



## ACTIVE FILTERS SYNTHESIS TO SIMULATED LADDER NETWORKS AND FEEDBACK CIRCUITS

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**Abstract:** An important feature in the design of passive and active filters networks is the method of the analysis employed. The purpose of this work is to synthesis two active low pass filters, the first is doubly terminated sixth order RLC passive ladder network and the second is multiple feedback network. The analyzing of the ladder network is achieved using graph-theoretic approach and the simulation is realized via MATLAB software tool version 8.1.0.604(R2010a) using Butterworth values for a corner frequency of  $\omega_c = 1$  MHz. A confirmation of results is achieved by nodal analysis and numerical analysis using Newton-Raphson method, respectively. The synthesized complete circuit requires 14 operational amplifiers and uses voltage subtractors and non-inverting integrators as building blocks with capacitors and inductors values evaluated from choosing  $R = 1$  k $\Omega$  as an arbitrary value. For the second network, an RC passive circuit is used, the feedback and feedforward transfer function are obtained using the transformed T-network. The elements values of an active network are computed by considering a corner frequency of  $\omega_c = 10$  kHz.

**Keywords:** Ladder filter, signal-flow graph, transfer function, active filter.

### تصميم دوائر المرشحات الفعالة لمحاكات الدوائر السلمية ودوائر التغذية العكسية

**الخلاصة:** ان الخاصية المهمة في تصميم دوائر المرشحات الخاملة والفعالة هي طريقة التحليل المتبعة. الغرض من البحث تصميم دائرتين لمرشح تمرير منخفض فعال الاولى هي الدائرة السلمية الخاملة التي تستخدم المقاومات والملفات والمتسعات ذات النهايتين ومن المرتبة السادسة، اما الدائرة الثانية فهي متعددة التغذية العكسية. تم تحليل الدائرة السلمية باستخدام أسلوب نظرية البيانات وطريقة المحاكات تم انجازها عبر اداة المحاكات البرمجية MATLAB النسخة 8.1.0.604(R2010a) باستخدام قيم Butterworth لتردد رنيني مقداره 1 ميكاهرتز. وقد تم التحقق من النتائج عن طريق التحليل العقدي والتحليل العددي باستخدام طريقة Newton-Raphson على التوالي. ان الدائرة المصممة الكلية تتطلب اربعة عشر مضخم عمليات وتستخدم طارحات الفولتية والمكاملات الغير مقلوبة كليات اساسية، وان قيم المكونات تم حسابها باعتماد قيمة مقاومة عشوائية مقدارها 1 كيلو أوم. واما الدائرة الثانية فهي تتكون من المقاومات والمتسعات وقد تم التحليل عن طريق حساب دالة الانتقال للتغذية العكسية والتغذية الامامية باستخدام تحويلات الشبكة T. وقد تم حساب قيم المكونات باعتبار التردد 10 كيلو هرتز.

## 1. Introduction

Numerous appropriate design methods involve analysis, curve approximation, and synthesis processes using proper components values can adequately describe the characteristics of filters' circuits [1]. Giving the filters specifications, the design can be performed for various active filters using approximation techniques [2].

The transfer function and frequency response are the most important aspects both in the design and applications of these filters in the field of communication, signal processing, and control systems [3-5].

A doubly terminated LC ladder structures was designed by symbolic maths [6] through the synthesis of the transfer function of seventh order bandpass filter. Active-RC filter synthesized from sixth order Chebyshev RLC low pass ladder filter is presented in [7], and the filter fabricated in CMOS process optimizes high frequency performance and minimizes current consumption.

In designing passive and active filters circuits, MATLAB software is flexible to achieve high performance filters using different methods [8-10]. The analog filter implementation using ladder filters has an advantage of low sensitivity in the passband [11]. The signal flow graph (SFG) is one of the formulation methods for characterizing the filter network, and the operational amplifiers building blocks can be used for the synthesis process [12].

### 2. Sixth Order Ladder Filter

A doubly terminated sixth order RLC low pass filter ladder network depicted in “Fig. 1” is chosen for analysis [13]. As shown the network contains two resistors, three inductors, and three capacitors with their Butterworth values for  $R_1 = R_2 = 1 \text{ k}\Omega$ , and corner frequency  $\omega_c = 1 \text{ MHz}$ .

