



SMART ANTENNA ADAPTIVE BEAM FORMING BASE ON NEURAL NETWORK WITH DIFFERENT TRAINING ALGORITHMS

Dr. Adheed Hassan Sallomi¹, *Sulaiman A.G Albane²

- 1) Assist Prof., Electrical Engineering Department, Al-Mustansiriyah University, Baghdad, Iraq.
- 2) Electrical Engineering Department, Al-Mustansiriyah University, Baghdad, Iraq.

(Received:24/08/2015; Accepted:8/12 /2015)

Abstract: In this paper an artificial Feed Forward Neural Network(FFNN) is used for smart antenna adaptive beamforming. Neural network is used to calculate the optimum weights that adapt the radiation pattern of the antenna by directing multiple narrow beams toward the desired users and nulling interference or unwanted users. Supervised learning algorithms were used to train the FFNN that used as adaptive beam former. The FFNN was trained using Levenberg-Marquardt (LM), Resilient Back propagation(RP) , Gradient Descent With Adaptive Learning Rate Backpropagation (gda) and Gradient descent with momentum and adaptive learning rate backpropagation(gdx).The simulation results are applied for uniform linear array with five antenna element and the spacing between element equal to half wavelength. The results show that the best system performance can be obtain when the network trained by Levenberg-Marquardt (LM) algorithm.

Keywords: smart antenna, fixed and adaptive beamforming, artificial neural network.

تكوين الشعاع المتكيف للهوائي الذكي اعتمادا على الشبكة العصبية الاصطناعية مع خوارزميات تدريب مختلفة

الخلاصة: في هذه البحث تم استخدام الشبكة العصبية الاصطناعية المتجهة الى الامام (FFNN) لتكوين الشعاع المتكيف للهوائي الذكي. أُسْتُخدمت الشبكة العصبية لحساب الأوزان المثلى التي تعمل على تعديل نمط إشعاع الهوائي عن طريق توجيه حزم اشعاع ضيقة متعددة باتجاه المستخدمين المطلوبين و الغاء الاشعاع باتجاه المستخدمين غير المرغوبين او غير المعنيين بالإشعاع . أُسْتُخدمت خوارزميات التعلم تحت الاشراف لتدريب الشبكة العصبية المتجهة الى الامام المستخدمة كمكون للإشعاع المتكيف. تم تدريب FFNN باستخدام الخوارزميات (LM,RP,GDA,GDX). ونتائج البحث طُبِّقت على هوائي خطي منتظم يتكون من خمسة عناصر التباعد بينها يساوي نصف الطول الموجي. أظهرت النتائج أن أفضل أداء للنظام يمكن الحصول عليه عندما تم تدريب الشبكة باستخدام الخوارزمية (LM).

1. Introduction

As the incremental demand for mobile communications is continuously growing, the requirements for best coverage, capacity improvement, and higher quality of transmission rises. In a conventional cellular system, the using of omni-directional or sectored antennas to make connection between the base station and the mobile user

*Corresponding Author sulaiman_kerbela@yahoo.com

moving between or with in cell lead to dissipate a lot of power as the most of power radiated doesn't focus in the desired directions. Beside the radiating signal also causes an interference with other undesired signals with in cell or co-channel cell [1, 2].

There are three major impairments cause a limitation in performance and capacity of wireless communication system which are: co-channel interference, delay spread and multipath [3]. With the growth of signal processing technologies, smart antenna systems promise an effective solution to the present wireless systems impairments while achieving reliable and robust high speed and high data rate of transmission. Smart antenna system can increase system capacity by directing narrow beams in the direction of desired user, while nulling other undesired users by increasing gain in the direction of desired receivers or desired signal sources and minimize gain toward direction the direction of known or potential interference sources. This allows for higher signal to interference (SIR) ratios, lower power consumption, and allows higher frequency reuse within the same cell [4, 5].

The ability of smart antenna to improve the performance of cellular communication system is due to the adaptive beamforming algorithms that used by signal processing unit of smart antenna system [6, 7]. Neural networks are widely using in the field of signal processing technology, mainly because of their fast convergence rates, general purpose nature, adaptive learning capability and new very large scale integration (VLSI) implementations [8]. Due to these inherent advantages, this paper introduces proposed model of using neural network to compute the optimum weights that allows adaptive array antenna to direct narrow beams in the directions of the desired users while simultaneously nulling undesired sources of interference. This lead to increase system capacity for the existing mobile communications systems.

The paper organization is as follow: first a brief overview about conventional (fixed weight) and adaptive beamforming is described. Then, a brief description for the artificial neural network is discussed. Next a proposed model of neural network adaptive beamformer and the derivation of the optimum array weights are presented. Finally the simulation results for the four training algorithm are presented and discussed.

2. Conventional (Fixed Weight) Beamforming

In a Fixed weight beamforming smart antenna use fixed predefined weights to study the signal arriving from a specific direction. Since it enhance the signal arriving from desired direction while mitigating signals from other directions, thus it is called the spatial matched filter[9].In the fixed weight beamforming approach the arrival angles does not change with time, so the optimum weight would not need to be adjusted[2].Some of fixed weights beamforming algorithm that used by antenna array such as: Maximum Signal-to-Interference Ratio, Minimum Mean-Square Error Method, Maximum Likelihood Method and Minimum Variance (MV) Method. Block diagram of fixed weight beamformer is shown in Fig.1 [10].

