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STRUCTURAL STUDY OF FRP COMPOSITE PILE UNDER AXIAL COMPRESSION LOAD USING FINITE ELEMENT METHOD

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Abstract: FRP-concrete- steel is a new hybrid composite structural member combines the privileges confined compressive strength of concrete and the ductility of steel to produce a lighter in weight member with considerable carrying capacity. In this research the FRP hybrid member was utilized as a pile which is a special case of FRP concrete filled pile with a hollow steel tube inside. F.E modeling were carried out for this new hybrid pile using the computer program ABAQUS, in which confined concrete model was included through the Concrete Damage Plasticity model. Verification of the model was implemented through the simulation of experimentally tested specimens, good agreement were achieved between experimental and F.E results. The F.E model was considered in studying the effects of FRP tube thickness, steel tube thickness, Hollowness ratio and stiffness of concrete. The results were presented in terms of axial load-axial strain curves. It has been found that the axial strength ratio increased as the stiffness of concrete, steel, and FRP increased while it decreased with the increase of hollowness ratio.

Keywords: *FRP, composite Pile, Finite Element, ABAQUS*

دراسة أنشائية لركيزة ألياف بوليميرية مسلحة مركبة تحت حمل ضغط محوري بإستخدام طريقة العناصر المحددة

الخلاصة: ان استخدام الالياف البوليمرية المسلحة مع الخرسانة مع والحديد كعنصر انشائي هجين مركب يجمع خاصية مقاومة الانضغاط العالية للخرسانة المحصورة مع ليونة الحديد للحصول على عنصر انشائي اخف وزنا وذو قابلية تحمل معتبرة. في هذا البحث تم استخدام هذا العنصر الانشائي كركيزة حيث يعتبر حالة خاصة من ركيزة الالياف البوليمرية المسحلة المملوؤة بالخرسانة مع انبوب حديد مجوف في داخلها. تم استخدام طريقة العناصر المحددة لتمثيل هذاالنوع من الركائز من خلال برنامج الحاسوب (ABAQUS) من خلال تمثيل الخرسانة المحصورة باستخدام نموذج الخرسانة المتضررة اللدنة (CDP) وللتأكد من هذا النموذج تم مقارنته مع نتائج فحوصات عملية ، حيث كانت هناك مطابقة جيدة بين نتائج طريقة العناصر المحددة ونتائج الفحوصات العملية . استخدم هذا النموج ومن خلال الطريقة المذكورة لدراسة تاثيربعض العناصر على سلوك الركيزة مثل : سمك طبقة الالياف البوليمرية المسلحة و سمك الانبوب الحديد و مقاومة الخرسانة للانضغاط بالاضافة الى نسبة الفراغ. تم عرض نتائج البحث على شكل علاقات بين الحمل المحوري مع الانفعال المحوري حيث لوحظ ان نسبة المقاومة تزداد بزيادة صلابة كل من الخرسانة والحديد والالياف البوليمرية المسلحة في حين انها تقل بزيادة نسبة الفراغ.

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1. Introduction

Timber, steel and concrete piles are the well-known types of piles considered as deep foundation alternatives long time ago, whenever piles are composed of more than a single material, like timber and steel or concrete and steel a composite pile is created.

The producers of pile foundation system offers several types of composite piles such as piles with steel core, concrete filled steel tubular piles, structurally reinforced plastic matrix piles, concrete-filled fiber reinforced polymer (FRP) piles, fiberglass piles, and lumber piles. Fig. (1), presents a typical cross sections for the FRP piles along with other composite piles. [1]

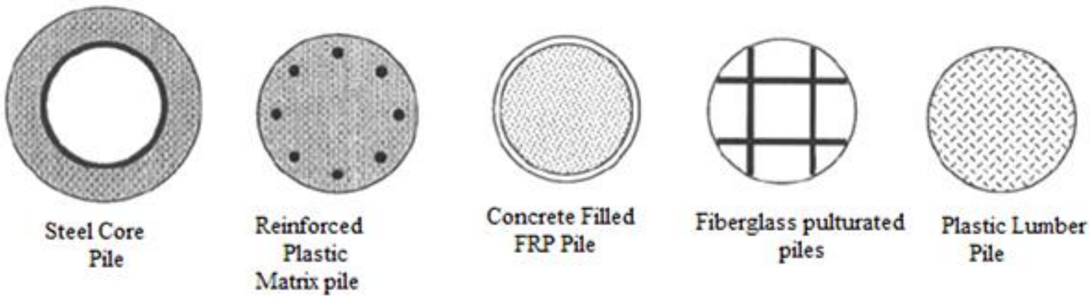


Figure (1), Typical cross sections of composite piles, [1].

Encasing concrete with steel or FRP tubes substitutes the reinforcing bars through providing high confinement stress to concrete resulting in high bearing capacity, high ductility which makes this type of structural members suitable for high rise building, bridge piers especially if seismic design is considered. Unlike conventional steel tubes FRP tubes can be used in corrosive soil and environmental condition without additional protection.

As an alternative for traditional cast-in-place or precast concrete pile existing in difficult environments, FRP piles have been studied in the last few decades by several researchers for example (Fam and Rizkalla) [2] studied the structural behavior of FRP pile under axial and flexural loading, while Mirmirn and Shahawy [3] studied the capacity and the behavior of FRPpiles under axial loading and under mechanical driving loads.

