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ANALYSIS OF THICK RECTANGULAR PLATES RESTING ON NONLINEAR WINKLER FOUNDATION

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Abstract: This research deals with the elastic behavior of thick plates resting on nonlinear Winkler foundation. Mindlin's thick plate theory is extended to include the effect of nonlinear elastic foundation. Finite difference program was developed to solve the differential equations for bending of thick rectangular plate on nonlinear foundation. The finite element (ANSYS program v12.1) method was used to analyze thick plates ANSYS program is used through the element SHELL 281 to represent the plate. The element has eight nodes with six degrees of freedom at each node. COMPIN39 element with two nodes was used to represent nonlinear soil behavior. The element has three translation degree of freedom at each node to represent the linear soil behavior COMBIN14 element was used, the element has two nodes with three translation degree of freedom at each node to represent linear soil behavior. The results obtained from finite difference technique are compared with the finite element results to check the accuracy of the solution. Good agreements between them are obtained.

Keywords: *Finite element, Finite difference, Thick plate, Nonlinear Winkler foundation*

تحليل الالواح المستطيلة السميكة المسندة على أسس ونكلرلاخطية

الخلاصة: هذا البحث يتعامل مع السلوك المرن للصفائح السميكة المسندة على أساس ونكلر اللا خطي.نظرية مندلن للالواح السميكة تم توسيع تطبيقاتها لتشمل تأثير الاساس المرن اللا خطي. أستخدمت طريقة العناصر المحددة (برنامجANSYS) لتحليل الالواح السميك. في طريقة العناصر المحددةتم أستخدام عنصر (SHELL281)في برنامج ANSYS لتمثيل الالواح. هذا العنصر يمتلك ثمان عقد وست درجات من الحرية لكل. وتم أستخدم عنصر (OMBIN39) لتمثيل التصرف اللا خطي للتربه الذي يمتلك عقدتين.هذا العنصر يمتلك ثلاث درجات حرية انتقالية لكل عقدة لتمثيل التصرف الاخطي للتربة.وتم استخدام عنصر (OMBIN14) لتمثيل التصرف الخطي للتربة, هذا العنصر يمتلك عقدتين مع ثلاث درجات حرية انتقالية لكل عقدلتمثيل التصرف الخطي للتربة. تم استخدام برنامج لتحليل الالواح السميكة الموضوعة على اسس مرنة لا خطية بواسطة الفروق المحددة وتم مقارنة النتائج مع طريقة العناصر المحددة.تم الحصول على توافق جيد بين النتائج باستخدام الطريقتين.

1. Introduction

A structural plate has a uniform or variable thickness that is small compared with the other dimensions. It is bounded by two parallel planes called faces and the distance between them called the thickness of the plate (h) [1].

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The thin plate theory ignores the effect of transverse shear and it may not give good results for plates with large thickness. In order to overcome this problem, various improved theories have been developed to include the effect of transverse shearing deformation. Reissner (1945) and Mindlin (1951) developed a theory for thick plates by allowing the normal line to the middle plane to rotate independently of slopes. And it assumes that the transverse shear distribution are constant in the thickness and therefore required a shear correction factor to correct the errors between the actual and assumed transverse shear distribution.

Al-Allaf (2005) [2] studied the problems of linear elastic behavior of thick isotropic and orthotropic rectangular and circular plates resting on Winkler elastic foundations with both normal and frictional resistance subjected to transverse loads. The finite element method is used to solve the problems. The results show good agreement with problems previously solved by other researchers using finite difference method.

Mohsin (2007) [3] studied the large deflection behavior of thick and thin rectangular plates resting on Winkler elastic foundation by finite difference technique Good results are obtained when compared with the finite element method.

Ahmed (2008) [4] studied the linear elastic behavior of thick plates resting on two parameter elastic foundations. The finite difference method was used to solve the problem of thick plates and the results were compared with other analytical and numerical methods to check the accuracy of the developed analysis.

Baltacioglu et al. (2011) [5] analyzed of a rectangular laminated thick plate resting on nonlinear elastic foundation. Shear deformation theory is based on the formulation of the plate and the shear correction factor is considered to be 5/6. The results were compared with other analytical and numerical methods to check the validity of the method.

Ozdemir (2012) [6]analyzed an isotropic thick plate resting on elastic foundation represented by one parameter Winkler model using finite element method. The analysis used high order Mindlin plate element with 17 nodes to avoid shear locking problem in the plate.

2. Elastic Foundation

Winkler model are used to model the elastic foundations. This model assumes that the base is consisting of closely spaced independent linear springs, consequently as shown in Figure-1.

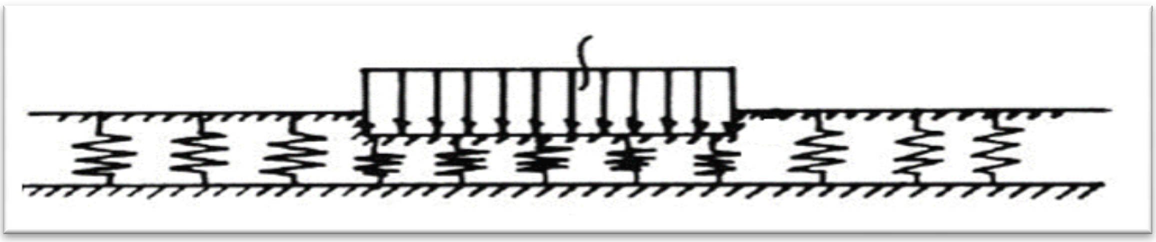


Figure -1 Surface displacement of Winkler model to uniform.[7]

Modulus of subgrade reaction is a conceptual relationship between soil pressure and deflection. It can be measured by using plate-load test. Using this test, a load-deflection curve is adopted. The modulus of subgrade reaction Kz can be calculated using:

$$K_Z=\frac{P}{W} \quad (1)$$

where

K_Z is the modulus of subgrade reaction

P is the applied pressure

W is the deflection

In this study, the linear and nonlinear behaviors are adopted. The nonlinear behavior is modeled using iterative values of K_z. A typical p-w diagram was taken from a plate loading test which was carried out on a soil in Maysan as shown in the report below.

