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AN EFFICIENET ALGORITHM FOR STATIC STATE ESTIMATION IN ELECTRIC POWER SYSTEM

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Abstract: This paper presents an efficient static state estimator that is suitable for real time monitoring of the power system. The proposed algorithm includes a weighted least square (WLS) method based on rectangular coordinates system. A predictor-corrector iterative technique is used in solving the state estimator model which is described by a set of nonlinear equations relating measured quantities and the state variables. The proposed algorithm is enhanced by selecting the allocation the PMUs meters optimally by using Particle Swarm Optimization (PSO). The proposed algorithm is applied to the IEEE-14 bus and IEEE-30 bus test systems. The obtained results reveal the significant contribution of the proposed algorithm in optimal estimate of the static state in terms of number of iterations for convergence, execution time and accuracy.

Keywords: *State estimation, rectangular coordinates , Phasor measurement units, predictor-corrector.*

خوارزمية كفوءة لتخمين الحالة الساكنة في منظومة القدرة الكهربائية

الخلاصة: يقدم هذا البحث خوارزمية جديدة كفوءة لتخمين الحالة الساكنة لمنظومة القدرة ملائمة للمراقبة الزمنية الحقيقية. تتضمن الخوارزمية المقترحة طريقة العامل التربيعي الاصغر الموزون بالاستناد على استخدام الاحداثيات المتعامدة لمتغيرات الحالة. تم استخدام تقنية التنبؤ والتصويب العددية في حل نموذج مخمّن الحالة الموصوف بمجموعة من المعادلات غير الخطية التي تربط الكميات المقاسة و متغيرات الحالة. تم تعزيز الخوارزمية باختيار المواقع المثلى لاجهزة قياس زاوية الطور باستخدام طريقة تقنية أمثلية الحشد الجزيئي . تم تطبيق الخوارزمية المقترحة على منظومة معهد المهندسين الكهربائيين القياسية ذات (14) عقدة و (30) عقدة. وقد أظهرت النتائج المستحصلة المساهمة المهمة للخوارزمية المقترحة في التخمين الامثل للحالة الساكنة لمنظومة القدرة بدلالة عدد التكرارات اللازمة لاقتراب الحل، زمن التنفيذ والدقة.

1. Introduction

Static state estimation is an essential analysis utilized in the real-time monitoring of the power system. It is the cornerstone of the power system security analysis. State estimation determines the optimal static state of the system (voltage magnitude and phase angle) by processing the available measurements based on an appropriate system model. Extensive research have been carried out which addressed the various functions

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of the state estimation since the first attempt made by Schweppe [1,2]. Different approaches have been proposed covering network topology processing [3-10].
The improvement of the state estimator by maintaining system observability has been demonstrated in [11-14]. Several techniques have been introduced on bad data detection and elimination [15-19]. Optimal estimate of the state vector is attracting the attention of many researchers. The most commonly used method in obtaining the optimal solution vector is the weighted least square (WLS). The state estimator model is described by a set of nonlinear equations relating measured quantities and the state variables. Singh and Alvarado proposed a weighted least absolute value state estimator by using interior point methods [20]. An attempt to obtain the optimum state vector based on singular value decomposition was proposed by Madtharad [21]. State estimation by using bilinear approach by introducing auxiliary state variables and auxiliary measurements were presented in [22, 23]. A state estimator algorithm based on fast decoupled WLS was presented in [24]. Abbasi and Seifi [25] suggested a master-slave-splitting technique to estimate the state variables of global power system. State estimator algorithm using a direct non-iterative method was given in [26]. A probabilistic approach to power system state estimation was demonstrated in [27]. A regularized based state estimator was developed in [28] to overcome the numerical instability of ill-conditioned state estimation problems. Recently, it was found that the redundancy in measurements can be achieved by incorporating the phasor measurements unit (PMU) in power system monitoring [29-31]. Since the solution of a WLS state estimator is based on iterative technique, it is very important to develop an algorithm that determines the optimal state accurately in a short time and suitable for real time application.

In this paper an efficient algorithm is proposed to obtain the optimal static state vector by using the WLS based on the rectangular coordinates of the state vector. The proposed algorithm deployed a predictor-corrector numerical technique in determining the state vector. The proposed algorithm is integrated with another algorithm that identifies the optimal placement of PMU meters and conventional meters by using an artificial technique, namely Particle Swarm Optimization technique.

This paper is organized as follows: Section 2 reviews the traditional weighted least square method. The state estimator model based on rectangular coordinate is presented in section 3. Section 4 presents the elements of the proposed algorithm. In Section 5, a discussion of the results obtained by applying the proposed algorithm on typical test systems is presented. Finally, section 6 presents the conclusion.

