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MATHEMATICAL ANALYSIS OF SQUARE HOLE DRILLING MECHANISM

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Abstract: Square hole is produced by Press working, Broaching, Non-conventional machining process like Wire EDM, CNC machines, Laser cutting, etc. These methods which mentioned have a complexity of manufacture along with high cost. Creating a square hole using a drill, in addition to having an easier manufacturing process, reduce the cost and time. In this paper, mathematical model of triple cam rotation of constant width inside a square frame is discussed. So, by designing a drilling tool on cam it would be creating a square hole on the work piece. During the rotation of the cam, the center of rotation is not fixed and this is the major problem in this mechanism. There is need to a non-coaxial coupling to transmit the rotational motion. The result of Universal non-coaxial coupling has been used were shown that the rotational velocity of input and output shafts are equals. This coupling can be used for transmitting the rotational motion from the drill to the cam and drilling tool to get a square hole shape..

Keywords: *Square hole, Universal coupling, Reuleaux polygons, CNC machining method, broaching machining method*

التحليل الرياضي لآلية التثقيب المربع

الخلاصة: يمكن انتاج الثقب المربع بواسطة اعمال المكابس ,التخليق ,عمليات التشغيل الغير تقليدية مثل ماكنة القطع بسلك التفريغ الكهربائي ,المكائن المسيطرعليها بالحاسوب،القطع بالليزر, الخ. هذه الطرق التي ذكرت معقدة في التصنيع مع التكلفة العالية. ان تكوين ثقب مربع باستخدام المثقب، بالإضافة إلى كونها عملية تصنيع سهلة، هي ايضا اقل كلفة ووقت. في هذا البحث، وتمت مناقشة النموذج رياضي لدوران حدبة ثلاثية ذات عرض ثابت داخل إطار مربع. لذلك، من خلال تصميم أداة التثقيب على الكامة سيتم تكوين ثقب مربع على قطعة العمل. أثناء دوران الحدبة يكون مركز الدوران غير ثابت وهذه هي المشكلة الرئيسية في هذه الآلية. لذلك هناك حاجة إلى اقتران غير محوري لنقل الحركة الدورانية .نتيجة لاستخدام المقترن العام غير متحد المركزتبين أن سرعة دوران المدخلات والمخرجات متساوية. ولذلك فإن هذا الاقتران يمكن أن تستخدم لنقل الحركة الدورانية من المثقب الى الحدبة واداة التثقيب للحصول على ثقب مربع الشكل.

1. Introduction

Square holes are used excessively in the industry. Some of the couplings are used as examples of these applications. Since the square form of the shaft and hole, will produce full participation of couplings, accordingly prohibit freewheeling. There are assortments of processes for making square holes. One of these processes is creating a square hole by Computer Numerical Control (CNC) machine.

Alan Overy [1] used CNC machine, in spite of the high accuracy, cost too much. In this process, designed the hole plan on workpiece by CAD software, and then transferred to CNC machine, after becoming to G-Code.

Ben Fleming [2] used an Electrical Discharge Machine (EDM).In EDM process, an electrical conducting material is used to produce spark electrical discharges between an electrode to erode, in this machine hardness and strength are not considered [2].

K.Y. Song et al.[3], constructed the electrode to the shape of required hole, an electrical discharge is produced from electrode of the tool to a workpiece, the shape of electrode caused spark on workpiece, no physical contact that noticed between the workpiece and the tool electrode is produced in the EDM process.

U.Kokturk and E. Budak [4], make square holes on workpiece by using broaching machining process. Machining of external or internal complex shapes, broaching is usually used when difficult to produce by different processes like turning and milling. Primarily broaching was sophisticated for non-circular internal shapes, such as square hole and keyways.