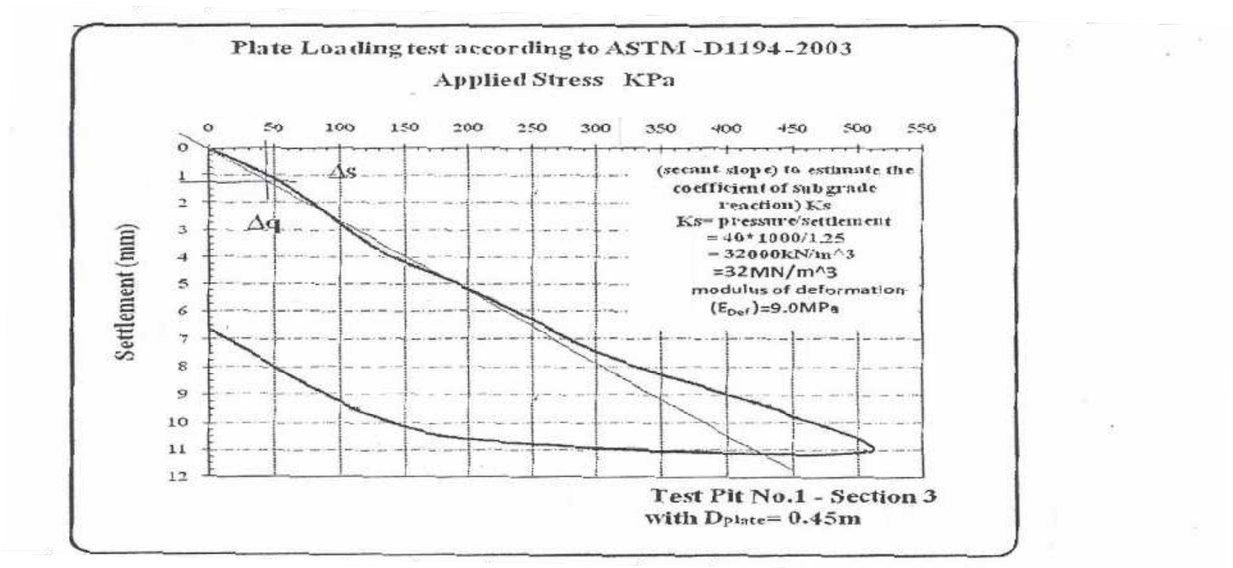


Figure -2 Surface displacement of Winkler model to uniform. [7]

3. Assumptions and governing equations for thick plate

The equations derived by Mohsen (2007) [3]are modified in the present study to include nonlinear Winkler foundation as follow:

$$Gc^2h\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial \psi_x}{\partial x}+\frac{\partial \psi_y}{\partial y}\right)+q+K_{z(w)}.w=0 \quad (2)$$

$$D\left(\frac{\partial^2 \psi_x}{\partial x^2}+\frac{(1+\upsilon)}{2}\frac{\partial^2 \psi_y}{\partial x\,\partial y}+\frac{(1-\upsilon)}{2}\frac{\partial^2 \psi_x}{\partial y^2}\right)-Gc^2h\left(\psi_x+\frac{\partial w}{\partial x}\right)=0 \quad (3)$$

$$D\left(\frac{\partial^2 \psi_y}{\partial y^2}+\frac{(1+\upsilon)}{2}\frac{\partial^2 \psi_x}{\partial x\,\partial y}+\frac{(1-\upsilon)}{2}\frac{\partial^2 \psi_y}{\partial x^2}\right)-Gc^2h\left(\psi_y+\frac{\partial w}{\partial y}\right)=0 \quad (4)$$

where G is the shear modulus, c^2 is the shear correction factor (c^2 =5/6 for rectangular cross sections), ψ_x ,ψ_y are the rotations of the transverse sections in xz-plane of the plate, w is the transverse deflection, h is the plate thickness, q is the transverse load per unit length, and K_z are the linear or nonlinear modulus of subgrade reaction in z directions.

4. Finite difference method

In applying the finite difference method, the derivatives in the differential equations under consideration are replaced by differences at selected points. These points are located at the nodes of a square or rectangular network (called the finite difference mesh). In the analysis of thick plates by this method, the coupled differential equations at each point (or node) should be replaced by coupled difference equations. By assembling the coupled difference equations for all nodes, a number of simultaneous algebraic equations equal to the number of nodes time the number of degrees freedom at each node are obtained and then solved for each iteration.

5. Finite element method

The finite element (ANSYS program v12.1) method was used to analyze thick plates ANSYS program is used through the element SHELL 281 to represent the plate. The element has eight nodes with six degrees of freedom at each node: translations in the nodal x, y, and z-directions and rotations about the nodal x, y, and z-directions. COMPIN39 element with two nodes is used to represent nonlinear soil behavior. The element has three translation degree of freedom at each node to represent the linear soil behavior COMBIN14 element is used, the element has two nodes with three translation degree of freedom at each node to represent linear soil behavior[8].Table -1 shows the number of elements that have been used in this research.

Table -1: No. of elements and structural component

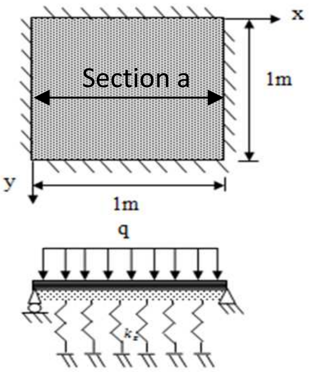
Element Type	No. of elements	Structural component
SHELL281	100	Plate
COMPIN14	441	Linear soil behavior
COMPIN39	441	Nonlinear soil behavior

6. Verification

The numerical results obtained from finite element and finite difference analyses have been compared with available numerical results to check the accuracy of the methods used in this study. The results are plotted together to show the agreements between them.

6.1 Simply supported thick plate on Winkler foundation

A square plate of side lengths (a = b =1.0 m), thickness (h = 0.25 m), having Young's modulus (E=24*106kN/m2), Poisson's ratio (υ = 0.2) and subjected to a uniformly distributed load (q=60 kN/m2) is considered as shown in Figure 3 .The plate is on Winkler foundation (K_z= 5000 kN/m3), represented by spring model. In the present study, this problem is analyzed by using the finite element method as shown in figures 4 and 5 and compared with finite difference method solved by Mohsin (2007) [3]. The results of central deflection and central moment are given in Table -2 to check the accuracy of the solutions. The table indicates that the results are in excellent agreements for both deflection and bending moment. Also, figures 6 and 7 show the deflection curves and the bending moment diagrams obtained from the present finite elements and Mohsin (2007) [3]. They are in excellent agreements also.



$q=60$ kN/m^2

$h=0.25$ m

$E = 24*10^6$ kN/m^2

$\upsilon = 0.2$

$K_z = 5000$ kN/m^3

Figure -3Simply supported thick plate on Winkler foundation.

Table -2: Comparison between the present finite element and Mohsin (2007) [3]for simply supported thick plate on Winkler foundation.

