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Mathematical Approach to Optimum Drip Irrigation Subunit Design

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Abstract

To increase the effectiveness of irrigation application, emission homogeneity, and economic return in the production process, drip irrigation system design is considered a crucial step. The goal of this study is to create a mathematical solution for the impartial function, which is a function of the subunit's total annual cost and combination of hydraulic emission uniformity, to obtain a relationship between the subunit's area and lateral and manifold lengths. The impartial function is expressed in terms of fundamental design parameters such as emitter and lateral spacing as well as manifold diameter. Mathematical techniques are then utilized to select the ideal number of laterals and manifolds for a given subunit. The results showed that the impartial function of the mathematical model increased by 6% to 64%, while the total annual cost decreased by 6% to 66% compared to the numerical model proposed in the previous study. The hydraulic emission uniformity values of the mathematical model were 89% to 97% for all cases used in this study. In general, the proposed model obtained a minimum total cost with reasonable hydraulic emission uniformity.

Keywords: Drip Irrigation, Impartial Function, Manifold Length, Optimum Design, Subunits Area.

1. Introduction

In Iraq, drip irrigation is regarded as more advanced than other conventional surface irrigation techniques. The pressure variation along laterals and manifolds determines the hydraulic performance of drip irrigation modules and systems. Changes in elevation along the flow direction and frictional energy loss are the primary causes of pressure variation [1]. A lot of studies have been done in the past 50 years to create an effective drip irrigation design methodology.

Low-pressure drip systems can be used to calculate irrigation uniformity according to a method established by Myers and Bucks [2]. For emitter discharge in low-pressure drip irrigation, their method relied on uniform dispersion. Bucks and Myers [3] created a lateral pipe design technique based on their earlier research [2]. In order to ascertain irrigation intervals, emitter spacing and flow rates, and system hydraulic properties, Bresler [4] provided design equations and validated data in their study. Keller and Karmeli [5] presented design charts for lateral, submain, and main lines for different field slopes, shapes, and pipe sizes along with design examples.

In order to achieve good consistency, Wu and Gitlin [6] created a drip lateral-line design process that permits the use of varied sizes of emitters throughout the system. A graphical approach was devised by Wu and Gitlin [7] to ascertain the uniformity of application along a lateral line as a function of lateral slope, lateral length, lateral flow rate, and inlet pressure. Valiantzas [8] described a method for performing a mathematical analysis of drip lateral lines under the assumption that emitter flow rates vary along the lateral pipe. An accurate analytical hydraulic analysis of multi-size laterals was proposed by Valiantzas [9]. Subsurface drip irrigation field studies conducted over an extended period of time have assessed changes in system uniformity [10], which is critical for the system's expected life expectancy as well as the chance to deliver nutrients consistently by chemigation [11].

Numerous studies were carried out to ascertain the homogeneity of emissions from drip irrigation systems [11]–[20]. Pitts et al. [12] used a computerized model to forecast emitter discharge uniformity and analyze drip irrigation laterals. Chen et al. [13] used the homogeneity coefficient to categorize the water distribution level. A technique for designing micro-irrigation laterals based on average emitter discharge and the necessary water application uniformity was provided by Kang and Nishiyama [14]. They examined the hydraulics of flow in the lateral using the finite element approach.

In order to forecast the hydraulic component of emission uniformity, Noori and Al-Thamary [15] created a theoretical formula and contrasted field and design emission uniformity. They came to the conclusion that carrying out a thorough design for the drip subunit is not necessary in order to use the created formula to compute the hydraulic emission uniformity.

A drip irrigation system usually requires relatively high initial investment and considerably low operational and maintenance costs. These costs depend mainly upon system design. Total annual costs are the sum of fixed and pumping costs. Fixed costs include initial cost, taxes, and insurance charges [18]. Pumping or energy cost is relatively small with a trickle irrigation system since the operating pressure and applied discharges are lower than those required with other types of pressurized systems. Most of the research that has been conducted so far either emphasizes minimizing total cost. But very limited effort has been performed to combine both hydraulic emission uniformity and total annual cost in one impartial function [21]-[24]. Therefore, an optimal design of a drip irrigation system is a design that gives maximum attainable emission uniformity and minimum total cost.

Abdullah [24] integrated the overall yearly cost of drip irrigation subunits and hydraulic emission uniformity into a single impartial function. Abdullah [24] utilized optimization techniques to select the ideal spacing between emitters and laterals in order to maximize hydraulic emission uniformity and minimize total annual cost. She did this by expressing the impartial function in terms of fundamental design parameters, such as manifold diameter, emitter spacing, and lateral spacing. Abdullah [24] demonstrated that the values of the impartial function increase up to a certain limit and then begin to decline when the distance between emitters and laterals increases. The overall yearly cost reduces as hydraulic emission uniformity rises and subsequently begins to rise. But the difference between the rise in hydraulic emission uniformity and the increase in the total annual cost was very small.

Limited effort has been performed to derive lateral and manifold length relationships with subunit areas based on an optimal mathematical solution of the impartial function that combined both hydraulic emission uniformity and total annual cost. The objective of this research is to develop a mathematical solution to the impartial function, which is a function of the combination of hydraulic emission uniformity and the total annual cost of the subunit, to generate a relationship between lateral length and manifold length to the area of the subunits based on the optimum solution of the impartial function to minimize the total cost.

