matlab (version 9.0) can be used to simulated the suggested LQR controller and LQR controller with integral effect with different examples. Two higher order transfer function are tested to explain the improve properties of the suggested method on the controlled examples.

The two examples of the higher order transfer function with controller are given as following:

### Example 1:

Consider the fourth order transfer function given in [11]:

$$G_n(s) = \frac{2400 + 1800s + 496s^2 + 28s^3}{240 + 360s + 204s^2 + 36s^3 + 2s^4} \quad \dots (23)$$

By applying the proposed method, the reduced order model is:

$$R_2(s) = \frac{19.13 + 9.5825s}{1.913 + 2.393s + s^2} \quad \dots (24)$$

Applying the LQR controllers to the reduced model. The values of the parameters are located in the linear model of the form  $\dot{x}(t) = Ax(t) + Bu(t)$ , with  $\dot{y}(t) = Cx(t) + Du(t)$ , whose values for  $A$ ,  $B$ ,  $C$  and  $D$  are these:

$$A = \begin{bmatrix} -2.393 & -1.913 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$C = [2.395 \quad 4.782] \quad D = [0]$$

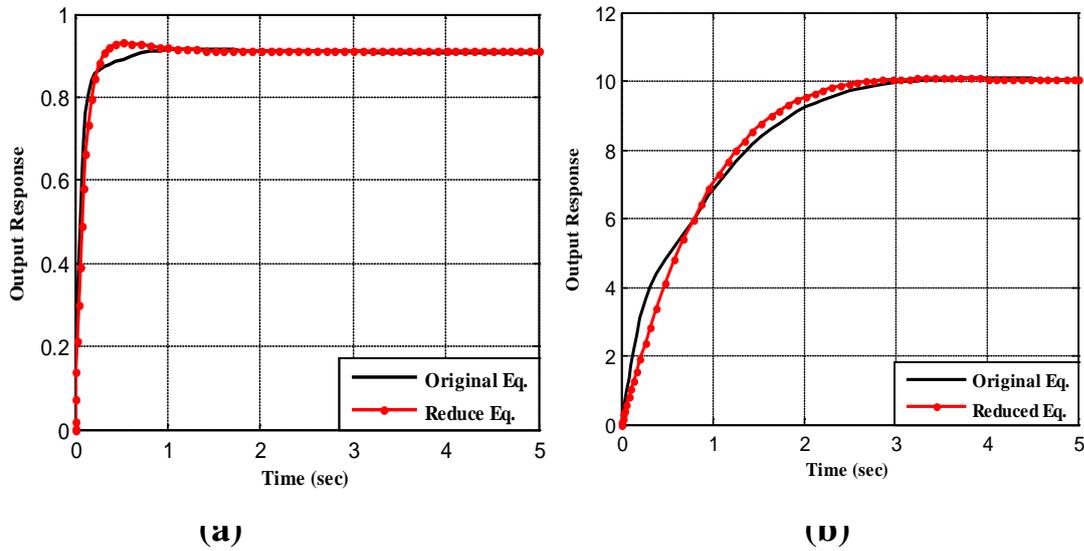
From the above matrices, the matrix  $Q$  and the matrix  $R$ , the solution of LQR problem is found with the `lqr` command in control toolbox of Matlab (version 9.0), thus:

$$Q = \begin{bmatrix} 5.7389 & 11.4570 \\ 11.4570 & 22.8723 \end{bmatrix} \quad R = [1]$$

