

Original Research

# RAYLEIGH FADING CHANNEL ESTIMATION BASED ON GENERALIZED REGRESSION NEURAL NETWORK

\*Emad Ahmed Hussien, Ghanim Abdulkareem

*Electrical Engineering Department, College of Engineering, Mustansiriyah University, Baghdad, Iraq*

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**Abstract:** With the rapid development of wireless communication, 5G is gradually growing into a large-scale basic Internet that supports various industries in the whole society. The substantial expansion of its service scope poses many challenges for the underlying technology, especially for the crucial component of the physical Layer-Orthogonal Frequency Division Multiplexing (OFDM). Recently, Neural Networks (NNS) have attracted extensive attention due to their excellent performance in computing vision and natural language processing. Its strong universality also provides new development space for traditional communications. This manuscript conducts an in-depth study on channel estimation for OFDM systems and explores the possible application of a Generalized Regression Neural Network (GRNN) to estimate the Channel Impulse Response (CIR) attenuated by AWGN and Rayleigh fading system. Moreover, three traditional channel estimation algorithms, i.e., LS, MMSE, and LMMSE, are derived by mathematics. In addition, this thesis illustrates several typical neural networks in detail, including their internal structure, parameter updating process, and related optimization algorithms.

**Keywords:** *Block-pilot; comb-pilot; least square; linear minimum mean square error; Rayleigh fading; time domain estimation*

## 1. Introduction

The effectiveness of wireless communication systems is greatly influenced by channel estimates. Contrary to traditional orthogonal

frequency division multiplexing (OFDM), the orthogonality requirement of quadrature amplitude modulation (QAM) systems only holds in the actual field, making it more difficult to estimate the channel in QAM over Rayleigh fading channel and AWGN. Numerous channel estimation techniques are provided, and the effectiveness of each is assessed using numerical simulations. In 2011 Meha G. and Fadia N. developed a pilot pattern proposed; pilot-aided form (3) and pilot-aided form (1) of the channel estimate discovered under frequency selective fading channel for both MMSE and LS estimators [1].

Several studies have relied on artificial neural networks to estimate the channel this is since current needs for high-speed communication cannot be fulfilled by conventional techniques. To decrease the cost of the computation, an approach uses a simple interpolation layer in place of the transposed convolutional layer seen in other networks [2].

Radosveta S. and Mete Y. use a fully connected deep neural network to implement channel

\*Corresponding Author:  
[dr.emadeng@uomustansiriyah.edu.iq](mailto:dr.emadeng@uomustansiriyah.edu.iq)

estimation. The method of data-aided estimating is used [3].

A study suggests utilizing deep learning to rectify the LS estimate mistake (DL). Simulation outcomes reveal that the suggested DL-based strategies outperform both Less complicated LS and MMSE channel estimation schemes than precise MMSE [4].

A method for channel estimate that could be used with both slow and fast-fading OFDM systems was put out by Yang Guangxi et al. in 2015. The suggested method used comb type and involved reducing ICI by including LPF in the transformation [5] [6]. In 2017, Ye, Li, Juang, and colleagues proposed a deep-learning neural network for estimating the channel and symbol recognition for the OFDM system [7] [8].

In this research, a brief introduction to GRNN will be introduced in addition to discussing the effect of Rayleigh fading to BER on the M-QAM-OFDM system on the channel. The estimation will depend on two strategies according to the domain dialed with (frequency or time domain). A Comb-type pilot is a frequency domain channel estimation that uses two estimator types: LS and MMSE whereas a Block-type pilot is a time domain depending on three strategies TD, TDD, and TDQ will be discussed for comparison.

## 2. Generalized Regression Neural Networks (GRNN)

Because it has so many uses in so many facets of life, artificial intelligence (AI) has a big effect on the nowadays research. Artificial Neural Networks ANNs are one of the key components of AI. Regression and approximation, prediction and forecasting, classification, recognition, identification, and other tasks may all be performed using ANNs. ANNs are useful because they can learn from the data and have

global approximation capabilities. In some cases, a feed-forward neural network with enough hidden neurons and at least one hidden layer may approximate any continuous function. In single-pass associative memory feed-forward ANNs, GRNN employ normalized Gaussian kernels as activation functions in the hidden layer. As seen in Fig. 1, GRNN is composed of input, hidden, summation, division, and output layers.