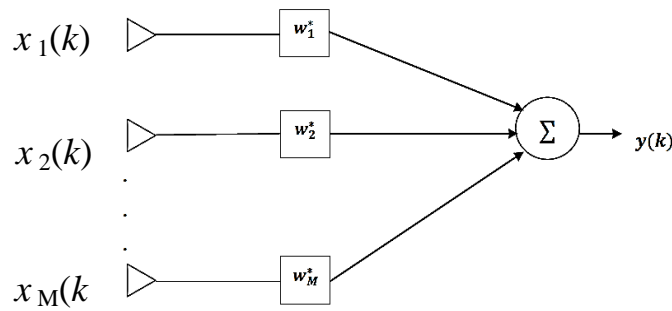


Figure 1. Fixed Weight Beam former

The weighted array output of antenna array can be given in the following form [11]:

$$y(k) = w^H . x (k) \tag{1}$$

where:

$$x(k) = a_0s (k) + [a_1 a_2 \dots a_N] . [i_1(k) i_2(k) \dots i_N(k)]^T + n(k)$$

$$x(k) = x_s(k) + x_i(k) + n(k) \tag{2}$$

$w = [w_1 w_2 \dots w_M]^T$ = weights of Array

$x_s(k)$ = desired vector of signal

$x_i(k)$ = vector of interfering signals

$n(k)$ = zero mean Gaussian noise for each channel

a_i = steering vector of M -element array for θ_i direction of arrival

3. Adaptive Beamforming

Adaptive Beamforming is a signal processing technique in which signal of each user is multiplied by complex weight vectors to adjust the magnitude and phase of the signal from each antenna element to direct narrow beam pattern in the desired direction and nulling toward the interferer directions. The signals received by different elements of an antenna array combined to form a single output. Classically, this is achieved by decreasing the mean square error (MSE) between the desired output and the actual array output [12]. Fig.2 shows the block diagram of adaptive beamformer.

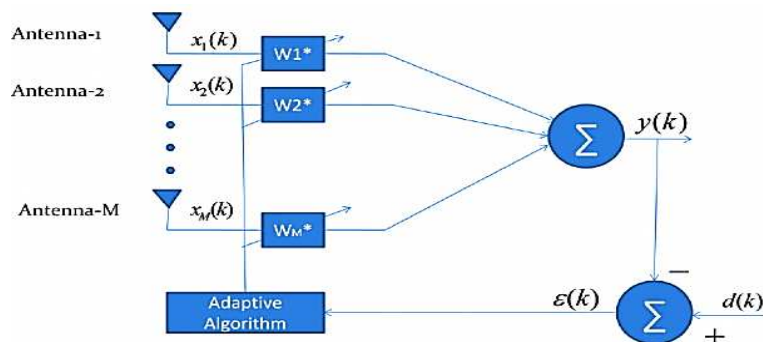


Figure 2. Adaptive Beam former

4. Artificial Neural Network

Artificial neural networks (ANNs) are artificial intelligence applications that attempt to simulate the activities of the human brain in its structure and functionalities in a mathematical model. In artificial neural nets the "artificial" is included to distinguish the computer-based systems from the biological neural network system ANNs have become very common in a broad domains including, industrial, medical and financial applications [13].

Also neural networks are widely used in the signal processing field mainly because of their general purpose nature, fast convergence rates, and its ability for storing experimental knowledge and making it available for use [8].

There are three features to characterize a neural network is [14]:

1. Architecture: the connection pattern of nodes between neurons.
2. The learning or training algorithm: the method of the weights calculations on the connections.
3. The activation function: the function that achieve an output for the input values received by a node.

5. Mathematical Model of Uniform Linear Array

It is assumed that there are K narrow-band incoherent plane waves, impinging a uniform linear array (ULA) with M ($M > K$) omnidirectional sensors spaced by distance d from directions $\{\theta_1, \theta_2, \dots, \theta_{K-1}\}$, which is between $[-90, 90]$. the geometry of linear array antenna is shown in Fig(3).

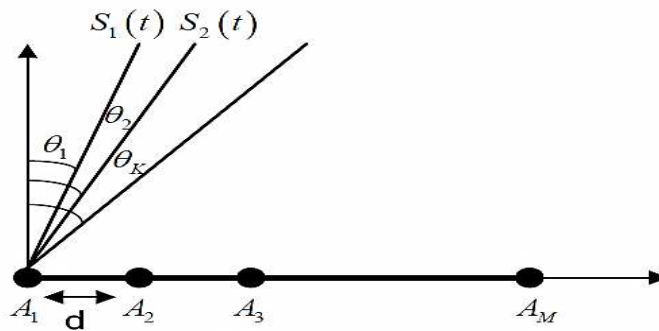


Figure 3. Geometry of Linear Array Antenna

The received signal at A_{th} antenna element is calculated by [15]:

$$X_i(k) = \sum_{m=1}^K s_m(k) e^{-j(i-1)k_m} + n_i(k) \quad (3)$$

$i=1,2,\dots,M$

Where: s_m are the signals coming from each received signal source .at i^{th} sensor, $n_i(t)$ is the noise received at each element of array antenna and

$$k_m = \frac{w_0 d}{c} \sin(\theta_m) \quad (4)$$

Where: d , is the spacing between the array elements, w_0 is the angular frequency and c , is light speed in free space.