2. Traditional WLS Technique

The vector of measured quantities is related to the state vector by the following non-linear system of equation [32]:

$$[z] = [h_{(x)}] + [e] \tag{1}$$

where:

$[z]$: Measurement vector
$[h_i(x)]$: nonlinear function describes the i^{th} measurement in terms of the state variables
x : system state vector (voltage magnitudes and angles)
$[e_i]$: the error of the i^{th} measurement
The best solution of state estimate vector *x* may be determined by minimizing the sum of weighted squares of residuals

$$\min \mathrm{J}(\hat{x}) = \sum_{i=1}^{m} \frac{{e_i}^2}{{\sigma_i}^2} \tag{2}$$

where:
$[w_i] = [R_i]^{-1} = \left[\frac{1}{{\sigma_i}^2}\right]$
$[R_i]$: is a diagonal matrix whose elements are the variances of the measurement error.
$[w_i]$: weighting factor is defined by the inverse of the measurements variances. Consequently, measurements of a higher quality have smaller variances that relates to their weights. Equation (2) can be represented in matrix form:

$$\min \mathrm{J}(\hat{x}) = [z - h(\hat{x})]^{T} R^{-1} [z - h(\hat{x})] \tag{3}$$

The necessary conditions for a minimum is that :

$$\mathrm{g}(x) = \frac{\partial J(x)}{\partial x} = -H(x)^T R^{-1} [z - h(x)] = 0 \tag{4}$$

$$\text{Where, } [\mathrm{H}(x)] = \left[\frac{\partial \boldsymbol{h}(x)}{\partial x}\right]$$

$$x^{k+1} = x^k - \left[G(x^k)\right]^{-1} . \mathrm{g}(x^k) \tag{5}$$

$$G(x^k) = \frac{\partial g(x^k)}{\partial x} = \left[\mathrm{H}(x^k)\right]^T R^{-1} [z - h(x^k)] \tag{6}$$

$$G(x^k) = \left[\mathrm{H}(x^k)\right]^T R^{-1} \left[\mathrm{H}(x^k)\right] \tag{7}$$

$$[G(x^k)][\Delta x^{k+1}] = [\mathrm{F}(x_i)] \tag{8}$$

Where,
$[\mathrm{F}(x_i)] = [\mathrm{H}(x^k)]^T R^{-1} [z - h(x^k)]$
$\Delta x^{k+1}: deviation\ of\ state\ factor = x^{k+1} - x^k$
$[\mathrm{H}(x)]$ is the measurement Jacobian matrix of dimension ($m \times n$)
k : iteration index
x^k : is the state vector at iteration k
$[G(x)]$: the gain matrix

Equation(8) is solved iteratively for the state vector until Max $|\Delta x^k| < \varepsilon$ where ε is a very small value.

3. Formulation of the State Estimator by using Rectangular Coordinates

The rectangular coordinate system has a better conditioning of a WLS estimation process than for the polar coordinate system. Since the polar coordinate is represented by transcendental functions, the Taylor series expansion of these functions is an infinite one. The rectangular coordinate system is based on quadratic terms. This leads to a major simplification of an expansion in Taylor series for J(x) in the rectangular coordinates [33]. The bus voltage is described by the following rectangular form:

$$V_i = e_i + \mathrm{j} f_i \tag{9}$$

where, e_i , f_i: a real and imaginary part of the voltage at bus *i* respectively
The state vector of the system is described as $[\boldsymbol{x}]^T = [f_2 f_3 \dots f_n \; e_1 e_2 \dots e_n]$
The real and reactive power injection at bus i :

$$P_i = \sum_{j=1}^{N} (e_i e_j G_{ij} - e_i f_j B_{ij} + e_j f_i B_{ij} + f_i f_j G_{ij}) \tag{10}$$

$$Q_i = \sum_{j=1}^{N} (-e_i e_j B_{ij} - e_i f_j G_{ij} + e_j f_i G_{ij} - f_i f_j B_{ij}) \tag{11}$$

The real and reactive power flow from bus *i* to bus *j* :

$$P_{ij} = -{e_i}^2 G_{ij} + e_i e_j G_{ij} - e_i f_j B_{ij} - {f_i}^2 G_{ij} + e_j f_i B_{ij} + f_i f_j G_{ij} \tag{12}$$

$$Q_{ij} = {e_i}^2 B_{ij} - e_i e_j B_{ij} - e_i f_j G_{ij} + {f_i}^2 B_{ij} + e_j f_i G_{ij} - f_i f_j B_{ij} \tag{13}$$

The structure of the measurement Jacobian $[H]$ will be as follows :

$$[H] = \begin{bmatrix} \frac{\partial V}{\partial f} & \frac{\partial V}{\partial e} \\ \frac{\partial P_i}{\partial f} & \frac{\partial P_i}{\partial e} \\ \frac{\partial Q_i}{\partial f} & \frac{\partial Q_i}{\partial e} \\ \frac{\partial P_{ij}}{\partial f} & \frac{\partial P_{ij}}{\partial e} \\ \frac{\partial Q_{ij}}{\partial f} & \frac{\partial Q_{ij}}{\partial e} \end{bmatrix}$$

4. Proposed Algorithm

In the initial phase of the proposed algorithm, the PMU devices to be installed have to be placed optimally by using Particle Swarm Optimization technique. This optimal location will maintain a redundancy in the measurements and improve the observability of the system.