Regarding the concrete-filled FRP composite pile, it comprise of two main structural components: an FRP shell or tube, and a concrete infill without steel reinforcement. Among other thing the purpose of FRP shell is to keep concrete in place, providing confinement to the concrete, work as tensile reinforcement, and corrosion protection [2].

Teng et al. [4], proposed new hybrid FRP-concrete-steel composite tubular structural member, the new hybrid member composed of two tubes; outer FRP tube, inner steel tube and concrete in between. The proposal of this member was an attempt to combine the ductility of steel, compressive strength of concrete and high tensile strength of FRP in one high performance structural member. This member also can act effectively as a pile especially when cyclic or lateral loading is expected.

In this paper a three dimensional F.E model is generated using the ABAQUS software package for simulating the FRP concrete -steel composite piles under axial compression load.

2. F.E Simulation of FRP Composite Pile

The general principle of the F.E. method include the simulation of the actual object as an assemblage of a finite number of discrete elements joined together at nodal points [5].

In general, the three-dimensional F.E modeling consist of geometric modeling that includes model configuration, object representation and meshing, the other part is material modeling which includes the constitutive model used for the material modeling.

2.1 Geometric Modeling

A composite FRP pile with steel tube inside is considered for analysis, this pile have a length of (L), FRP tube thickness and diameter of (t_f) and (D_F), respectively , and steel tube thickness and diameter of (t_s) and (D_s), respectively. Fig. (1), shows cross and longitudinal sections through the considered composite pile.

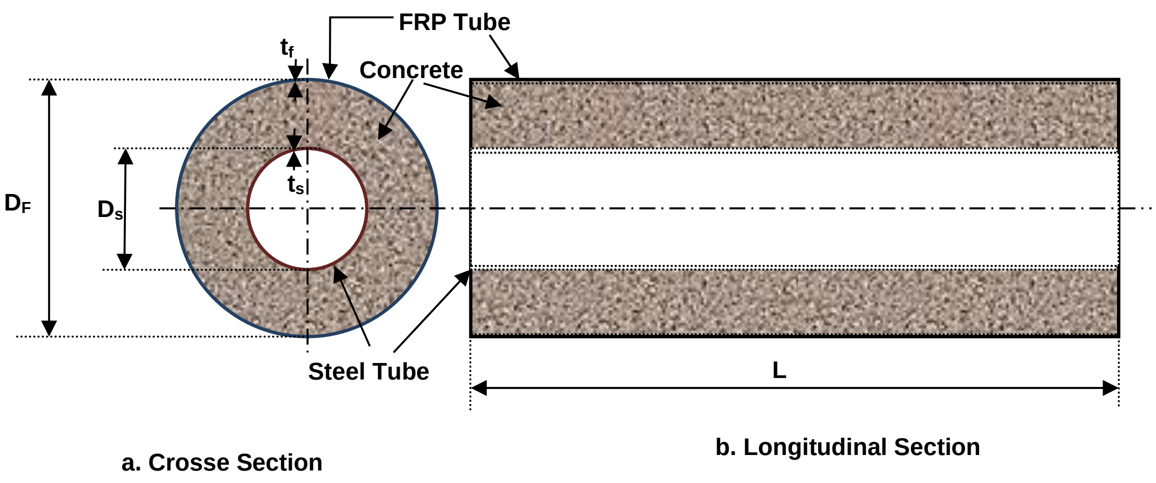


Figure (1) FRP Composite Pile details, (a) cross section, (b) Longitudinal section

The composite pile is discretized into F.Es using the meshing tools and elements library of ABAQUS software. SC8R (8-node hexahedron continuum shell element) is used to model the FRP and inner steel tubes. This type of continuum shell elements are general-purpose three-dimensional stress/displacement elements for use in modeling structures that behave as shell, this types of elements include three degree of freedom at each node. For modeling the concrete in between the two tubes, a three-dimensional eight-node element C3D8 with three degree of freedom at each node is considered. The final mesh size and elements number for each part of the FRP composite pile was concluded after a implementing a convergence study for the proper mesh size, accuracy of results and analysis time-cost.

2.2 Materials Modeling

2.2.1 Modeling of Confined Concrete

Confining concrete with steel reinforcements, steel tubes or FRP tubes is commonly used to delay the failure of concrete and improve its ductility, and hence increasing its compressive strength. Several researchers have studied the modeling of confined concrete through experimental research, theoretical analysis and finite element modeling.

The adoption of appropriate constitutive model for simulating concrete is important and may affect the overall behavior of the composite member. Concrete Damage Plasticity model (CDP) utilizes the principle of isotropic compression and tension plasticity to model the behavior of concrete. This model was proposed by Lubliner [6] for monotonic loading and has been developed later by Lee and Fenves [7] to consider the dynamic and cyclic loadings. CDP model is considered for modeling the infilled concrete of the FRP composite pile using ABAQUS material plasticity tools.

Mander et al. , [8] was a pioneer in modeling confined concrete members with spiral steel reinforcement, his model based on the axial compressive tests of concrete with a quasi-static strain rate and monotonic loading, columns of full scale were used to verify the results of this model, this model have been used in modeling concrete filled steel tubular members.