Array output can be written in matrix form as:

$$X(k) = AS(k) + N(k) \quad (5)$$

Where:

$$A = [a(\theta_1), a(\theta_2), \dots, a(\theta_k)] \quad (6)$$

And:

$$a(\theta_m) = [1 \ e^{-jk_m d} \ e^{-j2k_m d} \ \dots \ e^{-j(M-1)k_m d}] \quad (7)$$

$$k_m = \frac{w_0 d}{c} \sin(\theta_m) \quad (8)$$

$$X(k) = [x_1(k) \ x_2(k) \ \dots \ x_m(k)]^T \quad (9)$$

$$N(k) = [n_1(k) \ n_2(k) \ \dots \ n_m(k)]^T \quad (10)$$

$$S(k) = [s_1(k) \ s_2(k) \ \dots \ s_m(k)]^T \quad (11)$$

Where T, indicate the transpose of matrix and A is array steering matrix toward the direction of the incoming signal and is given by:

Array output y can be given in the following form:

$$y(k) = w^H X(k) \quad (12)$$

where:

$$w = [w_1 \ w_2 \ \dots \ w_M]^T \quad (13)$$

w : array weight, H: refer to transpose conjugate.

6. Proposed Model of Using ANN For Adaptive Beamforming

There are two steps of using neural network for smart antenna adaptive beamforming :

- **First Step** training network on the target (desired) phase vector
- **Second Step** training network on the desired output vector, the desired output vector is bipolar vector (all data sample are either 1 or -1).

The input of neural network in the first step of training is the phase vector of the received signals. The desired signals received by M array antenna overlapped with unwanted and noise signals as given in eq.1. The desired steering vector for the desired signal impinging antenna array from direction θ_m is given by:

$$A_d = [1 \ e^{-jk_m\theta_m} \ e^{-j2k_m\theta_m} \ \dots \ e^{-j(M-1)k_m\theta_m}] \quad (14)$$

While the steering vector of the received signals are actually combined with each other through receiving process. The steering vector of K plane waves signal (desired and interference signals) by M array antenna and is given by:

$$A_r = \sum_{m=1}^K e^{-j(i-1)k_m\theta_m} \quad (15)$$

$i=1,2,\dots,M$

The desired phase vector is given by :

$$\theta_d = \tan^{-1} \frac{\text{imag}(A_d)}{\text{real}(A_d)} \quad (16)$$

And the received phase vector is given by:

$$\theta_r = \tan^{-1} \frac{\text{imag}(A_r)}{\text{real}(A_r)} \quad (17)$$

The input of the neural network for the first step training is θ_r and the target that must be satisfy through this training is θ_d . the flowchart of neural network training is shown in Fig (4).

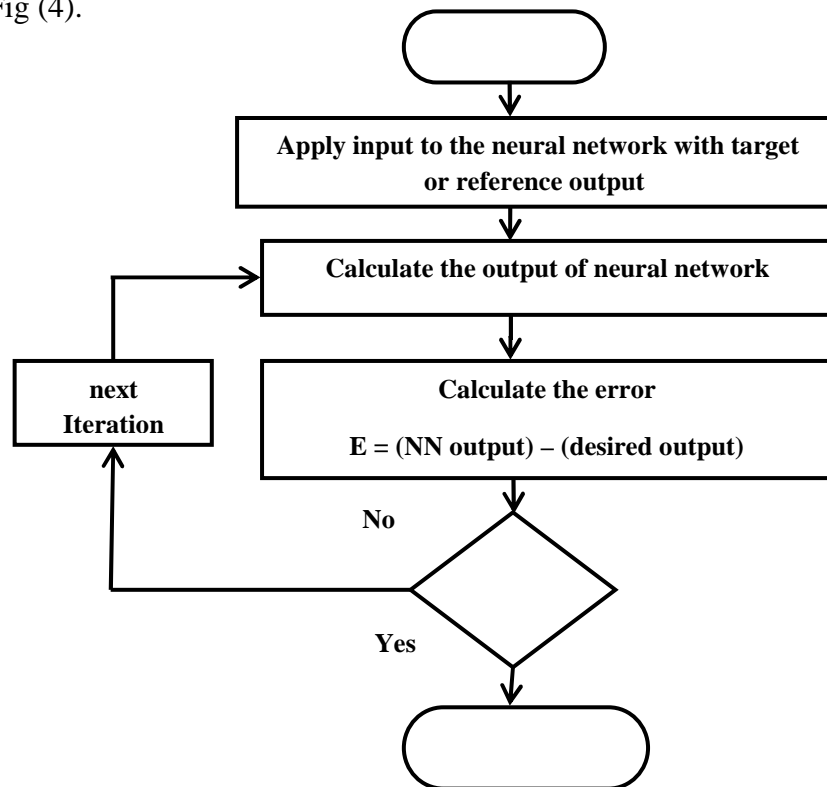


Figure 4. Flow Chart of NN Training Program

The second step of training network on desired output that must be verify by antenna array .in this step of training the input of neural network is :

$$X_u = |X_i(k)| \quad (18)$$

And the reference or desired output vector is D, D is bipolar vector:

$D = [-1 \ 1 \ -1 \ -1 \ 1 \ \dots \dots \dots]$, with n number of sample.

The proposed model of FFNN is shown in Fig (5) that consists of three layers, input layer has N nodes. For first step of training network on desired phase N equal to the number of antenna element (M), since the number of sample in the received phase vector are equal to M , while in the second step of training network on desired output vector (bipolar) N equal to the number of sample in the desired vector (n). The activation function in the hidden and output layer is tan sigmoid function. Hidden layer consist of five nodes and output layer has L nodes ($L=M$ in the first step of training, $L=n$ in second step of training). The activation function in the hidden layer is radial basis transfer function and the activation function at the output layer is linear transfer function. Chosen of these activation functions was due to that these functions gave better performance than others in the neural network training.

