APPROXIMATE SOLUTIONS FOR SOLVING TWO TYPES LINEAR INTEGRAL EQUATIONS BY USING BOU-BAKER POLYNOMIALS METHOD

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Abstract: In this paper, Bou-baker Polynomials Method [1] are used to find an approximate solution for linear second kind Volterra and Fredholm integral equations. These polynomials incredibly useful mathematical tools, because they are simply defined, can be calculated quickly on computer systems and represent a tremendous variety of functions. They can be differentiated and integrated easily.

Keywords: *Bou-Baker, Method.*

حل تقريبي لحل نوعين من المعادلات التكاملية الخطية باستخدام طريقة متعدد حدود بو- بيكر

الخلاصة: في هذا البحث استخدمت طريقة متعدد حدود بو- بيكر لإيجاد حل تقريبي لمعادلة فولتيرا وفريدهولم الخطية من النوع الثاني . أن متعدد الحدود تستعمل بشكل كبير في الطرق الرياضية وذلك لبساطة تعريفها ولسهولة وسرعة الحسابات فيها وتنوع دوالها . وهي قابلة للتفاضل والتكامل بسهولة.

1. Introduction

New methods are always needed to solve integral equations because no single method works well for all such equations. There has been considerable interest in solving differential and integral equations using techniques which involve Bou-baker Polynomials Method. The integral equation is an equation in which the unknown function y(x) appears under the integral sing. A linear integral equation is an integral equation which involves a linear expression of the unknown function .The general form of integral equation is given by[1][2]

$$h(x)y(x) - \int_{\Omega} k(x,t)y(t)dt = f(x) \qquad \dots \ (1)$$

determined, and when Ω is a finite interval [a,x] $\subseteq$ R

If the upper limit of the integral in equation (1) is variable then equation (1) is called Volterra integral equation. If the upper limite of the integral in equation (1) is

constant then equation (1) is called Fredholm integral equation. Now we can distinguish between two types of Volterra and Fredholm integral equations which are:

1. Volterra and Fredholm integral equation of the first kind when h(x)=0 in equation(1).

$$y(x) = f(x) + \int_a^x k(x,t)\,y(t)\,dt \qquad \text{… (2)}$$

$$y(x) = f(x) + \int_a^b k(x,t)\,y(t)\,dt \qquad \text{… (3)}$$

2. Volterra and Fredholm integral equations of the second kind when $h(x) \neq 0$ in equation(1).

$$y(x) = f(x) + \int_a^x k(x,t)\,y(t)\,dt \qquad \text{… (4)}$$

$$y(x) = f(x) + \int_a^b k(x,t)\,y(t)\,dt \qquad \text{… (5)}$$

In this paper, we can introduce approximation method to solve the following linear Volterra and Fredholm integral equation of the second kind by using   Bou-baker Polynomials method.

2. Bou-baker Polynomials Method

The Bou-baker polynomials of degree n are defined by [3].

$$B_n(t) = \sum_{p=0}^{\xi(n)} \left[\frac{(n-4p)}{(n-p)} c_{n-p}^p \right] (-1)^p \; x^{n-2p} \qquad \text{…(6)}$$

where

$$\xi(n) = \left\lfloor \frac{n}{2} \right\rfloor = \frac{2n + \left((-1)^n - 1\right)}{4}$$

Where $\xi(n) = \left\lfloor \dfrac{n}{2} \right\rfloor$ is denotes the floor function.

