

Comparative Performance of Point-to-Point Multiple Input Multiple Output System under Weibull and Rayleigh Fading Channels

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Abstract

Multiple-input multiple-output (MIMO) technologies use multiple antennas at the sender and the receiver to get the high data rate that the next-generation communication system needs. This paper compares MIMO systems using the Quadrature Phase Shift Keying (QPSK) modulation scheme for two-channel distributions of Rayleigh and Weibull types. The performance of the work is far with the Minimum Mean Square Equalizer (MMSE) in terms of Bit Error Rate (BER) for various antenna number situations on the transmitter and receiver sides. The MIMO module is carried out using MATLAB code. The channel noise will be a signal of random noise that is generated. To mitigate the impact of inter-channel interference, the MMSE approach employs inverse filtering at the receiver, and BER will be calculated. According to simulation results, the system's performance is primarily influenced by the number of antennas; it decreased as the number of antennas increased, and the BER of the Weibull channel decreased as the two-parameter value increased.

Keywords: Bit Error Ratio; Minimum Mean Square Error Equalizer; Multiple-Input Multiple-Output; Weibull distribution; Rayleigh distribution

1. Introduction

The signal spreads from the sender to the receiver through the communications channel, a physical path through which the signal travels. The statistical tools of the channel, such as Rayleigh, Weibull, etc., are used to construct the transmitter's channel encoder and modulator, as well as the receiver's channel demodulator and decoder. A signal experiences two basic flaws that change it from its original form as it travels across the communication channel. These flaws can be separated into predetermined forms (such as linear and nonlinear distortion, inter-symbol, etc.) [1].

The Multiple-input multiple-output MIMO system is one of the most powerful solutions for enabling high data rates and reliable wireless communication link quality [2]-[9]. MIMO systems have been incorporated into 5th generation (5G) communication networks and wireless protocols, including IEEE 802.11n, IEEE 802.16e, and LTE. With MIMO

techniques, the transmitting power and bandwidth can be kept constant while the spectral efficiency is enhanced and the error rate is decreased. Waloddi Weibull initially presented the Weibull distribution in 1937 to estimate the lifetime of machinery, and it gained widespread recognition in 1951 [10].

The Weibull distribution is used in many scientific fields today. For instance, it is a popular statistical model in failure data analysis and reliability engineering [11]-[12]. In addition, it is frequently used in radar systems to model the dispersion of the received signal level caused by some types of clutters. Regarding wireless communications, the Weibull distribution fits experimental fading channel measurements well in indoor and outdoor environments [13]-[14].

There has been a substantial amount of research on the BER performance of MIMO systems; however, only a few are listed here that are consistent with the viewpoint of this work.

For instance, in [15], the authors have employed linear detection, such as Zero-Forcing (ZF) and MMSE, with different decomposition techniques like Cholesky, QR, and Singular Value Decomposition (SVD) to enhance the performance. However, the authors attempt to examine the Bit Error Rate BER performance of the MIMO system using various correlation coefficients. The channel strength drops by more than 10 dB for every 10% rise in the correlation coefficient.

In [16], the authors have investigated ZF and MMSE equalizers for Binary Phase Shift Keying (BPSK) modulation. By analyzing the simulation result it shows that when the number of tap lengths increases, the BER in the ZF equalization will decrease. The performance of MMSE is superior to ZF equalization by comparing the BER versus SNR of two distinct types of equalizers.

In [17], the authors have analyzed the performance of ZF and MMSE equalizers for 2x2 and 4x4 MIMO wireless channels. The BER characteristics are simulated for various transmitting and receiving antenna numbers. According to the simulation results, the equalizer-based Zero-Forcing receiver can help with noise-free channels and effectively reduce ISI. Still, in terms of BER characteristics, MMSE is preferred to ZF.

The primary goal of this study is to simulate Rayleigh and Weibull channels using the MMSE for Quadrature Phase Shift Keying QPSK modulation for 2x2, 4x4, 16x16 and 25x25 MIMO spatial multiplexing wireless channels. Then, it also shows the BER versus $E_b N_0$ comparison of two different types of channels and tries to determine the best result for BER for a given $E_b N_0$.