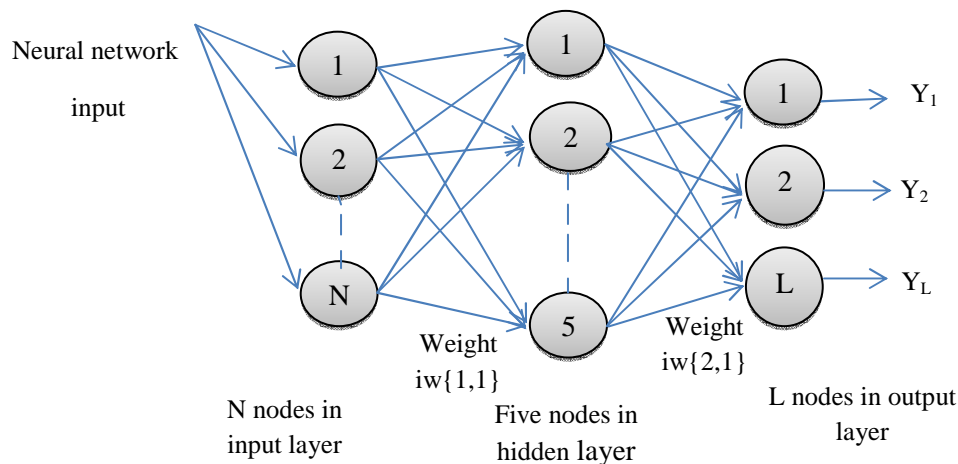


Figure 5 .Feed Forward Neural Network (FFNN) proposed model

where: $iw\{1,1\}$ refer to weight vectors (or weight matrix) between the input layer and hidden layer and $lw\{2,1\}$ refer to weight vectors between the hidden layer and output layer for phase training the input to the neural network is θ_r , θ_r take matrix form with row equal to the number of received signal and column equal to the number of antenna element (M). Each row in θ_r refer to the phase vector of each signal overlapped with other received signals. while the target that must be satisfy through neural network phase training is θ_d . so if the input is the first row of θ_r which represent the phase vector of the first desired signal that overlapped with other then the output is the first vector of θ_d which represent the desired phase vector of the first desired signal and the training is continue for other desired phase vector of the remaining desired received signals.

In the neural network training on desired output, NN input is X_u , where X_u is the specific desired signal (overlapped with other signals), if the input is X_u then the neural network target output must be D , target vector of the same signal (there is a

target output vector for each desired signal). Neural network training continues for other desired signal in the same way.

After achieving the desired phase and desired output vectors from the artificial neural network training. The optimum weight vector can be calculated from the neural network outputs from both training steps (phase training and training network on desired output vector) can be given as:

$$w(k) = Y(k) * x_s^\dagger \quad (19)$$

$Y(k)$ is the neural network output from the second step of training

$$x_s = e^{-j(i-1)k\theta_s} \quad (20)$$

Where θ_s is the output phase vector of the neural network from the first step of training (phase training).

In eq.19 \dagger refer to Moore-Penrose pseudoinverse of matrix to find the inverse of non-squared matrix (x_s) and it is given by [16]:

$$x_s^\dagger = (x_s^T x_s)^{-1} x_s^T \quad (21)$$

Beam pattern can be obtained (drawn) from the array factor equation:

$$AF = |w(k) \cdot e^{-j(i-1)\alpha(k)}| \quad (22)$$

$i=1, 2, \dots, M$

$\alpha(k)$ is the searching angle between $[-90^\circ, 90^\circ]$ with 1° as step size from -90 to 90.

7. Simulation Results

The simulation results of using Feed-Forward Neural Network Back-Propagation algorithm (FNNBP) to perform adaptive beamforming are implemented for four different Back-Propagation algorithms. It was assumed $M=5$, element spacing $d=\lambda / 2$, DOA= 30° , and the number of sample $n=500$.

7.1 Levenberg-Marquardt (LM) Training

The performance of Levenberg-Marquardt training for single target (one user with one direction of arrival), with 5 hidden neurons is shown in Fig (6) where the best phase training performance is $[6.0185e-25]$ at epoch 3 and the best desired o/p training performance is $[8.0838e-21]$ at epoch 4. The simulation result of beam pattern, linear and polar plot is shown in Fig (7).

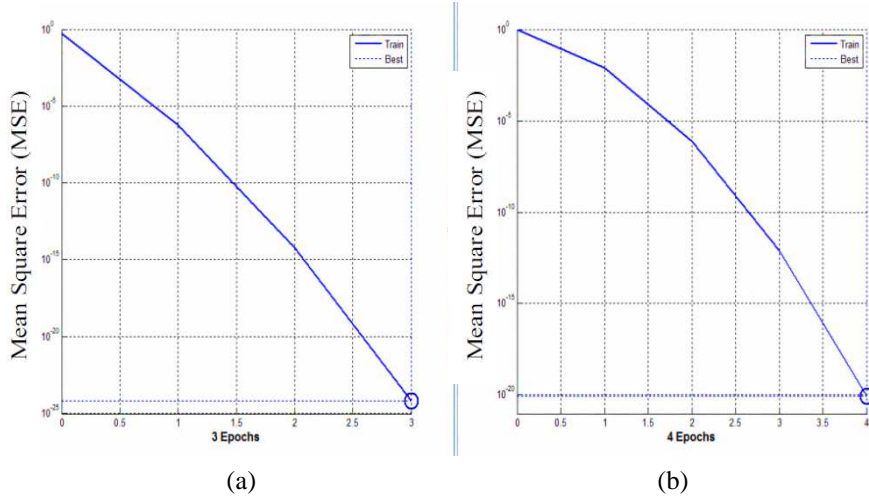


Figure 6: Performance of FFNN trained by LM algorithm (a) Phase training, (b) Training network on desired output