Here the standard Bou-baker polynomials are defined by

$$B_0(x) = 1$$
$$B_1(x) = x$$
$$B_2(x) = x^2 + 2$$
$$B_3(x) = x^3 + x$$
$$\vdots$$
$$B_m(x) = xB_{m-1}(x) - B_{m-2}(x) \qquad \text{for} \quad m > 2$$

3. Solution of Volterra and Fredholm Integral Equation with Bou-baker Polynomials Method.

In this section we consider Bou-baker polynomial approximation solution of the form

$$y(x) = \sum_{n=0}^{N} c_n B_n(x) \qquad -\infty < x \le b \le \infty \tag{7}$$

Where $B_n(x)$ $n = 0,1,2,\ldots$ denoted the Boubaker polynomials ,to find the approximate solutions for Voltarra and Fredholm integral equation, we will be introduced. Let us reconsider the Volterra integral equation of the second kind.

$$y(x) = f(x) + \int_a^x k(x,t) y(t) dt \qquad x \in [a, x] \tag{8}$$

And Let

$$y(x) = f(x) + \int_a^b k(x,t) y(t) dt \qquad x \in [a, b] \tag{9}$$

By using equation (7) applying the Bou-baker polynomials method for equation (8), we get the following formula.

$$\sum_{n=0}^{N} c_n B_n(x) = f(x) + \int_a^x k(x,t) \sum_{n=0}^{N} c_n B_n(t) \, dt \tag{10}$$

$$\left[B_O^n(t) B_1^n(t) B_2^n(t) \cdots B_n^n(t) \right] \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_n \end{bmatrix} = f(x) + \int_a^x k(x,t) \left[B_0^n(t) B_1^n(t) \cdots B_n^n(t) \right] \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_n \end{bmatrix} dt \tag{11}$$

Now to find all integration in equation (11) then in order to determine $c_0, c_1, \ldots, c_n$,we need n equations, Now choice $x_i, i = 1,2,3,\ldots,n$ in the interval [a,b],which gives

(n) equations. Solve the n equations by Gauss elimination to find the values $c_0, c_1, ..., c_n$.

In the smaller way we can solve Fredholm integral equation.

4. Numerical Examples and Results

In this section we consider the following examples on linear Fredholm and volterra integral equation, will be introduced.

Example 1:

Consider the following Fredholm linear equation of the second kind

$$u(x) = e^{-x} - \int_0^1 xe^t u(t)dt$$

with the exact solution $y(x) = e^{-x} - \dfrac{x}{2}$

Approximated solution for some values of x by using Bou-baker Polynomials Method and exact values of example1, and graph (1) are presented in Table (1) and fig(1).

Table(1) the results of example1

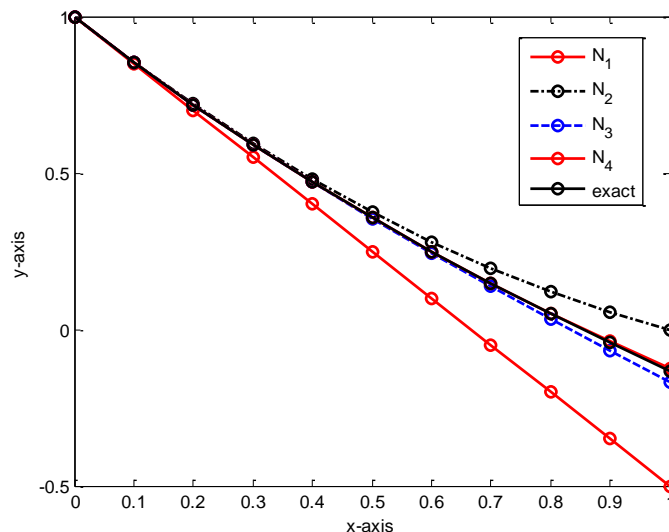| x	(Exact) $y(x)=e^{-x}-\dfrac{x}{2}$	Approximation $u(x)$ degree (n=1)	Approximation $u(x)$ degree (n=2)	Approximation $u(x)$ degree (n=3)	Approximation $u(x)$ degree (n=4)	L.S.E
0	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000
0.1	0.8548	0.8500	0.8550	0.8548	0.8548	0.0000
0.2	0.7187	0.7000	0.7 200	0.7187	0.7187	0.0000
0.3	0.5908	0.5500	0.5950	0.5905	0.5908	0.0000
0.4	0.4703	0.4000	0.4800	0.4693	0.4704	0.0001
0.5	0.3565	0.2500	0.3750	0.3542	0.3568	0.0003
0.6	0.2488	0.1000	0.2800	0.2440	0.2494	0.0006
0.7	0.1466	-0.0500	0.1950	0.1378	0.1478	0.0012
0.8	0.0493	-0.2000	0.1200	0.0347	0.0517	0.0024
0.9	-0.0434	-0.3500	0.0550	-0.0665	-0.0392	0.0042
1	-0.1321	-0.5000	0.000	-0.1667	-0.1250	0.0071