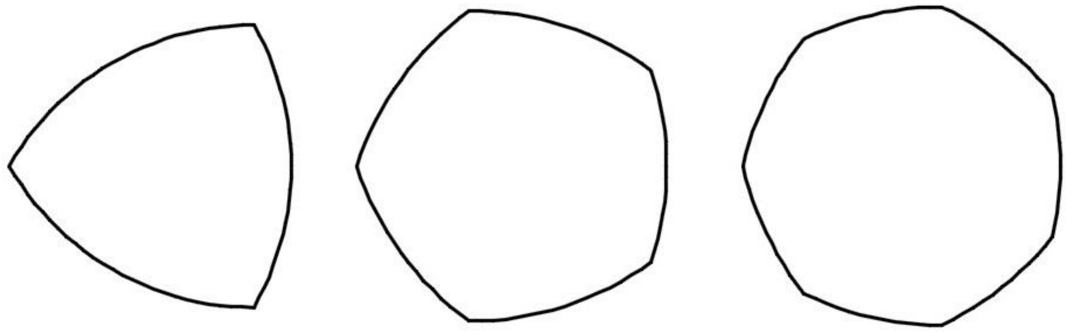
Oldham non-coaxial coupling has been studied and results were shown that the rotational velocity of input and output shafts are equals. So this coupling can be applied for transmitting the rotational motion from drill to the cam and bit.

2. Square Hole Drilling Technique

Square hole drilling technique, including that can have rotational motion for triangle-like cam inside square frame. To discuss the theoretical model for the cam, curves of constant width are proposed and explained. An example involved with vector space let us to gain arrival to large amount of information is provided by curves of constant width.Particularly, a special variety of curves of constant width in Reuleaux polygons give unforeseen results to practical questions; like a square hole drilling [5]. In the standard Euclidean plane, Euler is one of the earlier mathematicians who studies non-circular curves of constant width; e.g., the Reuleaux triangle was given by Reuleaux to Horn blower, the creator of the compound steam [6], theoretically Reuleaux triangle properties have very serious applications. Since a freely rotated Reuleaux triangle can generate a square shape, drilling holes of maximum square area that contact all four sides using curve of constant width can be applied, by connecting and creating non-uniform circular arcs to each Reuleaux polygons [7].

Some Reuleaux curves are illustrated in "Fig. 1". Width is a vertical distance across two parallel tangent lines and the non-circular curve, which has a constant and a plane shape in all directions.

The cam which rotates within the square frame is stationary to the body of the drill. The whole interior surface of square framework is covered by cam rotation within the square framework as cam is 360° rotated. Therefore, a whole square shape could be covered by a suitable bit designed on the cam and milling process applied. Universal or Hook and Oldham coupling "Fig. 2 and 3" are noted, off transmit torque between two off Richard Schmidt coupling "Fig. 4" [8].



Figutre 1.Curves of constant width

Neil Sclater and Nicholas [9] used Schmidt set that the center disk is independent to suppose its own center of rotation. Practically, all three disks rotate with same velocity.

Yutaka Nishiyama [10] noticed Oldham coupling can be confirmed all three disks rotate with same rotational velocity, thus it will be equal of driver shaft and driven shaft.

A first part in the current paper, the drilling of square holes and a Reuleaux triangle are mentioned earlier. The universal coupling applied for transmitting torque from drill to cam has been provided in the second part.

Figutre 2. Universal coupling

Figure 3. Oldham coupling

Figure 4. Richard Schmidt coupling

3. Reuleaux Triangle Acting as Drilling of Square Holes

The German engineer, In 19th century, Franz Reuleaux created a new form of rotating triangle then is known as a Reuleaux triangle. It is structured from the intersection of three disks of a circular shape, each having its center at the boundary of the other two. The outcome is the Reuleaux triangle illustrated in gray shape in"Fig. 5".

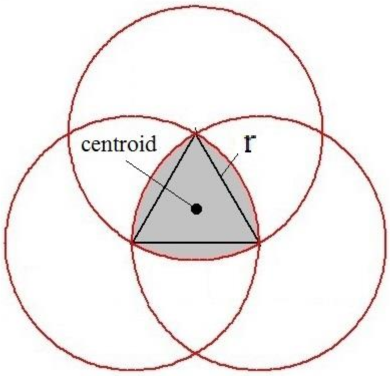


Figure 5. Construction of a Reuleaux Triangle

The centroid of rotating Reuleaux triangle will be driven by universal coupling constantly through a closed curve trajectory, then the Reuleaux triangle will sweep out a square with a small rounded corners. "Fig. 6" illustrates sample of driven and rotating Reulerax triangle inside square frame.