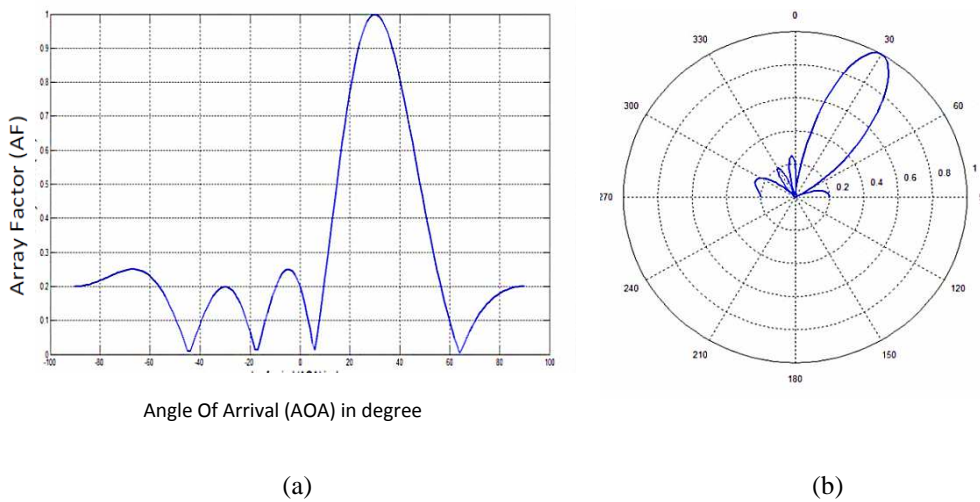


Figure 7 . Beam pattern using FFNN trained by LM algorithm (a) Linear plot, (b) Polar plot ($m=5$, $d=\lambda / 2$)

7.2 Resilient Back-Propagation (Rprop) Training

The performance of network training that updates weight and bias values according to the Resilient Back-Propagation (Rprop). for single target (one user with one direction of arrival), with 5 hidden neurons is shown in Fig (8) where the best phase training performance is $[1.04127e-11]$ at epoch 32 and the best desired o/p training performance is $[1.61704e-10]$ at epoch 25. The simulation result of beam pattern , linear and polar plot is shown in Fig (9).

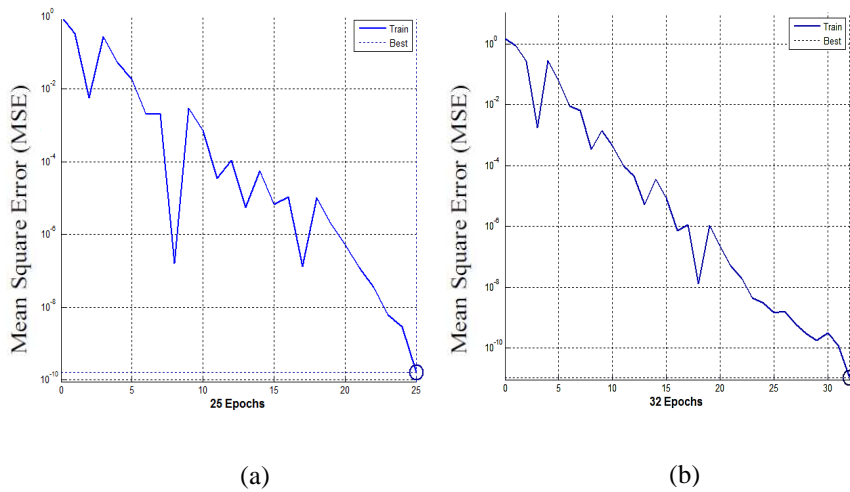


Figure 8 . Performance of FFNN trained by Rp algorithm (a) Phase training (b) Training network on desired output

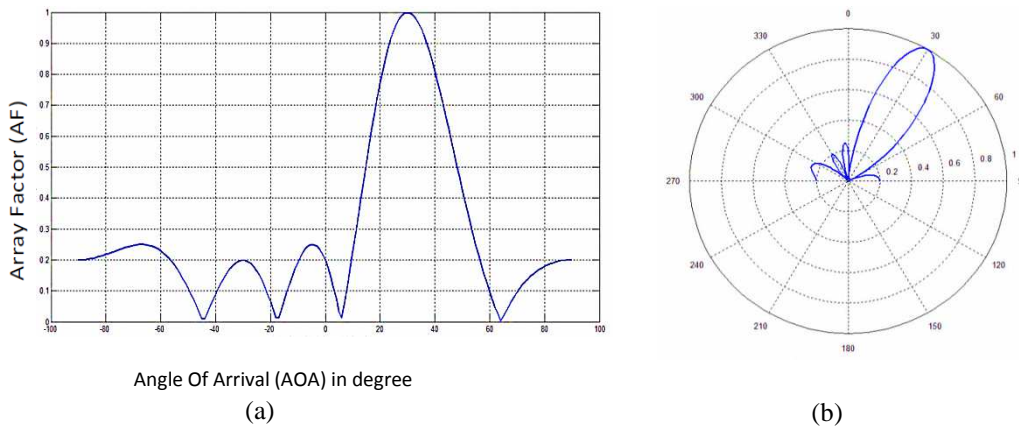


Figure 9.Beam pattern using FFNN trained by rp algorithm (a) Linear plot, (b) Polar plot. ($M=5$, $d=\lambda / 2$)

7.3 Gradient Descent with Momentum And Adaptive Learning Rate (Gdx) Training

The performance of network training that updates weight and bias values according to gradient descent momentum with adaptive learning rate for single target with 5 hidden neurons is shown in Fig (10) where the best phase training performance is [2.4099e-12] at epoch 63 and the best desired o/p training performance is [1.3189-10] at epoch 148. The simulation result of beam pattern, linear and polar plot is shown in Fig (11).

