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Mathematical Modelling of a Single Magneto-Rheological for Car Suspension System Using Modified Bouc-Wen Model

Siti Farizah Yaakub[1], Saifudin Hafiz Yahaya[2*], Mohd Shukor Salleh[3]

[1] Politeknik Merlimau, Merlimau, Melaka, Malaysia
[2,3] Fakulti Teknologi dan Kejuruteraan Industri dan Pembuatan, Universiti Teknikal Malaysia Melaka, Melaka, Malaysia

[1]https://orcid.org/0000-0002-8217-835X
[2]https://orcid.org/0000-0003-0525-6243
[3]https://orcid.org/0000-0003-4800-2470
[*]Email: saifudin@utem.edu.my

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Abstract

Magneto-Rheological (MR) automotive suspension represents the most advanced technology currently available for enhancing passenger comfort through intelligent fluid behaviour. This work employed a Modified Bouc-Wen model to create a mathematical representation of a single MR vehicle suspension and compared its ride performance to that of passive suspension systems. The model was formulated and solved using second-order linear differential equations, with MATLAB software utilized to visualize the results. The maximum vertical displacement for the single MR mathematical model in this work was 0.031 m, while the minimum was -0.0124 m. At the minimum position, the vertical displacements for the passive model were 0.0532 m and -0.0133 m. The results demonstrated that the vertical displacement variation was similar for both the mathematical model and the passive system. All displacements obtained from the mathematical modeling of the single MR system were close to its mean of 0.000875 m (nearly zero). There was reduced vibration when zero displacement was achieved. It was concluded that MR damper significantly improved car suspension performance compared to passive solutions. This paper developed a single MR vehicle suspension system using the Modified Bouc-Wen model in a Proton Preve.

Keywords: Bouc-Wen Equation, Magneto-Rheological Suspension System, Single Model, Vertical Displacement

1. Introduction

1.1 Suspension Systems

The suspension systems' objective is to give the passengers comfort when driving in different road profiles. Passive, active, and semi-active suspension systems are three types of car suspension systems. Passive suspension systems are used in today's vehicles, making it possible to achieve the highest level of ride comfort and quality in all driving situations [1].

This suspension system is made up of springs and dampers, the parameters of which are fixed by design and cannot be changed [2]. Thus, each vehicle must be equipped with a suspension system to absorb vibrations, especially on uneven road surfaces and to support heavy passenger weights [3]. The difficulty with passive suspension systems is the system cannot satisfy the comfort requirements when subjected to different road profiles [4]. Because the mobility of the car's body and wheels is limited, current passive suspension solutions cannot give better ride comfort. Car suspension systems are varied by the manufacturer, and they play a vital role in guaranteeing the car's safety functions [5]. To eliminate vibrations and shocks and ensure higher quality, a suspension system was used that connects the axle to the body. The quality of suspension systems may produce good car behavior and a high level of comfort regardless of the road profile [6].

Many automotive suspension system designs have been studied to overcome the limitations of passive systems. There have been several analytical and experimental investigations on suspension systems to improve ride quality and handling performance. Research results suggest that active and semi-active suspensions can improve performance over passive suspension systems [7]. Semi-active systems, such as Magneto-

Rheological (MR) suspension systems, which can create adjustable damping forces and a larger magnitude of yield stress, are then implemented to provide the greatest performance to the cars. From all of the systems, semi-active suspension systems allow modifying the dynamic properties with less energy consumption and mechanical complexity. Semi-active suspension systems were also less expensive, resulting in a better performance-to-cost ratio [8]. For semi-active suspension systems, MR is the best controller for vibrations [9]. The semi-active suspension is also used in an automobile which controls damping force and can customize the damping coefficient [10].

1.2 MR Systems

The MR suspension systems were the most appealing choice of semi-active vibration control systems [11]. The key benefits of MR suspension systems were that the systems used extremely little control power, were simple to build, responded quickly to control signals, and had relatively few moving components. The MR suspension systems have also attracted a lot of interest over the last two decades as a viable solution for semi-active control [12]. Automotive manufacturers prefer MR suspension systems due to their high strength, good controllability, wide dynamic range, fast response time, low energy consumption, and simple design [13].

Mohamed et al, also claimed that an MR suspension system in a semi-active system outperformed a passive system in terms of control performance [14]. The MR dampers are the most promising vibration control devices for intelligent suspensions today and are popular among automobile manufacturers due to their advantages such as high strength, good controllability, wide dynamic range, fast response speed, low energy consumption, and simple structure [15]. Furthermore, with the increasing need for comfortable and safe vehicle rides, intelligent suspension will be increasingly implemented in standard vehicles and engineering automobiles, resulting in a wider market for automobile suspension constructed of MR damper.