The behavior of FRP confined concrete has been extensively studied by many researchers, for example: Yu [9]; Yu and Teng [10]; Lam and Teng [11]. Design oriented models were considered in may models for representation of confined concrete, in this type of models experimental data are collected from test results along with using different regressions analyses to produce equations for stress strain relationships. At the other hand among others Teng et., al. [12], used test results directly to drive a closed form expressions considered explicitly the interaction between FRP jacket and concrete.

The refined Lam and Teng's, [11] design oriented model will be considered here in for modeling the behavior of FRP Confined Concrete, this model is shown in Fig.(2). This model was created depending on the following assumptions:

a. the stress strain curve composed of two parts, the first is parabolic and the second is linear;
b. the slope of the first parabolic part is the same as modulus of elasticity of unconfined concrete;
c. nonlinearity of the parabolic part is affected mainly by the existence of FRP plies;

d. the two parts meet smoothly without changing slope;
e. ultimate compressive strength and axial strain are recorded at the termination point of the linear part.

According to this model the stress strain relation can be described by the following forms:

$$\sigma_c = E_c \varepsilon_c - \frac{(E_c - E_{2c})^2}{4 f_o} \varepsilon_c^2 \qquad for \qquad 0 \le \varepsilon_c \le \varepsilon_t \tag{1}$$

and

$$\sigma_c = f_o + E_{2c}\, \varepsilon_c \qquad for \qquad \varepsilon_t \le \varepsilon_c \le \varepsilon_{cu} \tag{2}$$

where,

$\sigma_c =$ axial stress, ε_c = axial strain, E_{2c}= slope of the second part of the curve (linear part), f_o= stress at the intercept of stress axis with linear part of the curve, ε_{cu}= confined concrete ultimate axial strain, E_c= modulus of elasticity of unconfined concrete,

The transition from the parabolic part to the linear part occurs smoothly at (ε_t) which is described by :

$$\varepsilon_t = \frac{2 f_o}{(E_c - E_{2c})} \tag{3}$$

$\boldsymbol{E_{2c}}$ can be expressed by :

$$E_{2c} = \frac{f'_{cc} - f_o}{\varepsilon_{cu} - \varepsilon_t} \tag{4}$$

where, $\boldsymbol{f'_{cc}}$ is the compressive strength of confined concrete.

Lam and Teng [11], proposed that (f_o) is equal to unconfined compressive strength (f'_{co}), and they produced relations for predicting the confined compressive strength (f'_{cc}) and the ultimate axial strain (ε_{cu}). These relations have been refined later by Teng et. al, [14] depending on additional experimental data, below are the refined relations:

$$\frac{f'_{cc}}{f'_{co}} = \begin{cases} 1 + 3.5(\rho_k - 0.001).\,\rho_\varepsilon & if\ \rho_k \ge 0.01 \\ 1 & if\ \rho_k < 0.01 \end{cases} \tag{5}$$

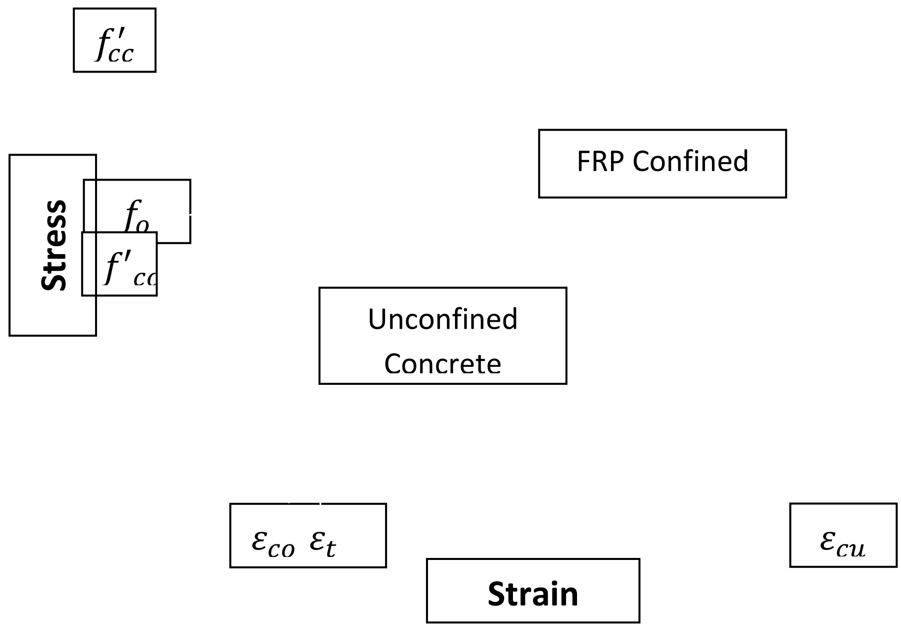


Figure (2) Confined and unconfined stress strain relations, [10].

$$\frac{\varepsilon_{cu}}{\varepsilon_{co}} = 1.75 + 6.5\,{\rho_k}^{0.8}\,{\rho_\varepsilon}^{1.45} \tag{6}$$

where,

$\rho_k \; = E_{frp}\, t_{frp} \,/\, (E_{seco} R_o)$, named confinement stiffness ratio,

$\rho_\varepsilon \; = \varepsilon_{h\,rup} \,/\varepsilon_{co}$, named as strain ratio,

E_{frp} = modulus of elasticity of FRP jackets,

t_{frp} = thickness of the FRP jacket,

$\varepsilon_{h\,rup}$= hoop strain of FPR at rupture under hoop tensile stress,

R_o = radius of the confined concrete,

ε_{co} = axial strain at the unconfined state,

$E_{seco} = \frac{f'_{co}}{\varepsilon_{co}}$ = secant modulus of the unconfined concrete

Generally, the design oriented models allow the use of data from experimental tests or considering design codes in finding other input parameters, as the case of modulus of elasticity, it can be taken as $\boldsymbol{E_c} = 4700\sqrt{f'_c}$ according to ACI-318-14 [14].