Variables	Mohsin (2007) [3]	Present study	% Difference
Central Deflection (m)	$9.64*10^{-6}$	$9.61*10^{-6}$	0.31 %
Central Moment (kN.m)	1.1	1.2	0.03 %
Max. Strain	$\pm 3.52*10^{-6}$	$\pm 3.8*10^{-6}$	7.9%

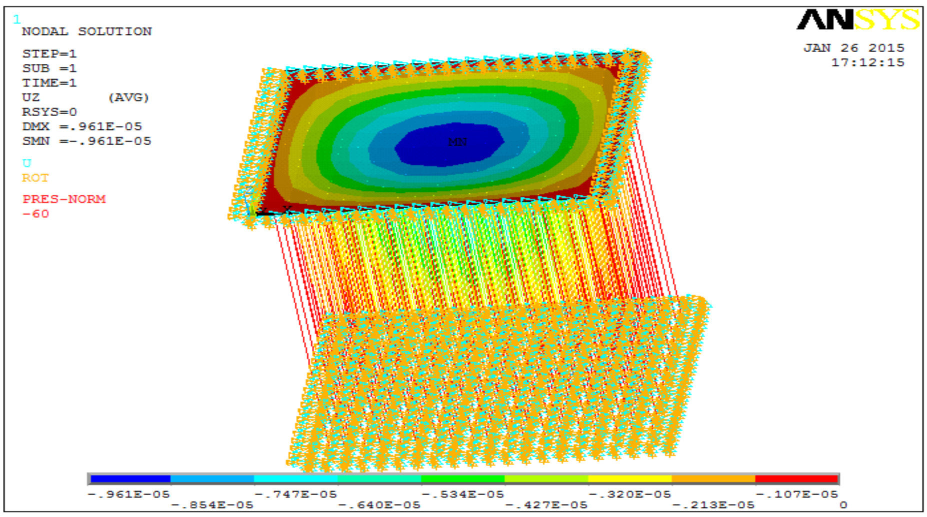


Figure -4 Deflection contours for simply supported thick plate on Winkler foundation. (m)

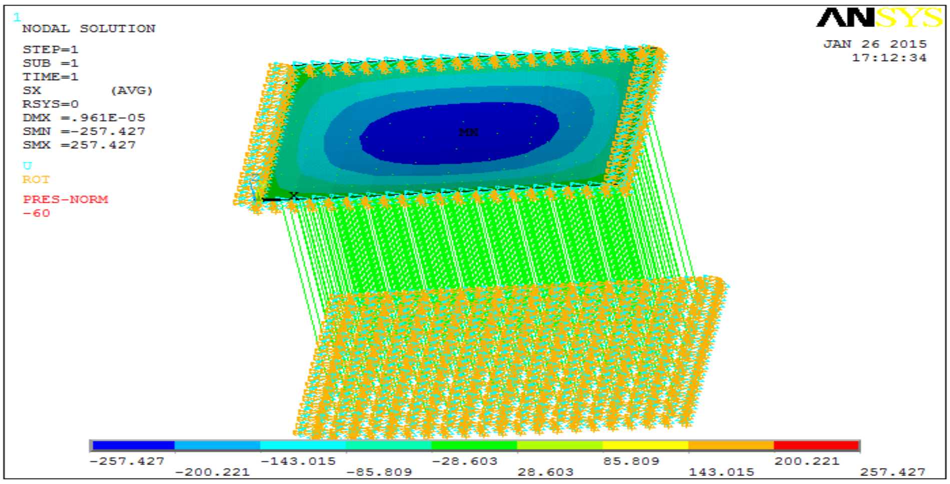


Figure -5Stress in x-direction contours for simply supported thick plate on Winkler foundation. (kN/m^2)

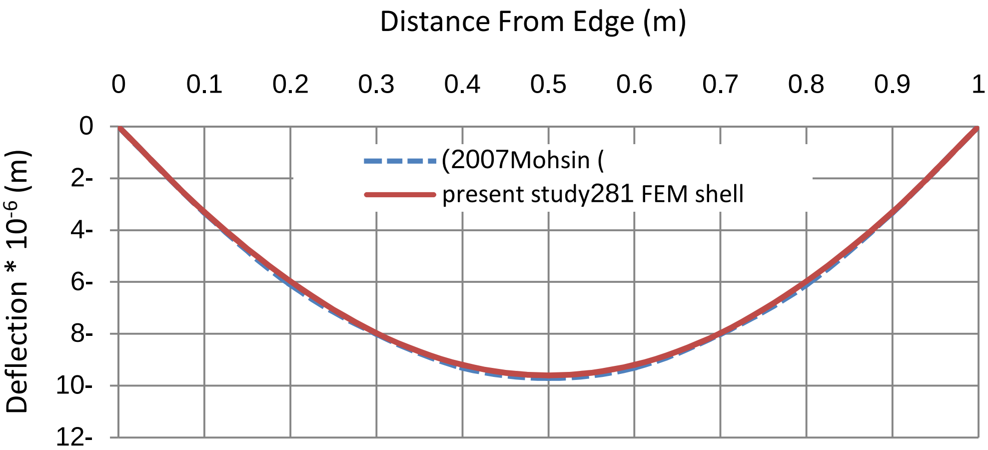


Figure -6 Deflection curves for simply supported thick plate on Winkler for section at mid length of a**.**

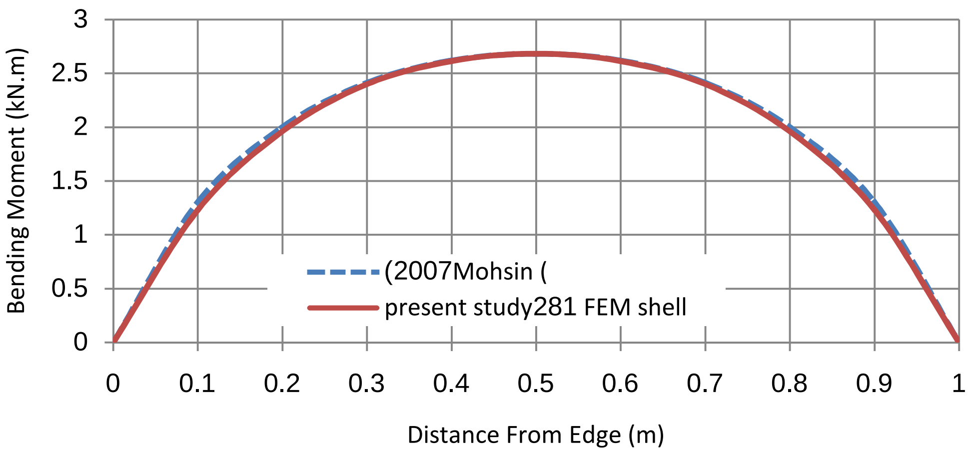
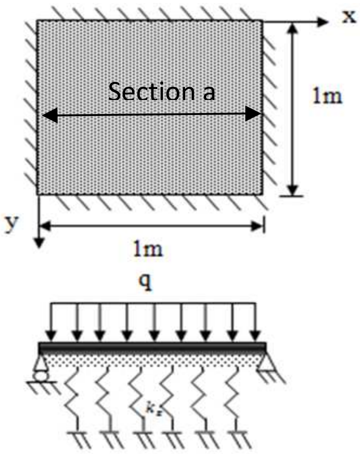


Figure -7 Bending moment diagrams for simply supported thick plate on Winkler foundation for section at mid length of a.