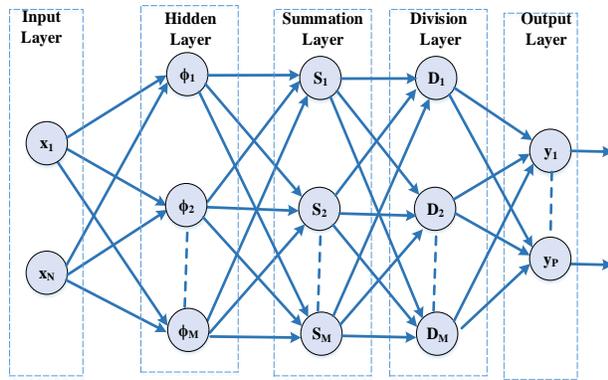
The GRNN learns every distinct pattern when it is trained. It is a single-pass network because of this and does not need a back-propagation technique. The GRNN will be able to generalize for new inputs once it has been trained with sufficient training patterns. Equation (1) below is used to calculate the GRNN's output [9] [10]:

$$\hat{Y} = \frac{\sum_{i=1}^N Y e^{(-D_i/2\sigma^2)}}{\sum_{i=1}^N e^{(-D_i/2\sigma^2)}} \quad (1)$$

where  $\hat{Y}$  is the training sample output,  $\alpha$  is the GRNN's smoothing value, and  $D_i$  is the Euclidean distance between the input  $X_i$  and the training sample input  $X$  which can be calculated as:

$$D_i = (X - X_i)^T (X - X_i) \quad (2)$$

The precision and speed of the GRNN training process are benefits. The large increase in the hidden layer, on the other hand, is one of the drawbacks of GRNN. However, this problem may be resolved by putting in place a unique algorithm that stores just the most important patterns, hence slowing the expansion of the hidden layer.



**Figure 1.** Generalized Regression Neural Network (GRNN) architecture.

### 3. OFDM System Description

In telecommunications, OFDM is a type of digital transmission and a method for encoding digital data on multiple carrier frequencies. OFDM, which is used in 4G/5G mobile communications, wireless networks, power line networks, digital television and audio transmission, and DSL internet access is a common technique for wideband digital communication [11]. Multiple input bits are encoded into a single data symbol in an OFDM system. Any digital modulation technique can be used to modify these N symbols. The symbols for the modulated data are given by  $X(0), X(1), \dots, X(N - 1)$ . For use with the Inverse Fast Fourier Transform (IFFT), the serial data symbols are transformed into parallel symbols. The IFFT procedure preserves the orthogonality of each subcarrier while mapping N symbols onto N subcarriers. The symbols for IFFT processed data are as follows:  $x(0), x(1), \dots, x(N - 1)$  and is represented mathematically as,

$$x(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X(k) e^{j2\pi kn/N} \tag{3}$$

Where  $x(n)$  is the  $n^{\text{th}}$  N-point IFFT sample and  $n$  ranges from 0 to N-1. Such an N number of IFFT processed samples are included in one transmitted OFDM signal. The multipath channel

taps are convolved with the currently transmitted OFDM signal. Some samples from an earlier OFDM symbol enter the convolving process and generate Inter Block Interference (IBI). At the beginning of each OFDM symbol, a cyclic prefix with a length higher than the number of channel taps is added to eliminate this IBI prefix. If assume that there are L channel taps, then the length of the cyclic prefix (G) must be bigger than L. Parallel samples are turned into a serial stream after a cyclic prefix. Transmitted samples with cyclic prefix have the shape  $x(n-G+1) \dots x(N-1)$ . AWGN is used to combine these transmitted samples. The samples that were received are expressed as,

$$y(n) = x(n) \otimes h(n) + w(n) \tag{4}$$

And because FFT is in the receiver, the equation will be:

$$Y(k) = X(k)H(k) + W(k) \tag{5}$$

Where  $X(k)$  indicates the transmitted pilot signal,  $W(k)$  is the channel noise (always assumed to be AWGN) and  $H(k)$  represents the impulse response of the frequency selective channel, that this research and previous research works to estimate [12]:

The whole OFDM operation for the transmitter and receiver are expressed as a flowchart shown in Fig.2 and Fig.3.