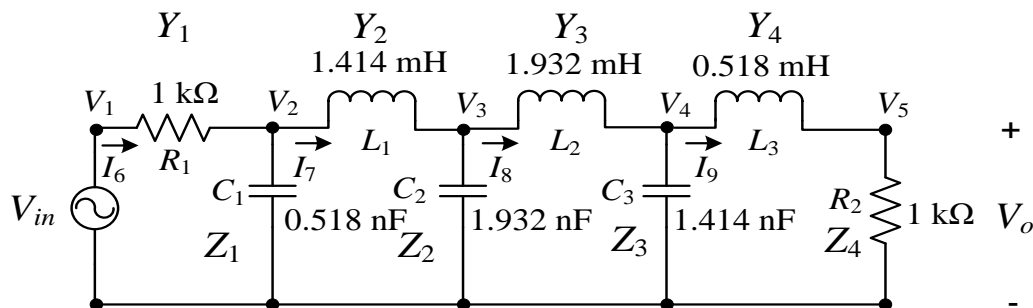


Figure 1. Doubly terminated ladder network.

The representation of the network using graph-theoretic approach is illustrated in the directed graph depicted in “Fig. 2” with a specific tree represented by solid lines.

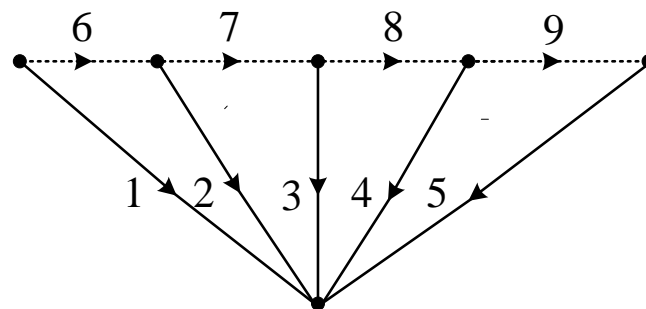


Figure 2. Network directed graph

The vertices are the voltage variables. The dotted lines which is the co-tree is the voltages of the branches while the tree is the currents branches. The voltages and currents branches can be listed in the following formulas;

Kirchhoff's voltage law:

$$V_6 = V_1 - V_2, V_7 = V_2 - V_3, V_8 = V_3 - V_4, V_9 = V_4 - V_5 \tag{1}$$

Kirchhoff's current law:

$$I_2 = I_6 - I_7, I_3 = I_7 - I_8, I_4 = I_8 - I_9, I_5 = I_9 \tag{2}$$

Expressing each branch in tree and co-tree in terms of impedances and admittances, respectively yields the following equations;

$$V_2 = (1/0.518 \times 10^{-9}s)I_2, V_3 = (1/1.932 \times 10^{-9}s)I_3, V_4 = (1/1.414 \times 10^{-9}s)I_4, V_5 = (1 \times 10^3)I_5 \tag{3}$$

$$I_6 = (1/1 \times 10^3)V_6, I_7 = (1/1.414 \times 10^{-3}s)V_7, I_8 = (1/1.932 \times 10^{-3}s)V_8, I_9 = (1/0.518 \times 10^{-3}s)V_9 \tag{4}$$

The impedances are

$$Z_1 = (1/0.518 \times 10^{-9}s), Z_2 = (1/1.932 \times 10^{-9}s), Z_3 = (1/1.414 \times 10^{-9}s), \text{ and } Z_4 = (1 \times 10^3)$$

and the admittances are

$$Y_1 = (1/1 \times 10^3), Y_2 = (1/1.414 \times 10^{-3}s), Y_3 = (1/1.932 \times 10^{-3}s), \text{ and } Y_4 = (1/0.518 \times 10^{-3}s)$$

### 3. Analysis

The signal flow graph (SFG) that corresponds to the directed graph is depicted in "Fig. 3".

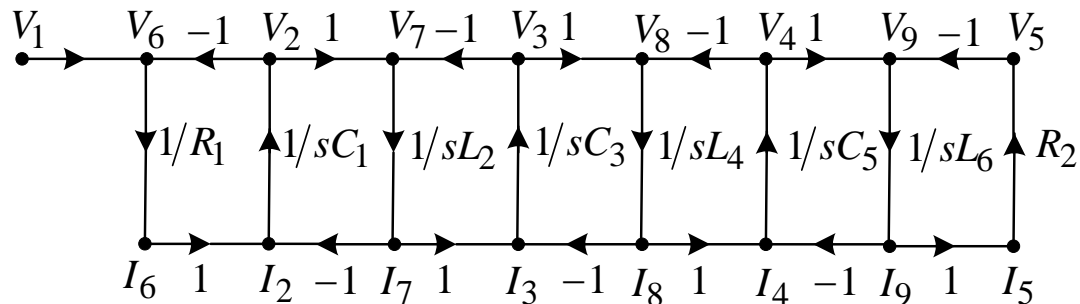


Figure 3. Signal flow graph.

