EXPERIMENTAL COMPARISON OF FLOW ENERGY LOSS IN TYPE-B AND -C TRAPEZOIDAL PIANO KEY WEIRS (PKWS)

Ali Qasim Rdhaiwi¹, Ali Khoshfetrat²*, Amirhossein Fathi³

¹Department of Water Resources Engineering, College of Engineering, Mustansiriyah University, Baghdad, Iraq
²Department of Civil Engineering, Isfahan (Khorasgan) Branch, Islamic Azad University, Isfahan, Iran
³Faculty of Civil and Environmental Engineering, Tarbiat Modares University, Tehran, Iran

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Abstract: As non-linear weirs with a high flow rate, Piano Key Weirs (PKWs) have attracted the attention of water engineers in recent years. Given the limited information available on the energy losses of these weirs, it is important to investigate the energy losses and discharge capacity of these weirs. In this research, two trapezoidal PKWs, i.e., type-B and -C, with a height of 0.2 m were used. The studied flow rates were 0.025, 0.03, 0.035, and 0.04 m³/s. The results showed that energy loss decreased by increasing the flow velocity and upstream depth. The average energy loss in the type-B trapezoidal PKW was about 10.9% lower than that in the type-C PKW. The type-B weir had a higher discharge coefficient of about 5.6% compared to that of the type-C weir. Finally, an equation was presented to calculate the energy loss of these two weirs with a correlation coefficient of 97.42%.

Keywords: Energy loss; Piano Key Weir (PKW); type-B; type-C; discharge coefficient

1. Introduction

Dams and reservoirs are among the most effective ways of providing long-term water storage and flood protection. They play a vital role in improving the water supply and electricity generation of any country [1-5]. Flood control must be guaranteed in such a way that the flood control or release can be facilitated without leading to any dangerous incidents. It is one of the basic aspects of dam protection since it can have significant effects depending on the available amount of water [6-7]. Introducing non-linear weirs is one of the most common solutions. These types of weirs enhance discharges while maintaining the length of a traditional linear weir [8]. Piano Key Weirs (PKWs) are non-linear weirs that evolved from labyrinth weirs. The first PKW was built on the Goulours Dam in France [9-11]. PKWs are presented in rectangular, triangular, and trapezoidal shapes and 4 types of A, B, C, and D. Many studies have been conducted on the impacts of hydraulic and geometrical parameters on the flow patterns of these types of weirs. The mentioned weirs have inlet and outlet keys with negative and positive slopes, respectively. Type-A, -B, -C, and -D PKWs have upstream and downstream fronts, an upstream front, a downstream front, and no fronts, respectively. As stated, piano key weirs are the best alternative to linear or other nonlinear weirs. The nonlinearity of these weirs

*Corresponding Author: khoshfetrat@khuisf.ac.ir
allows them to be used in rivers, drainage canals, and even dams. They are also a new alternative to concrete dams [10-12]. Many researchers have conducted valuable studies on the discharge coefficient of piano key weirs. For example, Kabiri-Samani and Javaheri. [13] found that the discharge coefficient is a function of the geometric parameters of the weir and that the discharge coefficient in free flow is significantly higher than the discharge coefficient in steady flow. Very few studies have been done on energy loss in them. Researchers, such as Ribeiro et al. [14], Khan et al. [15], Bieri et al. [16], and Epicum et al. [17], have conducted some studies on the energy losses of labyrinth and PKWs. Sajadi [18] performed experimental and numerical studies on the energy loss of a rectangular PKW and found that the presence of blocks in the outlet keys caused more energy loss. Al-Shukur and Al-Khafaji [19] carried out an experimental study on the energy loss of a rectangular PKW and discovered that energy loss increased by decreasing the slopes of the outlet keys. Naghibzadeh et al. [20] performed numerical and experimental studies on the energy loss of a rectangular PKW and found that the presence of steps and blocks enhanced energy loss. Eslinger and Crookston [21] conducted an experimental study on a rectangular PKW and presented empirical equations for energy loss at the bottom of the weir as a function of \( H/P \) and \( W_i/W_o \), in which \( H \) was flow depth plus kinetic energy; \( P \) was weir height; and \( W_i \) and \( W_o \) were widths of the inlet and outlet keys, respectively. Singh and Kumar [22] investigated the effects of different geometries of a type-B rectangular PKW on energy loss. They also studied the impact of the presence of 3 steps in the weir outlet keys on energy loss and found that they augmented energy loss. Fathi et al. [23], conducted an experimental study on the energy losses of flow over a trapezoidal piano key weir of type A. They found that the presence of steps increases the energy losses and reduces the flow velocity on the exit keys of the weir. They investigated the number of steps with different geometries. The energy losses in 5-, 10-, and 15-step weirs in their work were 15.73%, 24.93%, and 18.52% higher than those in a weir without steps. They concluded that the 10-step weir is the optimal weir.