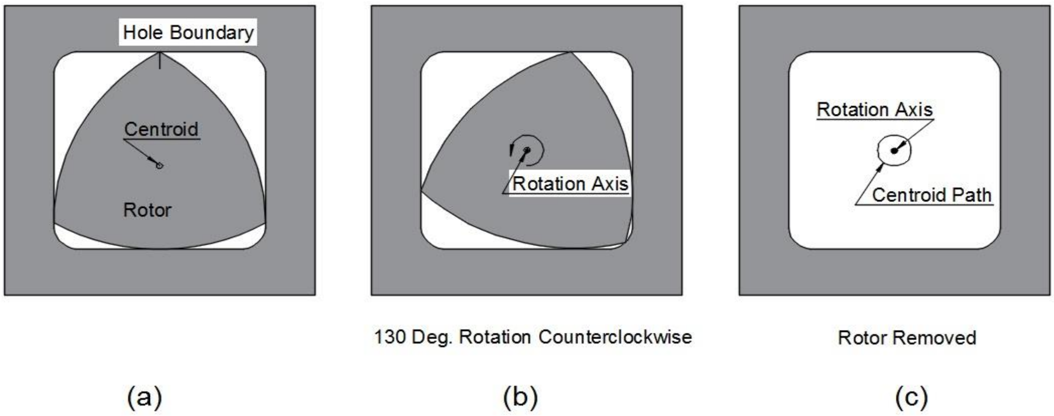


Figure 6. Sample of driven Reuleraux triangle for drilling nearly square holes

"Fig. 6" shows in (a) and (b) are two different rotor cases and (c) represents an approximate trajectory of centroid without the rotor to distinguish the path.

4. Universal Couplings for Torque Transmission

In square hole drilling mechanism, a non-coaxial coupling might be applied for torque transmission, due to center of rotation of cam is inconstant and is showed in a non-circular path. The universal coupling is vastly applied as non-coaxial coupling, which has the capable to transmit torque between parallel or non-parallel axes and is collected of two universal joints. A universal joint, which is designed in the 16th century as shown in "Fig. 7" was applied on the clock mechanism[11].

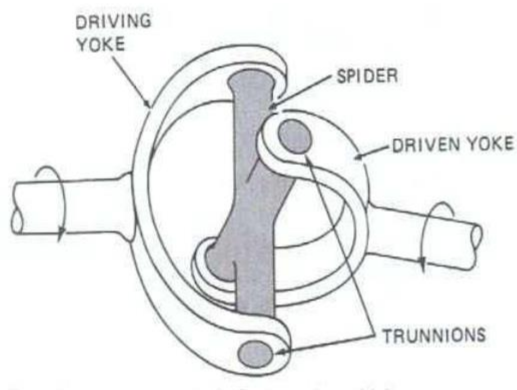


Figure 7. Universal joint in the 16th century

"Fig. 8" shows, a rotational angle of the driver axis ($\theta 1$) and the rotational angle of driven axis ($\theta 2$), the relation between the two angles is obtained in "(1)":

$$\tan\theta 2 = \frac{\tan\theta 1}{\cos\beta} \quad (1)$$

From "(1)", the ratio between the driver velocity and driven velocity is obtained in "(2)". In parameter $\theta 1$, a diagram of velocity ratio has also been plotted in "Fig. 9".

$$\frac{\omega 2}{\omega 1} = \frac{\cos\beta}{1-(\sin\beta.\sin\theta 1)^2} \quad (2)$$

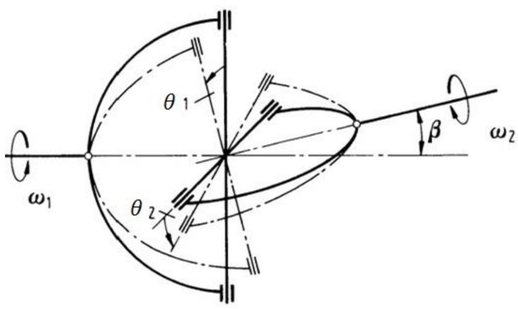


Figure 8. Universal joint angles in schematic shape.