4.1 Augmentation of the PMU in the Estimator Model

Let $[z_2]$ indicate PMU measurements, which contain voltage magnitudes, voltage angles, real and imaginary parts of current phasors. The matrix $[R_2]$ represents the measurement error covariance matrix of $[z_2]$. By adding the vector of PMU measurements $[z_2]$ to the vector of conventional measurement $[z_1]$ yields The new measurement set $[z]$.

$$[z] = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} z_1 \\ v_{PMU\_mag} \\ v_{PMU\_ang} \\ I_{PMU\_real} \\ I_{PMU\_Img} \end{bmatrix} \tag{14}$$

where, V_{PMU_mag} is the voltage magnitude measured by PMUs and V_{PMU_ang} is the phase angle measured by PMUs. I_{PMU_real} and I_{PMU_Img} are the real part and imaginary part of the current measured by PMUs.
Current phasor measurements can be included in the SE model by using the rectangular representation:

$$I_{PMU\_real} = e_i G_{ij} - e_j G_{ij} - f_i B_{ij} + f_j B_{ij} - \frac{B}{2} f_i \tag{15}$$

$$I_{PMU\_Img} = f_i G_{ij} - f_j G_{ij} + e_i B_{ij} - e_j B_{ij} + \frac{B}{2} e_i \tag{16}$$

Let $[h(x)]$ and $[h_2(x)]$ be the nonlinear equations of new measurement set $[z]$ and PMU measurements$[z_2]$,respectively. The new Jacobian matrix corresponding to the measurement set $[z]$ will be given as follows:

$$[H] = \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial h_1(x)}{\partial x} \\ \frac{\partial h_2(x)}{\partial x} \end{bmatrix} \tag{17}$$

Therefore, the state solution of the WLS state estimator can be written as follows:

$$[x_{i+1}] = [x_i] + [H^T\, R^{-1}\, H]^{-1}\, [H]^T\, [R]^{-1} [z - h(x_i)] \tag{18}$$

where, the error covariance matrix of measurement set[z] with mixing is

$$[R] = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} \tag{19}$$

4.2 Particle Swarm Optimization(PSO)

Eberhart and Kennedy presented a Particle swarm optimization (PSO) in 1995 as a good substitute to genetic algorithm (GA). PSO is one of the population based artificial intelligence (AI) algorithms. PSO concept was used to graphically simulate the smart and unforeseeable choreography of a bird flock, with the aim of detecting patterns that govern the ability to fly synchronously, and to suddenly alternate direction with a regrouping in an optimal formation. With this in consideration, it was realized that the conceptual model was, indeed a simple and efficient optimization algorithm. Each particle(individual) in the population can be considered as candidate solution. The location of each particle is symbolized by xy-axis site. The symbol v_x (the velocity of x-axis) represents velocity (displacement vector) while v_y represents (the velocity of y-axis). The location and the velocity information of the individual are used in modifying the location of the individual[34].
This modification can be expressed by the concept of velocity. The velocity of each individual is adjusted by the following equation:

$$v_i^{k+1} = w_i \; v_i^k + \; c_1 \, rand_1 \times (Pbest_i - s_i^k) + \; c_2 \, rand_2 \times (gbest_i - s_i^k) \tag{20}$$

where v_i^k is present velocity of individual i at iteration k, $rand_1$ and $rand_2$ are random numbers between 0 and 1, s_i^k is present site of individual i at iteration k, $Pbest_i$ is $Pbest$ of individual i, $gbest$ is $gbest$ of the group, w_i is weight function for speed of individual i, generally, the variation of the value of w_i is assumed to be linear from 0.9 to 0.4 as the iterative process continues. c_i is weight constants for cognitive and neighboring term.
Typically ,the weighting function is used in following equation (20):

$$w = w_{max} - \frac{w_{max} - w_{min}}{iter_{max}} \times iter \tag{21}$$

where w_{max} is the initial weight, w_{min} is final weight, $iter_{max}$ is maximum iteration number, $iter$ is iteration index. Using the above equation (21), diversification characteristics is progressively reduced. Based on equation (20), a specific velocity that progressively obtains close to pbests and gbest can be calculated. The present site (searching point in the solution space) can be adjusted by the following formula:

$$s_i^{k+1} = s_i^k + v_i^{k+1} \tag{22}$$

4.3 Optimal Placement Algorithm of PMU

Step1: Inputting traditional SCADA measurements which include bus voltages, line flows and power injections. Also inputting PMUs Measurements which include voltage magnitude, angle, real and imaginary parts of current. Inputting bus limit.
Step2: Initializing the PSO parameters. Setting up the set of PSO parameters such as, number of individuals (Number of variables (N)), acceleration constants ($C1$ and $C2$), maximum number of iteration, maximum and minimum of Inertia weight (W) and Population size .

Step3: Calculating the state estimation by using the traditional method (WLS).
Step4: Randomly creating an initial population of individuals (locations of PMU) ,and the positions and the velocities of the individuals. Setting the iteration counter $= 0$.
Step5: For each individual (location of PMU), if the bus number is within the limits, the state estimation is calculated using WLS method. Otherwise, that individual (location of PMU) is not feasible.
Step6: Recording and updating the best values. The two best values are stored in the searching process. Each individual moves in the direction related to its previously best solution it has reached so far which is stored as P_{best} . Another best value to be stored is the G_{best} , which accounts for the global best value achieved by neighboring individuals. P_{best} and G_{best} are the minimum value of the objective function. This step also updates P_{best} and G_{best} . Initially, the fitness of each individual is compared with its P_{best}. If the current solution is better than its P_{best} , then P_{best} is replaced by the current solution. Then, the fitness of any particle is compared with G_{best} . If the fitness of any individual is better than G_{best} , then G_{best} is replaced .
Step7: Updating the velocity and position of the location of PMU. Equation (20) is applied to update the velocity and position of the individual (location of PMU). A movement in the direction of chosen bus is represented by the velocity of an individual. Meanwhile, the position of the individuals is updated by applying Equation (22).
Step8: Checking End criterion. The end criterion is checked, if it is satisfied, the algorithm is stopped; otherwise, step 3-7 is repeated until the end criterion is satisfied.
In this work, the individuals are locations of PMUs as shown below:

$$P_{\text{particle}} = [X_1 \quad X_2 \quad \cdots X_n] \tag{23}$$

where,
n : is the number of PMU with selected bus, which limits according to the system size and limitations.
X : location of PMU.