6.2 Simply supported thick plate on nonlinear elastic foundation

A square plate of side length (a = 1.0 m), (b=1.0 m) thickness (h = 0.25 m), having Young's modulus ($E=24*10^6 kN/m^2$), Poisson's ratio (υ = 0.2) and subjected to a uniformly distributed load ($q=25\ kN/m^2$) is considered .The plate is on nonlinear elastic foundation $K_z= \alpha\ (\beta.e^{-\beta w})\ kN/m^3$,[where $\alpha =6.0109*10^4$And $\beta =0.768$] represented by Winkler spring model as shown in figure 8. In the present study, this problem is analyzed by using the finite elements and finite difference methods as shown in figures 9 and 10. The result of central deflection and central moment are given in Table -3 to check the accuracy of the used elements and the developed finite difference program. The table indicates that the results are in excellent agreements. In addition, figures 11 and 12 show the deflection curves and the bending moment diagrams obtained from the present finite element and developed program.



$q=25\ kN/m^2$

$h=0.25\ m$

$E =24*10^6 kN/m^2$

$\upsilon = 0.2$

$K_z= \alpha\ (\beta.e^{-\beta w})$

Where
$\alpha =6.0109*10^4$And $\beta =0.768$

Figure -8 Simply supported thick plate resting on nonlinear elastic foundation.

Table -3: Comparison between the present finite element and finite difference solutions.

Variables	Finite Element Solution	Finite Difference Solution	% Difference
Central Deflection (m)	$3.999*10^{-6}$	$3.999*10^{-6}$	0%
Central Moment (kN.m)	1.11	1.09	1.83 %
Max. Strain	$\pm3.55*10^{-6}$	$\pm3.488*10^{-6}$	1.77%

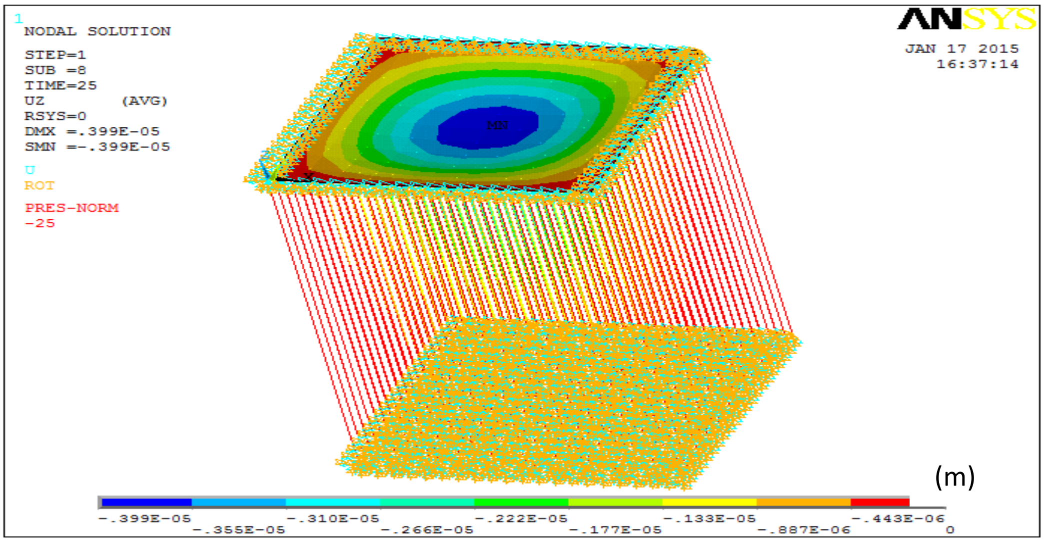


Figure -9 Deflection contours for simply supported thick plate on nonlinear elastic foundation.

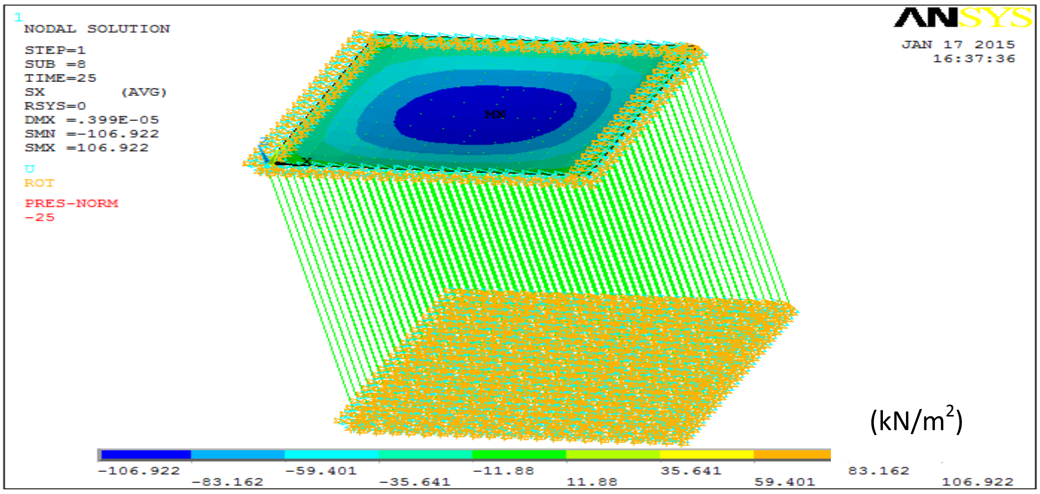


Figure -10 Stress contours in x-direction for simply supported thick plate on nonlinear elastic foundation

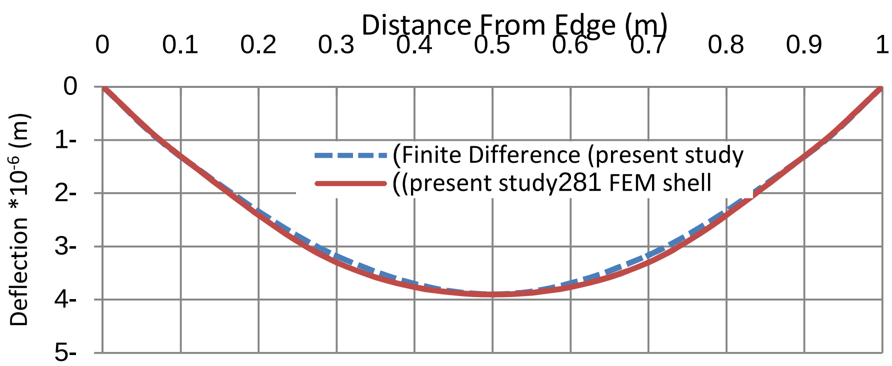


Figure -11 Deflection curves for simply supported thick plate on nonlinear elastic foundation for section at mid length of a.