2. Theoretical Procedure and Methodology

2.1. General Drip Irrigation Design

The three primary parts of a drip irrigation system are typically emitters, a distribution network, and a control head. The first portion is situated close to the farm inlet or water supply. It is made up of a control valve, chemical injection device, filter, and pumping unit. The distribution network, which consists of a network of pipes, is the second component. It takes water from the control head and sends it to the emitters at the proper pressure head. Manifolds, lateral lines, and main lines make up the network (Fig. 1). The energy dissipating of emitters, also known as drippers, are the final component of the system. To provide water to the soil close to the plants, the emitters are positioned at a predetermined distance apart [21].

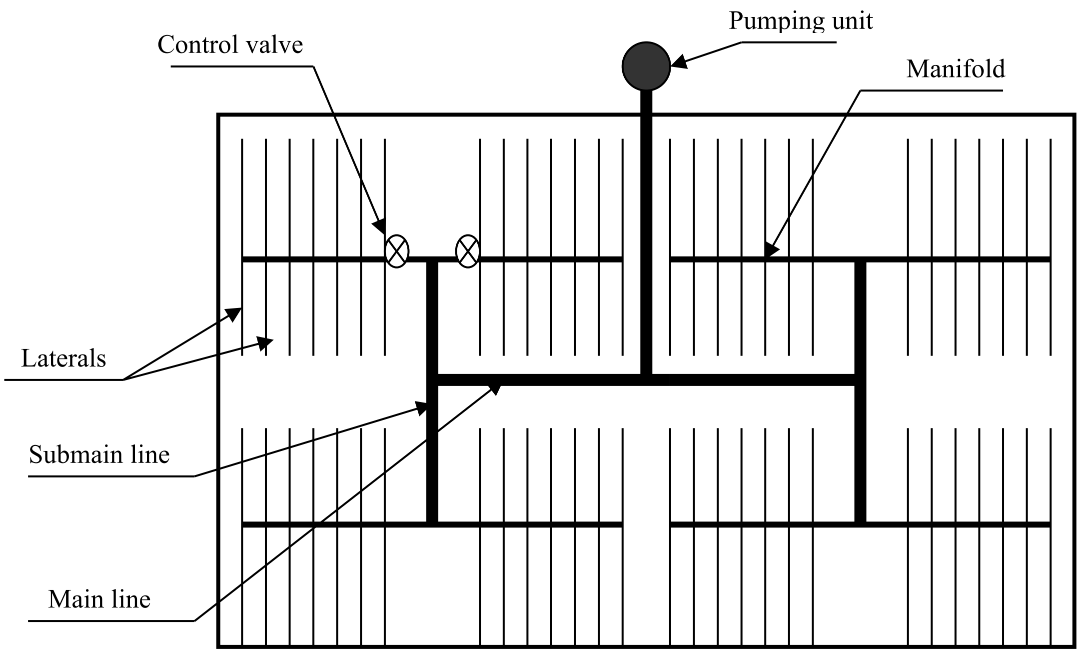

Figure 1. Typical drip irrigation system configuration.

Selecting the general emitter type that best meets the requirements of the area to be wetted is the first step after system layout. The specific emitter required is then chosen, and emitter discharge, pressure head, head loss in pipes, and emission uniformity are then calculated based on the system's required discharge, spacing, and other planning considerations [21]. As a function of pressure head, the emitter flow rate is often given as follows [25], [26]:

$$q_e = kh^x \quad (1)$$

where q_e is the flow rate of the emitter, L hr^{-1}, h is the emitter pressure head, m, k is an emitter type coefficient, and x is the emitter's discharge exponent that characterizes the flow regime. For fully turbulent flow, $x = 0.5$; for laminar flow $x = 1.0$; and for fully compensating emitters $x = 0.0$. Average emitter discharge, q_a, can be expressed as:

$$q_a = \frac{d_g}{t_a} S_e S_l \quad (2)$$

where d_g is the irrigation gross depth, mm, t_a is the irrigation application time, hrs, S_e is emitter spacing on the lateral, m, and S_l is lateral spacing on the manifold, m. (1) can be used to get the emitter flow rate pressure head, h_a, that produces the intended average emitter discharge. A lateral or manifold's maximum permitted head loss is determined by applying the 20% rule, which can be stated as follows [4], [27]:

$$h_{f\max} = 0.234\, h_a \quad (3)$$

where $h_{f\max}$ is the maximum allowable head loss along a lateral or manifold, m. Either the Darcy-Wiesbach or the Hazen-Williams friction head loss equations can be used to calculate the pressure head loss in a lateral or manifold. Since the flow in pipes is typically a transitional flow, the Hazen-Williams formula, which is stated as follows [25], [28], is frequently utilized:

$$h_f = 1.129x10^9 \left(\frac{Q}{C}\right)^{1.852} D^{-4.87} LF \quad (4a)$$

where Q is the pipe flow rate, m^3/hr, C is the Hazen-Williams friction coefficient, L is the pipe length, m, F is Christiansen's reduction factor [29], and h_f is the friction head loss, m. This accounts for the flow reduction along the pipe. The following formula is used to get the reduction factor, F:

$$F = \frac{\sum_{i=1}^{n} i^{1.852}}{N^{2.852}} \quad (4b)$$

where i is the number of openings (outlets), and N is the total number of openings.