The Constraints

The PSO procedures must comply with certain constraints: (i) each location of PMU is tested to verify if location number is between 2 and N bus in general (ii) only one PMU can be placed at each bus; (iii) 2 PMU cannot be located at buses at the edges of the same power line. Since PMU also gives information about current phasors in the lines connected to one bus, there is enough information to determine the voltage phasor at the other bus.

The Fitness Function (objective function)

In this work, PSO is implemented to find the optimal locations of PMUs through minimizing the following objective function:

$$FF_{min} = [R] - [H][G]^{-1}\,[H]^{\mathrm{T}} \tag{24}$$

The process will be continued until the maximum iteration and number of population reaches a prescribed value. Figure (1) shows the procedure of PSO technique. Table (1) illustrates parameters of technique used.

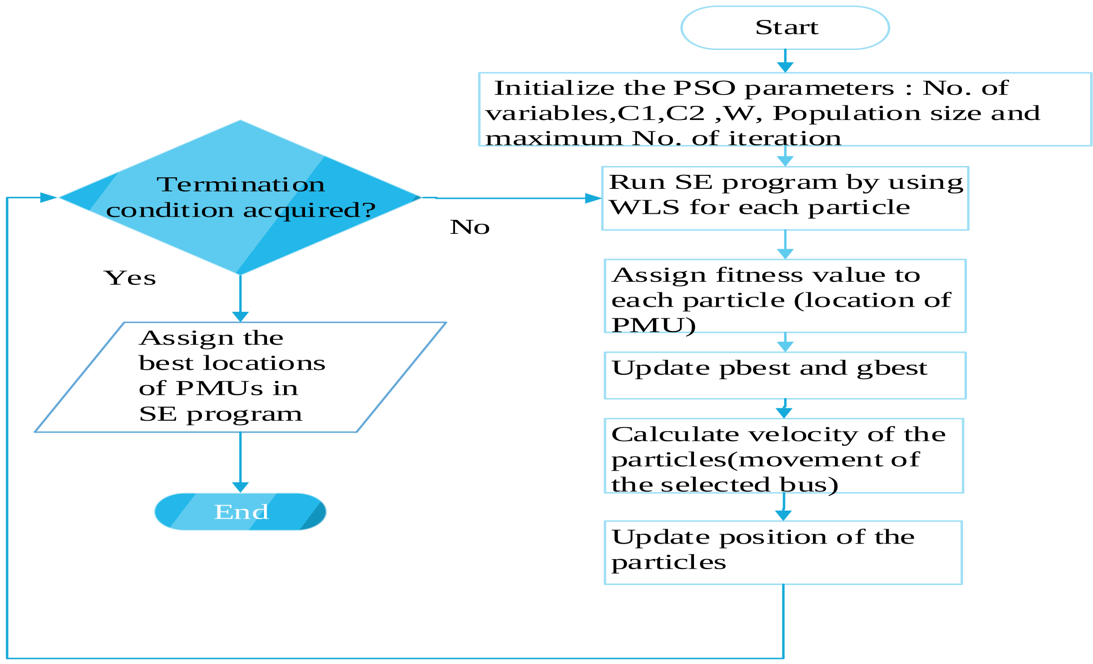

Figure 1. Flowchart of optimal PMUs location

Table 1. Parameter of PSO for Optimal PMUs Location

For Optimal PMUs Location (Parameter of PSO)	Number or Value
Number of variables (n) (Number of PMUs)	3 PMUs
Maximum Inertia weight (wmax)	0.9
Minimum Inertia weight (wmin)	0.4
Cognitive acceleration factor (C1)	1.5
Social acceleration factor (C2)	1.5
Population size (nPop)	25
Maximum number of iteration(Maxite)	100

4.4 Predictor-Corrector Iterative Technique

In a previous paper [35], the author applied successfully a predictor–corrector iterative technique in obtaining load flow solution. A brief description of the predictor –corrector iterative technique is given in Appendix A. The predictor –corrector iterative technique can be applied to equation (8) as follow:

$$[x_{i+1}] = [x_i] + 12\left[\mathrm{G}_{(x_i)} + 10\mathrm{G}_{(w_i)} + \mathrm{G}_{(y_i)}\right]^{-1}[\mathrm{F}(x_i)] \qquad (25)$$

Where,

$\left[\mathrm{G}_{(x_i)}\right]$: the gain matrix calculated at initial point.

$\left[\mathrm{G}_{(w_i)}\right]$: the gain matrix calculated at mid point.

$[\mathrm{G}_{(y_i)}]$: the gain matrix calculated at prediction point.
Equation(25) is solved iteratively for the state vector until Max $|\Delta x^k| < \varepsilon$ where ε is a very small value.

5. Results and Discussion

The algorithms are examined and tested on IEEE-14, IEEE-30 bus standard test system. In order to assess the performance of the state estimator, a power flow solution of each system is set as a benchmark for comparison. The Mean Square Error (MSE) is used as index to illustrate the accuracy of the proposed algorithm. MSE is defined in equation (26) as follows:

$$MSE = \frac{1}{N}\sum_{i=1}^{N}\left(|A_i{}^{true} - A_i{}^{est}|^2\right) \tag{26}$$

Where, N: number value of measurement.
$A_i{}^{true}$: true value of measurement i.
$A_i{}^{est}$: estimated value of measurement i.