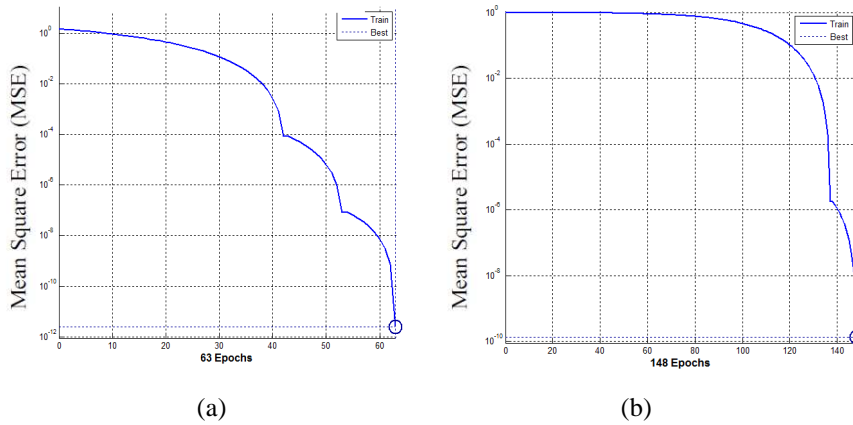


Figure 10. Performance of FFNN Trained By gdx Algorithm (a) Phase training, (b) Training network on desired output

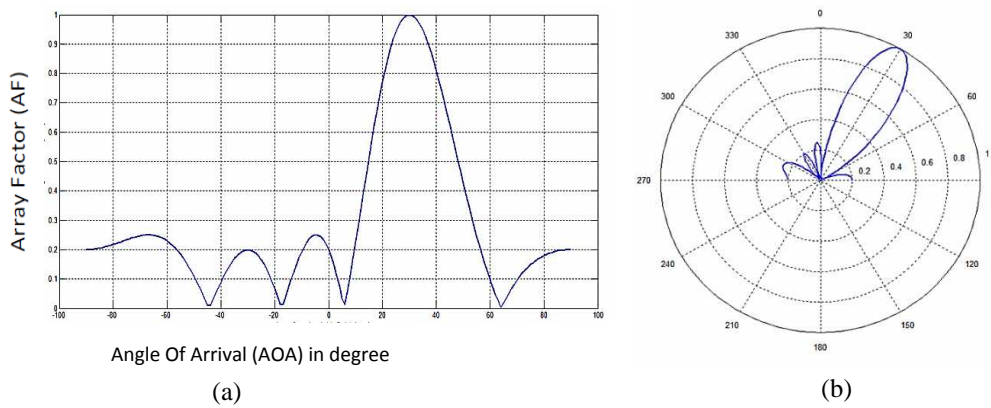


Figure 11. Beam pattern using FFNN trained by gdx algorithm (a) Linear plot, (b) Polar plot. ($M=5$, $d=\lambda/2$)

7.4 Gradient Descent with Adaptive Learning Rate (gda) Training

The performance of network training that updates weight and bias values according to gradient descent with adaptive learning rate for single target with 5 hidden neurons is shown in Fig (12) where the best phase training performance is $[1.53256e-11]$ at epoch 68 and the best desired o/p training performance is $[1.59668e-09]$ at epoch 158. The simulation result of beam pattern, linear and polar plot is shown in Fig (13).

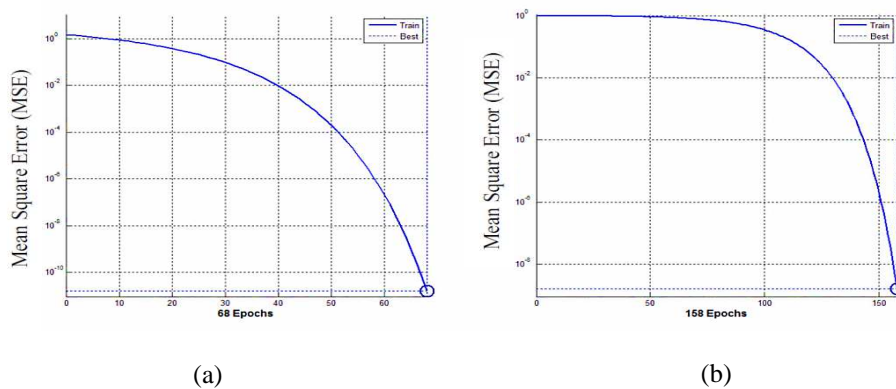


Figure 12. Performance of FFNN trained By gda algorithm (a) Phase training, (b) Training network on Desired Output