After reviewing the previous studies, little researches were found to have been done on energy loss of PKWs. Although valuable studies had been carried out by researchers in this field, there was no research on the amounts of energy loss in type-C and -B trapezoidal PKWs. Therefore, this research tried to address this issue. Also, an attempt was made to compare the flow, energy losses, and discharge coefficient of trapezoidal piano key weirs of type B and C simultaneously.

2. Dimensional Analysis

The amounts of energy upstream and downstream of weirs could be calculated by using Eqs. 1 and 2 and their amounts of energy loss could be measured by applying Eq. 3 [24]. In these equations, \( y \) and \( y_t \) are depths of the flow upstream and downstream (tailwater) of the weirs, respectively; \( P \) is weir height; \( V_1 \) and \( V_2 \) are flow velocities at the upstream and downstream parts of the weirs, respectively (By having the flow rate and measuring the flow depth upstream of the weir and tailwater and using the continuity equation, it is possible to obtain the flow velocity upstream and downstream of the weir); \( g \) is the acceleration of gravity; and \( E_1 \) and \( E_2 \) are specific energies at the upstream and downstream sides of the weirs, respectively.
\[ E_1 = P + y + \frac{v_1^2}{2g} \]  
\[ E_2 = y_t + \frac{v_2^2}{2g} \]  
\[ E_L = \frac{E_1 - E_2}{E_1} \]

The parameters affecting energy loss \((E_L)\) in the type-B and -C PKWs with fixed geometric characteristics can be written as the following function:

\[ E_L = f(\rho, \mu, \sigma, Y, P) \]  

Where \(\rho\) is water density; \(\mu\) is dynamic viscosity; \(\sigma\) is surface tension coefficient; \(Y\) is the depth of water on the weir plus kinetic energy; and \(P\) is weir height. Due to the high turbulence of the flow, the Reynolds number \((Re>4000)\) was removed. According to Table 1, at the lowest flow rate and the lowest flow velocity, the Reynolds number upstream of the weir is approximately 5,435. Due to the sufficient depth of the flow on the weir crest \((y>0.03m)\), the effect of surface tension and the Weber number were removed as well [25, 26]; hence, Considering the 3 repeated variables of \(\rho, P\) and \(Y\) and using Buckingham's Pi-theorem, energy loss is a function of dimensionless numbers as follows:

\[ E_L = f\left(\frac{Y}{P}\right) \]  

3. Materials and Methods

The experiments were carried out in a 10-m-long, 0.6-m-wide, and 0.8-m-high flume made of metal. The flow was supplied by two parallel tanks and a pump. Fig. 1 shows the locations of the weirs and flume installed in the laboratory.

The flow from the upstream tank entered the weir after passing 5.5 m. The flow would then return to the first tank after entering the experimental flume, passing through the flow straighteners, passing over the weir, and then passing through the end gate of the flume. The first and second tanks are connected by a pipe with a diameter of 130 millimeters (approximately 5 inches). After the pump (of the centrifugal type) was turned on, the flow was drawn into the beginning of the experimental flume by a pipe with the same specifications. The flume tilt system was adjusted by an electric motor and gearbox. The slope of the experimental flume was constant at 0.04. Type-B and -C trapezoidal PKWs with 3 keys and 2 half keys were employed. The weirs had the inlet and outlet key widths of \(W_i=0.215\) m and \(W_o=0.075\) m, respectively, the upstream and downstream lateral wall length of \(B_i=B_o=0.13\) m, lateral wall length of \(B=0.5\) m, width of \(W=0.6\) m, height of \(P=0.2\) m, and thickness of \(T_s=0.01\) m. The flow rates were 25, 30, 35, and 40 l/s. By setting an appropriate speed for the pump, the flow entered the laboratory flume at the studied rates. The upstream and downstream flow depths were measured with a needle depth gauge. The error of the pump and flow meter is \(\pm0.01\%\) and \(\pm0.001\) meter, respectively. The geometric specifications of both weirs were similar according to Fig. 2.
The upstream and tailwater depths were measured at the distances $2P$ and $8P$ from the weir, respectively [22]. Alternatively, the flow depths at the upstream and tailwater of the weir were measured at $4y$ and $10P$, respectively [23]. These two sentences and these distances are close to each other with very little error. The amounts of energy loss were calculated by using Eqs. 1, 2, and 3. As mentioned, a needle depth gauge was used for measurement. The flow depths of all the experiments were freely created in the free flow condition and the reservoir depth was not adjusted by the end gate of the laboratory flume. Table 1 shows the characteristics of the laboratory data. Due to experimental limitations, the dimensionless ratio of the $Y/P$ parameter was selected between 0.158 and 0.297.