5.1 IEEE-14 bus

The proposed algorithm is applied to determine the best estimate vector of IEEE-14 bus test system with a set of 41 conventional meters[36]. An additional three PMU meters have to be installed in the system to improve the observability.

By applying the PSO optimal placement algorithm to the system to select the optimal locations of the PMU meters, it is found that the best location of the meters are buses : 11, 12 and 14. The results of applying the proposed algorithm to the IEEE-14 bus test system are given in Tables (2,3) and Figures (2,3).

There is a clear discrepancy between the true bus bar voltages and the estimated voltages obtained by using traditional WLS, while the estimated voltages vector obtained by using the proposed algorithm is similar to the true voltages. It can be seen that the accuracy of proposed method is better as compared with conventional method (WLS). The MSE for voltage estimates in proposed method (0.000005) is less than WLS (0.00311) as shown from Table (6). Also from Table (7) it can be seen that MSE for bus angle in proposed method (0.00000004) is less than WLS (0.000655).

It is found that the proposed algorithm is converged within 7 iterations, while the traditional WLS algorithm is converged within 13 iterations. On the other hand, the execution time for proposed method less than WLS and that shown in Table (8).

Table 2.Actual voltages, estimated voltage, and errors for the 14 Bus system

Bus No.	Voltage Actual (p.u)	Voltage WLS Polar Form	Voltage Proposed Algorithm	Abs. Error WLS Polar Form	Abs. error Proposed Algorithm
1	1.06	1.0068	1.0621	0.0532	0.0021
2	1.045	0.9899	1.0459	0.0551	0.0009
3	1.01	0.9518	1.0093	0.0582	0.0007
4	1.01423	0.9579	1.0145	0.05633	0.00027
5	1.01724	0.9615	1.0179	0.05574	0.00066
6	1.07	1.0185	1.07	0.0515	0
7	1.05034	0.9919	1.0482	0.05844	0.00214
8	1.09	1.0287	1.0826	0.0613	0.0074
9	1.03371	0.9763	1.0335	0.05741	0.00021
10	1.03256	0.9758	1.0325	0.05676	0.00006
11	1.04748	0.9932	1.0474	0.05428	0.00008
12	1.0535	1.0009	1.0534	0.0526	0.0001
13	1.04711	0.9940	1.047	0.05311	0.00011
14	1.02131	0.9647	1.0212	0.05661	0.00011
MSE				0.00311	0.000005
No. of iter.	5	13	7		

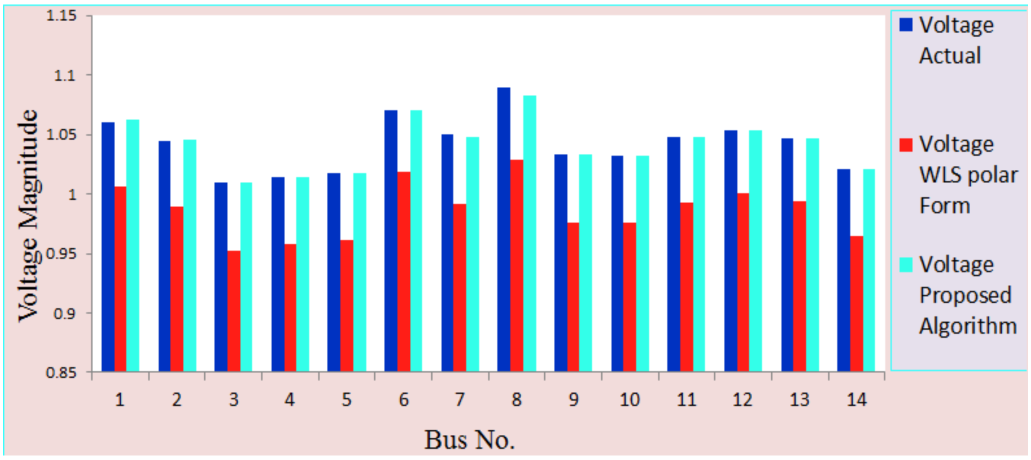

Figure 2. Comparison between actual and estimated values of the bus voltage magnitude

Table 3.Actual voltage angles, estimated angles, and errors for the 14 Bus system

Bus No.	Angle Actual (rad.)	Angle WLS (rad.)	Angle for Proposed Algorithm (rad.)	Abs. error WLS	Abs. Error for Proposed Algorithm
1	0	0	0	0	0
2	-0.08705	-0.09646	-0.08667	0.00941	0.00038
3	-0.2224	-0.2479	-0.22199	0.0255	0.00041
4	-0.17901	-0.19922	-0.17859	0.02021	0.00042
5	-0.15297	-0.17031	-0.15279	0.01734	0.00018
6	-0.25164	-0.28065	-0.2516	0.02901	0.00004
7	-0.23129	-0.25745	-0.23113	0.02616	0.00016

8	-0.23129	-0.25743	-0.23119	0.02614	0.0001
9	-0.25887	-0.2882	-0.25878	0.02933	0.00009
10	-0.26252	-0.2923	-0.26247	0.02978	0.00005
11	-0.25914	-0.28867	-0.2591	0.02953	0.00004
12	-0.26648	-0.29706	-0.26644	0.03058	0.00004
13	-0.26718	-0.29772	-0.26712	0.03054	0.00006
14	-0.28038	-0.31235	-0.2803	0.03197	0.00008
MSE				0.000655	0.00000004