A notion called emission uniformity (EU, %) has been put out to account for the consistency of emitter discharges across a subunit. The major challenge is that a drip irrigation system needs to have enough flow capacity to ensure that the least-watered areas are adequately irrigated. Solé-Torres [30] is expressed the emission uniformity of drip subunit as:

$$EU = EU_m * EU_h \quad (5)$$

where EU_m is emission uniformity due to manufacturing variation, %, and EU_h is emission uniformity due to pressure differences, %. Noori and Al-Thamary [15] developed a theoretical formula to predict the hydraulic part of emission uniformity, EU_h, and compared the design emission uniformity with field emission uniformity. Their developed formula was adopted in this study, and it is expressed as [15]:

$$EU_h = [1 - G_1\Gamma_{n_e} - G_2 n_e^{1.852}\Gamma_{n_l}]^{x} \quad (6a)$$

$$G_1 = \phi D_l^{-4.87} S_{eb} \quad (6b)$$

$$G_2 = \phi m^{1.852} D_m^{-4.87} S_{lb} \quad (6c)$$

$$\phi = \beta k^{1/x} q_a^{(1.852-\frac{1}{x})} \quad (6d)$$

$$S_{eb} = S_e + L_{ee} \quad (6e)$$

$$S_{lb} = S_l + L_{el} \quad (6f)$$

where D_l is lateral pipe diameter, mm, D_m is manifold pipe diameter, mm, m is a factor equal to 1 if the laterals extend on one side of the manifold and 2 if the laterals extend on both sides of the manifold, and C β is a coefficient equal to 0.07332 When C equals 150, Lee is the equivalent length of each emitter connection on the lateral, m, and Lel is the equivalent length of each lateral connection on the manifold, m. Wang et al. [31] simplified the calculation of Christiansen's reduction factor using an approximated approach by integrating a continuous function, and it is expressed as:

$$\Gamma_{n_e} = 0.351 n^{2.852} - 0.0486 \quad (7a)$$

$$\Gamma_{n_l} = 0.351\ n_1^{2.852} - 0.0486 \quad (7b)$$

$$n = n_e + 0.5 \quad (7c)$$

$$n_1 = n_l + 0.5 \quad (7e)$$

2.2. Mathematical Formulation of System Costs

The system costs in this study were split into two categories: fixed costs and variable costs. Interest charges and beginning costs make up the majority of fixed costs. Energy, lubrication, maintenance, taxes, insurance, repairs, and labor costs are examples of variable costs. A drip subunit's fixed costs consist of the total cost of the manifold, laterals, emitters, and fittings. Multiplying pipe lengths by pipe unit cost yields the total cost of the manifold and laterals. To calculate the costs of emitters, fittings, and connectors, multiply the quantity of each accessory by the unit cost. Therefore, the annual fixed cost is expressed as:

$$AFC = CRF * FC = CRF[\frac{L_l L_m}{S_e S_l} mC_e + \frac{L_l L_m m}{S_l} P + \frac{L_m}{S_l} C_c + L_m C_m] \tag{8}$$

where *AFC* is annual fixed cost, Iraqi Dinar (IQD), *CRF* is Capital Recovery Factor, *FC* is fixed cost of the drip subunit, IQD, C_e is unit cost of emitter, IQD, C_c is unit cost of lateral connections to the manifold, IQD, C_m is unit cost of the manifold pipe, IQD /m, and *P* is unit cost of the lateral pipes, IQD /m. The US dollar is officially equal to 1320 IQD.

This study is proposed to derive the optimal lateral and manifold lengths for a constant subunit area, A_s (where $A_s = m L_l L_m =$ m $S_e n_e S_l n_l$) based on minimum annual cost with maximum hydraulic emiision uniformity. Therefore, the impartial function could be converted to one variable, which is the number of emitters, n_e, by assuming the following:

$$n_l = \frac{A_S}{mS_e S_l n_e} = \frac{K}{n_e} \tag{9}$$

where $K = \frac{A_s}{mS_e S_l}$ is a coefficient. By combining (8) and (9), the annual fixed cost is expressed as:

$$AFC = E_1 + \frac{E_2}{n_e} \tag{10a}$$

$$E_1 = CRF[KC_e m + KS_e mP] \tag{10b}$$

$$E_2 = CRF[KC_c + KS_l C_m] \tag{10c}$$

The energy or pumping cost, which is mostly determined by the hydraulic design of the system, is a variable cost. Pumping costs can be calculated using the system's annual energy requirements. The total discharge of a subunit is the lateral pipe discharge times the number of laterals in the subunit. Then, the total dynamic head could be calculated based on head loss in the lateral and manifold pipes, minor losses in the subunits, and differences in ground elevation. Minor losses can be assumed as a percentage of the sum of the average pressure head and friction head loss. Therefore, the annual pumping cost, *APC*, for a subunit is expressed as:

$$APC = KW * T_a * U_c = E_3[aK + a_1 K \Gamma_{n_e} + a_2 K n_e^{1.852} \Gamma_{n_l}] \tag{11a}$$

$$E_3 = \frac{D_g}{d_g} t_a U_c \tag{11b}$$

$$a = \frac{2.722}{10^6 \varepsilon} m(1+R) \frac{q_a^{(1+1/x)}}{k^{1/x}} \tag{11c}$$

$$a_1 = \frac{0.5988}{10^6 \varepsilon} mS_{eb} D_l^{-4.87} (1+R) q_a^{2.852} \tag{11d}$$

$$a_2 = \frac{0.5988}{10^6 \varepsilon} m^{2.852} S_{lb} D_m^{-4.87} (1+R) q_a^{2.852} \tag{11e}$$

where *APC* is the annual pumping cost, IQD, T_a is the annual time of operation, which is equal to the time of application per irrigation, t_a, multiplied by the annual number of irrigations, U_c is unit energy cost, IQD/kW-hr, D_g is the annual gross depth, mm, ε is overall pumping-unit efficiency, %, and *R* is minor losses expressed as a percentage of the sum of the average pressure head and friction head loss. Then, the total annual cost is the summation of annual fixed cost and annual pumping cost expressed as:

$$TAC = AFC + APC = E_4 + \frac{E_2}{n_e} + J_1 \Gamma_{n_e} + J_2 n_e^{1.852} \Gamma_{n_l} \tag{12a}$$

$$E_4 = E_1 + aE_3 K \tag{12b}$$

$$J_1 = a_1 E_3 K \tag{12c}$$

$$J_2 = a_2 E_3 K \tag{12e}$$

where *TAC* is the annual total cost, IQD.