The use of magnetic fluid within a damping system was referred to as a magnetic fluid damper. Jacob Rabinow discovered MR fluid in 1948, and the US National Standards Bureau issued a patent for it in 1951 [16]. Many researchers have recently examined magneto-rheological (MR) fluids because their material properties can be modified by an applied electromagnetic field. Proper use of MR fluid technology requires that the base fluid has a low viscosity and does not vary with temperature. The MR fluids are an attractive class of smart materials with an unusual ability to change rapidly, nearly reversible, and large changes in apparent viscosity when an external magnetic field is applied [17]. The MR fluid is a carrier fluid containing micron-sized magnetically polarized particles, such as silicon or mineral oil [18]. When the fluid magnetic field was turned off, the oil behaved normally. The magnetic field is created when current flows through a magnetic coil. When this magnetic field is applied to the MR fluid, micrometer-sized iron particles dispersed in the fluid align along the magnetic flux lines, as seen in Fig.1 and Fig. 2.

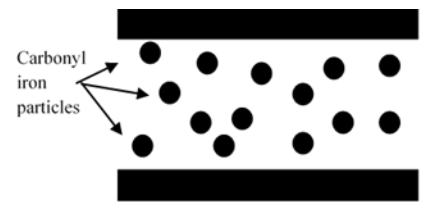

Figure 1. The MR Particle without Magnetic Field.

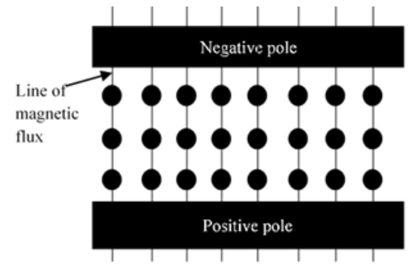

Figure 2. The MR Particle with Magnetic Field.

The solution for suspension protection can be particularly solve when the MR fluid damper appears. MR fluid had a higher yield stress than the electrorheological (ER) fluid, therefore it was used to create a variable damping force [19]. The damping force of an MR damper depends on the input current (voltage), relative speed, and movement of the piston and damper housing, so it cannot be directly controlled [20]. The damper controller remains a significant unresolved issue in determining the system's behavior.

The function of an MR damper is determined by how the MR fluid operates. The MR damper uses a controllable fluid and has the important ability to reversibly transform a free-flowing linear viscous fluid into a semi-solid fluid [21]. Several models have been created to simulate the behavior of MR suspension systems, including the Bingham model, Simple Bouc-Wen model, Modified Bouc-Wen model, Hyperbolic Tangent Function model, Nonlinear Viscous model, and Finite Element model. According to Yahaya et al, the Modified Bouc-Wen model is ideal for describing the behavior of MR dampers and can be used to create controllers and conduct experiments [22]. The Modified Bouc-Wen model has also successfully represented a wide range of nonlinear hysteretic systems.

According to all of the studies, the study of MR suspension systems is mainly focused on the intelligence of the systems themselves. However, mathematical modeling for a single MR suspension system utilizing the Modified Bouc-Wen model has not yet been investigated. Furthermore, there is less attention has been paid to the findings of the comparison between mathematical modeling and passive for quarter car Proton Preve. According to the review, no such investigation has been focused on the suspension system of the Proton Preve automobile.

Proton Preve is four -door compact saloon car developed by Malaysia automobile manufacturer, Proton. This model was claimed as the first global car by Proton and was launched on 16 April 2012. Proton Preve is also based on Proton's nextgeneration P2 platform [23]. In light of this, it is possible to view the work done to create and implement a single mathematical of MR systems with the Modified Bouc-Wen model for Proton Preve as an innovative and novel. To better understand and predict future behavior in the automotive industry, this model depicts a theoretical model of MR suspension systems.

The paper is organized into fifth sections, the first of which introduces the MR vehicle suspension system and the second of which describes the mathematical model for suspension systems of the passive vehicle as well as the new mathematical model solution in a single MR suspension system. Sections 3 compare vertical displacement for mathematical models and experimental work. The fifth section of this paper closes with a remarked conclusion.

The main objective of the research was to evaluate the displacement indicated by x(t), continued by plotting a graph of the MR single suspension system using MATLAB software and validating it with a graph from the passive model.