The remaining sections are organized as follows: The second section provides a theoretical foundation for the primary distribution channels under consideration. Section 3 depicts the achieved simulation outcomes. In section 4, some concluding observations are offered.

2. Theory

MIMO is a wireless technology that uses multiple transmitting and receiving antennas to increase an RF radio's data capacity. Many antennas send identical data over the same channel and bandwidth in a MIMO system. Each signal, therefore, reaches the receiving antenna through a unique path, resulting in more trustworthy data. Moreover, the data rate rises by a factor proportional to the number of base station antennas [18].

Multipath is a part of MIMO technology that looks at the behavior of radio waves. Because the transmitted information is interrupted at various points, such as when it hits walls, ceilings, and other surfaces, it receives the received signal at multiple times and angles. At first, multipath involved

interference, and due to interference, it gradually decreased wireless signals [19]-[20]. MIMO improves signal strength capture, connection reliability, performance, and diversity at the receiver. Consequently, MIMO combines data streams from different paths and at other times [21]-[22]. Typically, more antennas correspond to faster speeds. Three antenna wireless adaptors can reach 600 Mbps in speed. The speed of an adapter with two antennae is 300 Mbps. The router must have numerous antennas and support all 802.11n features for maximum performance.

2.1. A Channel Model and Performance Metrics

This section provides the MIMO system model's formal specification. The reader-relevant background is intended to set the stage for the next parts. Since the emergence of MIMO systems, interest in conventional linear detectors has increased. As a result, we also present the linear detection mechanism in this section.

Fig. 1 depicts the general MIMO system with N_T and N_R transmitting and receiving antennas, respectively.

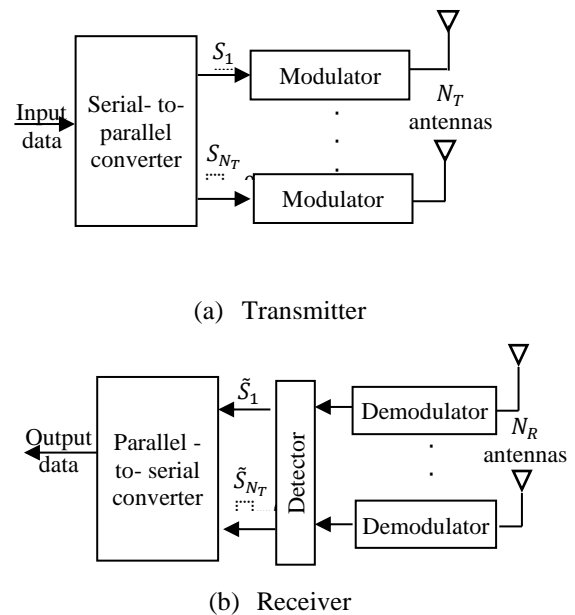


Figure 1. A communication system having several transmitting and receiving antennas

The signal model is depicted as follows:

$$r = Hs + n, \tag{1}$$

Where n is the $(N_R \times 1)$ additive white Gaussian noise (AWGN) vector with variance σ whose entries are independent identically distributed (IID), s is the $(N_T \times 1)$ transmitted signal vector and r is the received signal vector $(N_R \times 1)$. and H is the channel matrix between N_R antennas

and N_T antennas. This model assumes a flat fading channel, so each signal path can be represented by a single coefficient, which is represented in Eq. (2).

$$h_{n,l} = \sum_{i=1}^{N_R} \sum_{j=1}^{N_T} \alpha_{ij} \exp(j\theta_{ij}) \quad (2)$$

where, α_{ij} , θ_{ij} are the channel gain (the envelope), and phase, i , and j are the channel index. Furthermore, $h_{n,l}$ is the channel parameter between the l -th transmitter and the n -th receiver. The matrix representation of (2) can be written as shown in Eq. (3).

$$h_{n,l} = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1j} \\ h_{21} & h_{22} & \dots & h_{2j} \\ \vdots & \vdots & \vdots & \vdots \\ h_{i1} & h_{i2} & \dots & h_{ij} \end{bmatrix} \quad (3)$$

In the case of Rayleigh's fading channel α_{ij} has Rayleigh distribution, and in the case of Weibull fading channel α_{ij} has Weibull distribution

A MIMO detector determines the sent vector s based on the received vector r .