2.3. Developed Mathematical Formulation of Impartial Function

The impartial function proposed by Abdullah [24] is a combination of the total cost of drip irrigation subunits and hydraulic emission uniformity. The impartial function is expressed as:

$$Z = \frac{\alpha EU_h}{TAC} \tag{13}$$

where *Z* is the impartial function, and α is a scaling factor. Thus, maximizing Z inclusively means attaining the maximum possible hydraulic emission uniformity at a minimum possible total annual cost of the system. Abdullah [24] utilized optimization techniques to obtain an optimum solution or design.

This study proposes a mathematical solution to obtain an optimum solution or design. The problem encountered in this research is to attain maximum emission uniformity at the minimum possible system cost. As stated previously, the mathematical solution of the impartial function is based on one variable, which is the number of emitters, ne, to derive the optimal lateral and manifold lengths for constant subunit areas while the other parameters were constant. The optimum ne could be derived by differentiating the impartial function (i.e., (13)) with respect to ne as follows:

$$\frac{dZ}{dn_e} = \frac{\alpha * TAC * dEU_h - \alpha * EU_h * dTAC}{TAC^2} = 0 \qquad (14)$$

where *TAC* and EU_h are functions of ne and other parameters were constants. By combining (6) through (12) in (13) and simplifying (14), it can be rewritten as:

$$f(n_e) = xf_1(n_e)f_2(n_e) - f_3(n_3)f_4(n_e) = 0 \qquad (15a)$$

$$f_1(n_e) = E_4 + \frac{E_2}{n_e} + 0.351J_1n^{2.852} - 0.0486J_1 + 0.351J_2n_e^{1.852}n_1^{2.852} - 0.0486J_2n_e^{1.852} \qquad (15b)$$

$$f_2(n_e) = -G_1n^{1.852} - 0.65G_2n_e^{0.852}n_1^{2.852} + 0.09G_2n_e^{0.852} + KG_2n_e^{-0.148}n_1^{1.852} \qquad (15c)$$

$$f_3(n_e) = 1 - 0.351G_1n^{2.852} + 0.0486G_1 - 0.351G_2n_e^{1.852}n_1^{2.852} + 0.0486G_2n_e^{1.852} \qquad (15d)$$

$$f_4(n_e) = -\frac{E_2}{n_e^2} + J_1n^{1.852} + 0.65J_2n_e^{0.852}n_1^{2.852} - 0.09J_2n_e^{0.852} - J_2Kn_e^{-0.148}n_1^{1.852} \qquad (15e)$$

In order to find the solution for the number of emitters, n_e, from (15a), the parameter n_e can be derived directly from thegiven data of different parameters (see Table 1) using curve-fitting techniques and statistical methods by iteratively minimizing the error of (15a) relative to the value of $f(n_e)$, which is equal to zero. A methodology was developed to derive the parameter n_e from data given in Table 1 using an iterative solution aimed at minimizing the error between f(ne) = 0 and the functional solutions of (15a) using the generalized reduced gradient method [32]. Equations (6) through (15) were incorporated in Microsoft Excel Spreadsheet in and the parameter ne was determined using solver routine using the generalized reduced gradient method [32]. Based on the results of the optimal solution of parameter ne, the optimal lateral and manifold length relationships with the subunit areas were derived from data given in Table (1) and for different scenarios of changing the values of qa, x, k, and unit costs.

Table 1. Input data given for different scenarios of changing the values of q_a, x, k, and unit costs.

Parameter values	Case 1	Case 2	Case 3*	Case 4*	Case 5*	Case 6*
D_m, mm	50	38	75	100	63	150
D_l, mm	13	13	13	13	16	16
m	1	1	1	1	2	2
k	0.62	0.59	0.39	0.50	0.13	0.80
x	0.85	0.69	0.57	0.57	0.96	0.82
q_a, lph	3.38	3.75	1.38	0.75	6.33	4.22
L_m, m	100	80	90	70	120	180
L_l, m	40	55	60	20	80	96
L_{ee}, m	0.50	0.50	0.4	0.50	0.22	0.31
L_{el}, m	0.40	0.22	0.20	0.35	0.63	0.78
D_g, mm	600	500	400	900	800	1000
d_g, mm	18	20	12	18	48	38
t_a, hrs	12	12	24	24	18	36
C_m, IQD/m	6000	4000	5000	8000	4000	14000
C_e, IQD/no.	35	45	20	50	100	500
C_c, IQD/no.	25	60	50	35	120	250
P, IQD/m	20	25	20	25	150	300
CRF	0.150	0.145	0.150	0.140	0.155	0.147
R, %	0.12	0.15	0.15	0.14	0.13	0.10
U_c, IQD/kW-hr	200	180	180	800	100	1000
α	10^8	10^8	10^8	10^9	10^10	10^11
β	0.07332	0.07332	0.07332	0.07332	0.07332	0.07332
S_e, m	0.75	0.75	0.50	0.25	0.50	1.00
S_l, m	3.00	3.00	2.75	4.00	4.75	4.00
E_{ta}, %	90	95	85	90	95	90

*Data were taken from Abdullah [25].