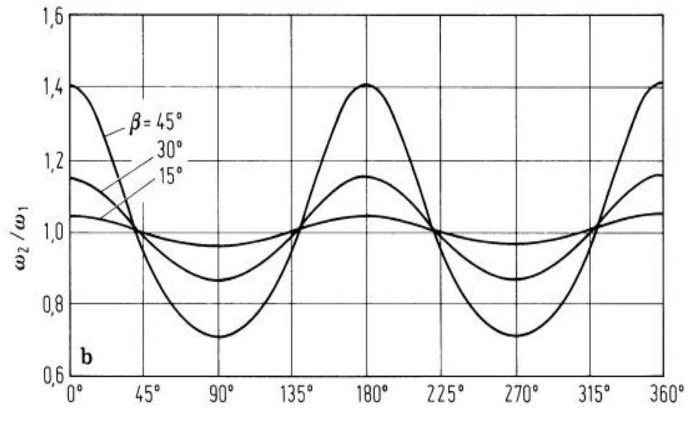


Figure 9. The ω2 ⁄ω1 ratio diagram in $\theta 1$

The driven axis acceleration is obtained from "(3)":

$$\alpha_2 = \omega_1^2 \frac{\cos\beta.\sin^2\beta.\sin 2\theta_1}{(1-\sin^2\beta.\sin^2\theta_1)^2} \qquad (3)$$

In the case illustrated in "Fig. 10", if two universal joints are applied in double mode, the universal coupling is able to transmit torque between off-axis shafts.

A relation between rotational angle of driver axis and rotational angle of driven axis in universal coupling may be given from "(4)":

$$\tan\theta 3 = \frac{\cos\beta 1.\tan\theta 1}{\cos\beta 2} \qquad (4)$$

Therefor driver and driven shafts rotational velocity are equal when driver and driven axes are off-axis [12].

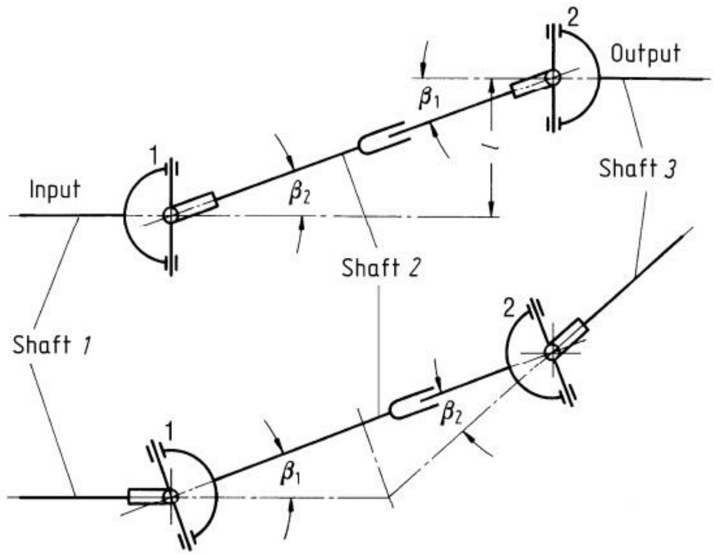


Figure 10. The universal coupling made of consecutive connection of two universal joints.

5. Mathematical Model for the Cam

According to the simplest curves of constant width, the circle, equations have been obtained, as shown in "Fig. 11", width of curve is calculated by the "(5)":

$$h(\phi) + h(\phi+\pi) = k \qquad (5)$$

Where (h) is perpendicular line tangent to trajectory curve, (k) is a constant and (ϕ) is the angle between horizontal axis and (h) line. Family of tangents be given from "(6)":

$$F(x, y, \phi) = x.\cos\phi + y.\sin\phi - h(\phi) \qquad (6)$$

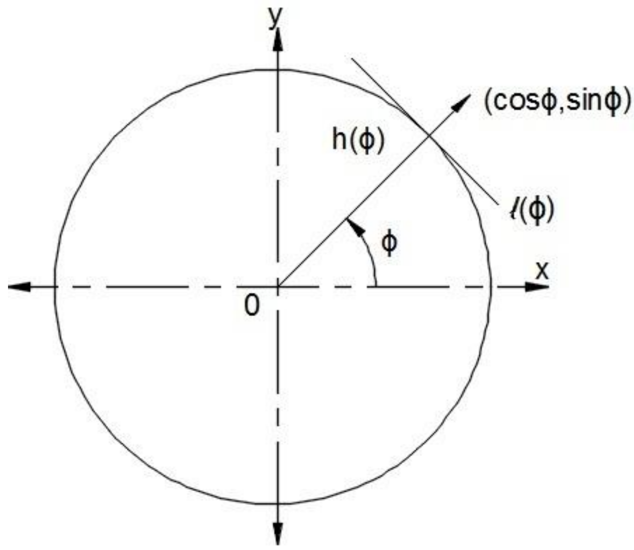


Figure 11. The simplest curve of constant width

By using "(7)", to find envelope of this family [13]:

$$F = \frac{\partial F}{\partial \phi} = 0 \qquad (7)$$

x and y are expressed in terms of rotational angle ϕ, using "(6)" and "(7)":

$$\left.\begin{aligned} x(\phi) &= h(\phi).\cos\phi - h'(\phi).\sin\phi \\ y(\phi) &= h(\phi).\sin\phi + h'(\phi).\cos\phi \end{aligned}\right\} \qquad (8)$$