The path value is

$$P = 4.995 \times 10^{35}/s^6$$

The first order loops' gain are given by

$$L_1 = -1/(5.18 \times 10^{-7}s), L_2 = -1/(7.324 \times 10^{-13}s^2),$$

$$L_3 = -1/(2.731 \times 10^{-12}s^2), L_4 = -1/(3.732 \times 10^{-12}s^2),$$

$$L_5 = -1/(2.731 \times 10^{-12}s^2), L_6 = -1/(7.324 \times 10^{-13}s^2),$$

$$L_7 = -1/(5.18 \times 10^{-7}s)$$

and the addition of these values yields

$$L_{s1} = -1/(2.59 \times 10^{-7}s) - 1/(2.68 \times 10^{-13}s^2)$$

The second order loops' gain are given by

$$L_1L_3 = 1/(1.414 \times 10^{-18}s^3), L_1L_4 = 1/(1.933 \times 10^{-18}s^3),$$

$$L_1L_5 = 1/(1.414 \times 10^{-18}s^3), L_1L_6 = 1/(3.793 \times 10^{-19}s^3),$$

$$L_1L_7 = 1/(2.683 \times 10^{-13}s^2), L_2L_4 = 1/(2.733 \times 10^{-24}s^4),$$

$$L_2L_5 = 1/(2 \times 10^{-24}s^4), L_2L_6 = 1/(5.364 \times 10^{-25}s^4),$$

$$L_2L_7 = 1/(3.793 \times 10^{-19}s^3), L_3L_5 = 1/(7.458 \times 10^{-24}s^4),$$

$$L_3L_6 = 1/(2 \times 10^{-24}s^4), L_3L_7 = 1/(1.414 \times 10^{-18}s^3)$$

$$L_4L_6 = 1/(2.733 \times 10^{-24}s^4), L_4L_7 = 1/(1.933 \times 10^{-18}s^3),$$

$$L_5L_7 = 1/(1.414 \times 10^{-18}s^3)$$

and the addition of these values yields

$$L_{s2} = 1/(2.683 \times 10^{-13}s^2) + 1/(1.094 \times 10^{-19}s^3) + 1/(2.68 \times 10^{-25}s^4)$$

The third order loops' gain are given by

$$L_1L_3L_5 = -1/(3.863 \times 10^{-30}s^5), L_1L_3L_6 = -1/(1.036 \times 10^{-30}s^5),$$

$$L_1L_3L_7 = -1/(7.327 \times 10^{-25}s^4), L_1L_4L_6 = -1/(1.415 \times 10^{-30}s^5)$$

$$L_1L_4L_7 = -1/(1.001 \times 10^{-24}s^4), L_1L_5L_7 = -1/(7.327 \times 10^{-25}s^4)$$

$$L_2L_4L_6 = -1/(2.001 \times 10^{-36}s^6), L_2L_4L_7 = -1/(1.415 \times 10^{-30}s^5),$$

$$L_2L_5L_7 = -1/(1.036 \times 10^{-30}s^5), L_3L_5L_7 = -1/(3.863 \times 10^{-30}s^5)$$

and the addition of these values yields

$$L_{s3} = -1/(1.34 \times 10^{-25}s^4) - 1/(2.589 \times 10^{-31}s^5) - 1/(2.001 \times 10^{-36}s^6)$$

The fourth order loops' gain is given by

$$L_1L_3L_5L_7 = 1/(2.001 \times 10^{-36}s^6)$$

The voltage transfer function is obtained by Mason's gain formula as,

$$VTF = \frac{4.995 \times 10^{35}}{s^6 + 3.861 \times 10^6 s^5 + 7.462 \times 10^{12} s^4 + 9.14 \times 10^{18} s^3 + 7.459 \times 10^{24} s^2 + 3.862 \times 10^{30} s + 1 \times 10^{36}} \quad (5)$$

Using MATLAB programming technique [14], the frequency response of the filter is depicted in "Fig. 4". It can be seen that the corner (resonance) frequency is  $\omega_C = 1$  MHz at  $-3$  dB.