3. Results and Discussion

In order to validate the mathematical model in (15a) to (15e) that was proposed in this study, several input data for six cases were presented in Table 1 by changing the values of q_a, x, k, D_m, D_l, L_m, L_l, L_{ee}, L_{el}, D_g, d_g, t_a, CRF, S_e, S_l, and unit costs. The average emitter discharge, q_a, ranges from 1.38 to 6.33 lph. The gross depth of irrigation, d_g, ranges from 12 to 48 mm. The emitter spacing on the lateral, S_e, and lateral spacing on the manifold, S_l, ranged from 0.25 to 1.00 m and from 3.00 to 4.00 m, respectively. The lateral pipe diameter, D_l, and manifold pipe diameter, D_m, were ranged from 13 to 16 mm, and from 38 to 150 mm, respectively. The unit cost of the emitter, C_e, the unit cost of the manifold pipe, C_m, and the unit cost of the lateral pipes, P, were from 20 to 500 IQD/no., from 4000 to 14000 IQD/m, and from 20 to 300 IQD/m, respectively, according to Abdullah [24] study.

Based on input values of Table 1, the relationships between numerical impartial function, Z, proposed by Abdullah [24], mathematical Z (proposed in this study from (15a) to (15e), numerical total annual cost, TAC, proposed by Abdullah [24], mathematical TAC (proposed in this study), numerical hydraulic emission uniformity, EU_h, and mathematical EU_h (proposed in this study) versus different average emitter discharges for cases # 1 and 2 were presented in Fig. 2. As expected, the values of mathematical and numerical Z increases as the values of the emitter discharge increase up to a certain limit then they were decreased. The optimum numerical and mathematical Z occurs at emitter discharge of 3.213 and 3.073 lph for cases #1 and #2, respectively. Higher values of mathematical Z in comparison to the the numerical one (Fig. 2). This results in low values of total annual cost for the proposed model in comparison to the one produced from Abdullah [24] study (numerical TAC). Similar results were obtained for other cases (see Fig. 3 and 4). A good match between numerical and mathematical values of Z, TAC, and EU_h was observed for case # 6 (Fig. 4). This was due to using higher values of unit costs (C_e, C_m, C_c, and P) compared to other cases. Based on the results of Fig. 2 through 4, the proposed model obtained minimum total cost with reasonable hydraulic emission uniformity, EU_h, compared to the one proposed by Abdullah [24].

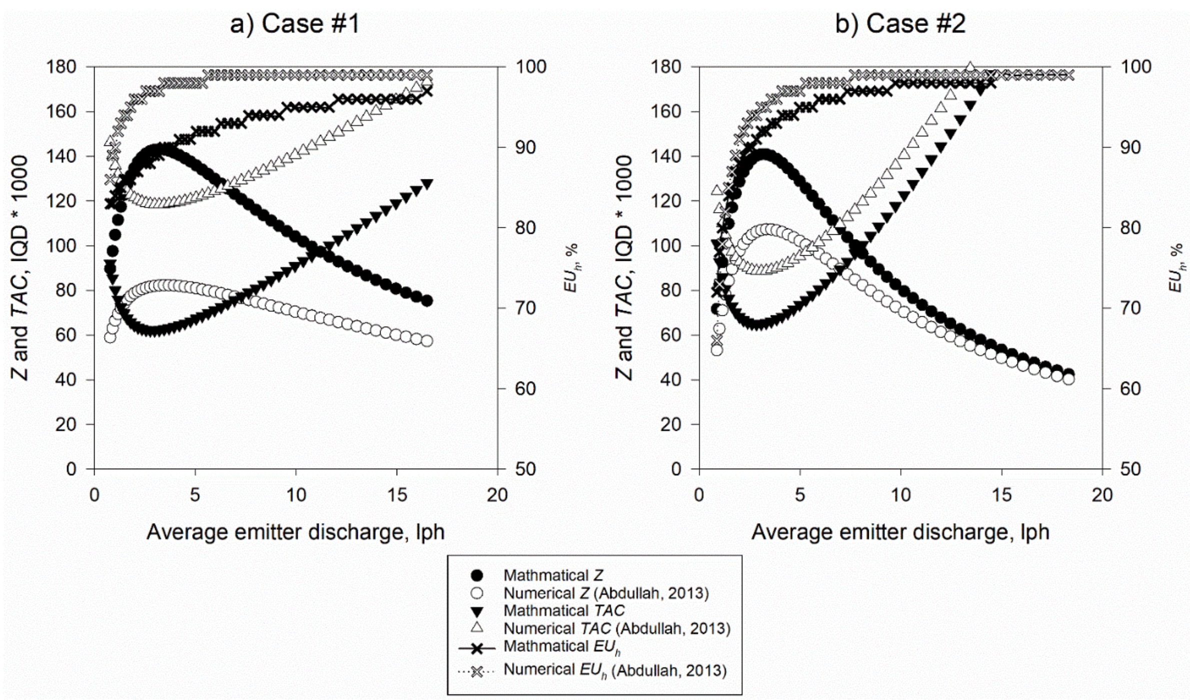

Figure 2. Relationships between numerical impartial function (Z) proposed by Abdullah [24], mathematical Z (proposed in this study), numerical total annual cost (TAC) proposed by Abdullah [24], mathematical TAC (proposed in this study), numerical hydraulic emission uniformity (EU_h), and mathematical EU_h (proposed in this study) versus different average emitter discharge for cases # 1 and 2.

A relationship between lateral lengths, L_l, to manifold lengths, L_m, that was derived from a mathematical impartial function (proposed in this study) for cases #1 through #6 was presented in Fig. 5. A linear relationship between L_l and L_m was obtained from Fig. 5 with a coefficient of correlation of more than 0.97. This linear relationship followed the following formula:

$$L_l = b - aL_m \tag{16}$$

where a is a coefficient proposed in this study ranging from 0.57 to 7.94, and b is a coefficient proposed in this study, ranging from 200 to 284. Almost all coefficients in cases #1 through #4 obtained similar results, while a slight divergence was obtained in cases #5 and #6 due to using high-cost values.