To different forms by choosing h(ϕ), equations are obtained of the assortment curves of constant width. Where h(ϕ), should satisfy "(5)". For example, by choosing h (ϕ) =1, equation is obtained for the simplest curves of constant width, the circle. By choosing $h(\phi) = a.\cos^2\left(\frac{3\phi}{2}\right) + b$, the curve width would be equal to a+2b [13],[14]. "Equation (8) can be written":

$$\left.\begin{aligned} x(\phi) &= \left(\frac{a}{2}\cos 3\phi + \frac{a}{2} + b\right).\cos\phi + \frac{3}{2}a.\sin 3\phi.\sin\phi \\ y(\phi) &= \left(\frac{a}{2}\cos 3\phi + \frac{a}{2} + b\right).\sin\phi - \frac{3}{2}a.\sin 3\phi.\cos\phi \end{aligned}\right\} \qquad (9)$$

The diagram of "(9)" is plotted in "Fig. 12", when a =1,2,3 and b =3.5a, using Matlab software, which has illustrated the profile of cam curves.

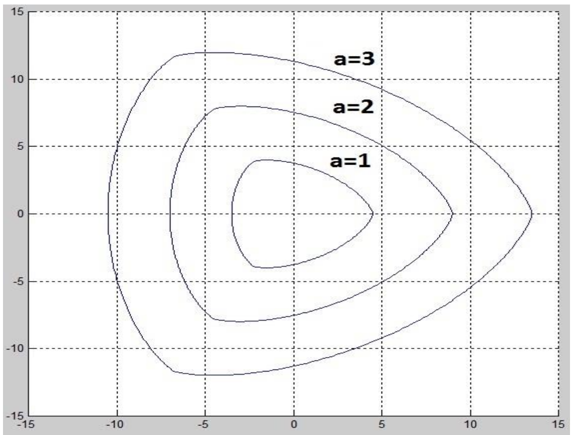


Figure 12. Diagram of cam curve

6. Calculations

In this paper, the side length of the square is 30 mm that will make calculations of a suitable cutting tool. For our calculating the following factors are determined:

6.1. Determination of Cam Dimensions

Cam dimensions are corresponding with a Reuleaux triangle which is a form consist of three non-concentric arcs, center of each arc at the vertex of an equilateral triangle. Firstly, to construct a Reuleaux triangle we fabricate an equilateral triangle of side '30 mm' shown in "Fig. 13". Now, from the center at one of the vertices, with a radius equal to'30 mm', draw an arc connecting the other two vertices. In a similar way, of an equilateral triangle draw arcs from each polygon vertex between the other two vertices. These three arcs form the 'Classic Reuleaux Triangle'. Constant width is one of its properties, so the estimated shape could be rotated totally around between two parallel lines unattached by distances, see "Fig. 13".

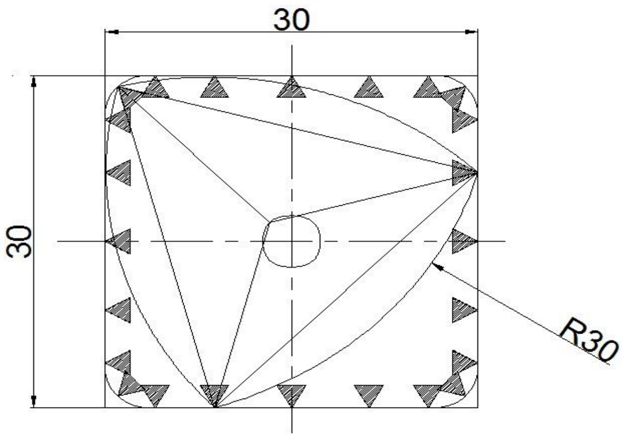


Figure 13. The locus of vertex of the Reuleaux triangle is almost a square and the path cutting tool on workpiece.