### 3.1 Confirmation

#### 3.1.1 VTF formula

The same result given in "(5)" can be obtained using nodal approach to the filter circuit [15]. The VTFC is the confirmed voltage transfer function given by

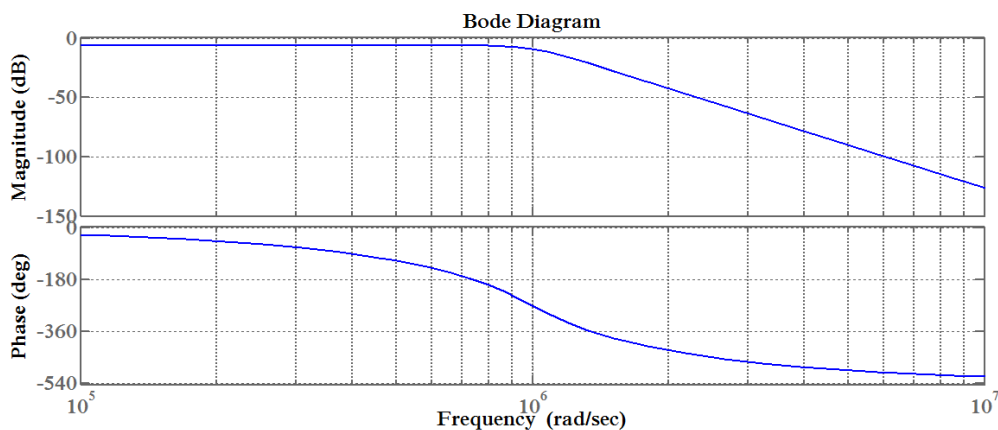


Figure 4. Frequency response of ladder filter.

$$VTFC = \frac{N(s)}{D(s)} \quad (6)$$

where

$$N(s) = R_2$$

and

$$\begin{aligned} D(s) = & R_1 C_1 C_2 C_3 L_1 L_2 L_3 s^6 + (R_1 C_1 C_2 C_3 L_1 L_2 + C_2 C_3 L_1 L_2 L_3) s^5 \\ & + (C_2 C_3 L_1 L_2 R_2 + R_1 C_1 C_2 L_1 L_3 + R_1 C_1 C_3 L_2 L_3 + R_1 C_2 C_3 L_2 L_3 \\ & + R_1 C_1 C_3 L_1 L_3 + R_1 C_1 C_2 L_1 L_2) s^4 \\ & + (R_1 R_2 C_1 C_2 L_1 + R_1 R_2 C_1 C_3 L_2 + R_1 R_2 C_2 C_3 L_2 + R_1 R_2 C_1 C_3 L_1 \\ & + C_2 L_1 L_3 + C_3 L_2 L_3 + C_3 L_1 L_3 + C_2 L_1 L_2) s^3 \\ & + (R_2 C_2 L_1 + R_2 C_3 L_2 + R_2 C_3 L_1 + R_1 C_1 L_3 + R_1 C_2 L_3 + R_1 C_3 L_3 \\ & + R_1 C_1 L_2 + R_1 C_2 L_2 + R_1 C_1 L_1) s^2 \\ & + (R_1 R_2 C_1 + R_1 R_2 C_2 + R_1 R_2 C_3 + L_3 + L_2 + L_1) s + (R_1 + R_2) \end{aligned}$$

Therefore

$$VTFC = \frac{1000}{s^6 + 3.861 \times 10^6 s^5 + 7.4572 \times 10^{12} s^4 + 9.1326 \times 10^{18} s^3 + 7.4557 \times 10^{24} s^2 + 3.8452 \times 10^{30} s + 9.9875 \times 10^{35}}$$

which is in closed agreement.

### 3.1.2 Corner Frequency

The value of the resonance frequency can be found by numerical analysis using Newton-Raphson method and MATLAB programming. The resonance frequency is calculated from

$$|VTF(j\omega)|^2 = \frac{1}{2} = \frac{1000}{(1 \times 10^{36} - 7.459 \times 10^{24} \omega^2 + 7.462 \times 10^{12} \omega^4 - \omega^6)^2 + (3.682 \times 10^{30} \omega - 9.14 \times 10^{18} \omega^3 + 3.861 \times 10^6 \omega^5)^2}$$

The evaluated value is 1.001827 MHz which gives a much closed result.

## 4. Realization of 6<sup>th</sup> Order Low Pass Leapfrog Ladder Filter

To synthesis the active filter, leapfrog realization, voltage subtractor and non-inverting integrator [16] are the two types of operational amplifier (op-amp) circuits used, both is depicted in “Fig. 5”. The implementation of branch weights and voltage relations in the SFG is accomplished according to the followings;

i- Each subtraction of voltages or currents is implemented using the circuit in “Fig. 5a” with  $R = 1 \text{ k}\Omega$  as an arbitrary choice. ii- Transmittance  $Y_1$  is replaced by two cascaded inverting op-amp stage, the second and third op-amp circuits in “Fig. 6a”, the input resistance is chosen arbitrarily to be  $1 \text{ M}\Omega$  and all other resistors are  $1 \text{ k}\Omega$ .