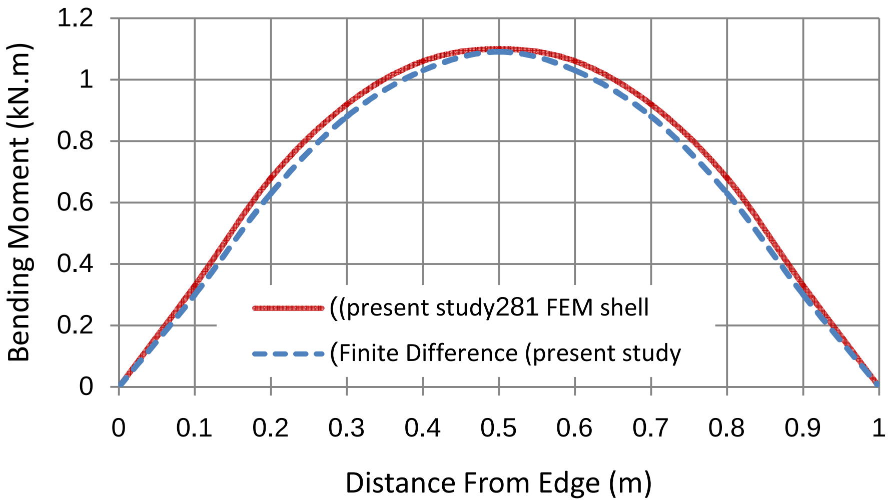


Figure -12 Bending moment diagrams for simply supported thick plate on nonlinear elastic foundation for section at mid length of a.

7. Parametric study

Several important parameters are studied and discussed in this section to show their effects on the behavior of thick plates resting on nonlinear elastic foundations. They are: the effect of plate thickness (h), edges boundary conditions, the type of loading (uniform or concentrated) and Elastic Foundation.

7.1 Effect of plate thickness (h)

To study the effect of plate thickness on the behavior (deflections and moments) of simply supported and fixed edges thick plate resting on nonlinear elastic foundations; the following values of thickness are considered (0.1, 0.15, 0.2, 0.25, 0.3, 0.35 and 0.4 m). Figure 13 and 14 show the variation of plate thickness with central deflections with fixed edges. From these figures, the central deflections will decrease while the central moment will increase as the plate thickness increased because the flexural rigidity (D) of plate will increase.

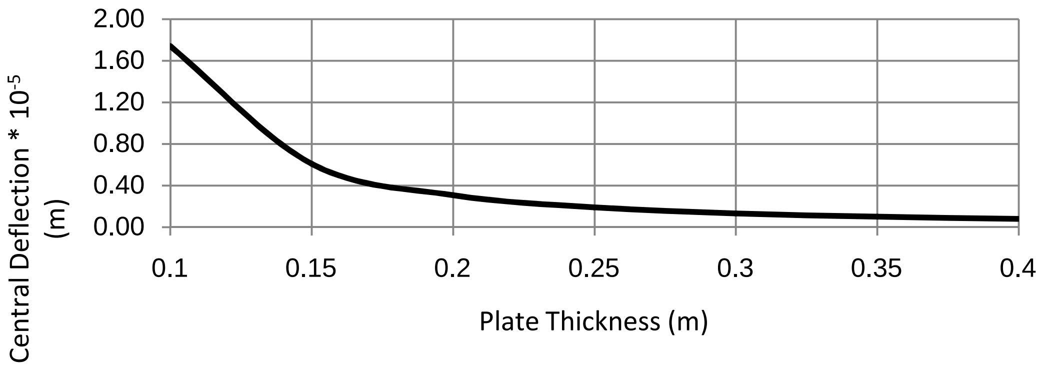


Figure -13 Effect of plate thickness on central deflections of fixed edges thick plate on nonlinear elastic foundation.

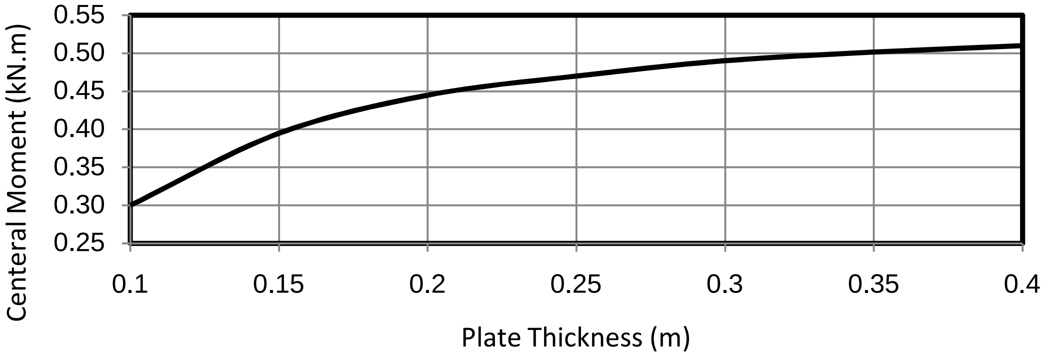


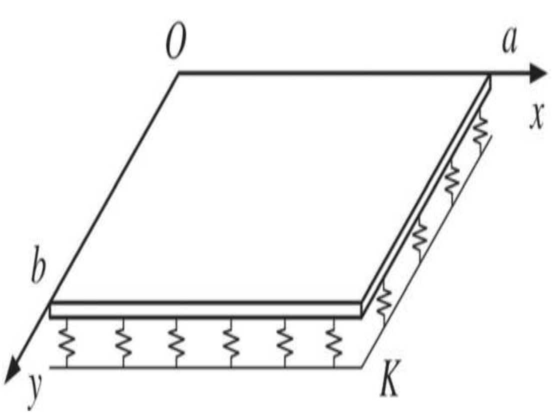
Figure -14 Effect of plate thickness on central moments of fixed edges thick plate on nonlinear elastic foundation.

7.2 Effect of boundary condition

A square plate of side length (a = 1.0 m), (b=1.0 m) thickness (h = 0.25 m), having Young's modulus ($E=24*10^6 kN/m^2$), Poisson's ratio (υ = 0.2) and subjected to uniform load ($q=25$ kN/ m^2) is considered. The plate is resting on nonlinear elastic foundation $K_z= \alpha\ (\beta.e^{-\beta w})$, [where α =6.0109*10^4And β =0.768] kN/m^3, represented by Winkler spring model as shown in Figure 15. In the present study, this problem is reanalyzed by using finite element method (ANSYS 12.1). For comparison, the results of maximum deflection and maximum moment are given in Table -4 for different boundary conditions. The maximum obtained deflection is found to be at the free edges and the maximum obtained central moment is found to be at the simply supported and fixed edges.

Table -4: Finite element solution for free edges thick plate

Variables	Central Deflection (m)	Central Moment (kN.m)	Max. Strain
Free Edges	$7*10^{-4}$	0.212	$\pm6.784*10^{-7}$
Simply Supported	$3.999*10^{-6}$	1.11	$\pm3.55*10^{-6}$
Fixed Edges	$1.91*10^{-6}$	0.55	$\pm4.33*10^{-6}$



$q=25$ kN/m^2

h=0.25 m

E =24*10^6kN/m^2

υ = 0.2

$K_z= \alpha\ (\beta.e^{-\beta w})$

where

α =6.0109*10^4andβ =0.768

Figure-15 Free edges thick plate resting on nonlinear elastic foundation.

