The (d50) Criterion in Estimating the Bedload in Earthen Channels

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Abstract

It is a common tradition to depend on (d50) of the bed-material to estimate the sediment load in earthen channels and consider it appropriate to yield results of acceptable accuracy.

The aforementioned concept is reviewed in this research through applications on a practical case study constituted nine existing channels, six of which are in the Middle-Euphrates area. Grain-size distribution analyses have been performed for the bed material of those six channels for the purposes of this research, whereas the available data for the other three were considered. The **Einstein [1950]** ***approach for estimating the bedload (as developed by*** **Al-Delewy [2003]** ***to make the approach fully amenable for solution by a computer) is used in this respect. Besides, some new additions were necessary to satisfy the requirements of the research.***

The bed material of each example channel has been classified into six size-classes. Estimates of bedload were performed, once based on (d50) and then for the classified soil; (PR) has been used to denote the ratio of the former to the latter.

Despite that no rigid conclusion can be abstracted from relatively limited applications, the research indicated that the (d50) criterion in some cases is not more than a very rough indicator, particularly with very fine or coarse beds, and especially with mild bed- slopes; the ratio (PR) ranged from (0 %) to (1718 %).

An indicator of the classified sizes, called texture coefficient, (TC), has been introduced in the research. Logical combinations of the involved effective parameters (namely, R, R', S, d50, and TC) have been established. On analyzing the computed values of the formed combinations with respect to (PR ***), it was found that (*** PR ***), in general and in a more pronounced fashion, varies successively with the parameter (*** $R' \times S / TC$ ***). A value of (*** $R' \times S / TC$ ***) around (200) (in the sense given in the text) makes the (d50) criterion quite reasonable in estimating the bedload.***

الخلاصـــــة

جرت العادة على الاعتماد على قياس (d50) لحبيبات تربة القاع في تخمين حمل الرسوبيات في القنوات الترابية، واعتباره مناسباً لإعطاء نتائج بدقة مقبولة.

في هذا البحث جرت مراجعة هذا المفهوم من خلال التطبيق على حالات عملية تضمنت تسع قنوات قائمة، ست منها ضمن منطقة الفرات الأوسط. لقد جرى تحليل نسجة تربة قاع تلك القنوات الست لأغراض البحث، في حين استخدمت البيانات المتوفرة بالنسبة للقنوات الثلاث الأخرى. واستعملت في تخمين حمل القاع طريقة [اينشتاين: ١٩٥٠] المطوّرة من قبل [الدلوي: ٢٠٠٣] ليكون الحل بالحاسوب ممكناً بشكل كامل، وأضيفت الى ذلك إضافات جديدة تطلّبتها طبيعة هذا البحث.

لقد تمّ تصنيف حبيبات تربة القاع لكل قناة الى ستة أصناف. وجرى حساب حمل الرسوبيات لكل قناة، مرة باعتماد (d50) وأخرى باعتماد الحجوم المصنفة، واستُعملت النسبة (PR) للدلالة على نسبة الأول الى الثاني.

ورغم عدم إمكان استخراج استنتاج قاطع بالاعتماد على حالات تطبيقية محدودة نسبياً، فإن البحث دلل على أن معيار (d50) قد لا يصلح في بعض الحالات إلا كدليل تقريبي جداً، خاصة مع التُرب الناعمة جداً أو الخشنة، وبالأخص مع انحدارات القاع الخفيفة، إذ تراوحت النسبة (PR) بين (صفر %) الى (١٧١٨ %).

لقد استحدث في البحث معيار سُمّيَ (معامل النسجة TC)، وتمّ تكوين مجاميع منطقية جديدة تربط العوامل المؤثرة (وهي TC, d50, S, R', R). وبتحليل قِيم تلك المجاميع وربطها مع النسبة (PR) وُجد بأن النسبة (PR) على العموم وبشكل أوضح نسبياً تتناسب طردياً مع المجموعة ($R' \times S / TC$)، وإن قيمة هذه المجموعة بحدود (٢٠٠) (بالصيغة المذكورة بالبحث) تجعل معيار (d50) مناسباً لتخمين حمل القاع.

1. Introduction

Man had faced the problems of erosion of and sedimentation in earthen channels since antiquity. The process of erosion-sedimentation in channels occasionally alters the bed slope, bed width, and side slope, creates bars, local bifurcation, and meanders, and sometimes leads to a distinguished change in the channel course. The course of the Euphrates River in its middle part (within Babil and Kerbala governates) is a good example in this respect. The present course of the river therein is distinctly far from its old one some centuries ago, whereas its older course is almost the present course of Shatt Al-Hilla.

The pioneer work of R.G. Kennedy [1] presented what has been commonly called the regime approach. It was a leading, scientifically-organized work in the field of erosion-sedimentation. However, besides being pure empirical, that work faced a major criticism in that it does not take into consideration any thing concerning the sediment itself.