Fig. (1). Comparison of the solutions of example (1)

Example 2:

Consider the following Volterra integral equation of the second kind:

$$y(x) = x + \int_0^x (t - x) y(t) dt$$

Which has exact solution $u(x) = \sin(x)$

When Bou-baker Polynomials Method is applied, Table(2) and fig(2) present the comparison between the approximate solution using Boubaker Polynomials Method and exact values.

Table(2) the results of example2

| x	(Exact) $u(x) = \sin(x)$	Approximation $u(x)$ degree (n=1)	Approximation $u(x)$ degree (n=2)	Approximation $u(x)$ degree (n=3)	L.S.E
0	0.0000	0.0000	0.0000	0.0000	0.0000
0.1	0.0998	0.1000	0.1000	0.0998	0.0000
0.2	0.1987	0.2000	0.2000	0.1987	0.0000
0.3	0.2955	0.3000	0.3000	0.2955	0.0000
0.4	0.3894	0.4000	0.4000	0.3893	0.0001
0.5	0.4794	0.5000	0.5000	0.4792	0.0002
0.6	0.5646	0.6000	0.6000	0.5640	0.0006
0.7	0.6442	0.7000	0.7000	0.6428	0.0014
0.8	0.7174	0.8000	0.8000	0.7147	0.0027
0.9	0.7833	0.9000	0.9000	0.7785	0.0052
1	0.8415	1.0000	1.0000	0.8333	0.0082
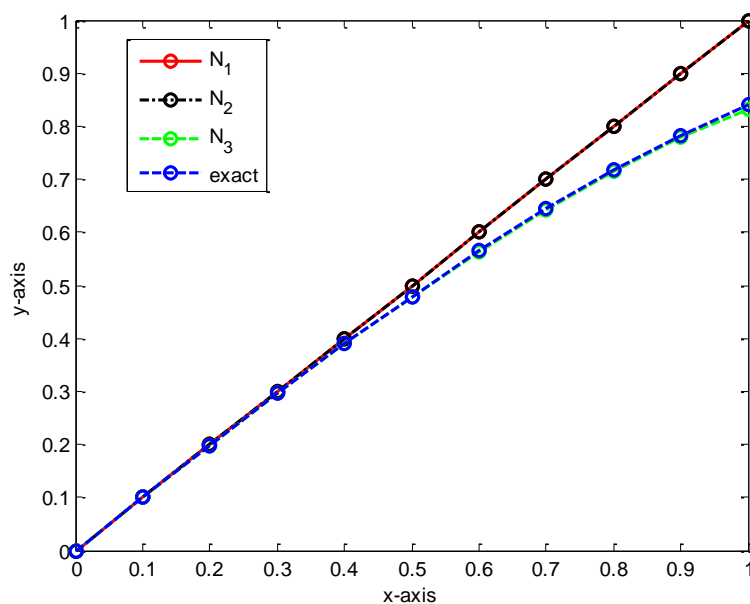
Fig. (2). Comparison of the solutions of example (2)

5. Conclusion

This paper presents the use of the Bou-baker Polynomials Method, for solving linear Voltarra and Fredholm integral equations of the Second kind.From solving some numerical examples the following points have been identified:

1. It is clear that using the Bou-baker Polynomials Method basis function to approximate when the n degree of Bou-baker Polynomials Method is increases the error is decreases.

2. We can see also from Table (1) and fig(1), Table (2) fig(2) that the approximation is good.

6. References

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