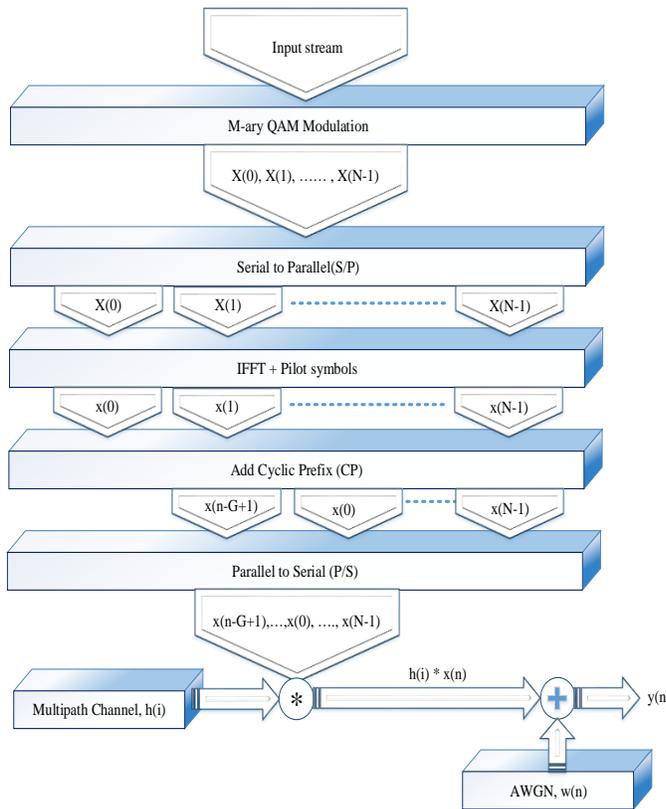


Figure 2. M-QAM OFDM Transmitter flowchart.

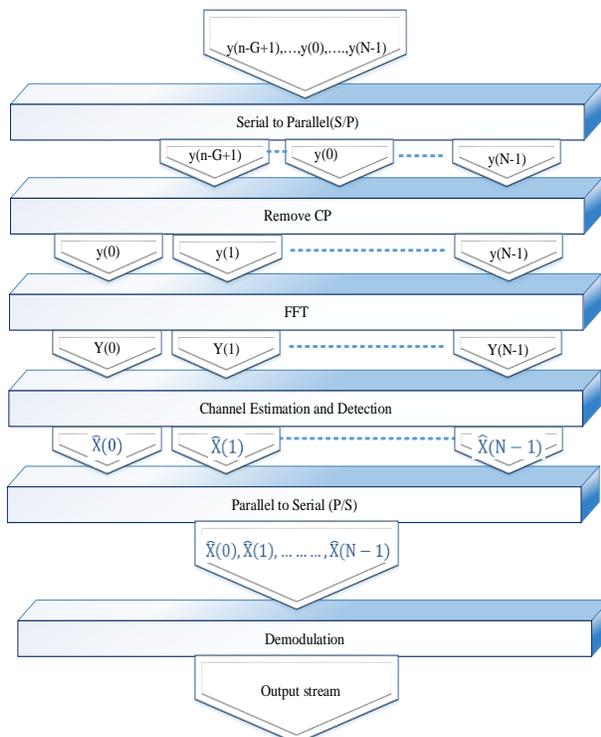


Figure 3. M-QAM OFDM Receiver flowchart.

### 4- Channel Estimation Techniques

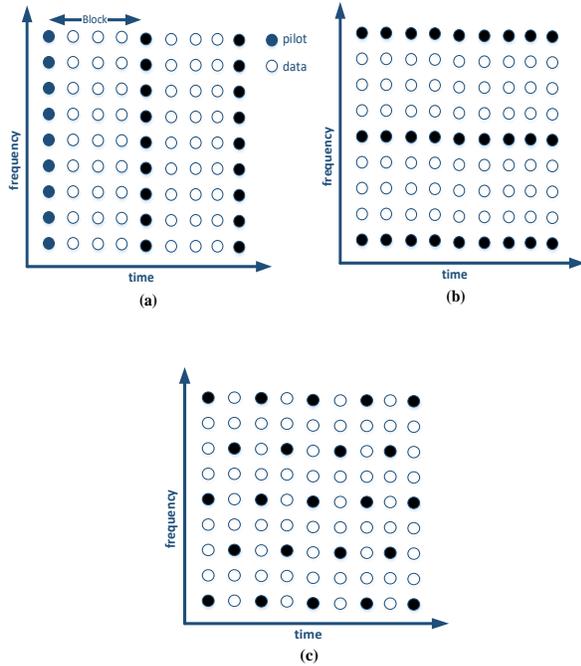
The channel estimate is a significant technological problem for mobile communication systems; the accuracy and speed of channel estimation have a direct impact on the system’s capacity and performance. However, pilot carriers cannot be utilized for data transmission, increasing the number of pilots in the signal lowers the available data rate. Any wireless communication system has this trade-off between pilot overhead and channel estimate quality. The location of the pilots within the OFDM symbol might vary depending on the application: Block-type pilot, Comb-type pilot, or Scattered pilot

Block-type pilot (Fig. 4a), meaning that each carrier holds a known value. The channel on each carrier is then estimated using these pilots, and the following OFDM symbols are equalized using the calculated channel coefficients. Block-type pilots also offer great accuracy in the frequency domain since they combine several data into a single estimate. Block-type pilots, however, are not effective for fast time-varying channels due to lower time domain resolution.