2.2. Channel Distributions Overview

The following is a quick summary of the random distributions used in this study:

In recent years, the Weibull distribution has been widely applied to multiple-path issues. The Weibull distribution is also used to solve life-testing challenges and reliability, such as determining the time to failure or life span of a component, measured from a predetermined time till failure [23]. The probability density function (PDF) of a random variable x is considered Weibull if and only if it satisfies the formula in Eq. (4).

$$f(x) = \begin{cases} \frac{\alpha}{\beta^\alpha} x^{\alpha-1} e^{-\left(\frac{x}{\beta}\right)^\alpha} & , x > 0 \\ 0 & elsewhere \end{cases} \quad (4)$$

In which the scale parameter β and the shape parameter α play their respective roles. The changing of either β or α results in a wide range of new curve densities. The pdf is nearly symmetric when $3 < \alpha < 4$. In certain circumstances, there are theoretical arguments for the applicability of the Weibull distribution, yet $f(x)$ provides an acceptable fitting to given data for certain values of α and β . The properties of this distribution provide a great deal of modeling freedom. Let X_1 and X_2 , random in-phase and quadrature variables with equal variances and zero means, be Gaussian distributed. The envelope in equation (5) is derived as a nonlinear function of the total of the variables.

$$x = \sqrt{X_1^2 + X_2^2} \quad (5)$$

When $\alpha = 1$, Pdf formed the exponential distribution, a specific case of the Weibull distribution [24]. A specific illustration of the two-parameter Weibull distribution is the Rayleigh distribution when $\alpha = 2$ and $\beta = \sqrt{2}\delta$ where δ is the scale parameter of the Rayleigh distribution, which is defined by [25] as shown in Eq. (6):

$$f(x) = \frac{x}{\delta^2} e^{-\frac{x^2}{\delta^2}} \quad (6)$$

x is the signal magnitude random variable. Here variance is δ^2 . The phase of the received signal's complex envelope is considered evenly distributed in $(-\pi, \pi)$. The envelope is provided by Eq. (7) when two independent random variables with a mean of zero and a variance of δ^2 are considered statistically Gaussian distributed.

$$x = \sqrt{X_1^2 + X_2^2} \quad (7)$$

Fig. 2 shows the Weibull probability density function for different β values

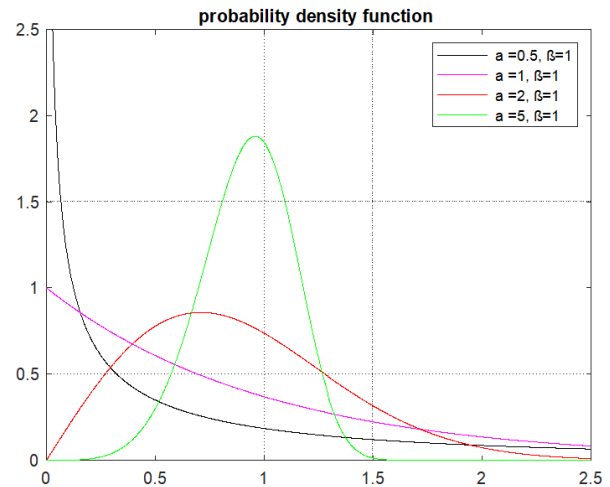


Figure 2. A Weibull probability density function

2.3 Minimum Mean Square Error Equalizer

The theory behind the MMSE detector is to minimize the mean-square error (MSE) that arises from comparing the estimated signal \hat{s}_{MMSE} with the transmitted signal s according to the equations (8), (9), and (10):