7.3 Type of loading

7.3.1 Concentrated load

To show the effect of loading type on the behavior of a square plate of side lengths (a = b= 1.0 m), thickness (h = 0.25 m), having Young's modulus ($E=24*10^6 kN/m^2$), Poisson's ratio (υ = 0.2) and subjected to concentrated load (P =q*a*b), where q=25 kN/m^2) at the center of the plate. The plate is resting on nonlinear elastic foundation $K_z= \alpha\ (\beta.e^{-\beta w})$ kN/m^3 where α $=6.0109*10^4$ and β=0.768, represented by Winkler spring model with simply supported edges. This problem is reanalyzed by using finite element method (ANSYS 12.1). The result of central deflection and central moment are given in Table -5, Figures 16 and 17show the deflection profiles and the bending moment diagrams. The concentrated load gives the highest values for deflection and bending moment in about (4-8) times.

Table -5: Effect of type of loading on deflection and moment for simply supported thick plate resting on nonlinear elastic foundation.

Variables	Central Deflection (m)	Central Moment (kN.m)	Max. Strain
Concentrated Load	$1.75*10^{-5}$	9.146	$\pm2.9267*10^{-5}$
Uniform Load	$3.999*10^{-6}$	1.11	$\pm3.55*10^{-6}$

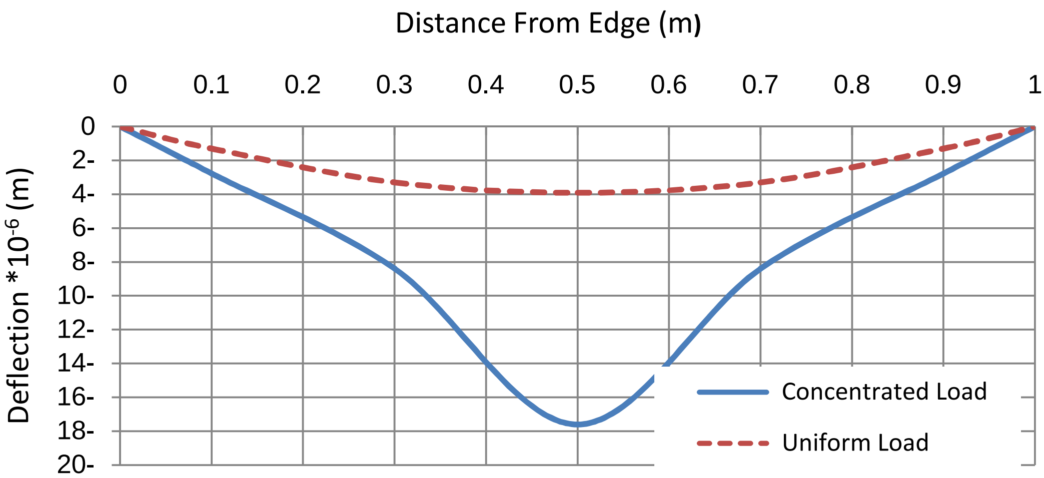


Figure 16 Deflection profiles for simply supported thick plate on nonlinear Winkler foundation.

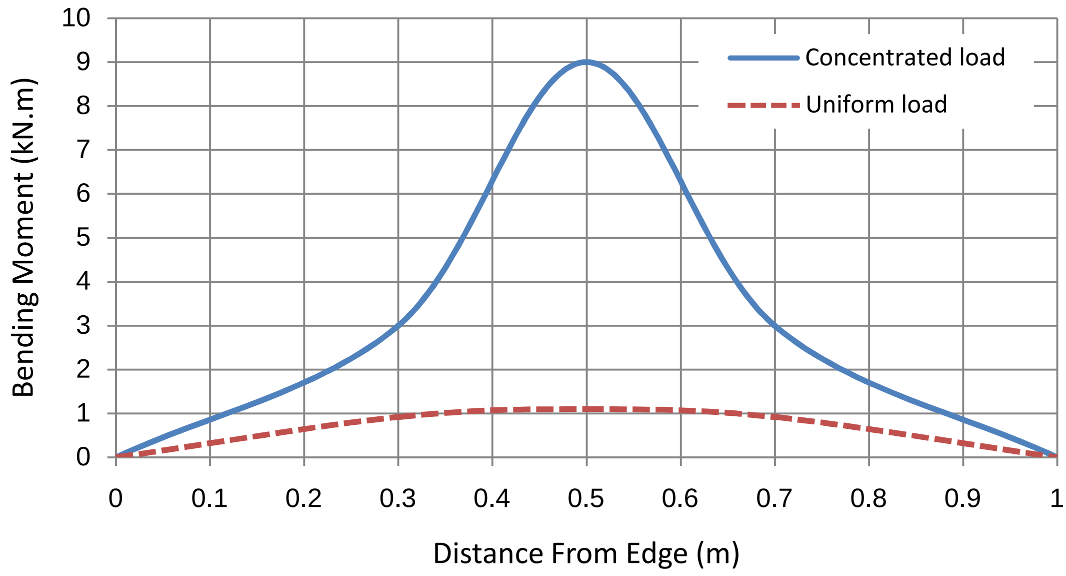


Figure 17 Bending moment diagram for simply supported thick plate on nonlinear elastic foundation.

7.3.2 Plate under Uniformly Distributed Load at the Middle of Plate

A square plate of side length (a = 1.0 m), (b=1.0 m) thickness (h = 0.25 m), having Young's modulus ($E=24*10^6 kN/m^2$), Poisson's ratio (υ = 0.2) and subjected to uniform distributed load ($q=25$ kN/ m^2) at the middle of the plate at the distance of (0.1 m) from the center of plate on each directions. The plate is resting on nonlinear elastic foundation $K_z = \alpha (\beta . e^{-\beta w})$, [where $\alpha = 6.0109*10^4$ and $\beta = 0.768$] kN/m^3. In the present study, this problem is reanalyzed by using finite element method (ANSYS 12.1). The result of central deflection and central moment are given in Table -6, Figures 18 and 19 show the deflection profiles and the bending moment diagrams.

Table -6: Effect of type of loading on deflection and moment for simply supported thick plate resting on nonlinear elastic foundation.