2. Mathematical Modelling: Passive System

In this paper, a mathematical model of an automobile suspension system is developed. To begin, the Proton Preve model was incorporated into a novel mathematical model for car suspension in passive systems. Mathematical modeling was accomplished using second-order linear differential equations.

The mathematical model was created by applying Newton`s Second law of motion to the car body in a vertical direction. The differential equation approach is used on automotive suspension systems, as well as a mathematical formulation based on a quarter automobile suspension system and a sinusoid road surface profile. The established passive mathematical model's link between vertical displacement and time was plotted using the MATLAB program. The validation procedure utilized a sinusoidal waveform pattern as shown in passive suspension as the graph reference.

The actual graph was derived from passive suspension systems, while the proposed model graph was derived from the mathematical modeling of a single MR. Whenever the proposed model's graph followed a structure analogous to that of the graph generated by the passive model, it was deemed acceptable. Due to the similarities in the patterns, the error rate was effectively zero. Section 5 will provide the model analysis that has been performed.

The work process flow diagram is displayed in Fig. 3. For both models, the passive suspension system and mathematical modeling of a single MR, every procedure was used. By applying the Modified Bouc-Wen model in the Proton Preve car, this study seeks to establish a model for Magneto-Rheological automobile suspension. The model was created by applying Newton's second law of motion along the vehicle's vertical axis. The differential equation method is also examined in this study, with its application to car suspension systems serving as the primary study object. The mathematical model is based on a sinusoid road surface profile and a suspension system for a quarter-scale automobile.

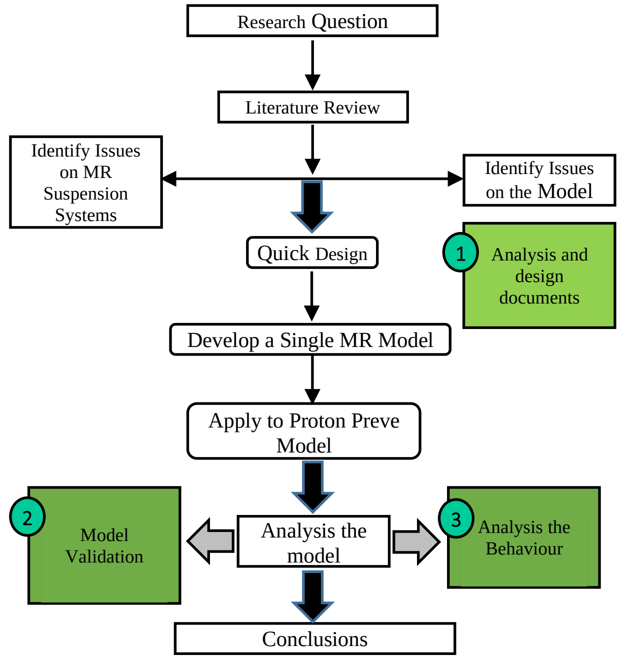

Figure 3. Work Process Flow Diagram [22].

2.1. The Process of Mathematical Modelling

Limiting the relative velocity of the body and its wheels, a passive suspension system ensures the desired ride quality is maintained [24]. Dampening devices, like hydraulic shock absorbers, are installed between the vehicle's frame and wheels to achieve this effect. The passive vehicle suspension design is based on the Ford Scorpio model.

The spring (k_p) and damper (c_p) linked to the wheel mass (m_1) and car body mass (m_2) are used in the model. Fig. 4 depicts the model.

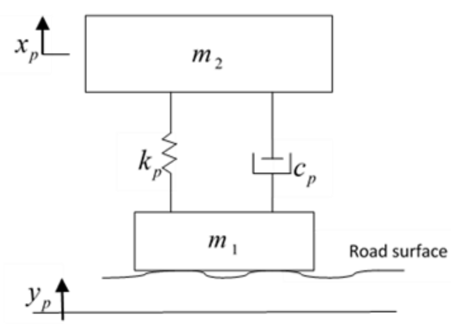

Figure 4. Suspension Model of Passive Systems in Car Body [25]

Apply Newton's second law of motion in the vertical direction to obtain the mathematical equation of motion. Let's assume the body and wheels are traveling vertically upwards, the spring and damper are given as

$$X_p(t) = x_p - y_p \tag{1}$$

Where:

X_p is the spring's extension;

x_p is the car body's vertical displacement beyond its balance position;

y_p is the wheel's vertical displacement.