Comb-type pilots (Fig. 4b), employ several carriers just for pilot transmission. The channel coefficients for non-pilot subcarriers must be estimated in this case via frequency-domain interpolation. High resolution in the time domain is offered by comb-type pilots, but accuracy in the frequency domain is subpar. They can therefore be used for time-varying channels.

Scattered-type pilot (Fig. 4c), attempts to balance time-resolution, frequency resolution, and pilot overhead. Scattered pilot systems can provide a better accuracy trade-off while still requiring time- and frequency-domain interpolation

algorithms. In the cellular LTE system, for instance, scattered pilots are employed.



**Figure 4.** Channel estimation Pilot methods: (a) Block-type. (b) Comb-type. (c) Scattered type.

To obtain the estimated CIR using the LS estimator ( $\hat{H}_{LS}$ ), equation (5) must be minimized using the LS algorithm cost function [13] [14]:

$$J(\hat{H}_{LS}) = (Y - X\hat{H}_{LS})^H (Y - X\hat{H}_{LS}) \quad (6)$$

For minimization, (6) should be derived for  $\hat{H}_{LS}$  and set to zero to get the simplified estimated CIR:

$$\hat{H}_{LS} = (X)^{-1}Y \quad (7)$$

The LS estimators are extremely simple to construct and do not require any prior knowledge of the channel data, but they have a significant mean-square error. Particularly in low SNR circumstances, the MMSE estimator performs significantly better than LS estimators. The MMSE estimator has a significant computational complexity issue, particularly if matrix

inversions are required each time the input data changes. The CIR for the MMSE estimator can be derived as [15] [16]:

$$\hat{H}_{MMSE} = R_{HH} [R_{HH} + \sigma_n^2 (XX^H)^{-1}]^{-1} \hat{H}_{LS} \quad (8)$$

Knowing the AWGN variance ( $\sigma_n^2$ ), the autocovariance matrix of H ( $R_{HH}$ ) can be calculated from:

$$R_{HH} = E\{H \cdot H^H\} = FR_{HH}F^H \quad (9)$$

Where F is the DFT matrix and  $(\cdot)^H$  is the Hermitian (conjugate transpose) operator.

To reduce the MMSE complexity, a modified MMSE estimator, known as Linear MMSE (LMMSE), has been proposed. The simplification is to replace the term  $[\sigma_n^2 (XX^H)^{-1}]$  with the term:  $[(\beta / SNR) \cdot I]$  to make (8) be:

$$\hat{H}_{LMMSE} = R_{HH} [R_{HH} + (\beta / \overline{SNR}) \cdot I]^{-1} \hat{H}_{LS} \quad (10)$$

Where  $\overline{SNR}$  is the average signal-to-noise ratio, I is the identity matrix and  $\beta$  is the constellation constant range from 0 to 10.

In the time domain, TD-LMMSE will be derived using (10) after converting CIR from frequency domain (H) to time domain (h):  $\hat{h}_{LS} = F^{-1} \hat{H}_{LS}$  where  $F^{-1}$  is the IDFT matrix to be:

$$\hat{H}_{TD} = FR_{hh} [R_{hh} + (\beta / \overline{SNR}) \cdot I]^{-1} \hat{h}_{LS} \quad (11)$$

Where:  $R_{hh} = h \cdot h^T$

In the same way, one can derive TD using the same strategy by ignoring channel covariance, put  $R_{gg} = h \cdot h^H$ , to get:

$$\hat{H}_{TDD} = FR_{gg} [R_{gg} + (\beta / \overline{SNR}) \cdot I]^{-1} \hat{h}_{LS} \quad (12)$$