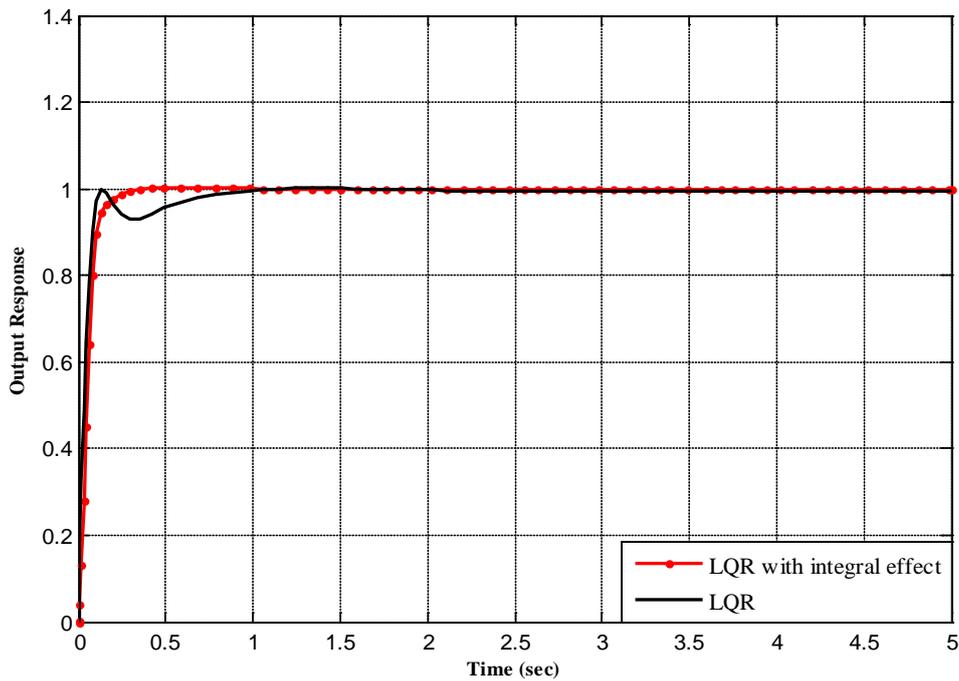
And the constant matrix of control  $k$  is calculated. Its value is:

$$K = [2.2759 \quad 4.3281]$$

**Figure (4)** shows the output response for the open loop and close loop connection for both original transfer function Eq. (23) and the reduced transfer function Eq. (24) under unit step input, **figures (5)** shows the step response of the higher order system with LQR and LQR with integral effect.



**Fig.(4): a) Output response for open loop connection of original and reduced linear transfer function. b) Output response for closed loop connection of original and reduced linear transfer function**



**Fig. (5): the step response of the higher order system with LQR and LQR with integral effect**

We can see from the figure above that both of the controllers make the system follow the desired input with zero steady state, but LQR with integral effect solved the problem of the oscillation of the LQR controller, this controller (LQR with integral effect) are tuned to obtain a response with the desired performance specification.

**Example 2:**

Consider the eighth order system transfer function given in [16]:

$$G_n(s) = \frac{194480 + 482964s + 511812s^2 + 278376s^3 + 82402s^4 + 13285s^5 + 1086s^6 + 35s^7}{17760 + 45952s + 46350s^2 + 24469s^3 + 7669s^4 + 1558s^5 + 220s^6 + 21s^7 + s^8}$$

.... (25)

The roots that have the smallest magnitude to obtain the denominator of the transfer function of the reduced order model is (  $-1.56 \pm j 6.21$  ).

By applying the proposed method (as detailed in section 2), the second order reduce model is obtained as:

$$R_2(s) = \frac{405.166+35.754s}{s^2+2s+37}$$

.... (26)

Applying the LQR controllers to the reduced model, the values of  $A, B, C$  and  $D$  are:

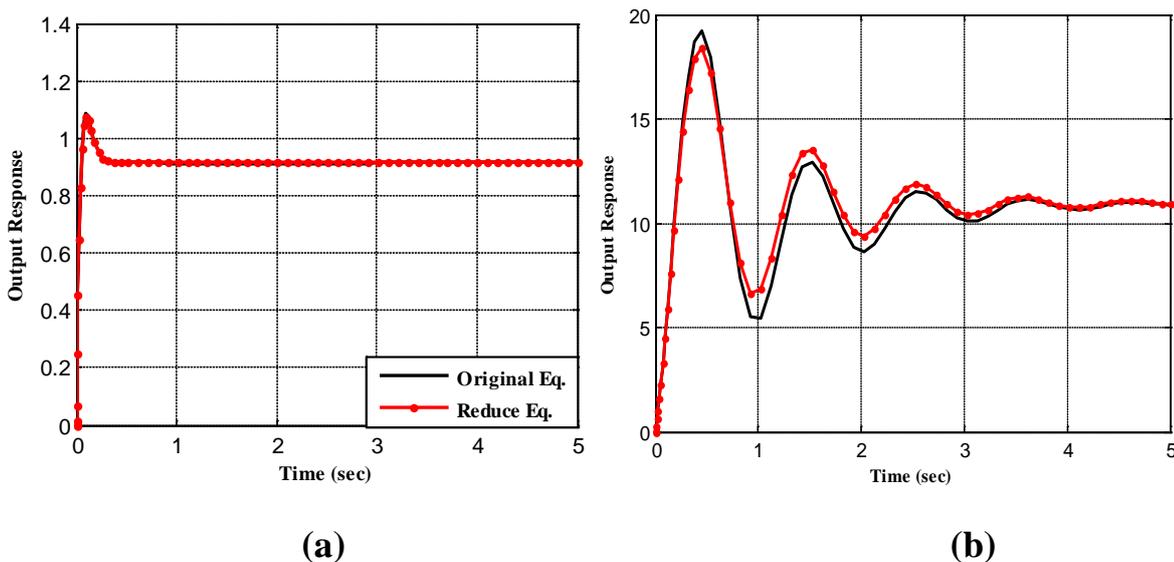
$$A = \begin{bmatrix} -2 & -4.625 \\ 8 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 0 \end{bmatrix} \quad C = [4.469 \ 6.33] \quad D = [0]$$