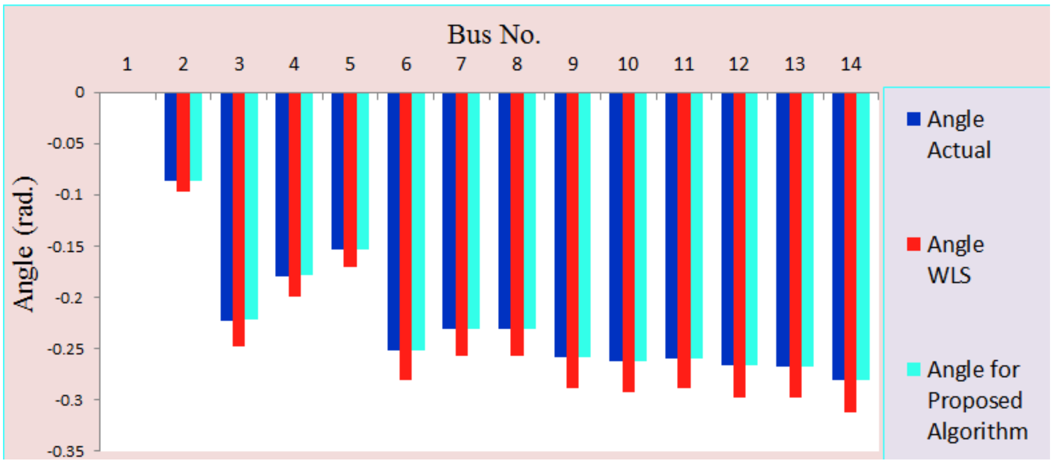

Figure 3.Comparison between actual and estimated values of the bus phase angle

5.2 IEEE-30 bus System

The set of measurement of the IEEE-30 bus system consists 93 conventional meters [36]. An additional three PMU meters have to be installed in the system to improve the observability . By applying the PSO optimal placement algorithm to the system to select the optimal locations of the PMU meters, it is found that the best locations of the meters are buses: 13, 26 and 30.

From the results of IEEE-30 bus test system in Tables (4,5) and Figures (4,5) reveal that the proposed method is more accurate than the conventional method (WLS). From Table (6) it can be seen that MSE for voltage estimates in proposed method (0.0000068) is less than WLS (0.00553). The MSE for bus angle in proposed method (0.0000017) is less than WLS (0.00185) as shown from Table(7). The number of iteration for proposed method (7 iter.) is less than WLS (12 iter.). On the other hand, the execution time for proposed method is less than WLS and that shown in Table(8).

Table 4.Actual voltages, estimated voltage, and errors for the 30 Bus system

Bus No.	Voltage Actual (p.u)	Voltage WLS polar Form	Voltage Proposed Algorithm	Abs. Error WLS Polar Form	Abs. Error Proposed Algorithm
1	1.06	0.9865	1.0556	0.0735	0.0044
2	1.043	0.97	1.0429	0.073	0.0001
3	1.01964	0.9474	1.0214	0.07224	0.00176
4	1.01041	0.9384	1.013	0.07201	0.00259

5	1.01	0.9335	1.009	0.0765	0.001
6	1.00958	0.9395	1.0143	0.07008	0.00472
7	1.00197	0.9287	1.004	0.07327	0.00203
8	1.01	0.9449	1.0194	0.0651	0.0094
9	1.03925	0.9667	1.04	0.07255	0.00075
10	1.02147	0.9472	1.0224	0.07427	0.00093
11	1.082	1.0093	1.0798	0.0727	0.0022
12	1.04959	0.9746	1.0495	0.07499	0.00009
13	1.071	0.9954	1.0709	0.0756	0.0001
14	1.03202	0.9559	1.0321	0.07612	0.00008
15	1.02508	0.9491	1.0254	0.07598	0.00032
16	1.03042	0.9555	1.0307	0.07492	0.00028
17	1.01876	0.9441	1.0194	0.07466	0.00064
18	1.01145	0.9352	1.012	0.07625	0.00055
19	1.00656	0.9306	1.0073	0.07596	0.00074
20	1.0095	0.9339	1.0102	0.0756	0.0007
21	1.00819	0.9328	1.009	0.07539	0.00081
22	1.01196	0.9372	1.0129	0.07476	0.00094
23	1.00855	0.9331	1.0093	0.07545	0.00075
24	0.99908	0.9231	0.9999	0.07598	0.00082
25	1.00318	0.927	1.0033	0.07618	0.00012
26	0.98525	0.907	0.9853	0.07825	0.00005
27	1.01445	0.9395	1.0146	0.07495	0.00015
28	1.00779	0.9398	1.0148	0.06799	0.00701
29	0.99442	0.9176	0.9946	0.07682	0.00018
30	0.98284	0.9051	0.983	0.07774	0.00016
MSE				0.00553	0.0000068
No. of iter.	6	12	7		