Table 1. Specifications of the tests performed

<table>
<thead>
<tr>
<th>Row</th>
<th>Type</th>
<th>$Q$ ($\frac{m^3}{s}$)</th>
<th>$P + y$ (m)</th>
<th>$y_t$ (m)</th>
<th>$V_1$ (m/s)</th>
<th>$V_2$ (m/s)</th>
<th>$Re$</th>
<th>$E_r = \frac{E_2}{E_1}$</th>
<th>$E_L$</th>
<th>$Y/P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>B</td>
<td>0/025</td>
<td>0.23</td>
<td>0.09</td>
<td>0.181</td>
<td>0.463</td>
<td>5435</td>
<td>0.44</td>
<td>0.56</td>
<td>0.158</td>
</tr>
<tr>
<td>2</td>
<td>B</td>
<td>0/03</td>
<td>0.237</td>
<td>0.1</td>
<td>0.211</td>
<td>0.500</td>
<td>7806</td>
<td>0.47</td>
<td>0.53</td>
<td>0.196</td>
</tr>
<tr>
<td>3</td>
<td>B</td>
<td>0/035</td>
<td>0.245</td>
<td>0.106</td>
<td>0.238</td>
<td>0.550</td>
<td>10714</td>
<td>0.49</td>
<td>0.51</td>
<td>0.239</td>
</tr>
<tr>
<td>4</td>
<td>B</td>
<td>0/04</td>
<td>0.253</td>
<td>0.125</td>
<td>0.264</td>
<td>0.533</td>
<td>13966</td>
<td>0.54</td>
<td>0.46</td>
<td>0.283</td>
</tr>
<tr>
<td>5</td>
<td>C</td>
<td>0/025</td>
<td>0.231</td>
<td>0.05</td>
<td>0.180</td>
<td>0.833</td>
<td>5592</td>
<td>0.37</td>
<td>0.63</td>
<td>0.163</td>
</tr>
<tr>
<td>6</td>
<td>C</td>
<td>0/03</td>
<td>0.2385</td>
<td>0.08</td>
<td>0.209</td>
<td>0.625</td>
<td>8071</td>
<td>0.42</td>
<td>0.58</td>
<td>0.204</td>
</tr>
<tr>
<td>7</td>
<td>C</td>
<td>0/035</td>
<td>0.247</td>
<td>0.06</td>
<td>0.236</td>
<td>0.972</td>
<td>11100</td>
<td>0.43</td>
<td>0.57</td>
<td>0.249</td>
</tr>
<tr>
<td>8</td>
<td>C</td>
<td>0/04</td>
<td>0.256</td>
<td>0.1</td>
<td>0.260</td>
<td>0.667</td>
<td>14583</td>
<td>0.47</td>
<td>0.53</td>
<td>0.297</td>
</tr>
</tbody>
</table>
4. Results and Discussion

Fig. 3 shows the flow through the trapezoidal type-C weir. As it is clear from the figure, the flow is transferred as an inclined jet from the outlet keys and a free-falling jet from the inlet keys towards the downstream side and outlet keys.

A weak hydraulic jump was also observed downstream of the weirs. In the type-C PKW, the flow without a submerged area was transferred from the outlet keys toward the downstream part. However, in the type-B PKW, the absorption area or flow bulge was formed due to the long path of the outlet keys, which increased the speed of the flow from the outlet keys. The high flow rate reduced energy loss. In the type-C weir and at the entrance of the outlet keys, the flow had a slight depression. Also, compared to the type-B weir, a stronger hydraulic jump was formed in front of the outlet keys and below the inlet keys in this weir due to the short length of the outlet keys. This hydraulic jump prevented the jets from falling, thus augmenting energy loss. Fig. 4 displays the interference of the flow from the inlet keys and the hydraulic jump created by the flow at the outlet keys. This flow interference caused a vortex and severe flow turbulence.

By using the general equation of weirs (Eq. 6), the discharge coefficients of the type-B and -C PKW were investigated. In this equation, the length of the weir crest or the channel width is used, as well as the upstream depth of the weir plus the equivalent height of the kinetic energy. This parameter is related to the type of weir and varies, as shown in Table 1. Fig. 5 displays discharge coefficient variations with regard to the ratio of flow depth upstream of the weir plus kinetic energy to the weir height. As can be seen, decreased discharge coefficients have resulted from increased flow rates. In Eq. 6, $Q$, $Cd$, and $L$ stand for the flow rate, discharge coefficient, and weir length, respectively. The discharge coefficient of the type-B weir is about 5.6% more than that of the type-C weir [27].

$$ Cd = \frac{Q}{\frac{2}{3}L\sqrt{2gy^2}} $$

(6)

Fig. 6 demonstrates the effects of the flow rates per unit width ($q$), which is a function of the flow velocity and depth in relation to changes in the downstream and upstream energies of the weirs. As can be seen, the amount of downstream energy increases as the discharge per unit width is enhanced with an upward trend. The downstream energy in the type-B
A parameter that affects energy loss is the ratio of total flow height to the weir height as shown in Fig. 7. As depicted in this figure, the type-C weir has undergone a more energy loss. The average amounts of energy loss in the type-B and type-C trapezoidal PKWs are about 51.5 and 57.8%, respectively. In general, the amount of energy loss in the type-B weir is about 10.9% less than that of the type-C weir. As the flow rate increases and the dimensionless ratio $Y/P$ increases, the energy loss in all weirs decreases.