Variables	Central Deflection (m)	Central Moment (kN.m)	Max. Strain
Uniform Load at the middle	$0.433*10^{-6}$	0.209	$\pm 6.688*10^{-7}$
Uniform Load	$3.999*10^{-6}$	1.11	$\pm 3.55*10^{-6}$

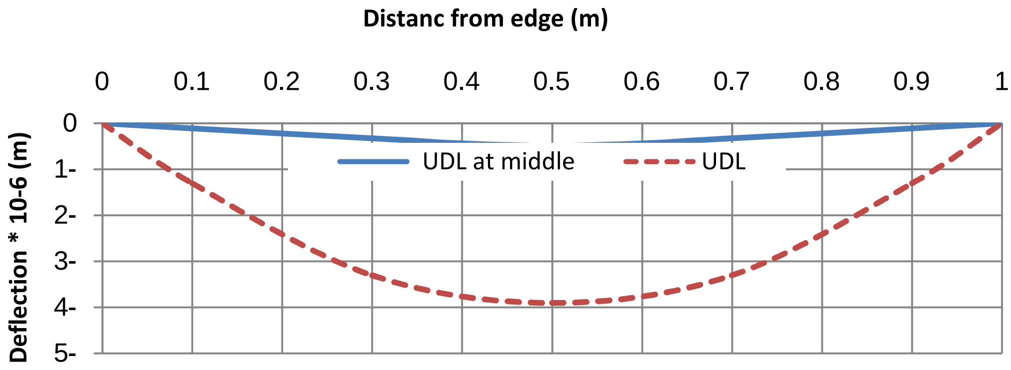


Figure 18 Deflection profiles for simply supported thick plate on nonlinear Winkler foundation.

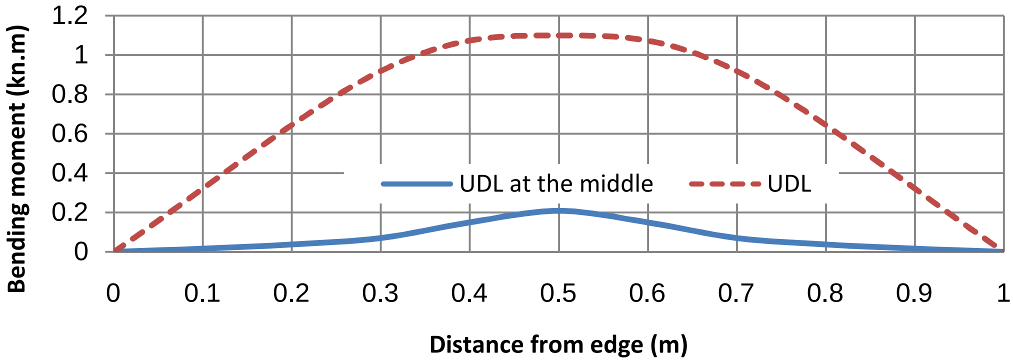


Figure 19 Bending moment diagram for simply supported thick plate on nonlinear elastic foundation.

7.3.3 Plate under Line Loads in the Two Directions

A square plate of side length (a = 1.0 m), (b=1.0 m) thickness (h = 0.25 m), having Young's modulus ($E=24*10^6 kN/m^2$), Poisson's ratio (υ = 0.2) and subjected to line loads (25 kN) in the two directions of the plate. The plate is resting on nonlinear elastic foundation $K_z= \alpha\ (\beta.e^{-\beta w})$, [where α =$6.0109*10^4$ and β =0.768] kN/m^3. In the present study, this problem is reanalyzed by using finite element method (ANSYS 12.1). The result of central deflection and central moment are given in Table -7, Figures 20 and 21 show the deflection profiles and the bending moment diagrams.

Table -7: Effect of type of loading on deflection and moment for simply supported thick plate resting on nonlinear elastic foundation.

Variables	Central Deflection (m)	Central Moment (kN.m)	Max. Strain
Line load in two directions	$0.271*10^{-3}$	91.8	$\pm 2.9376*10^{-4}$
Uniform Load	$3.999*10^{-6}$	1.11	$\pm 3.55*10^{-6}$

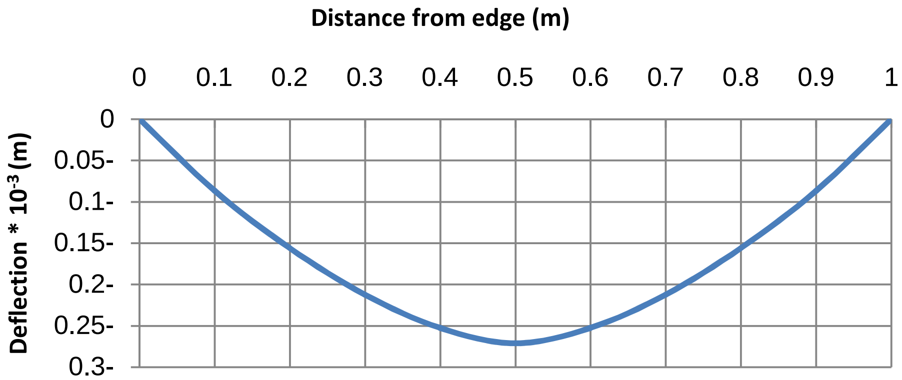


Figure 20 Deflection profiles for simply supported thick plate on nonlinear Winkler foundation.

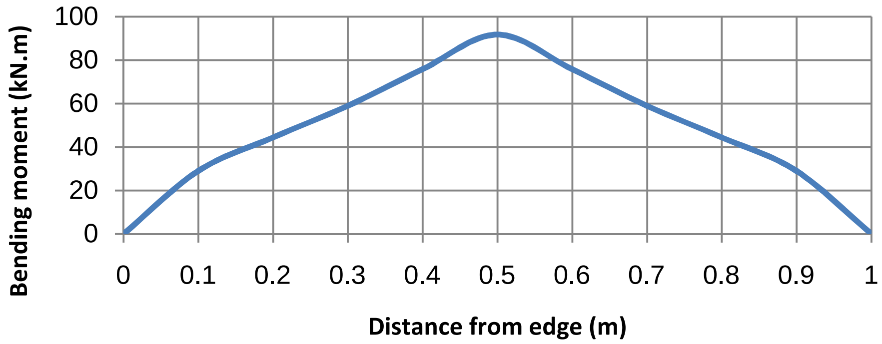


Figure 21 Bending moment diagram for simply supported thick plate on nonlinear elastic foundation.

7.4 Elastic Foundation

A square plate of side length (a = b =1.0 m), thickness (h=0.25 m) having young's modulus ($E=24*10^6 kN/m^2$), Possion's ratio (υ = 0.2) and subjected to uniform load ($q=15\ kN/m^2$) with simply supported edges, the plate is resting on linear Winkler foundation (K_z=10000) and nonlinear Winkler foundation [$K_z= \alpha\ (\beta.e^{-\beta w})$ where α =24.88 , $\beta=5.138*10^2$]. The deflection and moment profiles shown on figure 18 and 19 indicate that the nonlinear Winkler foundation gives higher values than linear Winkler foundation with maximum difference of (18.8%) in central deflection and (20.19 %) in central moment , because of the cumulative deflection with load increments' and reduced modulus subgrade reaction . The result of central deflections and moments are given in Table -8.