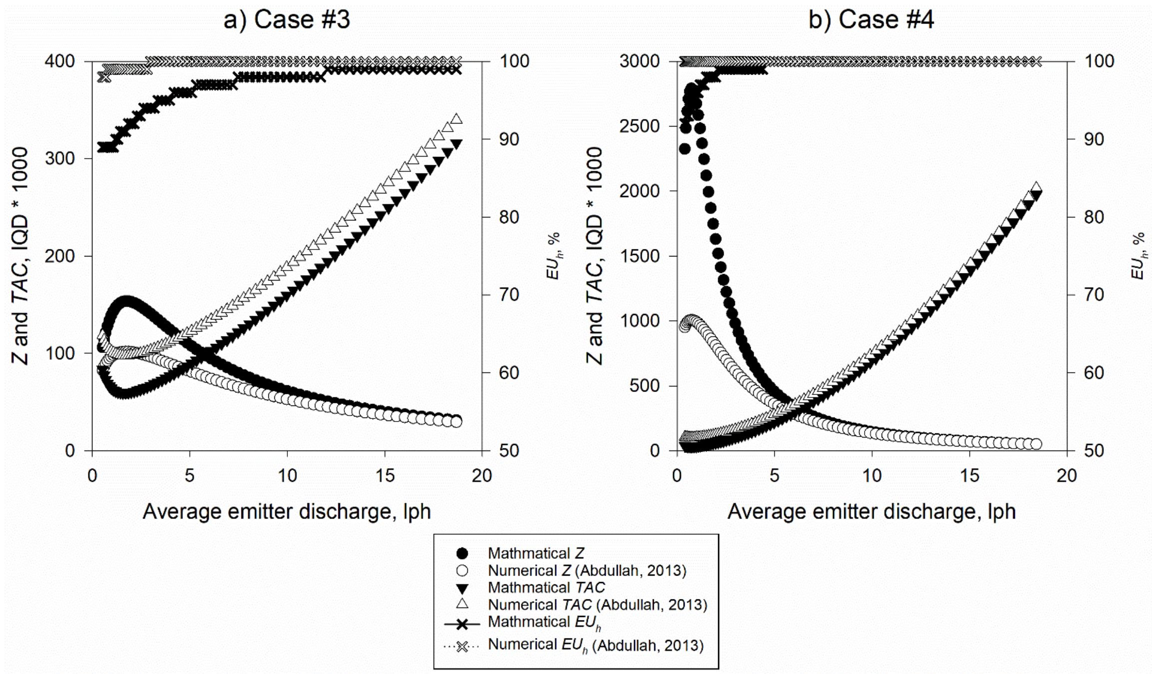

Figure 3. Relationships between numerical impartial function (*Z*) proposed by Abdullah [24], mathematical *Z* (proposed in this study), numerical total annual cost (*TAC*) proposed by Abdullah [24], mathematical *TAC* (proposed in this study), numerical hydraulic emission uniformity (EU_h), and mathematical EU_h (proposed in this study) versus different average emitter discharge for cases # 3 and 4.

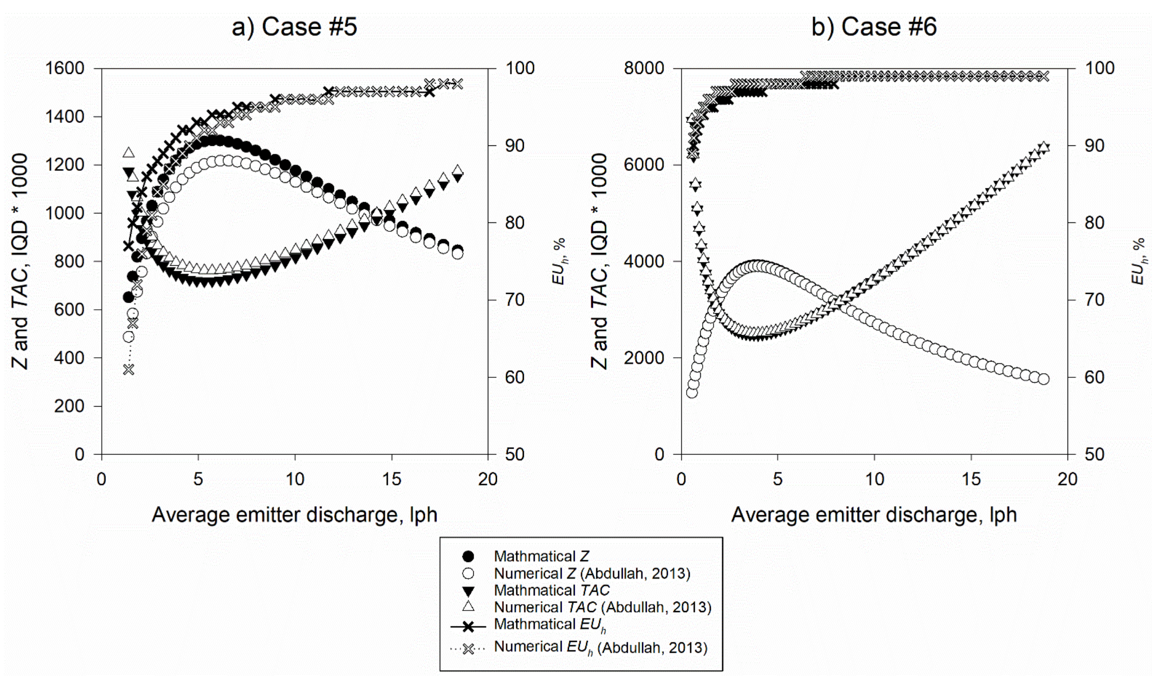

Figure 4. Relationships between numerical impartial function (*Z*) proposed by Abdullah [24], mathematical *Z* (proposed in this study), numerical total annual cost (*TAC*) proposed by Abdullah [24], mathematical *TAC* (proposed in this study), numerical hydraulic emission uniformity (EU_h), and mathematical EU_h (proposed in this study) versus different average emitter discharge for cases # 5 and 6.