When external forces are applied to the mechanical elements of a spring, the spring can become deformed. The relationship between acting force f_p and displacement x_p expressed as

$$F_1 = k_p x_p(t) \quad (2)$$

and the spring force gives as

$$F_1 = k_p\left(x_p - y_p\right) \quad (3)$$

Where:

F_1 is a spring's force;

k_p is a constant parameter;

$x_p - y_p$ is an extension of the spring.

The linear damping force is dealt with by this passive mechanism. From Fig. 4, a resistance for the motion of a damper is expressed as

$$\frac{dX_p(t)}{dt} = \frac{d(x_p - y_p)}{dt} \quad (4)$$

First order differential in Equation (4) can be converted as

$$v_p = x_p'(t) - y_p'(t) \quad (5)$$

Therefore, the damper force expresses as

$$F_2 = c_p v_p(t) \quad (6)$$

Where:

F_2 resists the damper's force;

c_p is the damping coefficient;

v_p the damper's relative velocity.

The Newton law of motion is implied in Fig. 5. The indication of Newton's second law of motion is

$$f = ma \quad (7)$$

Where:

m is the mass;

a is the acceleration;

f is the force.

Another way to express Newton's second law is as a second-order ordinary differential equation, such as

$$m_p''(t) = -F_1(t) - F_2(t) \quad (8)$$

where is $m_p''(t)$ the total force for the damper?

Fig. 5 depicts the modeled and simplified version of Fig. 4.

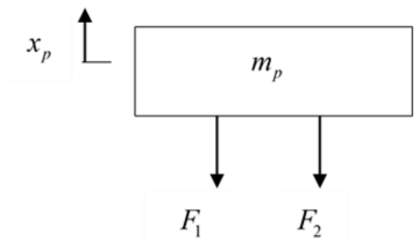

Figure 5. Car Model for Damper and Spring Forces [24].

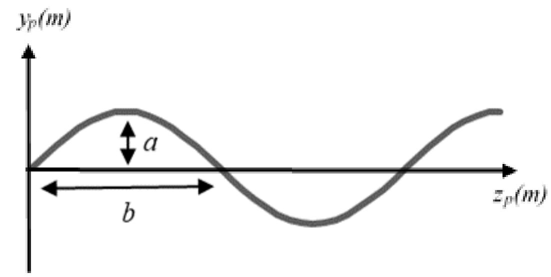

Figure 6. Sinusoidal-Shaped Profile.

According to Nanthakumar et al. [26], the profile of the road surface was characterized as a sinusoidal curve, which can be seen in Fig. 6. The road's sinusoidal profile is defined as

$$y = \alpha \sin\,(\alpha z) \quad (9)$$

where α represents the sinusoidal amplitude.
Assume that the car is traveling at an average horizontal speed, V along the road profile as

$$z_p = Vt \quad (10)$$

Therefore, substitute equation (10) into Equation (9) to yield

$$y_p(t) = \alpha sin\frac{\pi Vt}{b} \quad (11)$$

y_p is the vertical displacement of the wheel caused by the road surface (measured concerning a fixed horizontal reference line) z_p is the horizontal displacement and b represents the wave cycle.
Differentiate of $y(t)$ to yield

$$y_p'(t) = \alpha\frac{\pi V}{b} cos\frac{\pi Vt}{b} \quad (12)$$

The values for the parameter of the car are then used to validate the suggested model. The Proton Preve automobile model has the following parameters [24]:

$m_p = 290kg$; $k_p = 16812Nm^{-1}$;

$c_p = 1000Nsm^{-1}$; $V_p = 14ms^{-1}$;

$b = 2m$; $a = 0.1m$.

From Equation (8), it follows that

$$m_p''(t) = -k_p\left(x_p - y_p\right) - c_p\left(x_p'(t) - y_p'(t)\right) \quad (13)$$

$$m_p''(t) + c_p x_p'(t) + k_p x_p(t) = k_p y_p + c_p y_p'(t) \quad (14)$$

Substitute Equations (11) and (12) into Equation (14) to yield

$$290x''(t) + 1000x'(t) + 16812x(t) = 1681.2\sin(7\pi t) + 700\pi\cos(7\pi t) \quad (15)$$