$$\hat{s} \triangleq \arg \min_{H \in \mathbb{C}^{N_R \times N_T}} \mathbb{E} \|n\|^2 \quad (8)$$

$$\hat{s} = \arg \min_{H \in \mathbb{C}^{N_R \times N_T}} \mathbb{E} \|r - Hs\|^2 \quad (9)$$

$$\hat{s}_{MMSE} = (H^T H + \sigma^2 I_{N_T})^{-1} H^T r \quad (10)$$

The noise effect is taken into account in MMSE. I_{N_T} corresponds to the identity matrix. The MMSE detector's output can be produced by Eq. (11).

$$\hat{s} = [x_{MMSE}] \quad (11)$$

So, consider both the noise and the interference from other symbols.

2.4 Modulation

In this study, The data bits are translated into a symbol in Quadrature Phase Shift Keying (QPSK), and each symbol has two bits of data selecting one of four possible carrier phase shifts ($0, \pi/2, \pi$ or $3\pi/2$).

3. Simulation Model

The system was designed using the following algorithm steps

- Introduce the input signal by generation of random binary sequences
- Assign the signal to each antenna
- Modulate the transmission signal using the QPSK scheme
- Transmit the signal through two types of channels

First, Rayleigh's channel

Applying (2) with α_{ij} has Rayleigh distribution and θ_{ij} is uniform in the interval $(-\pi, \pi)$

1. Add Gaussian White Noise to the channel: applying Eq. (1)
2. Decoding the received signal using MMSE Equalization as in (10)
3. Demodulate the equalized signal using QPSK demodulation
4. Calculate the BER using Eq. (12).

$$BER = \frac{\text{Count of Error Bits}}{\text{Count of Total transmitted bits}} \quad (12)$$

Second, the Weibull channel

Again, applying (2) with α_{ij} has Weibull distribution and θ_{ij} is uniform in the interval $(-\pi, \pi)$ and performs the above steps (1-4)

Two fading channels are performed, and BER is calculated by MMSE equalization

Finally, the simulated value is done.

4. Simulation Results

To generate the channel matrix, MATLAB m-file is used to validate the model and simulate the effects of several distribution types (Rayleigh and Weibull) for a MIMO system under flat fading.

Determine the BER for a variety of $E_b N_0$ values (0 to 25) dB. The simulation is conducted for several N_R and N_T pairs, as specified in Table 1.

4.1 Rayleigh Distribution

The Rayleigh distribution is the first distribution considered. The bit error rate for the Rayleigh distribution is computed for each scenario in Table 1 shows a variety of $E_b N_0$ (0 to 25) dB, each of the four scenarios is represented by BER curves in a distinct color with a unique indicator sign.

Table 1. transmitter/receiver antennas number

CASE NO.	The transmitter antenna number (N_T)	The Receiver antenna number (N_R)
1 st	2	2
2 nd	4	4
3 rd	16	16
4 th	25	25

The obtained findings are depicted in Fig. 3. From examining Fig.3 and the first curve ($N_T = 2, N_R = 2$), it is evident that the BER decreases as $E_b N_0$ grows.

For the 2nd case ($N_T = 4, N_R = 4$), with more transmitter and receiver antennas, the BER drops for the same values of $E_b N_0$ as in the first case.