Beside improving the empirical coefficients of Kennedy, the scientists and investigators that followed him in this field focused on introducing one or more of the basic characteristics of the sediment, namely, type, shape, size-distribution, and quantity.

In respect to the sediment type, the majority of the works dealt with non-cohesive soils. This type is more amenable to analytic or semi-analytic analysis whereas cohesive soils require special treatment that takes into consideration the forces of cohesion. With regard to the shape of soil particles, sphericity is commonly assumed. However, some investigators introduce some sort of a shape factor to consider the effect of non-sphericity.

Whenever the size of the sediment originating from a soil mixture is needed, a common practice is to consider the soil bulk to be reasonably represented by its (d50), the size for which (50%) of the soil by weight is finer. Few of too many to mention are reviewed hereinafter.

G. Lacey made several contributions concerning the regime approach. In his first work, [2], he introduced the Lacey silt factor (f_L) which is a function of (d50).

T. Blench, alone or sharing others, also made several contributions in the same field. He divided Lacey's (f_L) into two factors, the bed factor (F_B) and the sides factor (F_S) [3]. The former is a function of both (d50) and the quantity of sediment whereas the latter depends on the type of the soil of the channel bed.

D. B. Simons and M.L Albertson [4] introduced a table of empirical coefficients that consider basically the type of bed soil, to a lesser extent the amount of sediment, but not the size of the sediment.

The (d50) represents a governing parameter in any analytic or semi-analytic approach in sediment transport. Consequently, all sediment-transport equations involve (d50) as a basic parameter. A leading example in this respect is the work of A. Shields in 1936 {Quoted from [5]} and the famous plot known as the Shields diagram.

The two most familiar works in this respect are the bedload equation of E. Meyer-Peter and R. Muller of 1948 {Quoted from [6]} and the Einstein [1950] approach to estimate the sediment load [7]. The latter allows for investigating the sediment transport whether as bedload, suspended load, or total bed-material load, and for different fractions of a soil bulk.

The Einstein [1950] approach to estimate the bedload has been developed so that it became completely amenable for solution by a computer [8]. This research aims at using the developed Einstein [1950] approach to investigate the extent of accuracy in using (d50) in bedload calculations for some practical cases.

2. The Einstein (1950) Approach

Estimating the sediment load in an existing channel by the Einstein [1950] approach (or its developed version) requires certain physical and hydraulic data. Such data are: channel bed-width (B), side slope (Z), bed-slope (S), design discharge (QX) and its corresponding flow depth (D), beside water density (ρ) and kinematic viscosity (υ), and gravitational acceleration (g). As for the channel bed-material, the approach requires soil relative density (S_S) and the grain sizes (d35) and (d65), beside (d50) (which is the keystone in this research when considering the soil bulk). However, the comprehensive sediment-load calculations require a detailed grain-size distribution analysis. According to that analysis, the bed material would be divided into a number of classes of grain sizes, (is), and determine for each class its representing nominal size (di) and the percentage from the sample mass, (ib), each class occupies.

The Einstein [1950] approach involves preliminary calculations and two categories of basic calculations. The preliminary calculations involve determining the cross-sectional area of flow, (A), the wetted perimeter, (P), hydraulic radius, (R), and average flow-velocity, (U). The first category of basic calculations involves calculating parameters which are related to (R'), the part of the hydraulic radius with respect to grains. The second involves calculating parameters which are related to the nominated grain size, (d50) or (di), as the case may be.

The goal is to obtain a good estimate of Einstein's intensity of transport for individual grain–size, (Φ_*). The estimate of the annual bedload could be obtained as:

$$Qsy\ (m^3 / year) = \{ \Phi_* \ [(S_S - 1)\ g\ d^3]^{1/2}\ (ib)\ (P)\} \{86400 \times 365\} \quad \textbf{(1)}$$

When calculations are made for the soil bulk then (d = d50) and (ib = 100% = 1). However, if the calculations are for a certain class (is) of the soil bulk, then (d = di) associated with its respective (ib). This will yield (is Qsy) in respect to the considered grain-size class (is). In this case, the respective (Qsy) concerning the considered channel would be: Qsy = $\sum$ is Qsy.

The first category of basic calculations starts by specifying the value of (R') corresponding to the considered hydraulic data. Such a value is not directly available but through a trial-and-error approach. Instead of that, a simple iterative optimization approach is used in the research. It proceeds as follows:

1. Assume a reasonable value of (R'), say (AR').
2. Use the developed Einstein [1950] approach to calculate the corresponding flow velocity (CU).
3. With the existing flow velocity (U), define a convergence parameter (α) as :

$$\alpha = \left| \frac{CU - U}{U} \right| \times 100 \quad \textbf{(2)}$$

4. The optimization process aims at minimizing (α), subject to: ($0 < AR' < R$). Specify a reasonable tolerance limit ($\alpha 1$). The iteration ends when ($\alpha \le \alpha 1$). At such a state, ($R' = AR'$).
5. If step [4] is not satisfied, repeat the calculations with a new value of (AR').