Keller and Bliesner [33] developed a relationship between L_l and L_m versus subunit area, A_s, based on optimum design for drip system design in a general manner. They assumed the system cost as a function of design factors. Then, they solved for minimum system cost based on trial-and-error results. Their relationship is expressed as follows [33]:

$$L_m = A_s^{0.55} \tag{17a}$$

$$L_l = A_s^{0.45} \tag{17b}$$

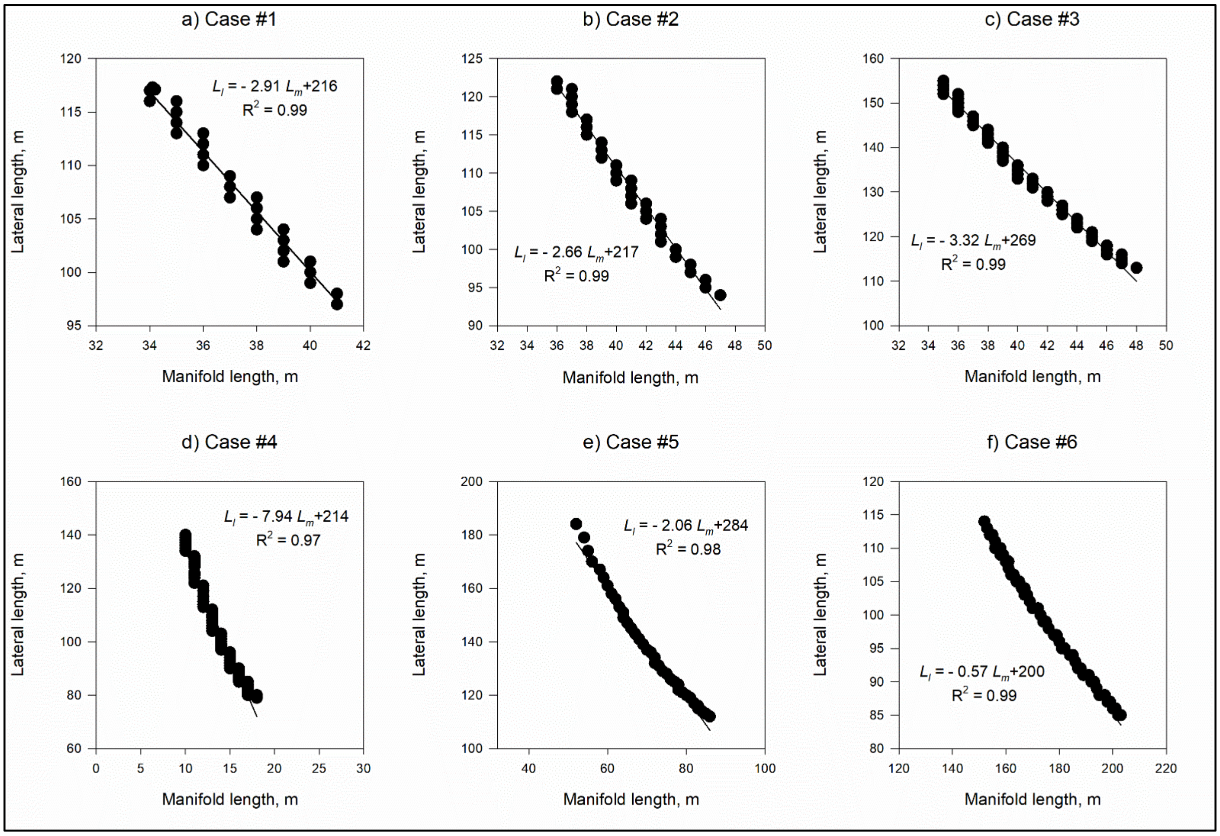

Figure 5. Relationships between lateral length (L_l) and manifold length (L_m) developed from the mathematical model proposed in this study for: a) case #1, b) case #2, c) case #3, d) case #4, e) case #5, and f) case #6.

Fig. 6 presents a comparison of the proposed relationship between Ll and Lm versus As to the one developed by Keller and Bliesner [33]. A power relationship between L_m versus A_s for the proposed study (mathematical model) similar to the one developed by Keller and Bliesner [33] was obtained from Fig. 6a. This relationship with a coefficient of correlation of 0.99, is expressed as followed:

$$L_m = 0.009\, A_s^{1.003} \tag{18a}$$

However, a weak power relationship between L_l versus A_s for the proposed study (mathematical model) was obtained in comparison to the one developed from Keller and Bliesner [33] from Fig. 6b. This relationship with almost no coefficient of correlation is expressed as follows:

$$L_l = 111\, A_s^{-0.003} \tag{18b}$$

The different results in the relationships between the proposed model and the one developed from Keller and Bliesner [33] were due to the values of Ll around 100 m, while the ones developed from Keller and Bliesner [33] ranged from 10 to 90 m. This was due to the limitation in lateral length for the numerical model compared to no limitation for L_l in the proposed model (mathematical model).