By ignoring the smoothing matrix  $[R_{hh} [R_{hh} + (\beta / \overline{SNR}) \cdot I]^{-1}]$ , TDQ can be put simply:

$$\hat{H}_{TDQ-LMMSE} = F \hat{h}_{LS} \tag{13}$$

Where:

$$\hat{h}_{LS} = F^{-1} \hat{H}_{LS}$$

Finally, the theoretical calculation for CIR was used as a comparison method with the rest of mentioned methods to find out how close the error is to the lowest error, where:

$$\hat{H}_{Theory LS} = (\beta / \overline{SNR}) \tag{14}$$

### 5. BER of AWGN-Rayleigh Fading Channel

Large-scale fading and small-scale fading, two different kinds of fading effects, are features of mobile radio channels. Small-scale fading is sometimes referred to as Rayleigh or Rician fading (PDF) according to reflecting pathways since the received signal envelope is represented by a Rayleigh or a Rician probability density function when a large number of reflected routes are encountered. Rayleigh fading is the term for fading that occurs when there are several distinct reflecting paths but no dominant line-of-sight (LOS) propagation channel. The fading is Rician distributed if there is also a dominating LOS route. The Probability Density Function (PDF) for Rayleigh fading is given by [17]:

$$f_R(r) = \frac{r}{\sigma_h^2} e^{-\frac{(p^2+q^2)}{2\sigma_h^2}} = \frac{r}{\sigma_h^2} e^{-\frac{(r^2)}{2\sigma_h^2}} \tag{15}$$

Where  $\sigma_h^2$  is the Rayleigh distribution variance.

Now, the theoretical BER for the M-QAM modulation scheme with noise is given by [18]:

$$P_b^{AWGN} = 4 \left(1 - \frac{1}{\sqrt{M}}\right) Q \left( \sqrt{\frac{3 \log_2 M E_b}{M-1 N_0}} \right) \tag{16}$$

For AWGN noise if known that the variance is  $\sigma_w^2$ , BER will be:

$$P_b^{AWGN} = 4 \left(1 - \frac{1}{\sqrt{M}}\right) Q \left( \sqrt{\frac{3 \log_2 M E_b}{M-1 2\sigma_w^2}} \right) \tag{17}$$

BER in the presence of Rayleigh fading channel computed as:

$$p_b = \int_0^\infty p_b^{AWGN} \cdot P_\gamma(\gamma) d\gamma \tag{18}$$

Where:  $P_\gamma(\gamma) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right)$ ,  $\gamma = h^2 \frac{E_b}{N_0}$  is the effective bit energy to noise ratio and  $\bar{\gamma} = 2\sigma_h^2 \frac{E_b}{N_0}$  is the instantaneous bit energy to noise ratio. Substitute (17) in (18) yields:

$$P_b = \frac{1}{2} \left[ 1 - \frac{\sqrt{2\sigma_h^2 \frac{E_b}{2\sigma_w^2}}}{\sqrt{1+2\sigma_h^2 \frac{E_b}{2\sigma_w^2}}} \right] \tag{19}$$

### 6- The method of work

The data was modulated using the M-QAM modulation technique and converted to the frequency domain in a parallel situation. After that, these converted data are reconverted to time domain using the IFFT technique and sent across the propagation channel model. The sent signal is frequently subjected to several phenomena connected to the propagation environment that emerges during the reception during the signal propagation between the transmitter and the receiver. In reality, a signal is made up of a number of constituent signals, each of which follows a unique path and has a unique propagation duration and amplitude. It displays phase changes that can recombine in both beneficial and harmful ways, which results in the signal completely disappearing. The latter effects, sometimes referred to as "fading", may have an impact on how well mobile cellular networks function.

The method is to learn off-line GRNN to the modified MMSE equation:

$$\hat{H}_{GRNN} = \frac{1}{N} (\beta / \overline{SNR}) \left( \frac{I}{I + (\beta / SNR)} \right) \quad (21)$$

Where  $\overline{SNR}$  is the average signal-to-noise ratio, and  $\beta$  is constellation constant.

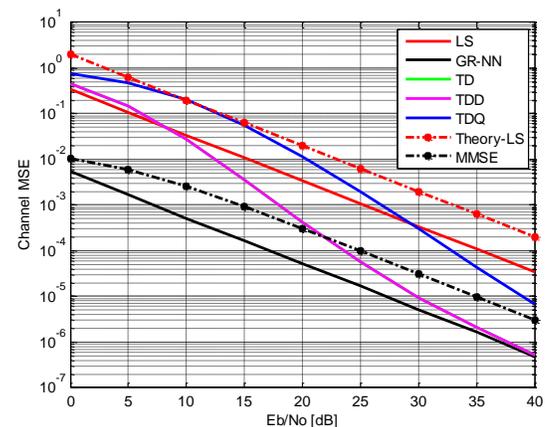
The trained GRNN will be placed online in the system to recover the lost and faded data to get minimum MSE and the best BER performance. The design is very efficient to convey and recover data from the transmitter to the receiver in an efficient manner while taking into account various impacts of the noisy channel, it is crucial in this context to evaluate the performance of these systems. Any wireless network's transmission strives to be error-free while taking into account the significant revolution in data applications. The network is therefore expected to exchange a huge volume of user data. So, the applied algorithm that will be translated to MATLAB program:

1. Read the input parameters after initialization parameters.
2. Grouping and creation of random data.
3. M-QAM Modulation, Pilot insertion.
4. Defining and initializing the estimating techniques.
5. Compute the time domain signal using the IFFT signal.
6. Parallel to serial conversion, Insert Guard interval assessing.
7. Convolute the transmitted signal with Rayleigh fading and add AWGN noise.
8. At the receiver, remove the guard interval and the FFT process.
9. Calculating channel estimation values.
10. Learn and Train GRNN
11. Compare the result of GRNN with the actual CIR
12. Forcing the resulting value to be as GRNN CIR to recover data.

13. Performance assessment and comparison of MSE.

## 7- Simulation Results

In this section, a comparison of the mentioned methods with the proposed GANN method will be discussed. This is based on four parameters: Modulation order, FFT size, cyclic prefix length, and constellation constant ( $\beta$ ). The modulation order, which is defined as the number of symbols that are communicated using the digital communication system, is the first parameter studied in this research. The channel MSE has been calculated for 16-QAM and 32-QAM modulation orders. Fig. 5 depicts the performance of the seven approaches for 16-QAM modulation order.

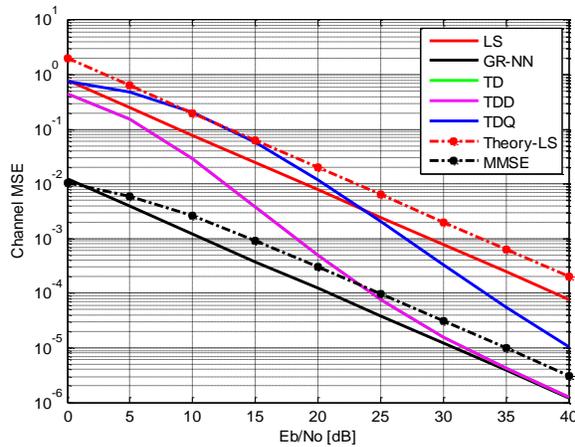


**Figure 5.** MSE characteristics for size of FFT=64 for 16 QAM

As seen in Fig. 5, the best performance is obtained by employing the GRNN, MMSE and TDD approach, with the best value less than  $10^{-6}$  obtained at 40 dB. Unlike previous estimating methods, this approach works well with low SNR values. The worst is an LS and theory-LS method value larger than  $10^{-5}$  at 40 dB. There are places of intersection between the curves, as shown in

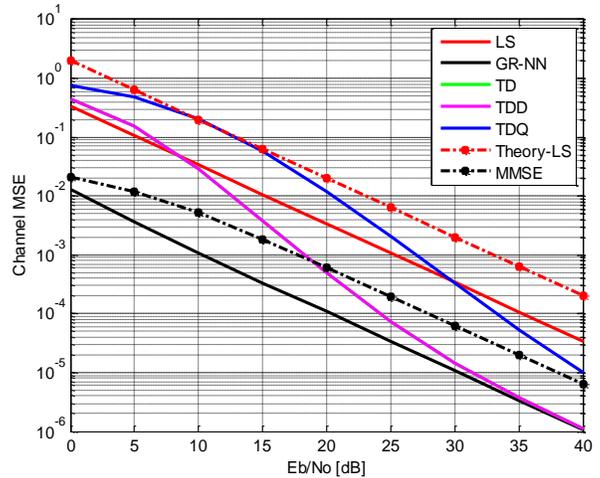
Fig. 5, indicating that the two approaches attain the same channel MSE performance at this point.

The second modulation order value is 32 QAM (shown in Fig. 6), GRNN and TDD approaches share the lead in the results. Overall, the GRNN method is the best of all methods at all SNR values.