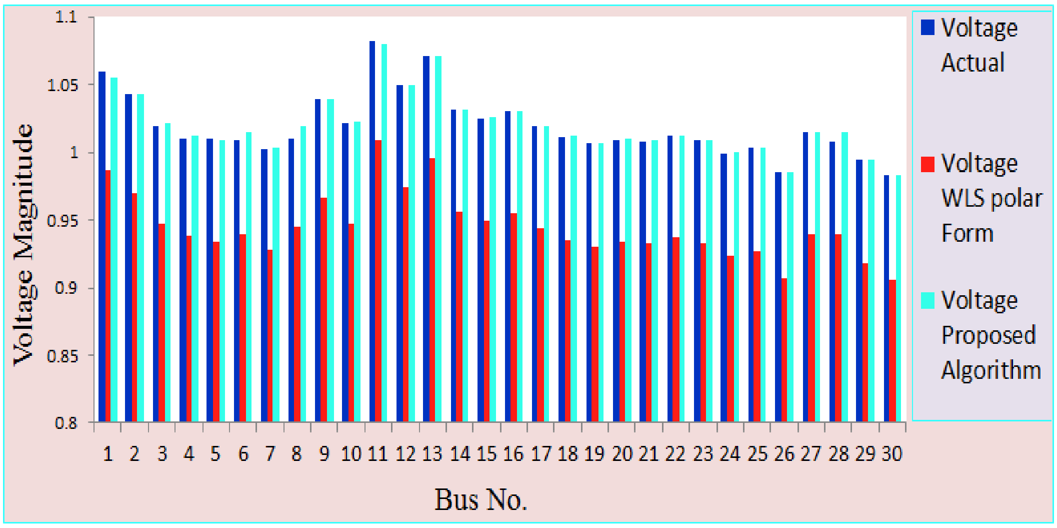

Figure4.Comparison between actual and estimated values of the bus voltage magnitude

Table 5.Actual voltage angles, estimated angles, and errors for the 30 Bus system

Bus No.	Angle Actual (rad.)	Angle WLS (rad.)	Angle Proposed Algorithm (rad.)	Abs. Error WLS	Abs. Error Proposed Algorithm
1	0	0	0	0	0
2	-0.09345	-0.1093	-0.0941	0.01585	0.00065
3	-0.13144	-0.1543	-0.1329	0.02286	0.00146
4	-0.16204	-0.1903	-0.1636	0.02826	0.00156
5	-0.24738	-0.2879	-0.2477	0.04052	0.00032
6	-0.193	-0.2269	-0.1951	0.0339	0.0021
7	-0.22453	-0.2626	-0.2258	0.03807	0.00127
8	-0.20629	-0.2437	-0.2096	0.03741	0.00331
9	-0.24547	-0.2877	-0.2472	0.04223	0.00173
10	-0.2735	-0.3202	-0.275	0.0467	0.0015
11	-0.24547	-0.2877	-0.2472	0.04223	0.00173
12	-0.26397	-0.3088	-0.2643	0.04483	0.00033
13	-0.26397	-0.3088	-0.2643	0.04483	0.00033
14	-0.27928	-0.3266	-0.2797	0.04732	0.00042
15	-0.2794	-0.3269	-0.2801	0.0475	0.0007
16	-0.27271	-0.319	-0.2737	0.04629	0.00099
17	-0.27696	-0.3241	-0.2784	0.04714	0.00144
18	-0.28984	-0.3389	-0.2906	0.04906	0.00076
19	-0.29262	-0.3422	-0.2936	0.04958	0.00098
20	-0.28886	-0.3379	-0.2899	0.04904	0.00104
21	-0.28305	-0.3313	-0.2845	0.04825	0.00145
22	-0.27892	-0.3266	-0.2804	0.04768	0.00148
23	-0.28326	-0.3315	-0.2847	0.04824	0.00144
24	-0.2845	-0.333	-0.2857	0.0485	0.0012
25	-0.28051	-0.3277	-0.2807	0.04719	0.00019
26	-0.28805	-0.3361	-0.2882	0.04805	0.00015
27	-0.27325	-0.3193	-0.2734	0.04605	0.00015
28	-0.20449	-0.2407	-0.207	0.03621	0.00251
29	-0.29509	-0.3449	-0.2952	0.04981	0.00011
30	-0.31079	-0.3633	-0.3109	0.05251	0.00011
MSE				0.00185	0.0000017

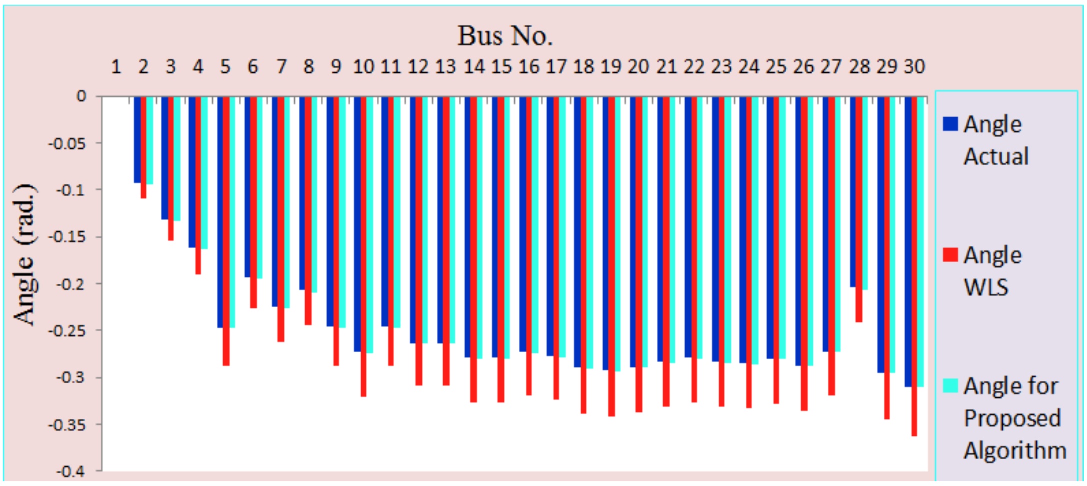

Figure 5.Comparison between actual and estimated values of the bus phase angle

Table 6. Comparison of the estimation accuracy in bus voltage magnitude

Test System	Mean Square Error (MSE) in bus voltage magnitude	
	WLS in Polar Form	Proposed Algorithm
IEEE-14 bus test System	0.00311	0.000005
IEEE-30 bus test System	0.00553	0.0000068