Use the nonlinear form to solve Equation (15) such as in [24]

$$X_p(t) = X_1(t) + X_2(t) \quad (16)$$

where X_2 is a complementary solution and X_1 is a general specific solution. Moreover, the following equations are used to solve X_1:

$$290m_a^2 + 1000m_a + 16812 = 0 \quad (17)$$

$$m_a = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (18)$$

$$m_a = -1.72 \pm 7.42i \quad (19)$$

Because Equation (19) contains the complex conjugate's root, the solution for X_a can be found as

$$X_1(t) = e^{-1.72i}[a\cos(7.42)t + bsin(7.42)t] \quad (20)$$

The following equation is used to develop X_2 becomes

$$X_2(t) = k_p y_p(t) + c_p y_p'(t) \quad (21)$$

Substitute Equation (11) into Equation (21) to yield

$$X_2(t) = k_p \alpha sin\left(\frac{\pi Vt}{b}\right) + c.\alpha\left(\frac{\pi t}{b}\right)cos\left(\frac{\pi Vt}{b}\right) \quad (22)$$

Using data from the Proton Preve automobile model [23] to produce such as

$$X_2(t) = \propto cos(7\pi t) + \rho sin(7\pi t) \quad (23)$$

Where:

$$\propto = c_p\alpha\left(\frac{\pi t}{b}\right) \text{ and } \rho = k_p\alpha$$

Differentiate Equation (23) to yield

$$\dot{X}_2(t) = 7\pi\rho cos(7\pi t) - 7 \propto (7\pi t) \quad (24)$$

and

$$\ddot{X}_2(t) = -49\pi^2 \propto cos(7\pi t) - 49\pi^2\rho sin(7\pi t) \quad (25)$$

Substitute Equation (23), Equation (24), and Equation (25) into Equation (17) to express

$$290[-49\pi \propto cos(7\pi t) - 49\pi\rho sin(7\pi t)] + \pi\rho cos(7\pi t)7\pi \propto sin(7\pi t)$$

$$+16812[\propto sos(7\pi t) + \rho sin(7\pi t)$$

$$= 1681.2sin(7\pi t) + 700\pi cos(7\pi t) \quad (26)$$

Solve Equation (26) to obtain $\propto$ and ρ such as

$$-107992.92\gamma - 26389.38\alpha = 2500 \quad (27)$$

Then,

$$\propto = -0.02 \text{ and } \rho = -0.01$$

By substituting a value of $\propto$ and ρ thus

$$X_2(t) = -0.02(7\pi t) - 0.01sin(7\pi t) \quad (28)$$

Substitute Equations (20) and (28) into Equation (16) to produce

$$X_p(t) = e^{-1.72i}[mcos(7.42)t + nsin(7.42)t] - 0.02\cos(7\pi t) - 0.01sin(7\pi t) \quad (29)$$

To determine m and n, the following initial conditions are used:

$$X(0) = 0 \;\&\; X'(0) = 0 \quad (30)$$

Thus,

$$m = 0.02 \;\&\; n = 0.35 \quad (31)$$

Substitute the values, m, and n into (29) to depict them as

$$X_p(t) = e^{-1.72i}[0.02cos(7.42)t + 0.35sin(7.42)t] - 0.02\cos(7\pi t) - 0.01sin(7\pi t) \quad (32)$$

2.2. Mathematical Modelling of MR Single Suspension System

The new mathematical model will be introduced for a single MR suspension system, and the model will be solved using the Modified Bouc-Wen model. Mathematical modeling was developed using second-order linear differential equations. The relation between vertical displacement and time settlement was then plotted on a graph generated by converting the mathematical model using the Proton Preve parameter through the MATLAB program. The validation of a mathematical model employing dispersion models such as mean and standard deviation between a single MR and a passive model.

Fig. 7 depicts a quarter-vehicle model with a single MR suspension system. The Modified Bouc-Wen model can be used to describe the behavior of a single MR suspension system. This model was a prominent mathematical description for hysteretic systems such as MR systems [27]. Other parameters must be included in the model as shown in Fig. 7 to obtain more tidy findings [28].

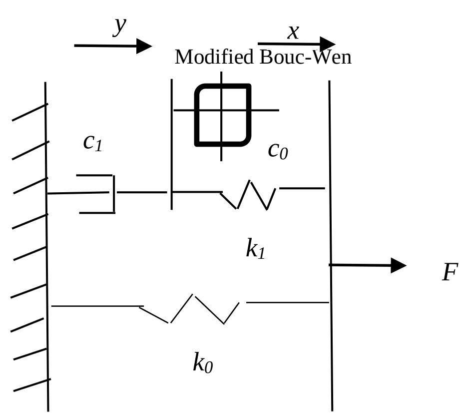

Figure 7. MR Single Model.

Where:

c_0 and c_1 are the damping;
k_0 and k_1 are the spring;
F is the damping force;
x is the vertical displacement of the car body;
y is the vertical displacement of the wheel.