In this figure, the experimental data of Fathi et al. [23] were used. Since the trapezoidal piano key weir type-A was studied in their work, their data were used for comparison. As can be seen, the energy loss in the trapezoidal piano key weir type A is approximately 13.8% less than the trapezoidal piano key weir type-C and approximately 3.2% less than the trapezoidal piano key weir type-B. The reason for the lower energy loss in their work may be the simultaneous presence of a hanging edge in the upstream or downstream, the different geometry of the weir, or the presence of strong turbulences due to the impact of the outflow from the inlet and outlet keys.

The following equation was presented to calculate the energy loss. In this equation, $K$ is a coefficient that takes into account the effect of the PKW type (B and C). The values of this coefficient and correlation coefficient are presented in Table 2.

$$E_L = K(1.5771 \left(\frac{Y}{P}\right)^2 - 1.4728 \left(\frac{Y}{P}\right) + 0.8282) \quad (7)$$
Table 2. Calculation of coefficient $K$

<table>
<thead>
<tr>
<th>Row</th>
<th>Type</th>
<th>$K$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>B</td>
<td>0.89</td>
<td>0.961</td>
</tr>
<tr>
<td>2</td>
<td>C</td>
<td>1</td>
<td>0.973</td>
</tr>
</tbody>
</table>

Fig. 8 shows the calculated (Eq. 7) and observed values of energy loss, which can be used with an accuracy of ±5% and a correlation coefficient of 98.26% according to the drawn fitting line. In this figure and in (Eq. 7), the experimental data of Fathi et al. [23], were used. (Eq. 7) is also applicable for the energy loss type-A with an acceptable error of (±12%). The coefficient $K$ for the trapezoidal piano key weir type-A is 0.6.

![Energy loss comparison graph](image)

**Figure 8.** Observed and computed energy loss values

### 5. Conclusions

The results of the tests conducted on type-B and -C PKWs with a height, lateral wall length, and width of respectively 0.2, 0.5, and 0.6 revealed that the energy loss in the type-B PKW was about 10.9% less than that of the type-C PKW. Also, the discharge coefficient of the type-B weir was about 5.6% more than that of the type-C weir. Because the discharge coefficient of weirs is of greater importance, energy loss is vital for reducing the amount of scouring and risk of weir overturning. One of the parameters affecting energy loss is the outflow from weir outlet keys. The turbulent flow and hydraulic jump in the type-C weir were relatively stronger than those of the type-B weir, thus causing more energy loss. Energy loss was seen to decrease by increasing the flow depth, velocity head upstream of the weir, and flow rate. The following equation was presented with a correlation coefficient of 98.26% for energy losses of the two mentioned weirs. The results of the comparison showed that the energy loss in the type-B and -C weirs is higher than that of the type-A weir. In the future, the scour rate downstream of these two weirs can be investigated and compared.

### Conflict of interest

The authors confirm that the publication of this article causes no conflict of interest.

### Contribution Statement of Author

Ali Qasim Rdhaiwi, Ali Khoshfetrat, and Amirhossein Fathi: proposed the research problem. Authors Ali Qasim Rdhaiwi and Ali Khoshfetrat.: developed the theory and performed the computations. Authors Ali Khoshfetrat and Amirhossein Fathi.: checked the theoretical analysis methods and supervised the results of this research. The authors checked and discussed the results and contributed to this work to present the final manuscript.

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### Nomenclature

- **B**     lateral wall length     m
- **B_i**  upstream overhanging length     m
- **B_o**  downstream overhanging length     m

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Cd: discharge coefficient  
E₁: Upstream energy  
E₂: Downstream energy  
Eₐ: Energy loss  
Fr: Froude number  
g: acceleration of gravity  
K: Fixed coefficient  
L: weir length  
P: weir height  
Q: discharge  
Re: Reynolds number  
Ts: wall thickness  
V₁: Flow velocity upstream of the weir  
V₂: Flow velocity downstream of the weir  
W: weir width  
Wᵢ: inlet key width  
Wₒ: outlet key width  
We: Weber number  
γ: depth of water on the weir  
γₜ: tailwater depth  
Y: depth of water on the weir plus kinetic energy  
ρₛ: sediment density  
ρₚ: water density  
μ: dynamic viscosity  
σ: Surface tension

References