The voltage subtraction using voltage subtractor, item-i, at the input of the SFG together with implementation of  $Y_1$  is depicted in “Fig. 6a”, and further reduction yields “Fig. 6b”.

iii- Transmittance  $Z_1$  is represented using “Fig. 5b”. With arbitrary choice of  $R = 1\text{ k}\Omega$ , the capacitor value is calculated from  $RC = 0.518 \times 10^{-9}$ , this gives  $C = 0.518\text{ pF}$ . Transmittance  $Y_2$  is implemented similarly with the same resistor value but with  $C = 1.414\text{ }\mu\text{F}$ .

iv- The transmittances  $Z_2, Z_3, Y_3,$  and  $Y_4$  are represented using item-ii.

v- The transmittances  $Z_4$  is represented using “Fig. 6a” by interchanging resistors values of  $1\text{ k}\Omega$  with that of  $1\text{ M}\Omega$  and vice versa.

The complete simulated circuit for ladder filter with a total of fourteen op-amps is depicted in “Fig. 7”.

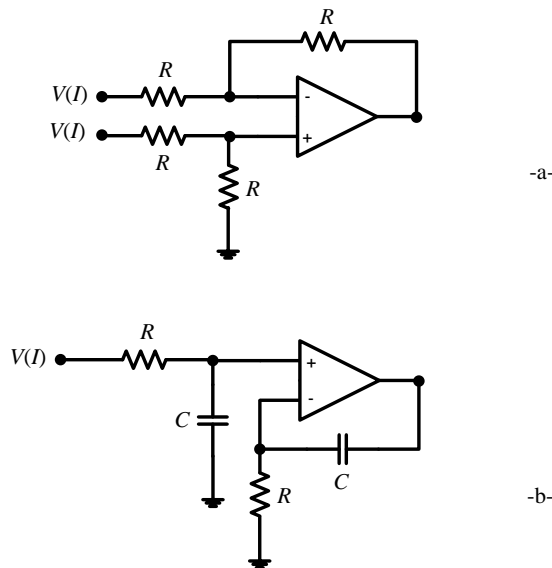


Figure 5. Op-amp circuits, a- voltage subtractor, and b- non-inverting integrator.

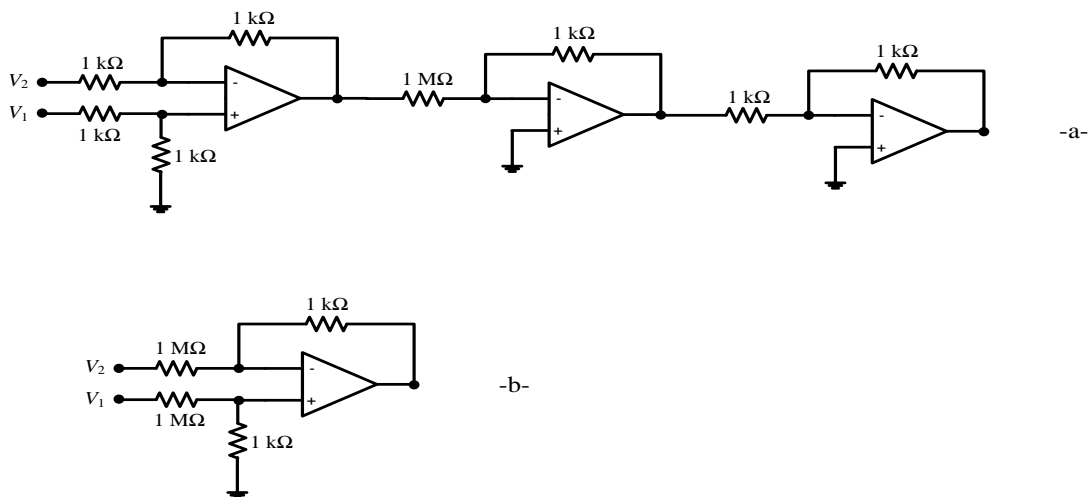


Figure 6. Representing of a- voltage subtraction and b- transmittance  $Y_1$ .