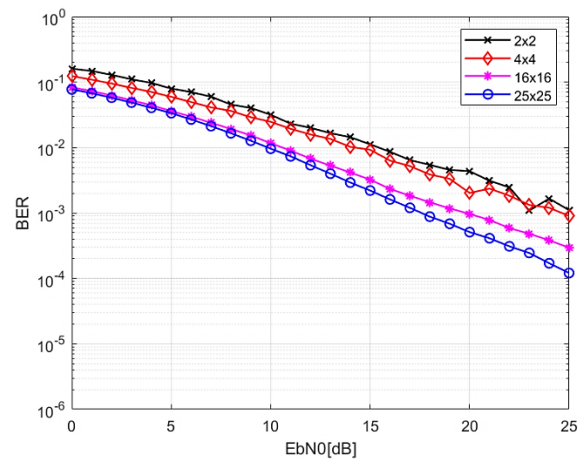


Figure 3. The bit error rate vs $E_b N_0$ for Rayleigh distribution

The 3rd case ($N_T = 16, N_R = 16$) shows a lower BER for the identical $E_b N_0$ values as the first and second cases. The 4th case ($N_T = 25, N_R = 25$) demonstrates that, in comparison to earlier cases, the BER is lower for the same values of $E_b N_0$. It is obvious that, despite the tiny value of $E_b N_0$, The BER has still improved compared to the earlier situations.

4.2 Weibull Distribution

Three alternative sets of β and α are used in the Weibull distribution. For each scenario in Table 1, within a broad scope of $E_b N_0$ (0 to 25) dB, the BER of the system is

calculated for each set of Weibull distribution parameters. The unity value for shape and scale parameters makes up the first set. Fig. 4 shows the results that have been reached.

Fig. 4 shows how the BER changes depending on how many antennas are used. The BER decreases as the number of antennas on both the transmitter and receiver sides increases.

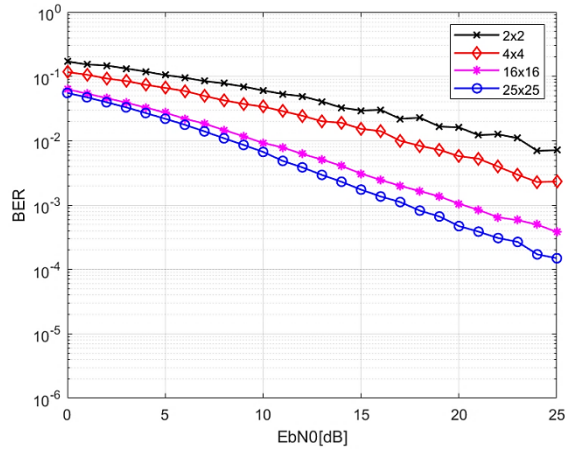


Figure 4. The bit error rate vs E_bN_0 for Weibull distribution (with $\beta=1, \alpha=1$)

This is similar to how the Rayleigh distribution works. Compared to the shown in Fig. 3, the BER performance with the Weibull density function (with $\beta=1, \alpha=1$) is better than that with the Rayleigh density function for the 1st to second cases. At the same time, it is higher in other cases (3rd and fourth).

($\beta=1$ & $\alpha=2$) make up the second set of parameters. Fig.5 shows the results that were reached. BER with Weibull distribution is about the same as BER with Rayleigh distribution, based on the results shown in Fig. 3.

The third set of parameters is ($\beta=1$ & $\alpha=3$). Fig.6 shows the outcomes that were reached. The BER lowers as the number of antennas on the transmitter and receiver sides grows. However, in this case, the BER goes down a lot for the same E_bN_0 and the number of antennas for transmitter and receiver (since α grew by 1 compared to their previous levels).

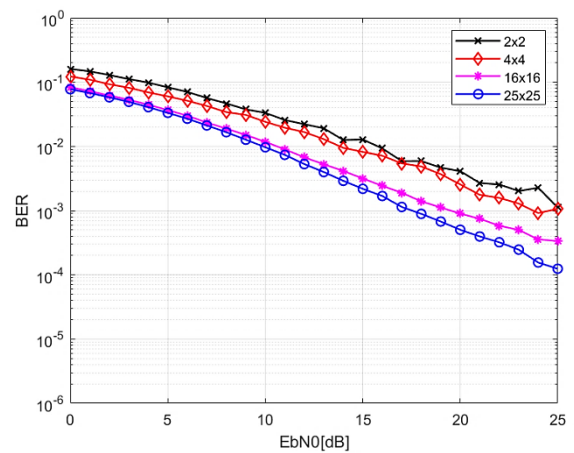


Figure 5. The bit error rate vs E_bN_0 for Weibull distribution (with $\beta=1, \alpha=2$)