Assume that when the movable magnet is extended, both the car body and wheels move vertically upwards, a mathematical is formulated from a differential equation in displacement function that can be linear or nonlinear in terms of magnetic field and electric current.

By applying Newton's second law in Equation (7), the model can be derived as

$$m_1\ddot{u}_1 = k_1(u_0 - u_1) + C(\dot{u}_0 - \dot{u}_1) + k_0(u_2 - u_1) + F \quad (33)$$

$$m_2\ddot{u}_2 = -k_1(u_2 - u_1) - F \quad (34)$$

Where:

m_1 for vehicle axle mass;
m_2 for vehicle body mass;
u_0 for vertical displacement of the road disturbance;
u_1 for vertical displacement of the vehicle axle;
u_2 for vertical displacement of the vehicle body;
k_0 is the damper's constant spring tension;
k_1 is the style spring constant;
C is the tire damping coefficient;
F for the damping force exerted by the MR damper.

From Equation (33) and Equation (34), the force generated from the damper can be expressed as follows by using Modified Bouc-Wen Model

$$F(t) = \alpha z + c_0(x - y) + k_0(x - y) + k_1(x - x_1) \quad (35)$$

with

$$\dot{y} = \frac{1}{c_0 + c_1}[\alpha z + k_0(x - y) + c_0 x] \quad (36)$$

and

$$\dot{z} = -\gamma|\dot{x} - \dot{y}||z|^{n-1}z - \beta(\dot{x} - \dot{y})|z|^n + \delta(\dot{x} - \dot{y}) \quad (37)$$

Where:

x_1 is the initial displacement;
c_0 and c_1 are the damping coefficients;
k_0 and k_1 are spring constants of damper in suspension systems;
α The parameter is the scaling value of the Bouc-Wen model;
γ, $\beta$and δ control the range and shape of the hysteresis loop;
$\dot{z}$ is hysteretic of z.
The effect of the magnetic field is caused by the voltage applied to the windings of the MR damper and can be explained as follows

$$\alpha = \alpha(u) = \alpha_a + \alpha_b u \quad (38)$$

$$c_0 = c_0(u) = c_{0a} + c_{0b} u \quad (39)$$

$$c_1 = c_1(u) = c_{1a} + c_{1b} u \quad (40)$$

where u is a first-order filter defined as

$$\dot{u} = -\varphi(u - V) \quad (41)$$

with

φ is the time constant for the filter;

V is the voltage that is applied for the current driver.

The estimated parameters at 1 Hz excitation for the Modified Bouc-Wen model are $r = 4.0\ mm^{-2}$ (no applied current) and $r = 0.1\ mm^{-2}$ (current applied), $A = 180$, $k_0 = 0$, $\beta = 0$ and $n = 2$ [29].

Numerical calculations of the Modified Bouc-Wen model were computed using MATLAB and are presented in Section 3.

3. Results

The main objective of the paper was to evaluate the displacement indicated by $x(t)$, continued by plotting a graph of the MR single suspension system using MATLAB software and validating it with a graph from the passive model.

Fig. 8 indicates the oscillation of the car body's vertical displacement is damping for the Proton Preve car. The line indicates passive systems using the data from the experimental and the dash-dot indicates passive systems using the model [22].

Table 1. Dispersion Model for Passive Suspension Systems Using Simulation and Experimental

Items	Experimental	Simulation
Standard Deviation	0.001	0.0025
Min Value	-0.016	-0.013
Max Value	0.002	0.005

From Table 1, at 0.002m, the displacement is maximum and minimum at -0.016m for the experimental. The maximum displacement for modeling suspension systems is 0.005 and the minimum at -0.013m. For stability, both of the graphs become stable at 4s. The standard deviation produced by the experimental work is 0.001 while the standard deviation from the mathematical model is 0.0025 [22].

As can be seen in Fig. 9, the blue line represents the results of a mathematical modeling of an MR single system, whereas the red dotted line represents the results of a model of a passive system.

Fig. 9 displays the similarity between the graph patterns of the passive model and the MR single model. For the stability region, both graphs showed that the systems began to stabilize at t = 4 s and reached full stabilization at t = 6 s. When comparing the passive model's graph to the single MR model's mathematical modeling, it is clear that their consistency at t = 4 s is very close. Compared to passive systems, the oscillation of the graph was closest to 0 m for single MR systems when the systems were initiated at t = 5 s.