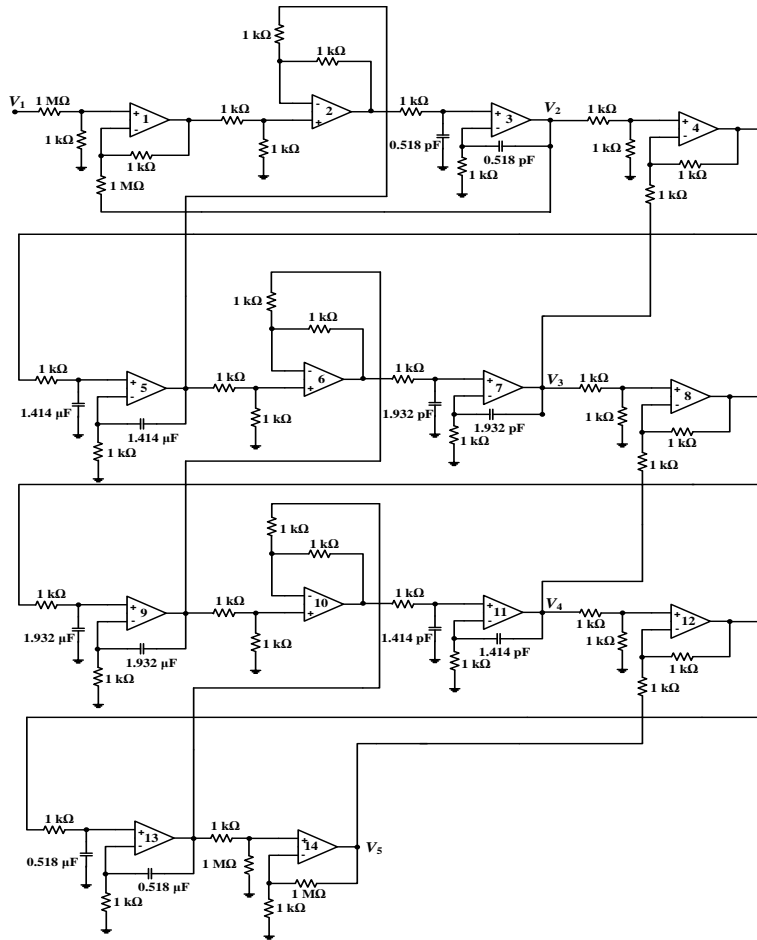


Figure 7. Circuit realization of sixth order low pass ladder filter.

### 5. Multiple feedback Filter

This is the second circuit to be synthesized, it is a second order multiple feedback active low pass filter. An RC circuit that considered in analysis is depicted in “Fig. 8”, the transfer function of this circuit can be found from feedback and feedforward transfer functions [17].

#### 5.1.1 Feedback transfer function

Setting  $V_2 = 0$  to the circuit of “Fig. 8”, the circuit appears as depicted in “Fig. 9”. The internal T-circuit is converted to the impedances  $R_{a1}$ ,  $R_{b1}$ , and  $R_{c1}$  which can be computed as follows

$$R_1 \parallel \frac{1}{sC_2} = \frac{R_1}{sC_2R_1 + 1}$$

$$r_1 = \frac{R_1R_3}{sC_2R_1 + 1} + \frac{R_1R_2}{sC_2R_1 + 1} + R_2R_3 \tag{7}$$

The impedances  $R_{a1}$  and  $R_{c1}$  are



$$R_{a1} = \frac{r_1}{R_1 \parallel \frac{1}{sC_2}} = \frac{sC_2R_1R_2R_3 + R_1R_2 + R_1R_3 + R_2R_3}{R_1} = \frac{a}{R_1} \tag{8}$$

$$R_{c1} = \frac{r_1}{R_3} = \frac{sC_2R_1R_2R_3 + R_1R_2 + R_1R_3 + R_2R_3}{sC_2R_1R_3 + R_3} = \frac{a}{sC_2R_1R_3 + R_3} \tag{9}$$

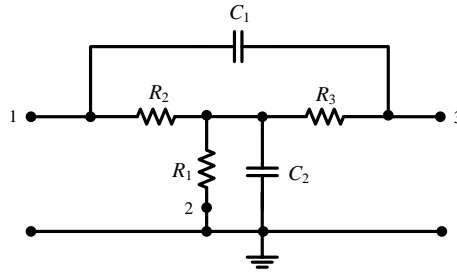


Figure 8. RC circuit.

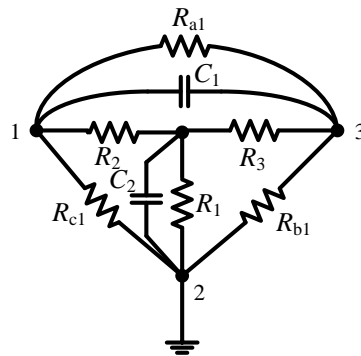


Figure 9. Circuit with  $V_2 = 0$ .