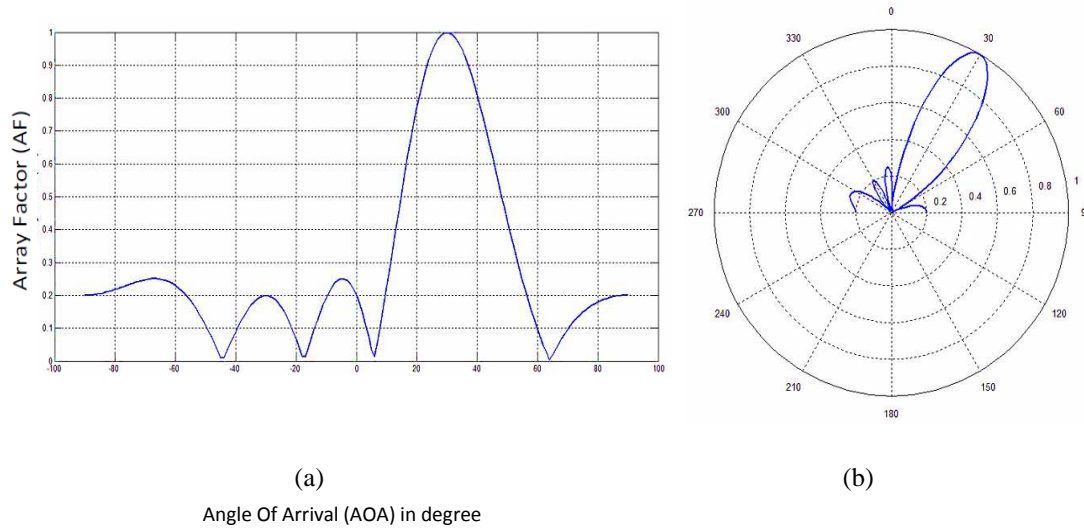


Figure.13 Beam pattern using FFNN trained by gda algorithm (a) Linear plot,(b)Polar plot ($M=5, d=\lambda / 2$)

As mentioned in previous simulation results are implemented with same information inputs to the smart antenna system ($M=5, d= \lambda / 2$, DOA= 30° $n=500$) in order to make comparison between the performance of each learning Back-Propagation algorithms. Tables (1) and Table (2) show the best performance for each Feed-Forward Neural Network Back-Propagation algorithms (FNNBP), in this work the performance of Levenberg-Marquardt training is the best performance as compared with the performance of other training algorithms as it considers the fastest training function and it is the default training function for Feed- Forward Neural Network. This method tends to be less efficient for large networks (with thousands of weights), since it require more memory and more computation time for these cases.

Table (1) show the simulation result of best performance of phase training while Table (2) shows the simulation result of best performance of training FFNN on desired output and Table (3) shows the optimum weights for each training algorithm

Table 1. Simulation results of best training phase performance for each training algorithm

<i>algorithms</i>	<i>Best performance</i>	<i>epoch</i>
lm	6.019e-25	3
rp	1.04127e-11	32
gdx	2.41e-12	63
gda	1.5336e-11	68

Table 2. Simulation results of best performance of training FFNNB on desired output

<i>algorithms</i>	<i>Best performance</i>	<i>epoch</i>
lm	8.0838e-21	4
rp	1.61704e-10	25
gdx	1.3189-10	148
gda	1.59668e-09	158

Table 3. Optimum weights for FFNNB algorithms that used to form the adaptive beam pattern

<i>W</i>	<i>FFNNB</i> (<i>lm</i>)	<i>FFNNB</i> (<i>rp</i>)	<i>FFNNB</i> (<i>gdx</i>)	<i>FFNNB</i> (<i>gda</i>)
1	2.094e-5 - 0.24999i	2.042e-5 - 0.25i	2.1445e-5 -0.2499i	2.2207e-5 - 0.249i
2	0.25+2.0943e-5i	0.2499+2.205e-5i	0.2499 + 2.094e-5i	0.25 + 2.0943e-5i
3	-2.0944e-5 + 0.25i	-2.0429e-5+ 0.2499i	-2.1445e-5+ 0.249i	-2.221e-5 + 0.25i
4	-0.25-2.0944e-5i	-0.2499 - 2.2053e-5i	-0.25 - 2.0944e-5i	-0.25 - 2.094e-05i
5	2.09439e-05- 0.25i	2.04290e-5 - 0.2499i	2.14447e-5 - 0.249i	2.221e-5 - 0.2499i

8. Conclusion

In this paper a model of smart antenna adaptive beamforming based on artificial neural network was introduced. The weights were computed using a FFNN trained by four different back propagation learning algorithms. The network was successful in directing narrow beam toward the desired user while simultaneously nulling interferences. The simulation result was implemented for $M=5$, $d=\lambda / 2$, $n=500$ and showed that the performance of Levenberg-Marquardt (LM) training is the best performance as compared with the performance of other training algorithms as mentioned in the section of simulation results. It is worth mentioning that the most previous researches are used radial basis function neural network for antenna array adaptive beamforming with single training algorithm and with more complexity ways in the hybridization process between the artificial neural network and antenna array such as the researches that given in [8,15,17,18 ,19].