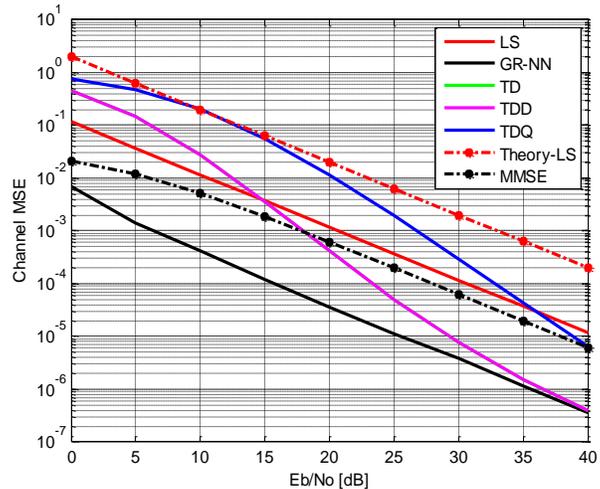


**Figure 6.** MSE characteristics for size of FFT=64 for 32 QAM

The FFT size is the second factor investigated in this research. The FFT size values chosen are 32 and 64. Fig. 7 and Fig. 8 depict the MSE performance of the seven approaches when the FFT size is set to 32 and 64 respectively. As usual, the GRNN approach produced the greatest attainable performance since it had the lowest MSE values among the other six estimating methods. For high SNR levels, both GRNN and TDD function well. When SNR is less than 20 dB, MMSE and GRNN are preferable to TDD method.



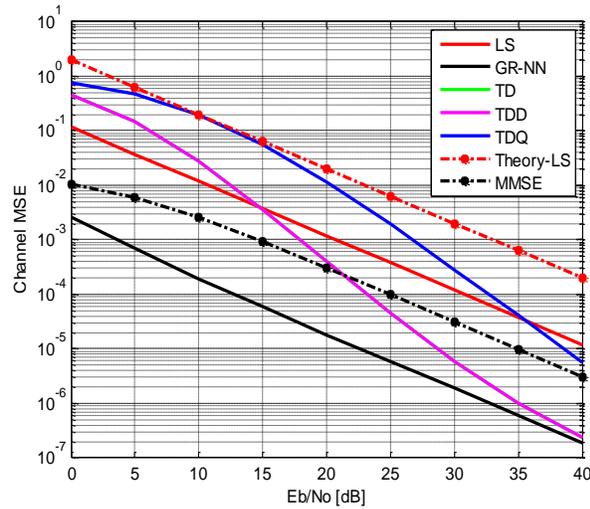
**Figure 7.** MSE characteristics for size of FFT=32 for 64 QAM



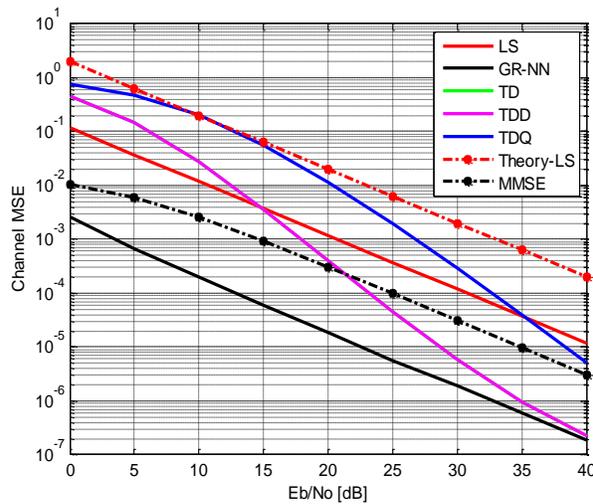
**Figure 8.** MSE characteristics for size of FFT=64 for 64 QAM

The length of the Cyclic Prefix (CP) is the third considered parameter that is utilized between two symbols to avoid Inter-Symbol Interference(ISI). The CP used for investigation are: 4 and 12; Fig. 9 is produced when CP=4. As seen in Figure 9, the GRNN approach still produces the greatest results, while the theory LS method produces the poorest. The remaining five approaches perform differently for the SNR range under consideration. TDD and GRNN have comparable results. when SNR = 23 dB At SNR = 24.5 dB,

LS and MMSE perform the best methods after GRNN.