Table 7. Comparison of the estimation accuracy in bus phase angle

Test System	Mean Square Error (MSE) in bus phase angle	
	WLS in Polar Form	Proposed Algorithm
IEEE-14 bus test System	0.000655	0.00000004
IEEE-30 bus test System	0.00185	0.0000017

Table 8. Execution time in seconds

Test System	CPU time in (Sec.)	
	WLS in Polar Form	Proposed Algorithm
IEEE-14 bus test System	0.039422	0.021803
IEEE-30 bus test System	0.15386	0.063552

6. Conclusions

An algorithm to obtain the optimal estimate of the state vector has been presented. The formulation of estimator model was based on rectangular coordinates of the state variables. A predictor-corrector technique has been used for solving the nonlinear model of the estimator. The proposed algorithm was enriched by placing the PMU meters optimally. The applications of the proposed algorithm on test systems are given in the paper. The results obtained reveal the superiority of the developed algorithm compared to the conventional algorithm in terms of the execution time, accuracy and the number of iterations for the system to be converged. The proposed algorithm is very promising for real time monitoring application.

7. Abbreviations

SCADA	Supervisory Control and Data Acquisition
PSO	Particle Swarm Optimization
WLS	Weighted Least Square
PMUs	Phasor Measurement Units
GA	Genetic Algorithm
SE	State Estimation
AI	Artificial Intelligence
FF	Fitness Function

IEEE	Institute of Electrical and Electronic Engineers
MSE	Mean Square Error
WLAV	Weighted Least Absolute Value
LMS	Least Median of Squares

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Appendix **A**

iterations as compared with the traditional numerical methods. For the following nonlinear algebraic equation:

$$\mathrm{F}(x) = 0 \tag{A.1}$$

Assuming that *x* be the simple zero of this equation. Equation (A.1) can be written as[37,38] :

$$(x) = \mathrm{F}(x_i) + \int_{x_i}^{x} \mathrm{F}'(t)dt \tag{A.2}$$

The second term of equation (A.2) involves multiple integrals and can be approximated with average of midpoint and Simpson quadrature formulas, then we have:

$$\int_{x_i}^{x} \mathrm{F}'(t)dt = \frac{x-x_i}{2}\, \mathrm{F}'\left(\frac{x_i+x}{2}\right) + \frac{x-x_i}{12}\left[\mathrm{F}'(x_i) + 4\mathrm{F}'\left(\frac{x_i+x}{2}\right) + \mathrm{F}'(x)\right] \qquad \text{A.3}$$

Substituting (A.3) into (A.2) we have:

$$\mathrm{F}(x) = \mathrm{F}(x_i) + \frac{x-x_i}{2}\, \mathrm{F}'\left(\frac{x_i+x}{2}\right) + \frac{x-x_i}{12}[\mathrm{F}'(x_i) + 4\mathrm{F}'\left(\frac{x_i+x}{2}\right) + \mathrm{F}'(x) \qquad \text{A.4}$$

Since, $\mathrm{F}(x) = 0$ then

$$[x] = [x_i] + 12\left[G_{(x_i)} + 10G_{(w_i)} + G_{(y_i)}\right]^{-1}[\mathrm{F}(x_i)] \qquad \text{A.5}$$

$$[w_i] = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = \frac{1}{2}\begin{bmatrix} y_1 + x_1 \\ y_2 + x_2 \\ \vdots \\ y_n + x_n \end{bmatrix}, [x_i] = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, [y_i] = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \qquad \text{A.6}$$

It is an implicit way since $[\boldsymbol{x}_{i+1}]$ occurs on both sides of the equation. To carry out this implicit way, one has to calculate the approximate solution implicitly, which is itself a difficult problem. To overcome this defect, the prediction and correction method is typically used. By using formula (A.6), a two-step iterative method for solving the system of nonlinear equation (A.1) can be obtained. For a given x_0, the approximate solution $[\,\boldsymbol{x}_{i+1}]$ is calculated by iterative scheme as follows:

$$y_i = x_i - \frac{\mathrm{F}(x_i)}{\mathrm{F}'(x_i)} \qquad \text{A.7}$$

$$x_{i+1} = x_i - \frac{12\,\mathrm{F}(x_i)}{\mathrm{F}'(x_i) + 10\mathrm{F}'\left(\frac{x_i+y_i}{2}\right) + \mathrm{F}'(y_i)} \qquad \text{A.8}$$

In a compact form, equation A.9 can be written as:

$$[x_{i+1}] = [x_i] + 12\left[\mathrm{G}_{(x_i)} + 10\mathrm{G}_{(w_i)} + \mathrm{G}_{(y_i)}\right]^{-1}[\mathrm{F}(x_i)] \qquad \text{A.9}$$