6.2. Calculation Percentage of the Cutting Area

A percentage of cutting area is a ratio between actual cutting area to the total area of the form. The desired shape is a square, which does not be completely cut off due to a small area at the four corners of the square. These areas are out of the reach of the cutting tool and may be calculated.

By moving the cutting tool which is in the form of a triangle inside the square shape and observe the triangle movement at the four corners of the established square. These movement consist of drawing curve, which represents the top of the triangle movement at the corners of the square shape, therefore were taken (10) points represent this movement and it was drawing "Fig. 14".

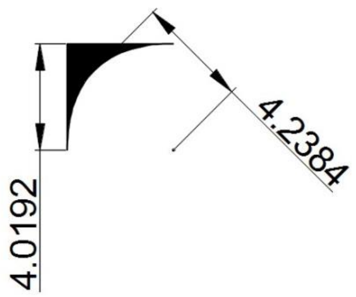


Figure 14. Residual area at the corner

Using CAD software to calculate an area of this region and found to be equal to 2.7668 mm^2.Therefore:

$$\text{the cutting area percentage} = \frac{\text{area of square shape} - 4 * \text{residual area at the corner}}{\text{area of square shape}} * 100\%$$

$$= \frac{900 - 4 * 2.7668}{900} * 100\% = 98.77\%$$

6.3. Determination of Eccentricity of the Centroid of Reuleaux Triangle from its Original Center of Rotation

The geometric centroid does not move along a circle as shown above, nor does it remain constant. Actually, rotational path of curve is collected by (4) arcs of an ellipse.

"Fig. 14" illustrate the ellipse has the parametric equations in the lower-left quadrant for bounding square of side length.

6.4. Universal Coupling

This is the base of the revolution movement necessary for the tool to follow the square path and generate the required dimension. The primary function of this coupling is to provide the rotational motion as output, which is getting as an input form the machine. The secondary function is to provide a link which allows for the centric movement of the Reuleaux guiding triangle which in turn will guide the square cutting tool.

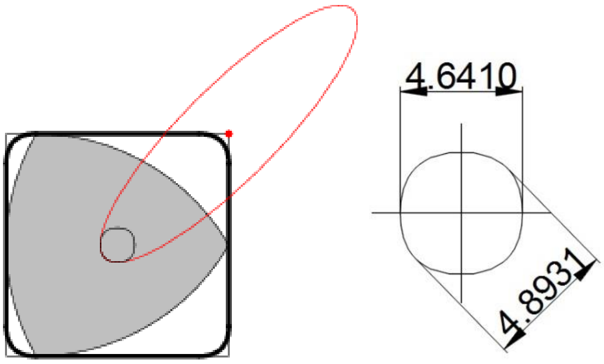


Figure 15. Center movement and governing ellipse in the third quadrant.

6.5. Determination of Guiding Plate Dimensions

One of the most important plates, which contain a square frame. In this frame the Reuleaux triangle cam rotates. This plate is considered as a guiding of cam movements. The dimensions of a square frame that the same dimensions of desired hole. Square frame dimensions are shown in "Fig. 15".

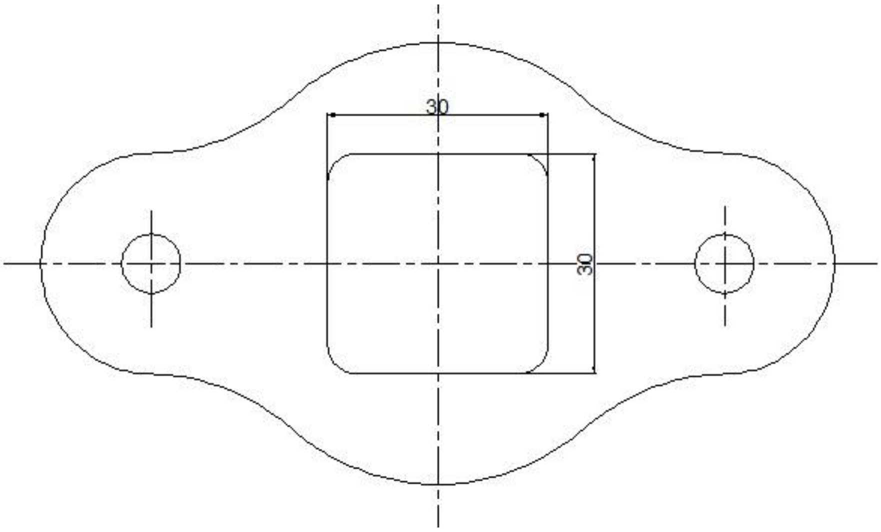


Figure 16. Guiding plate dimensions

6.6. Determination of Cutting Tool Dimensions

This is the main component on which the working of the whole mechanism is dependent, this part which has to actually trace the square and cut the material according to its movement. The base figure for the cutting tool is same as the guiding Reuleaux but it is further modified to keep a factor of safety for certain cutting parameters.