With the previous matrices, the weight matrices  $Q, R$  and the constant matrix of control  $k$  are:

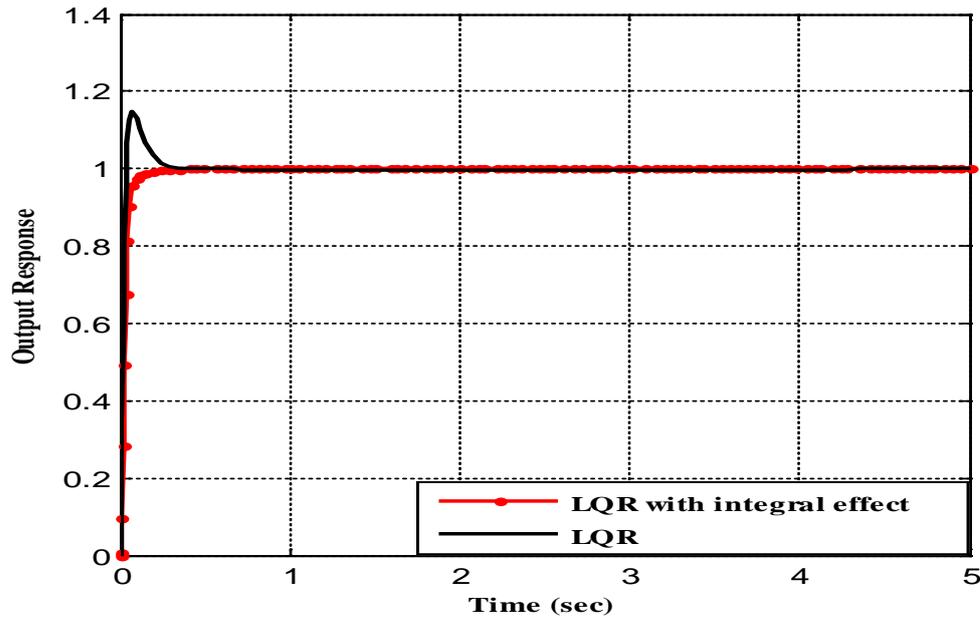
$$Q = \begin{bmatrix} 19.9737 & 28.2900 \\ 28.2900 & 40.0689 \end{bmatrix} \quad R = [1]$$

$$K = [5.3707 \ 5.7782]$$

Figure (6) shows the output response for the open loop and close loop connection for both original transfer function Eq. (25) and the reduced transfer function Eq. (26) under unit step input, **figures (7)** shows the step response of the higher order system with LQR and LQR with integral effect.



**Fig.(6): a) Output response for open loop connection of original and reduced linear transfer function. b) Output response for close loop connection of original and reduced linear transfer function**



**Fig. (7): The step response of the higher order system with LQR and LQR with integral effect**

Figure (7) show that the controllers make the system follow the input with zero steady state, LQR with integral effect solved the problem of the oscillation of the LQR controller.

## **6. Smith Predictor:**

A higher order system is reduced to a low order form plus a time delay [10]. It is known the smith predictor (SP) control structure. The block-diagram for the proposed control scheme is shown in Fig.(8).

Where:

$G_h(s)$ : transfer function for the higher order.

$G_r(s)$ : reduced second order model.

$e^{-ds}$ : the actual time delay.

$e^{-d_p s}$ : the predicted time delay.

With Matlab\Simulink, two higher order examples are tested LQR controller with smith predictor connection, to show the stability when the difference between the actual and the predication time delay become large. The purpose for selected two tested examples is to illustrate the ability of the suggested method in reduce any higher order transfer function to second order transfer function. Note the parameters of the LQR controller are obtained for the reduced order model.