The parallel combination of  $R_{a1}$  with  $1/sC_1$  yields

$$R_{t1} = R_{a1} \parallel \frac{1}{sC_1} = \frac{a}{sC_1a + R_1} \tag{10}$$

The feedback transfer function  $V_1/V_3$  is

$$\frac{V_1}{V_3} = \frac{R_{c1}}{R_{t1} + R_{c1}} \tag{11}$$

Substituting “9” and “10” into “11” gives

$$\frac{V_1}{V_3} = \frac{s^2C_1C_2R_1R_2R_3 + sC_1(R_1R_2 + R_1R_3 + R_2R_3) + R_1}{s^2C_1C_2R_1R_2R_3 + s[C_2R_1R_3 + C_1(R_1R_2 + R_1R_3 + R_2R_3)] + R_1 + R_3} \tag{12}$$

### 5.1.2 Feedforward transfer function

Setting  $V_3 = 0$  to the circuit of “Fig. 8”, the circuit of “Fig. 10” is obtained. The internal T-circuit is converted to  $\pi$ -circuit with impedances  $R_{a2}$ ,  $R_{b2}$ , and  $R_{c2}$ . Let

$$r_2 = \frac{R_1 R_3}{sC_2 R_3 + 1} + \frac{R_2 R_3}{sC_2 R_3 + 1} + R_1 R_3 \quad (13)$$

The impedances  $R_{a2}$  and  $R_{c2}$  are

$$R_{a2} = \frac{r_2}{R_3 \parallel \frac{1}{sC_2}} = \frac{sC_2 R_1 R_2 R_3 + R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3} = \frac{a}{R_3} \quad (14)$$

$$R_{c2} = \frac{r_2}{R_1} = \frac{a}{sC_2 R_1 R_3 + R_1} \quad (15)$$

and

$$R_{t2} = R_{c2} \parallel \frac{1}{sC_1} = \frac{a}{sC_1 a + sC_2 R_1 R_3 + R_1} \quad (16)$$

The feedforward transfer function  $V_1/V_2$  is

$$\frac{V_1}{V_2} = \frac{R_{t2}}{R_{a2} + R_{t2}} \quad (17)$$

Combining “15”, “16”, and “17” yields

$$\frac{V_1}{V_2} = \frac{R_3}{s^2 C_1 C_2 R_1 R_2 R_3 + s[C_2 R_1 R_3 + C_1(R_1 R_2 + R_1 R_3 + R_2 R_3)] + R_1 + R_3} \quad (18)$$

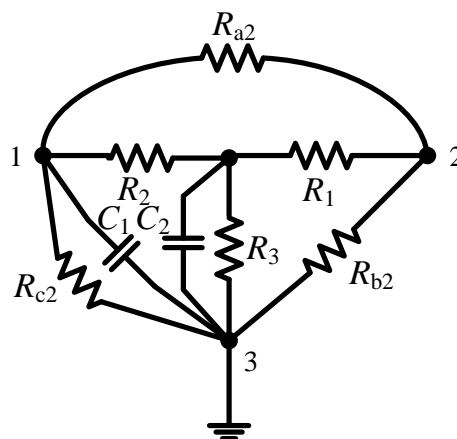


Figure 10. Circuit with  $V_3 = 0$ .

From “12” and “18”, the transfer function of the RC circuit is

$$TF = -\frac{V_1/V_2}{V_1/V_3} = -\frac{1/C_1 C_2 R_1 R_2}{s^2 + s\left(\frac{1}{C_2 R_1} + \frac{1}{C_2 R_2} + \frac{1}{C_2 R_3}\right) + \frac{1}{C_1 C_2 R_2 R_3}} \quad (19)$$

which is a low pass filter. The multiple feedback active low pass filter that uses the RC circuit is depicted in “Fig. 11”.

### 5.2. Filter design

At  $\omega_c = 0$ , the gain in “19” is  $TF(0) = -R_3/R_1$ . Using Butterworth polynomials for second order filter [18], if  $R_1 = R_2 = 1 \Omega$  then  $R_3 = 10 \Omega$ , the capacitors values can be found from “19” as

$$1.414 = \frac{1}{C_2} \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)$$

$$C_2 = 1.485 \text{ F}$$

and since

$$1 = \frac{1}{C_1 C_2 R_2 R_3}$$

then

$$C_1 = 0.067 \text{ F}$$

The elements values for impedance scaling to  $10^3$  and frequency scaling to 10 kHz are

$$R_1 = R_2 = 1 \text{ k}\Omega, R_3 = 10 \text{ k}\Omega, C_1 = 6.7 \text{ nF}, C_2 = 0.148 \text{ nF}$$