2.2.2 Modeling of Steel Tube

The traditional elastic-plastic model is used to model the constitutive behavior of steel tube material. The parameters need to be defined for modeling steel tube, are: Modulus of Elasticity, Poisson's ratio, and yield stress. Isotropic hardening is considered as the flow rule after yielding of steel tube occurs. it is worth mentioning here that the inner steel tube was restrained against local buckling outward because of the existence of the external concrete

2.2.3 Modeling of FRP Tube

Linear elastic behavior is considered for modeling the fiber reinforced polymer material, the FRP tube continue to retain elastic strains until it fail when ultimate strain is reached.

2.2.3. Modeling of Interaction

There are two interfaces existed in the F.E mesh, which are the concrete to the inner steel tube interface and the concrete to the FRP outer tube interface. Both interfaces are modeled using pairs of contact surfaces. The nodes on these interfaces are connected through the contact surfaces which can model infinitesimal sliding and friction between the concrete and the tubes [15]. The friction coefficients used in the interface model for FRP tube and steel tube are; μ= 0.75 and μ= 0.5, respectively. This type of interface allow the nodes on the interfaces surfaces to either contact or separate but not to penetrate each other.

3. Verification of the F.E Analysis

The model of FRP composite pile created using ABAQUS F.E program shall be verified first throughout using this model in solving experimentally tested problem by Yu [9], whom implemented an experimental program included testing of three pairs of hybrid FRP-concrete-steel tubular columns, all columns had an outer diameter of 152.5mm, a height of 305mm, and the same steel tube inside with a diameter of 76.1mm and thickness of 3.2 mm. These Columns were surrounded with FRP tubes of different thicknesses outside, which had reinforcement fibers only in the hoop direction. He conducted tensile tests on steel coupons; it was found that the steel tube had a yield stress of 352.7 MPa, an ultimate tensile strength of 380.4 MPa and a Young's modulus of 207.28 GPa. The FRP tubes were prepared by the wet lay-up process; the FRP used had a nominal thickness of 0.17mm per ply, a tensile strength of 2300 MPa and a Young's modulus of 76 GPa based on this nominal thickness. The elastic modulus, compressive strength and ultimate strain of the concrete were 30.2 MPa, 39.6 MPa and 0.002628 respectively. Single ply of FRB tube was used for the first pair of columns, while two and three plies were used for the second and third pairs of columns, respectively.

FRP confined concrete model was considered in modeling part of Yu 's [9] work which included three pairs of FRP composite compression members of different outer tube thickness under axial loading condition. The results were presented for the six specimens in terms of axial load-axial strain curves, each pair showed similar behavior and very close results. Fig. (3), shows a comparison between Yu's experimental results and the ABAQUS F.Es results. It is quite obvious that even the F.E results are slightly higher than experimental results but still there is a very good agreement between them.Verification can be made also by checking the deformed shape of the specimens after failure, Fig. (4.a) shows one of the experimentally tested specimens by Yu [9] with two plies FRP tube after failure with fracture of FRP tube at the middle of the specimen, while this is also occurred in the F.Es modeling, as show in Fig. (4b and 4c), maximum stresses noticed at the middle of FRP tube, as well as fracturing of concrete and maximum lateral displacement of concrete also occurred at mid height of the specimen. The steel tube showed no local buckling with low stress level as compared to concrete and FRP parts.