Dimensions of cutting tool are less than the dimensions of the square frame which is rotating in it that located in guiding plate by 0.1 per side to facilitate movement. The tool dimensions are shown in "Fig. 16".

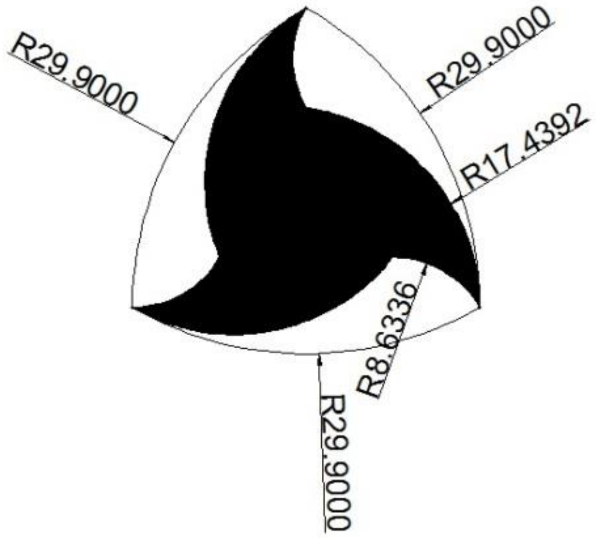


Figure 17. Cutting tool dimensions.

7. Design square hole equipment

The square hole equipment as designed is shown in "Fig. 18".

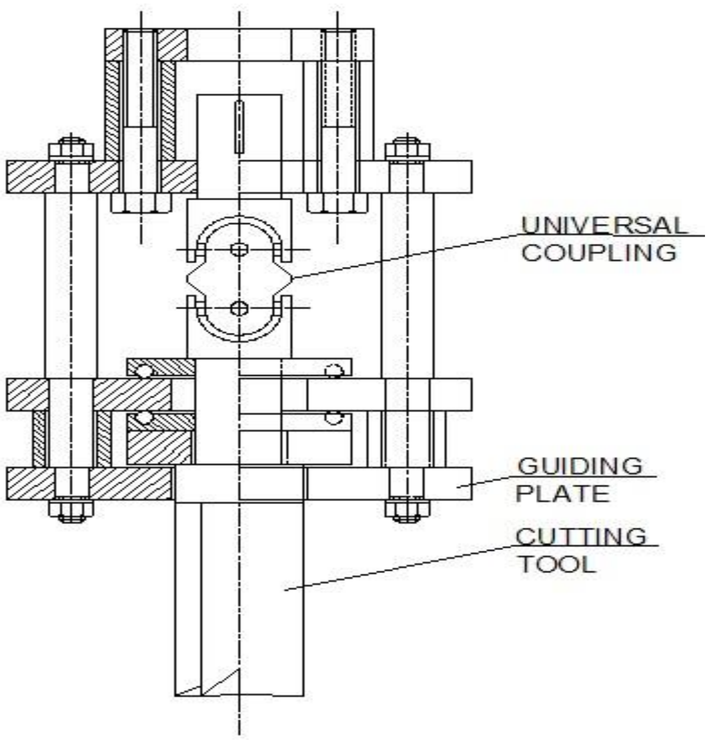


Figure 18. Square hole equipment

8. Conclusions

The cutting tool after installation and proper assembly is found to be exact up to 98.77%. That is, it is able to cut a square shape with the same dimensions as that of the cutting tool with nearly 98.77% area of the original square. The residual 1.33%, which is not cut is existent in an arcs form in the four corners of the square.

According to the calculations, a curve of constant width can have rotational motion in a shape of the curve of constant width is rotate in a square frame and therefore, it would be making a square hole on the workpiece by designing a cutting tool on the cam.

In a mechanism of square guiding, Reuleaux triangle is rotated, so cutting tool give rise to a rotated motion making cut of accurate square hole.

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