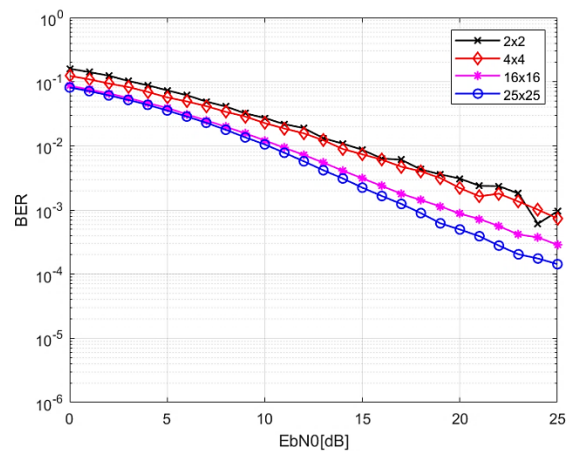


Figure 6. The bit error rate vs E_bN_0 for Weibull distribution (with $\beta=1, \alpha=3$)

($\beta=1$ & $\alpha=4$) make up the fourth pair of assessment parameters. Fig.7 displays the outcome. Analyze Fig. 7, and it is obvious that the decreasing amount of the BER performance is related to the increasing number of antennas in the transmitter and receiver for the same E_bN_0 , yet there is a slight decrease in the BER in Fig. 7 compared to that given in Fig.4, 5, and 6 for the same E_bN_0 and the number of antennas for the transmitter and receiver. The BER with the Weibull density function ($\beta=1, \alpha=4$) is less than the BER with the Rayleigh density function when compared to Fig. 3.

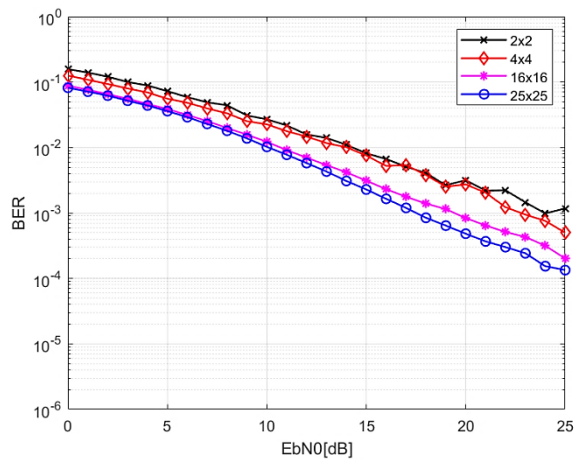


Figure 7. The bit error rate vs $E_b N_0$ for Weibull distribution (with $\beta=1$, $\alpha=4$)

5. Conclusions

The performance of MIMO systems is examined in this work in broadband wireless communication systems. The system investigated under the Rayleigh and the Weibull fading channels using QPSK modulation and the MMSE Equalizer at the receiver. The Monte-Carlo simulation is used for the system analysis using MATLAB simulation software. The results show that the BER performance for the Rayleigh and Weibull channels is roughly the same, with $\beta=1$ and $\alpha=2$. Changes to the evaluation parameters of the Weibull density function, assuming the same $E_b N_0$ In dB, the number of antennas for the transmitter and receiver resulted in different BER values due to the effect of generating the H matrix. The simulation findings show that the bit error rate of the MIMO system is improved by increasing the number of base station antennas. For example, increasing the antennas from 2*2 to 25*25 improves system gain by more than 10 dB at BER = 10^{-3} .

Conflict of interest

The authors declare that no conflicts of interest are associated with this manuscript's publication.

Author Contribution Statement

Azhar Hussein Neama.: proposed the research problem, developed the theory, and performed the computations.

Ghanim A. Al-Rubaye.: verified the analytical methods and supervised the findings of this work.

Both authors discussed the results and contributed to the final manuscript.

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