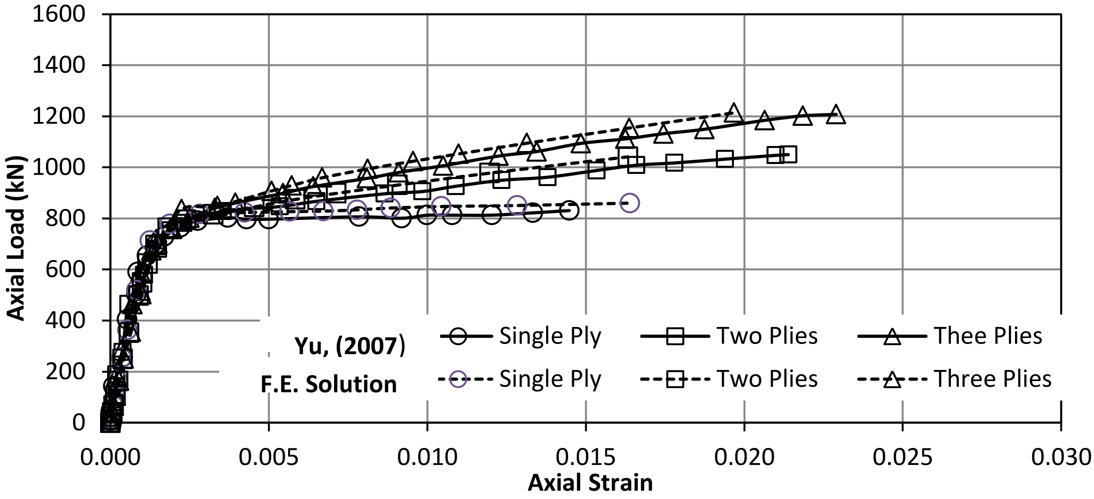
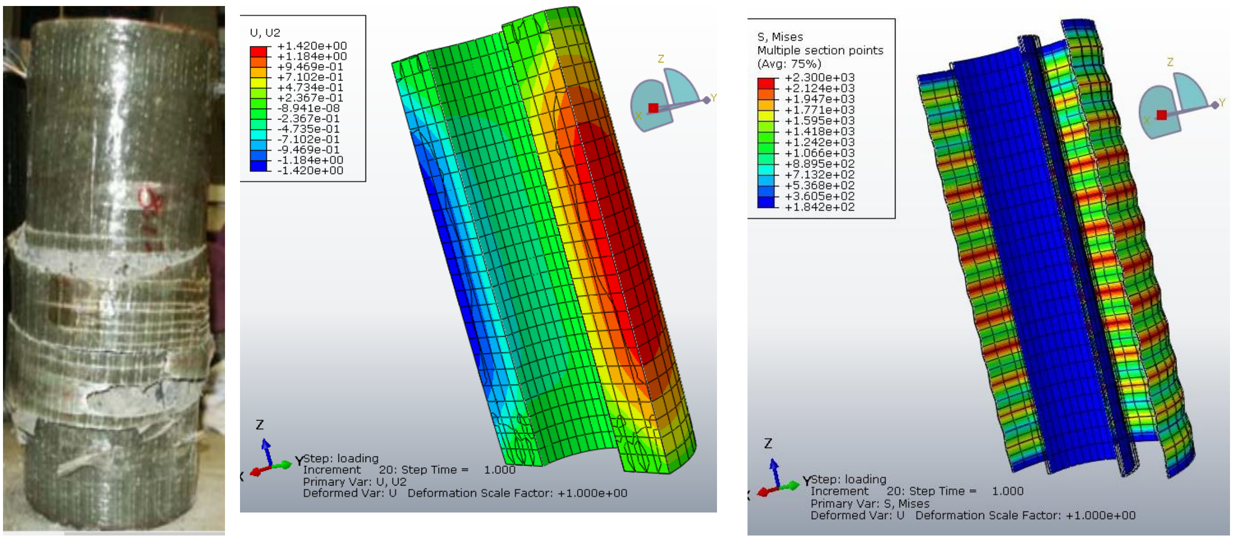


Figure (3) Comparison between Yu,[8], and F.E model



a) Yu, (2007) Specimen

b) laeral displacement in concrete part (F.E)

c) Stresses in FRP and steel tubes (F.E)

Figure (4) Comparison between experimental and F.E model deflection shape.

4. Behavior of FRP Composite Pile under Axial Compression

In this section a study of the structural behavior of FRP composite piles is conducted to asses the effect of certain parameters on the axial strength of the pile. The dimension of the piles considered herein are the same of those tested by Yu, [9] which adopted in verification of the F.E model. In order to extend results into full scale the axial strength of the composite pile is studied in terms of axial strength ratio, which is a dimensionless parameter can be defined as P_o / (P_{co}+ P_s), where P_o is axial capacity of the FRP composite pile recorded as the highest value of axial load reached by the specimen before failure in the F.E program that is resulted from applying an axial compression at the upper surface of FRP composite pile . P_{co} is the axial capacity of concrete alone which can be calculated as ($P_{co}= A_c * f'_c$), in which A_c and f'_c are cross sectional area and compressive strength of unconfined concrete, respectively. P_s is the axial strength of steel section alone which can be calculates as ($P_s = A_s * f_y$),where A_s and f_y are cross sectional area and yield stress of steel tube.

4.1 Effect of FRP Tube Thickness

The thickness of the FRP tube is a key parameter in providing confinement to concrete and increasing it axial strength. Some of FRP tube are created using number of piles of certain thickness, in other cases FRP tube are manufactured with the required thickness. In this study piles of thickness of 0.17mm are considered to change the thickness of the FRP tube. Fig. (5) shows the axial load-axial strain curves of FRP composite piles of different FRP tube thicknesses,