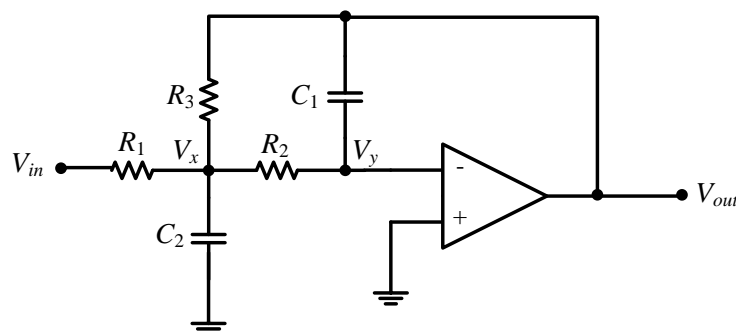


Figure 11. Multiple feedback active low pass filter.

## 6. Discussions and Conclusions

The synthesis of first circuit, the doubly terminated RLC low pass ladder filter, is performed using SFG. The implementation of the filter is achieved by selecting

resistors values, and the capacitor values are found from network function of the op-amp. An appropriate change in frequency and magnitude can made the components value of practical one. The computed value of corner frequency obtained from numerical analysis differs slightly, approximately (0.001 kHz), due to high power coefficients. The ladder circuit can also be designed using inverting op-amps (summers and integrators).

Active simulation to second multiple feedback active low pass filter is also accomplished. The filter is synthesized by computing the elements values at a certain specifications. The design allows the using of practical components values and Chebyshev polynomial.

## 7. References

1. Deliyannis, T., Sun, Y., and Fidler, J.K. (1999). "Continuous-Time Active Filter Design". Electronic Engineering Systems Series, CRC Press LLC.
2. Aremu, O.A., Lyiade, O.J., and Ajadi, D.A. (2013). "Design and Construction of Low-and High-Pass Active Filters Using Butterworth and Chebychev Techniques". The Pacific Journal of Science and Technology. Vol.14, No. 1, pp. 139-149.
3. Baschiroto, A. (2005). "Active-RC Filters". Analog Filters for Telecommunications. Universita' degli Studi di Bologna.
4. Paarmann, L.D. (2003). "Design and Analysis of Analog Filters: A Signal Processing Perspective". Kluwer Academic Publishers.
5. Michel Ruel, P.E. (2003). "Using Filtering to Improve Performance". The Instrumentation, Systems and Automation Society.
6. Alexander, J.C. and Esther, R.V. (2011). "A Review and Modern Approach to LC Ladder Synthesis". J. Low Power Electron. Appl. pp. 20-44.
7. Baoyue, W., Hongkun, L., Yunfeng, W., and Haiying, Z. (2012). "A 6th order wideband active-RC LPF for LTE application". Journal of Semiconductors, Vol.33, No.6.
8. Kelebekler, E. and Yildiz, A.B. (2007). "Analysis of Passive and Active Filters Using Modified Nodal Approach". 1-4244-1055-X/07/ IEEE, 5<sup>th</sup> International Conference-Workshop, Compatibility in Power Electronics.
9. Javed, M., Aftab, H., Qasim, M., Sattar, M. (2008). "RLC Circuit Response and Analysis (Using State Space Method)". IJCSNS International Journal of Computer Science and Network Security, Vol.8, No. 4, pp. 48-54.
10. Wilamowski, B.M. and Gottiparthi, R. (2005). "Active and Passive Filter Synthesis using MATLAB". Int. J. Engng Ed. Vol. 21, No. 4, pp. 561-571.
11. Dimopoulos, H.G. (2012). "Analog Electronics Filters: Theory, Design and Synthesis". ACSP. Analog Circuits and Signal Processing. Springer.
12. Chen, W.K. (2009). "Fundamentals of Circuit and Filters". The Circuits and Filters Handbook. Third Edition. Taylor & Francis Group, LLC.
13. Bowick, C. (1982). "RF Circuit Design". Butterworth-Heinemann print.
14. Otto, S.R. and Denier, J.P. (2005). "An Introduction to Programming and Numerical Methods in MATLAB". Springer-Verlag London Limited.

15. Brown, T.A. (2013). "Designing LC Ladder Filters with Lossy Elements". Master of Science thesis- Auburn University.
16. Scott, H. (2003). "An Analog Electronics Companion: Basic Circuit Design for Engineers and Scientists". Cambridge University Press.
17. Gobind, D. (1976). "Principles of Active Network Synthesis and Design". John Wiley & Sons.
18. Pactitis, S.A. (2007). "Active Filters: Theory and Design". CRC Press. Taylor & Francis Group.