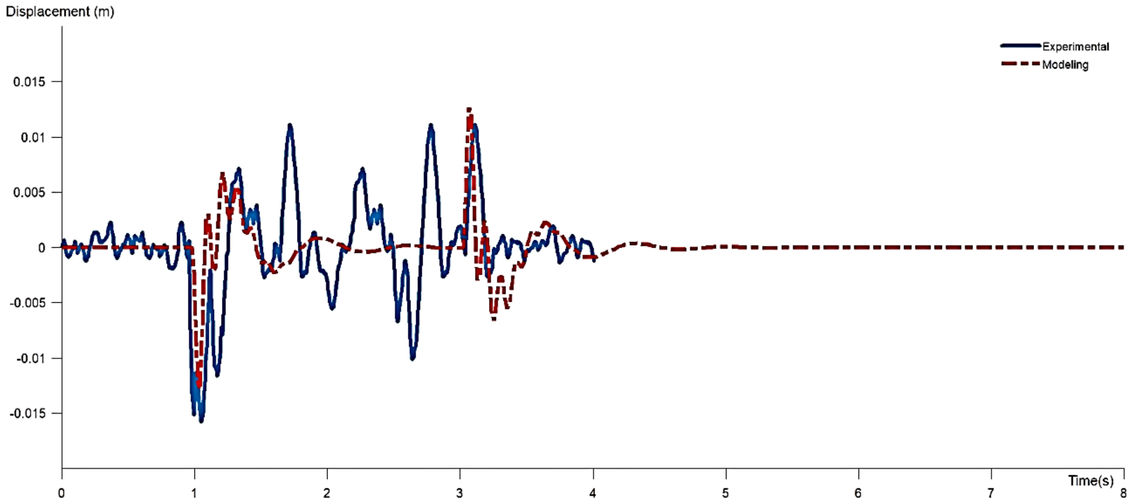

Figure 8. Displacement between Experimental and Simulation for Passive Systems [22].

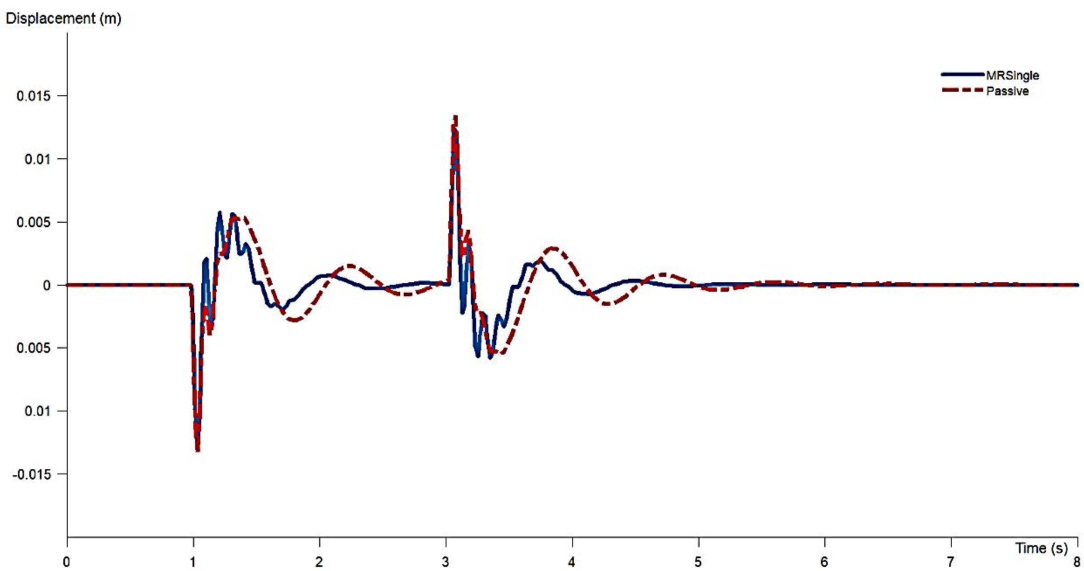

Figure 9. Displacement between MR Single and Passive Systems**.**

The results of the mathematical modeling of the MR single and passive suspension systems are shown in Table 2 for $x(t)$ in meters with a time (t) in seconds. This work looks at the mathematical model of a single MR suspension as well as the passive suspension systems. The MATLAB application is used to analyze the model's equation systems. Using the quarter Proton Preve automobile model, a passive and novel mathematical model of a single MR suspension system is created. The vehicle body's vertical displacement to its equilibrium position was $x(t)$, where $x(t)$ has been identified as the difference in spring movement of the car wheel.