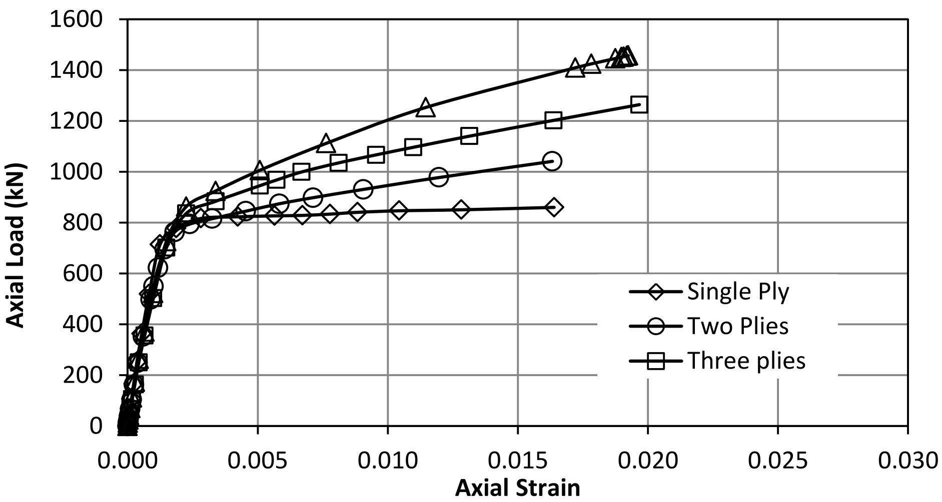


Figure (5) Axial stress-axial strain of composite FRP pile with different FRP tube thicknesses,

Fig. (6) shows a linear increase in axial strength of the FRP pile with the increase of confinement provided by FRP tube, about 20% gain in strength noticed when an additional ply was added to FRP pile with single ply FRP tube. Three plies of FRP increased the axial strength by 21% with respect to FRP tube of two plies.

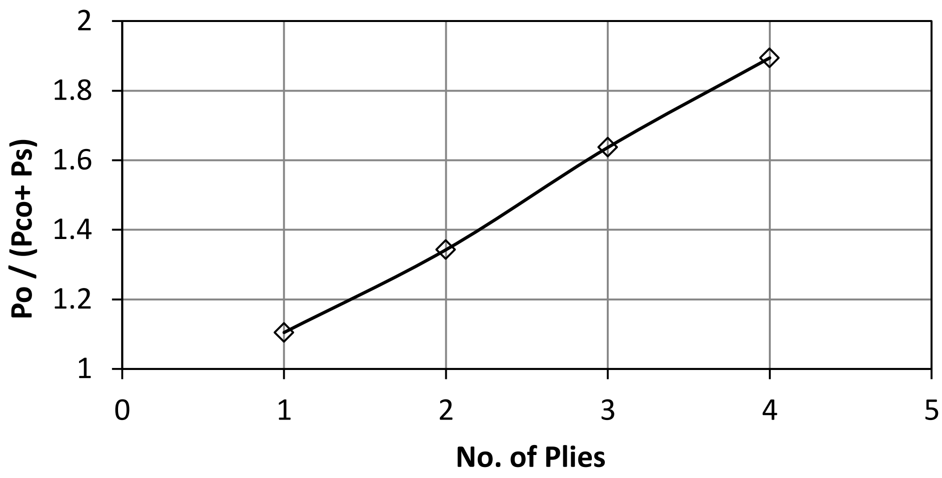


Figure (6),Variation of Axial strength Ratio of FRP composite pile with thickness of FRP tube,

4.2 Effect of Hollowness Ratio

The hollowness ratio (H) is an indication of how large is the empty hole inside the composite FRP piles is, where a pile with (H=0) is a concrete filled FRP tube with no hole inside, and a pile with (H=1) is a completely hollow pipe pile. Hollowness ratio also indicates the cross sectional area of concrete and can be calculated as the ratio of the inner tube diameter to the outer tube diameter ($H=D_s / D_{FRP}$), where D_s is the diameter of steel tube and D_{FRP} is the diameter of FRP tube. In this study five values of Hollowness ratio are considered as : 0.3, 0.4, 0.5, 0.6, and 0.7.

Fig. (7) shows the axial load-axial strain of FRP composite pile for the specified Hollowness ratios, whereas Fig. (8) present the variation of the axial strength ratio with the Hollowness ratio, in this figure the axial strength decreases as the Hollowness ratio increases, as this can be attributed to the decrease in the area of concrete. The decrease of strength is approximately linear with the increase of Hollowness ratio, about 4% of strength is lost with each change from a Hollowness ratio to another.

4.3 Effect of Steel Tube Thickness

The Inner steel tube acts as a longitudinal reinforcement and as a replacement for the inner void, therefore it should has an enough thickness to keep concrete triaxially confined from inside, this confinement can be lost if local buckling occur in this tube. An investigation of the effect of the inner steel tube thickness on the axial strength of the composite FRP pile has been made by considering four different thicknesses of (2.0, 2.5, 3.2, and 4.0) mm while keeping the FRP tube thickness constant at the value of 0.51mm (three plies). Fig. (9) shows the axial load-axial strain curves for FRP piles with the considered thicknesses, it can be seen that the pile with the least thickness (2.0 mm) failed with axial strain smaller than of other piles, as this can be referred to the failure of the inner steel. The effect of the inner steel tube is obvious as it increased the axial strength of the FRP composite pile increases, where, about 19% improvement in strength achieved as the thickness of the steel tube increased from 2.0 mm to 4.0 mm.