**Figure 9.** MSE characteristics for size of FFT=64 for 64 QAM and CP=4

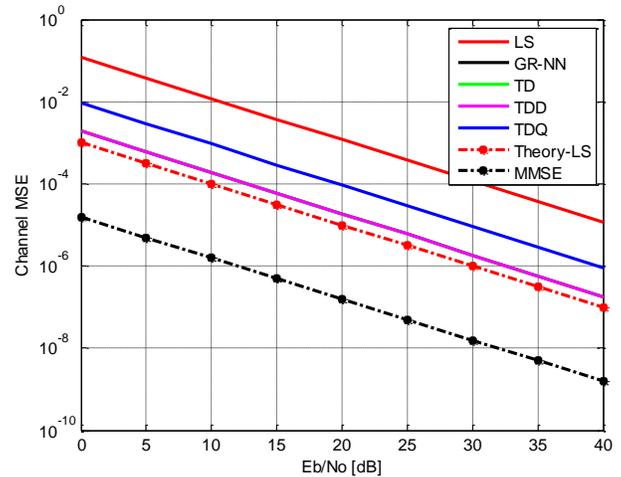


**Figure 10.** MSE characteristics for the size of FFT=64 for 64 QAM and CP=12

The next CP length considered is 12 (shown in Fig. 10), increasing the value of CP does not affect any of the techniques. As a result, the curves in Fig. 10 are identical to those in Fig. 9. For both CP settings, the obtained channel MSE value is the same.

The final parameter investigated in this work is the constellation coefficient ( $\beta$ ). Two values are assigned to this variable to assess its influence on

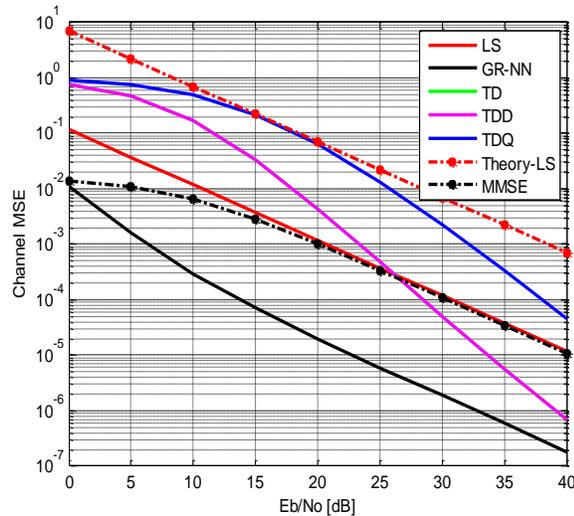
MSE performance for the estimation techniques under consideration. Fig. 11 depicts the MSE performance when  $\beta=0.001$ .



**Figure 11.** MSE characteristics for size of FFT=64 for 64 QAM and  $\beta=0.001$

Choosing  $\beta=0.001$  resulted in completely parallel curves for the technique. This implies that their MSE performance will never be the same. Some curves, however, do not appear in the graphic, indicating that they are identical to one of the other curves. At SNR=40 dB, the maximum attainable performance is attained utilizing the theoretical LMMSE approach with a minimum MSE of  $10^{-9}$ .

The lowest feasible performance is reached using the GRNN approach, with a minimum MSE of less than  $10^{-4}$  attained at SNR=40 dB. Choosing  $\beta=2$ , as seen in Fig. 12, can notice that the best method is still GRNN with a minimum MSE of  $10^{-6}$  at SNR= 40dB.



**Figure 12.** MSE characteristics for size of FFT=64 for 64 QAM and  $\beta=2$

## 8-Conclusion

This paper systematically studies the OFDM system based on a Generalized Regression neural network. The performance of the system channel estimation has been completed, and the training data collection of the OFDM system based on MATLAB a2021 toolbox, the optimization selection of the network training parameters, and the pilot design have been completed. The study found that the channel estimation algorithm based on GRNN proposed in this paper learns the characteristics of the channel through offline channel data, and can achieve channel estimation performance similar to that of the LMMSE algorithm with comparable computational complexity of the LS algorithm.

Parameters such as the optimal learning rate in the network training phase and the number of neurons in the hidden layer can be used in the online channel estimation phase, and the channel estimation algorithm based on GRNN only needs a small number of pilots, so the frequency band utilization of the system is higher. It is worth noting that, compared with convolutional neural networks, recurrent neural networks, and long-

term short-term memory networks that uses a gradient descent algorithm which leads to slow network training or even stagnant phenomenon, but GRNN is simpler to implement than other networks. In addition, the structural parameters of the GRNN are related to the application scenario and the data set used in the previous offline training stage.

So far, there is no unified and perfect theoretical method to deduce. Calculating the optimal network structure can generally only be obtained from experience. It will be a future research direction to further explore the application of other neural network models in OFDM system channel estimation.

## Conflicts of interest

The authors declare that there is no conflict of interest in the publication of this article.

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## Author Contribution Statement

In creating and revising this paper, all authors contributed. The research issue and results of this study were proposed by the authors. All authors worked together to develop the manuscript's introduction and structure. They also talked about the findings and contributed to finish the manuscript.

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