Table 3 shows that the passive system model yielded a maximum $x(t)$ of 0.0532 m and a minimum $x(t)$ of -0.0133 m. Maximum and minimum values $x(t)$ for a single MR system were 0.031 m and -0.0124 m, respectively. The graphs' highest displacement showed that the spring was expanding, while its minimum displacement showed that the spring was shrinking. These decreases in $the\ x(t)$ value also demonstrated that the car ride comfort was growing greatly as the spring's responses were used to stabilize the car's body.

Table 2. Displacement with Time between MR Single and Passive Suspension Systems

Time(s)	MR Single	Passive
0.0	0.0000	0.0000
0.2	0.0000	0.0000
0.4	0.0000	0.0000
0.6	0.0000	0.0000
0.8	0.0000	0.0000
1.0	-0.0124	-0.0133
1.2	0.0006	-0.0003
1.4	0.0031	0.0053
1.6	-0.0003	-0.0016
1.8	-0.0009	-0.0027
2.0	0.0006	-0.0011
2.2	0.0015	0.0003
2.4	0.0007	-0.0003
2.6	-0.0002	-0.0005
2.8	0.0001	-0.0005
3.0	0.0003	0.0000
3.2	0.0001	-0.0014
3.4	-0.0048	-0.0053
3.6	-0.0001	-0.0024
3.8	0.0025	0.0002
4.0	0.0012	-0.0006
4.2	-0.0005	-0.0013
4.4	0.0002	-0.0011
4.6	0.0007	0.0001
4.8	0.0006	-0.0001
5.0	-0.0001	-0.0002
5.2	0.0000	-0.0004
5.4	0.0000	-0.0001
5.6	0.0000	0.0000
5.8	0.0000	-0.0002
6.0	0.0000	-0.0001
6.2	0.0000	0.0000
6.4	0.0000	0.0000
6.6	0.0000	0.0000
6.8	0.0000	0.0000
7.0	0.0000	0.0000

Table 3. Dispersion Model between MR Single and Passive Suspension Systems

Items	MR Single	Passive
Standard Deviation	0.0022	0.0025
Mean Value	0.00013	0.00088
Min Value	-0.0124	-0.0133
Max Value	0.0031	0.0053

In this work, the dispersion model was utilized to determine the model's stability and to assess data accuracy, whether convergent or divergent. A smaller standard deviation showed that the result was generally nearer the mean [30]. The passive model yielded a standard deviation of 0.0025, but the single MR mathematical modeling produced a standard deviation of 0.0022. Thus, the value obtained from the mathematical modeling of the single MR was lower than that of the passive model. A single MR modeling yielded a tendency for all $x(t)$ to approach the mean value which was 0.00013 (almost zero) compared to the passive model which is 0.00088. Less vibrations occurred when all $x(t)$ converged to zero. It proved that the single MR mathematical modeling had a good suspension system that was able to quickly absorb road impact and reduce vibration in a variety of road profiles while keeping the tires in contact with the road. Moreover, MR suspension systems provide good passenger comfort and vehicle stability, and the systems have a high potential for future development

There were a few limitations in this work such as using a quarter car model for Proton Preve. Therefore, the full-car model of Proton Preve can be considered for future work. Car handling and ride comfort performance can be studied concurrently when applying the full car model.

4. Conclusions

This work effectively introduced the mathematical modeling of a single MR suspension system using the Proton Preve model. This model was built with second-order differential equations and supported with MATLAB software. The dispersion model of riding performance was utilized to analyze the graph behavior. The displacement of the car and the stability of the systems were examined for modeling accuracy using the model from passive systems. The findings showed that the vertical displacement behavior of MR and passive suspension systems was the same. When the vertical displacement of the model was approximated to zero, the vibration for the systems was minimized, giving comfort and enhanced car control. The mathematical modeling of MR single applied in the Proton Preve model has proven significantly.

Future work may provide a double Magneto-Rheological (MR) suspension system based on a Modified Bouc-Wen model. A comprehensive automotive model can be modeled using mathematical modeling for a double MR suspension system. Therefore, it would be more interesting to evaluate the outcomes in terms of the car system's comfort and stability further.

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Conflict of interest

The manuscript has not been published elsewhere and is not under consideration by other journals. All authors have approved the review, agree with its submission, and declare no conflict of interest in the manuscript.

Author Contribution Statement

Siti Farizah Yaakub proposed the research problem, developed and solved the model, ran the simulation, confirmed their validity and efficacy, and wrote the original draft. Saifudin Hafiz Yahaya verified data curation, validation, and supervision. Mohd Shukor Salleh reviewed the manuscript. All authors discussed the results and contributed to the final manuscript.

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