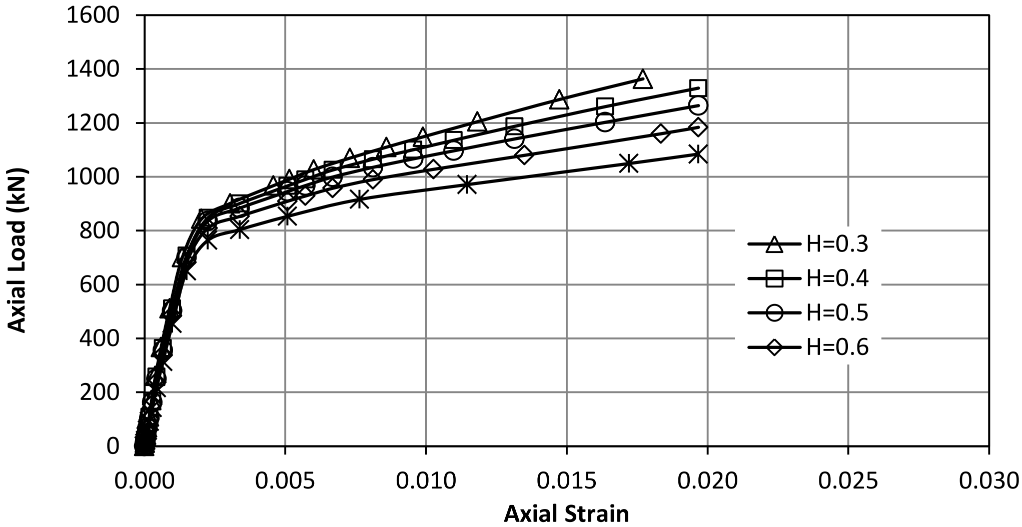


Figure (7) Axial stress-axial strain of composite FRP pile of different hollowness ratios

Figure (8),Variation of Axial strength Ratio of FRP composite pile with hollowness ratios,

Fig. (10) shows the variation of the axial strength ratio with the thickness of the inner steel tube relative to the FRP tube thickness, in this figure the strength ratio first increased when thickness changed from 2.0 mm 2.5 mm, but then it started to decrease with the increase of the tube thickness, this is resulted from the increase in the cross sectional area of steel which leads to an increase in the axial strength of steel (P_s).

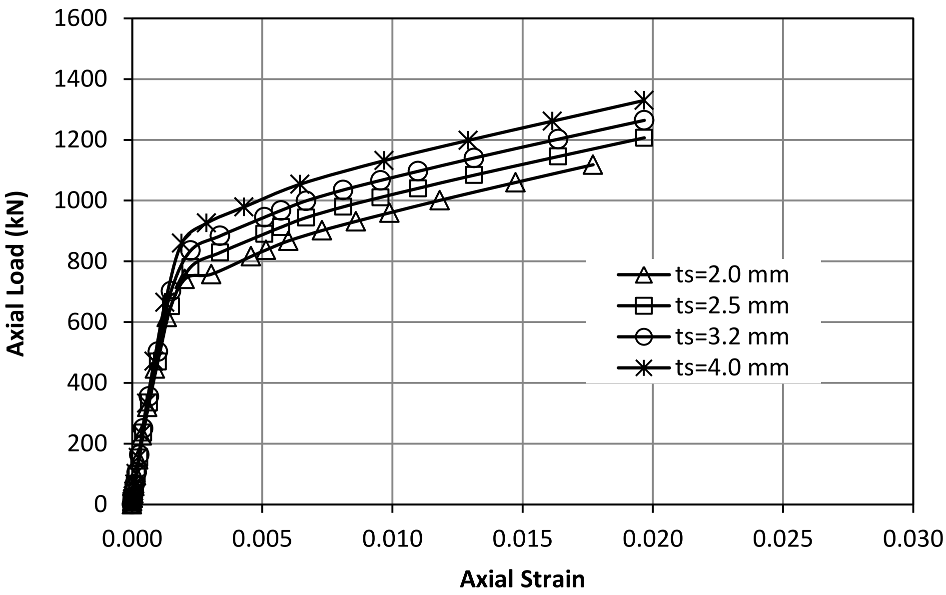


Figure (9) Axial stress-axial strain of composite FRP pile of hollowness ratios

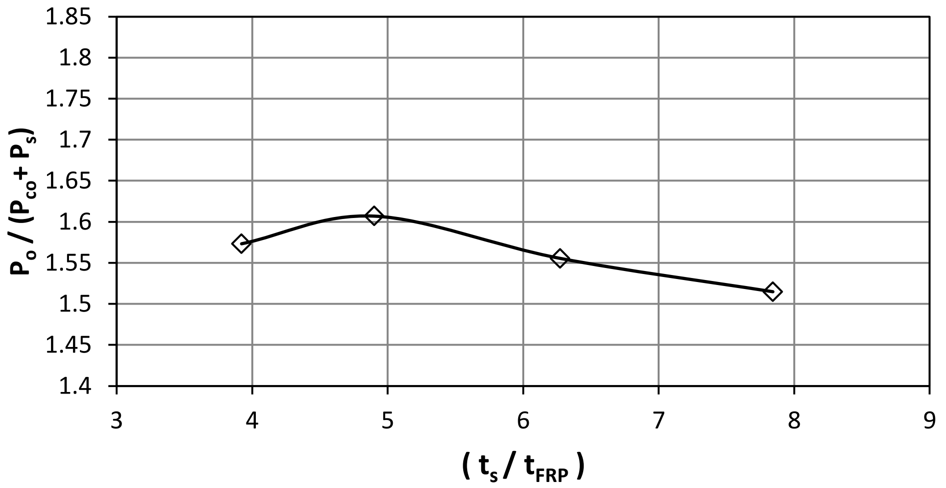


Figure (10),Variation of Axial strength Ratio of FRP composite pile with different steel tube thicknesses,