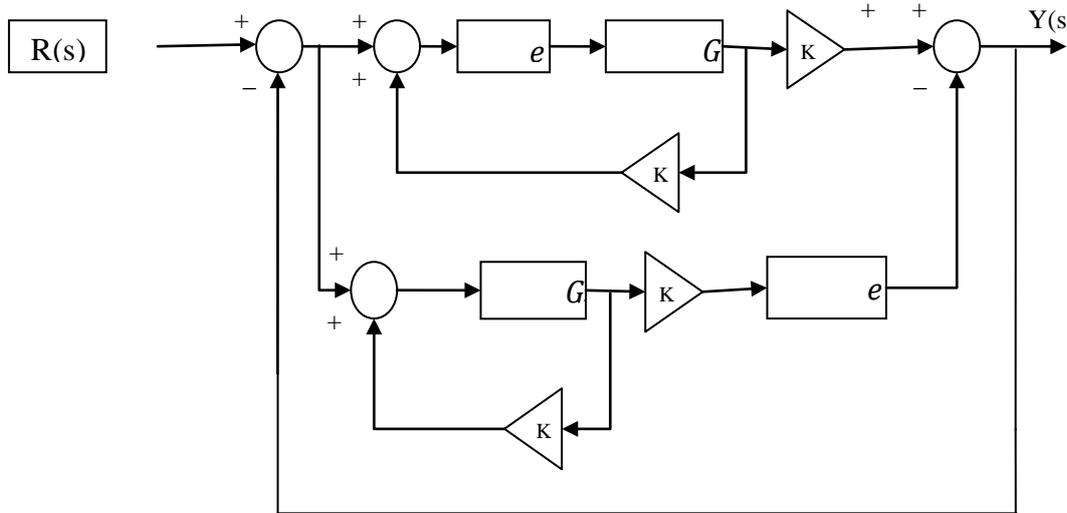


Fig. (8): the LQR controller with smith predictor

**Example (3):**

consider the fourth order transfer function which is given in Eq. (23), the reduced second order model is given in Eq. (24), the simulation results for this example with time delay for example:  $d = 0.4$  sec. and different predicted time delay  $dp = (0.3, 0.6)$  sec. are shown in Fig. (9).