Fig. 7 presents a comparison of relationships between subunit area, annual pump cost (*APC*), and annual fixed cost (*AFC*), at the optimum impartial function and minimum total annual cost of the mathematical model. As expected, both *APC* and *AFC* increased as subunit area increased, with higher values for annual fixed costs. The maximum costs were 890,000 and 1,600,000 IQD for APC and AFC, respectively, at a subunit area of 35,000 square meters.

4. Conclusions

In this research, a mathematical solution of the impartial function, *Z*, was developed to design drip irrigation systems. The impartial function is a function of hydraulic emission uniformity to the total annual cost of the subunit. Also, relationships between lateral length and manifold length, and between the area of the subunits and the optimum solution of the impartial function, were developed. Basic design parameters such as emitter and lateral spacing and manifold diameter, and optimum number of lateral and manifold for a specific subunit were derived based on optimum design for the impartial function. The results showed that the values of mathematical and numerical Z (developed from Abdullah [24] *'s study)* increase as the values of the emitter discharge increase up to a certain limit, then they decrease. Higher values of mathematical *Z* in comparison to the numerical one were obtained. The proposed model obtained minimum total cost with reasonable hydraulic emission uniformity, EU_h, compared to the one proposed by Abdullah [24]. A linear relationship between L_l and L_m was obtained with a coefficient of correlation of more than 0.97. A power relationship between L_m versus A_s for the proposed study (mathematical model), similar to the one developed by Keller and Bliesner [33], was obtained. This research presented an ecumenical result compared to the previous studies with a mathematical model to design drip irrigation systems.

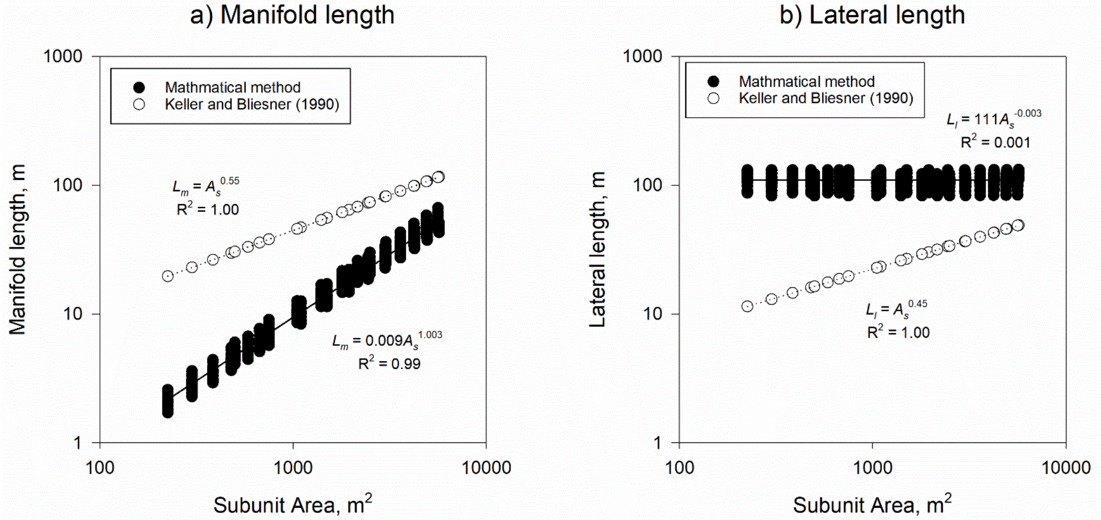

Figure 6. Comparison of relationships between subunit area versus a) lateral length (Ll) and b) manifold length (Lm) that developed from the mathematical model (proposed in this study) and the one developed from the Keller and Bliesner [33] approach for all cases proposed in this study.

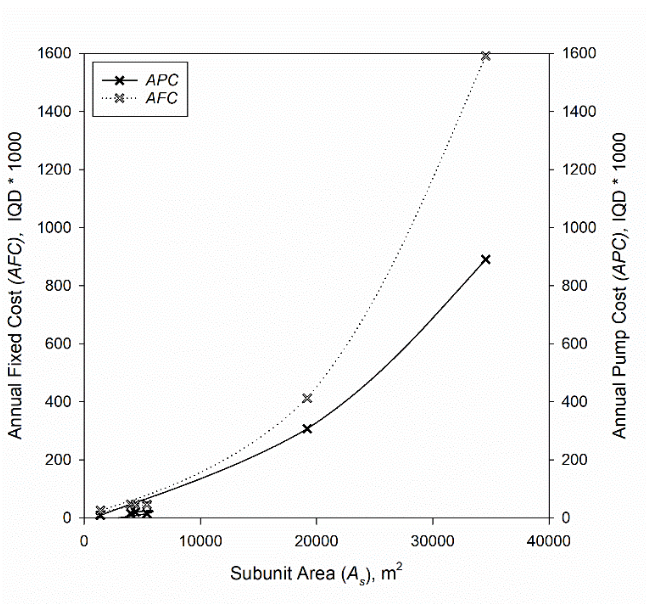

Figure 7. Comparison of relationships between subunit area versus annual pump cost (*APC*) and annual fixed cost (*AFC*) at optimum impartial function and minimum total annual cost of the mathematical model.

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Conflict of interest

The authors declare that there are no conflicts of interest regarding the publication of this manuscript.

Author Contribution Statement

Abdul-Sahib T. Al-Madhhachi proposed the research problem, developed the theory, performed the computations, discussed the results, and contributed to the final manuscript.

Safa N. Hamad verified the analytical methods and supervised the work's findings.

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