Table -8: Comparison between thick plate resting on linear and nonlinear elastic foundation.

Variables	Linear foundation	Nonlinear foundation	% Difference
Central Deflection (m)	$2.33*10^{-6}$	$2.77*10^{-6}$	18.88 %
Central Moment (kN.m)	0.634	0.762	20.19 %
Max. Strain	$\pm2.0288*10^{-6}$	$\pm2.4334*10^{-6}$	19.9%

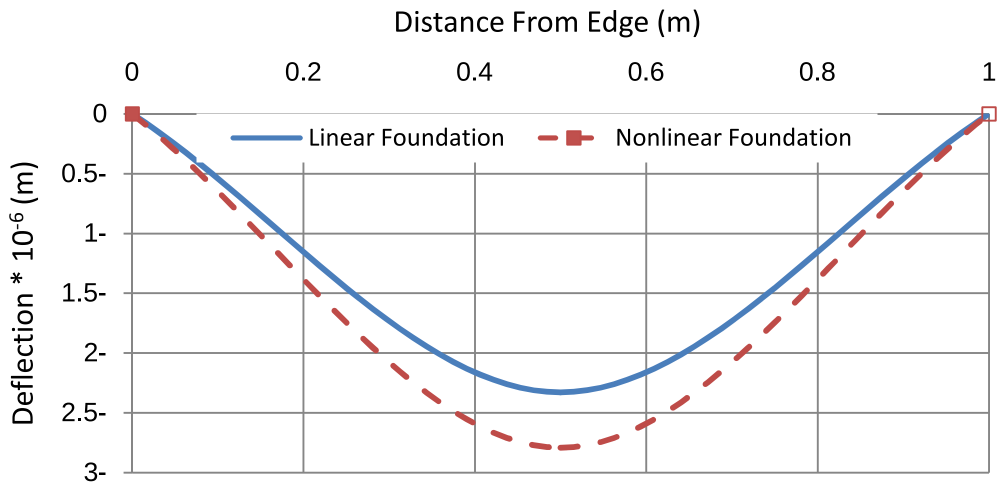


Figure-18 Deflection curves for a simply supported thick plate resting on linear and nonlinear elastic foundation for section at mid length of a.

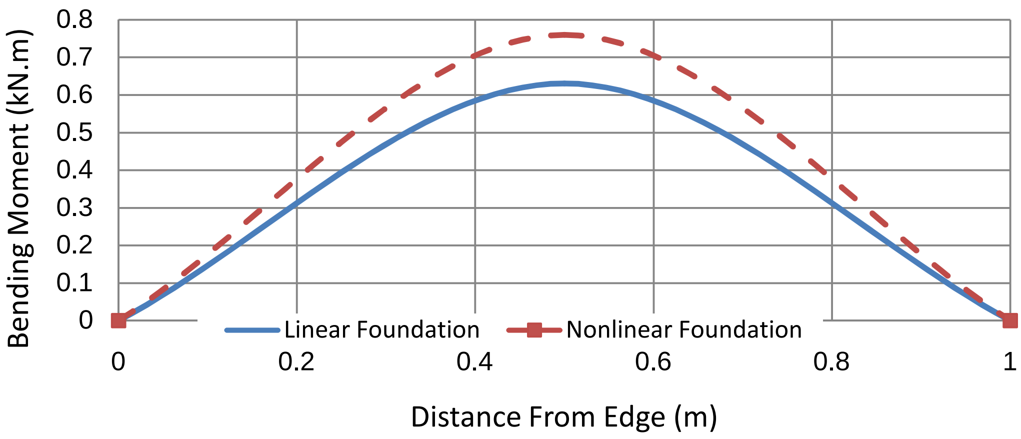


Figure 19 Bending moment diagrams for a simply supported thick plate resting on linear and nonlinear elastic foundation for section at mid length of a.

8. Conclusions

From this study, the main conclusions are as given below:

1. The results obtained from the present study, the finite element method have been compared with the available numerical results to check the accuracy of used elements. Good agreements are obtained between these methods. The results show that the used method gives good estimation of the behavior of plates. It is found that the maximum difference in deflection between finite element and the available numerical methods is (2.68%), while in bending moment the maximum difference is (3.77 %). The differences in central deflection and bending moment from the present study finite element and the finite difference methods for the case of simply supported thick plate resting on nonlinear elastic foundation are (0 %) and (1.83 %) respectively, while (2.68 %) and (3.77 %) respectively for the case of fixed edges.

2. The effect of thickness (h) on the behavior of plate resting on nonlinear elastic foundation shows that: the deflection at the center is decreased as the thickness increased, because the flexural rigidity (D) of plate will be increased as the relation between deflection and flexural rigidity is inversely proportional. The moment at the center is increased as the thickness is increased, because the flexural rigidity (D) of plate will be increased as the relation between moment and flexural rigidity is directly proportional. It is found that by increasing the thickness (h) from (0.2 to 0.4) m the deflection is decreased by by (73.8 %) and the moment is increased by (14.6 %).
3. The types of load effect show that: when the plate is subjected to concentrated load at the center of the plate [P=q* (a*b)], where q=25 kN/m^2) instead of uniform load, the deflection, moment and maximum strain will be increased by (337.6 %) , (723.96 %) and (724 %) respectively. While when the plate in subjected to uniform distributed load at the middle of plate ,the deflection, moment and maximum strain will be decreased by (89.17 %),(81.17 %) and (81.16 %) respectively. While when the plate is subjected to line load in both directions , the deflection , moment and maximum strain will be increased by (6676.69 %), (8170.27 %) and (8174.9 %) respectively, for simply supported edges.
4. The effect of boundary conditions show that: when the edges of the plate are considered free and keep the uniform load constant at (25 kN/m^2), the central deflection is increased by (99.4 %),while the central moment and maximum strain is decreased by (423.58%) and (423 %) respectively, if compared with simply supported edges. While the central deflection is increased by (99.727 %) and both the central moment and maximum strain is decreased by (159.4%) and (538.26 %) respectively, if compared with fixed edges.
5. The maximum deflection, bending moment and maximum strain will be increased by (18.88%) , (20.19%) and (19.19 %) respectively for the simply supported thick plate when the foundation is modeled as nonlinear Winkler model rather than linear Winkler model because of the cumulative deflection with load increments' and reduced modulus subgrade reaction.

Abbreviations

A	Smaller dimension of the plate.
b	Larger dimension of the plate.
c^2	Correction factor for transverse shear.
D	Flexural rigidity of plates.
E	Modulus of elasticity of plates.
G	Shearing modulus for plates.
H	Plate thickness.
K_z	Modulus of subgrade reaction in z-direction.
q(x,y)	Pressure.
P	Concentrated load.
x, y, z	Cartesian coordinates.
u, v	Displacements in x and y-directions.
W	Displacement in z-direction.

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