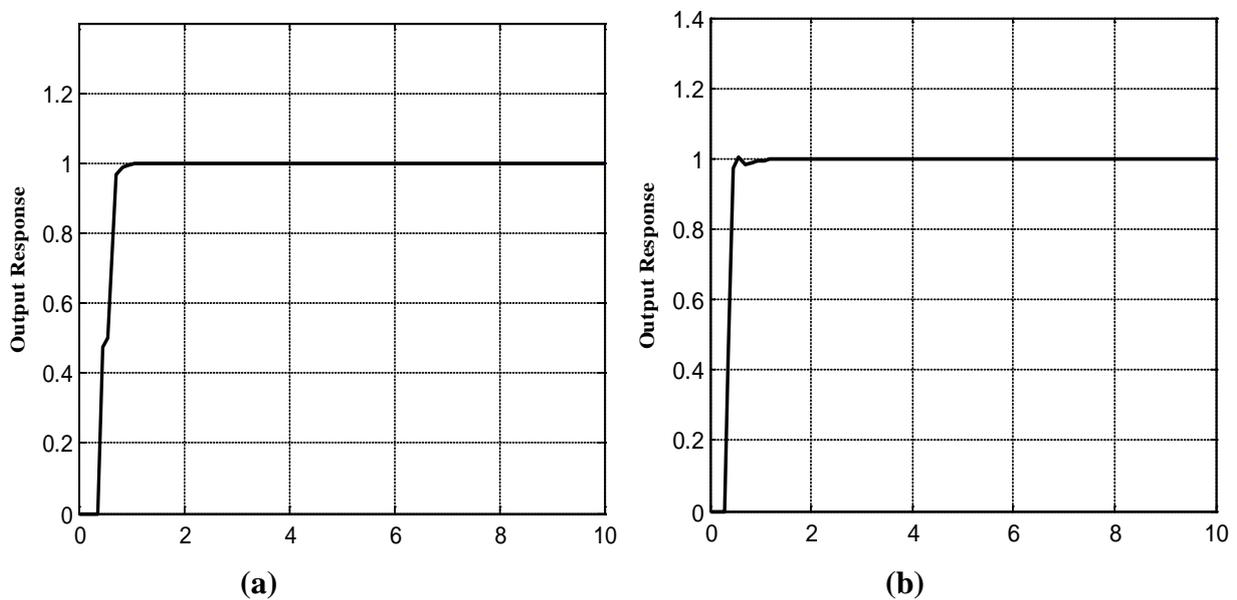
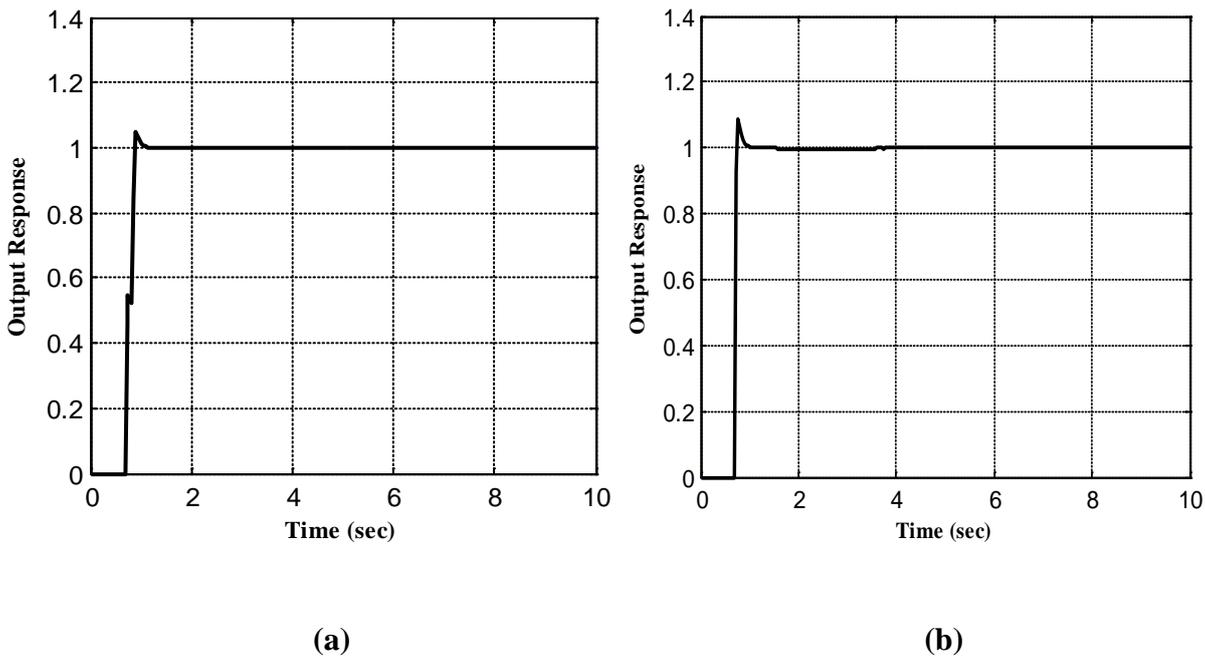


Fig. (9): the output response for Ex.3. with  $d=0.4$  sec., (a): with controller and  $dp=0.3$  sec., (b): with controller and  $dp=0.6$  sec.

**Example (4):**

consider the eighth order transfer function which is given in Eq. (25), the reduced second order model is given in Eq. (26), The simulation results for this example with time delay for example:  $d = 0.7$  sec. and different predicted time delay  $dp = (0.7, 1)$  sec. are shown in Fig. (12).



**Fig. (10): the output response for Ex.4. with  $d=0.7$  sec., a): with controller and  $dp=0.7$  sec., b): with controller and  $dp=0.9$  sec.**

The response of the two simulated examples (3 and 4) with (LQR and smith predictor) when  $d$  is determined and different  $dp$  values, show that these controller maintain the system stability but some times the oscillation increase when the difference between the actual  $d$  and the predicted time delay  $dp$  increase and hence this can be led to make the system unstable.

**7. Conclusion:**

A mixed model reduction has been proposed for higher order stable plant. The suggested approach is evaluated for Linear Single-Input Single-Output (SISO) system, the results proved to be better than the other proposed methods. LQR with and without integral effect, are designed and compared to research a more appropriate control method. The simulation results demonstrate that both of these controllers are effective and suitable for improving the characteristics of system response. According to the results, LQR controller with integral effect method give the better performance compared to LQR without it.

Otherwise, A LQR controller scheme with smith predictor is suggested in this paper, since the suggested scheme is considered to enhance the proposed method, therefore it is applied the same examples that are presented in this paper. The simulation results with this proposed show accurate and faster for the tested examples.

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