4.4 Effect of Concrete Compressive Strength

Concrete is the main part in sustaining the compression load. In this study, compressive strength of concrete is considered as the variable parameter while keeping the FRP tube thickness, FRP stiffness, steel stiffness, and steel tube thickness constant as well as considering a hollowness ratio of 0.5. As expected the axial strength and the axial strength ratio increased with the increase in concrete compressive strength as shown in Fig. (11) and Fig. (12). While keeping the stiffness of FRP is constant, almost there is a linear relationship between axial strength ratio and the stiffness of concrete (E_c) which is calculated in terms concrete compressive strength according to [13]. Increasing f'_c from 28 MPa to 48 MPa revealed an enhancement of about 30% in the axial strength ratio.

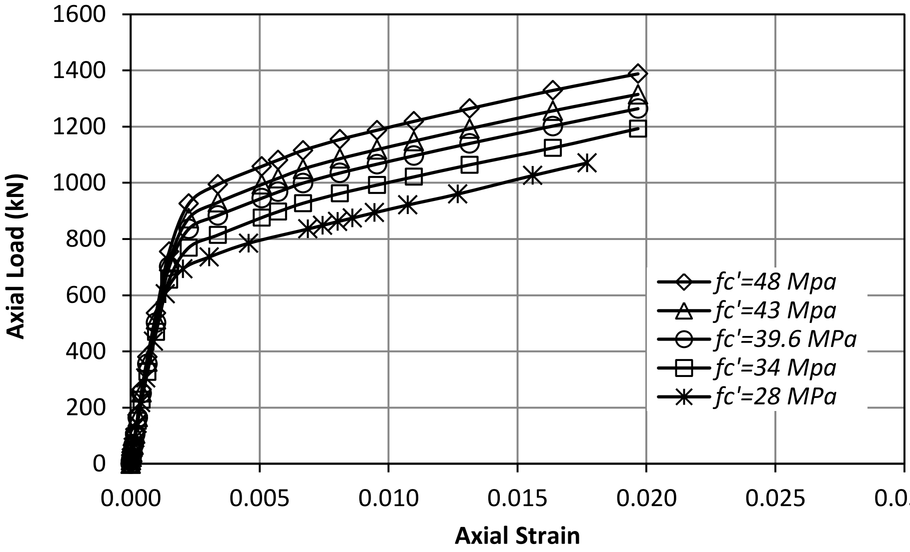


Figure (11) Axial stress-axial strain of composite FRP pile of different compressive strength values of unconfined concrete

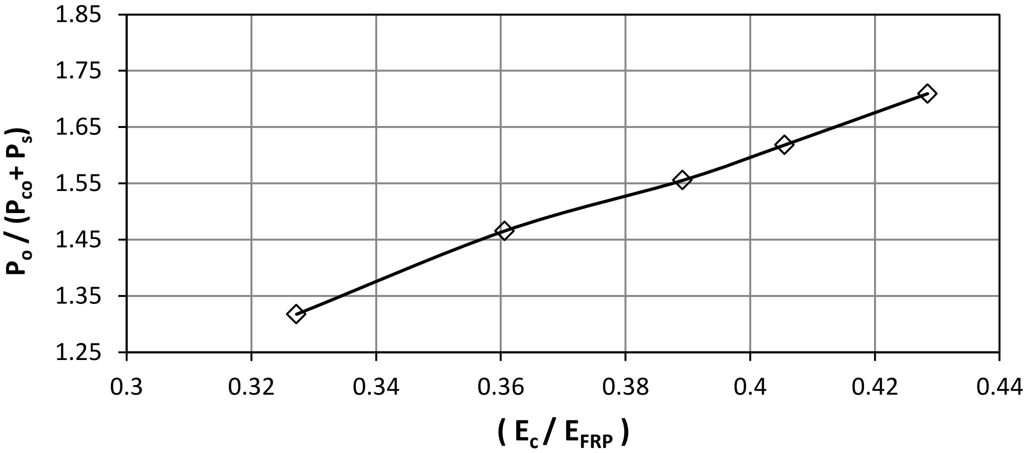


Figure (12),Variation of Axial strength Ratio of FRP composite pile with stiffness ratios.

5. Conclusions

F.E simulation of FRP composite pile with steel tube inside, ABAQUS computer program has been used considering the Concrete Damage Plasticity for concrete modeling with the refined Lam and Teng [11] model for modeling the confined concrete, this mode has been verified against experimentally tested specimens. Different parameters were studied throughout this study; the following conclusion can be drawn:

1. In general FRP composite piles are failed when FRP tube start tearing resulting in loss of confinement, 21% increase was achieved when increasing the thickness of FRP tube from 0.34 mm to 0.51mm.
2. Cross sectional area of concrete represented Hollowness ratio is an important factor affecting the axial strength of FRP composite pile, increasing the

hollowness ratio from 0.3 to 0.7 resulted in losing about 16 % of the axial strength.

3. Increasing the stiffness of steel tube by increasing its thickness and increasing the stiffness of concrete by increasing its compressive strength led to an increase in the axial strength of the FRP composite pile, 30% increase in the axial strength ratio was achieved by increasing the concrete compressive strength from 28 to 48 MPa.

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