Abbreviations

<i>Symbol</i>	<i>Definition</i>
$y(k)$	Array output of antenna array
w	Weights of Array
$x_s(k)$	desired vector of signal
$x_i(k)$	Vector of interfering signals
$n(k)$	Zero mean Gaussian noise for each channel
a_i	Steering vector of M -element array
M	Number of antenna element
θ_i	Direction of arrival angle
Net	Summation part of neural network
C	Speed of light
$X_i(k)$	Received signal at i^{th} antenna element
s_m	Steering vector of the received signals
d	Spacing between the array elements
w_0	The angular frequency

$X(k)$	Array output in matrix form
A	Array steering matrix
θ_m	Direction of arrival angle of desired signal
A_d	Desired steering vector
K	Number of the received plane wave signals
A_r	Steering vector of the received signals
θ_d	Desired phase vector
θ_r	Received phase vector
X_u	Neural network input
D	Neural network desired output
θ_g	Output phase vector of neural network
AF	Array Factor
$\alpha(k)$	Searching angle

References

1. RK Jain, Sumit Katiyar and NK Agrawal. (2011). "Smart Antenna for Cellular Mobile Communication". VSRD International Journal of Electrical, Electronics & Communication, Department of Electrical & Electronics Engineering, Singhania University, Jhunjhunu, Rajasthan, INDIA, Vol. 1, No.9, pp.530-541.
2. Hung Tuan Nguyen. (2005). "Multiple Antenna Systems for Mobile Terminals". PhD. Thesis. Department of Communication Technology, Aalborg University, Denmark.
3. Jack H.Winters AT and T Labs. (1998). "Smart Antennas For Wireless Systems". IEEE Personal Communication, ISSN :1070-9916, pp.23-27.
4. Frank Gross. (2005). "Smart Antenna for Wireless Communications". Mcgraw-hill, September.
5. Murray, B.P. and Zaghloul, A.L. (2014). "Survey of Cognitive Beamforming Techniques". IEEE. Dept. of Electrical and Computer Engineering, Virginia Tech, Blacksburg, VA, USA, ISBN: 978-1-4799-3119-4.
6. Nwalozie G.C, Umeh K.C, Okorogu V.N and Oraetue C.D. (2012). "Performance Analysis of Constant Modulus Algorithm (CMA) Blind Adaptive Algorithm for Smart Antennas in a W-CDMA Network", International Journal of Engineering Science and Innovative Technology (IJESIT), ISSN: 2319-5967, Volume 1, Issue 2, pp.246-254.
7. Xu-Bao Shun and Shun-Shi Zhong. (2004). "An Adaptive Beamforming Approach Using Online Learning Neural Network". IEEE, School of Communication and Information Engineering Shanghai University, China, ISBN: 0-7803-8302-8, pp.2663-2666.
8. A. H. El Zooghby, M. Georgiopoulos and C. G. Christodoulou. (1999). "Neural Network-Based Adaptive Beamforming for One- and Two-Dimensional Antenna Arrays". IEEE, Electrical and Computer Engineering Department, University of Central Florida, Orlando, FL 32816 USA, pp.1891-1893.

9. Heikki Koivo and Mohammed Elmusrati. (2009). "*Smart Antennas, Systems Engineering in Wireless Communications*". John Wiley and Sons, Ltd. ISBN: 978-0-470-02178-1, pp 261-302.
10. Mohammad Tariqul Islam and zainol Abidin Abdul Rashid. (2006). "*MI-NLMS Adaptive Beamforming Algorithm For Smart Antenna System Applications*". Journal of Zhejiang University Science, Department of Electrical, Electronics and System Engineering, Faculty of Engineering, University of Kebangsaan Malaysia, ISSN 1009-3095, pp.1709-1716.
11. Ivica Stevanović, Juan R. Mosig and Anja Skrivervik. (2003). "*Smart Antenna Systems for Mobile Communications*". Laboratoire d'Électromagnétisme et d'Acoustique Ecole Polytechnique Fédérale de Lausanne CH-1015 Lausanne Suisse January.
12. Jian Li and Petre Stoica. (2005). "*Robust Adaptive Beamforming*". JOHN WILEY & SONS, INC, ISBN: 10 0-471-67850-3.
13. D.M. Rodvold, D.G. McLeod, J.M. Brandt, P.B. Snow, and G.P. Murphy. (2001). "*Robust Adaptive Beamforming Introduction to Artificial Neural Networks for Physicians: Taking the Lid Off the Black Box*". Wiley-Liss, Inc., Vol. 46, Issue 1, pp.39–44.
14. Laurene Fausett. (1994). "*Fundamentals Of Neural Networks Architectures, Algorithms and Applications*". Prentice-Hall, ISBN: 0133341860, 9780133341867.
15. Ahmed H. El Zooghby, Christos G. Christodoulou, and Michael Georgiopoulos. (2000). "*A Neural Network Based Smart Antenna for Multiple Source Tracking*". IEEE, TRANSACTIONS ON ANTENNAS AND PROPAGATION, Electrical and Computer Engineering Department, University of Central Florida, Orlando, FL 32816 USA., Vol. 8, No. 5, pp.768-775.
16. Arthur Albert. (1972). "*REGRESSION AND THE MOORE-PEN ROSE PSEUDOINVERSE*". Department of Mathematics Boston University, Boston, Massachusetts.
17. Wenqiang Guo, Tianshuang Qiu, Hong Tang and Wenrong Zhang. (2007). "*Performance of RBF neural networks for array processing in impulsive noise environment*". Dalian University of Technology, Dalian 116024, Xinjiang University of Finance and Economics, Urumchi 830012, PR China, pp.168-178.
18. MURAT GUREKEN. (2009). "*Neural Network Based Beamforming For Linear And Cylindrical Array Applications*". Master thesis, Department of Electrical and Electronics Engineering, Middle East Technical University.
19. Zehua Fan, Alex Grey and Jackson Wilson. (2014). "*Adaptive beamforming technique based on neural network*". Engineering Technology and Applications – Shao, Shu & Tian (Eds), Library of Huazhong University of Science & Technology, Wuhan, Hubei, China, ISBN 978-1-138-02705-3, pp.213-217.