It is to be noted that a successful choice of the initial value of (AR') and its revisions during the iteration process is necessary to avoid cyclic or non-convergent solutions and to decrease the number of iterations for a convergent solution.

3. The Case Study

Nine-somehow different-practical examples have been chosen to form the case study. These are:

1. Iskendariya Canal, (Km 0.000).
2. Great – Musaiyab Canal crossing Baghdad – Hilla highway, (Km 11.200).
3. Beni – Hasan Canal, (Km 18.000).
4. Mahaweel Canal just upstream Mahaweel City, (Km 8.000).
5. Shatt Al-Hilla at the upstream of Hilla City, (Km 40.000).
6. Euphrates River within Hindiya City, (Km 626.000).
7. Tigris River in the vicinity of Baghdad City.
8. Diyala River in Qara Teppa area.
9. The example river reach in [5].

The hydraulic data concerning example channels (1) through (6) have been obtained from [9]; all data concerning channels (7) and (8) have been obtained from [10]; those for channel (9) are available from [5].

The research aimed at more realistic information with respect to channels (1) through (6) through performing a real bed-material analysis. However, all these channels are continually flowing. This made extracting soil samples from the existing beds impractical. Consequently, after locating an accessible location on each one, two soil samples have been taken from that location (personally by the researcher); one sample has been extracted from the bare sides just above the existing water surface whereas the other from the latest ruins of excavation heaped in the vicinity of the location. The twelve samples were analyzed by the researcher for a detailed grain-size distribution (at the Soil Lab of the Civil Engineering Department, with acknowledged assistance of a member of the laboratory staff), first by standard sieve analyses and the remainder by the hydrometer test.

The basic bed-materials parameters of the case study required for the research, namely, S_S, d35, d50, and d65, are given in **Table (1)**. The hydraulic data concerning the case study and usable in the research are given in **Table (2)**.

The first six examples represent channels with fine-textured and mildly-slopped beds. The seventh has a medium-textured bed and a moderate bed-slope. The eighth has a coarse-textured bed and a moderately-steep bed-slope. The ninth is a channel reach with a medium-textured bed and a steep bed-slope.

Table (2) The case study: summary of hydraulic data
[Sources of data are mentioned in the text]

Channel		*B (m)*	*Z (H :V)*	*S (cm/km)*	*Q (m^3/s)*	*D (m)*
1	Iskandariya Canal	6.10	1.5	10	6.5	1.60
2	Gt.Musaiyab Canal	20.65	1.5	9	58.7	2.99
3	Beni – Hasan Canal	7.05	2	10	28.6	2.34
4	Mahaweel Canal	9.30	1	13	9.75	1.48
5	Shatt Al – Hilla	56.75	3	8	305	5.20
6	Euphrates River	203	1	8	1500	9.07
7	Tigris River	300	1	15	3000	7.51
8	Diyala River	320	1	35	2000	3.75
9	Graf example	91.46	1	70	573	1.96

4. Application

In estimating the bedload, the following have been adopted in the research:

(1) With respect to grain-size distribution analysis:

a. The least grain size considered in the analysis is (0.01 mm).

b. The bed material of each channel has been divided into six classes.

c. Linear interpolation is used to define the respective class size between two grain sizes.

When a class covers more than two grain sizes, a weighted mean is calculated as :

$$\text{is di} = \sum (\text{di . ib}) / \sum \text{ib} \quad \text{.. (3)}$$

The results of grain-size distribution analysis are summarized in **Table (2)**

(2) For water: $\rho = 1000\ kg / m^3$ and $\upsilon = 1 \times 10^{-6}\ m^2 / s$; $g = 9.8\ m / s^2$.

(3) The iteration process mentioned before to determine the value of (R') that corresponds to the existing (U), i.e., to the specified (QX), requires an assumed initial value of (R') to start with; however, a conclusive guide in this regard is unavailable. Consequently, a conservative value of (AR'=0.99R) has been chosen. Besides, some practical mathematical measures have been set in the computer program to prevent cycling and to decrease the number of iterations.

Moreover, the tolerance limit for convergence [as defined in Eq.(2)] that has been adopted in the research was ($\alpha 1 = 0.5\ \%$).

5. Results and Analysis

1. The finally adopted values of (R') are given in Column (4) of **Table (3)**.

2. Final results of bedload estimates are given in **Table (3)** too. Those based on (d50) are given in Column (5). Those for classified bed-materials are given in Column (6). Column (7) shows the ratio of the former to the latter as a percentage, (PR). The respective values of (PR) indicate noticeable differences; the ratios ranged from (0 %) to (1718 %).

It is to be noted that in respect to the example channels of the case study, channel (8) is unique, being with a coarse-textured bed. Channel (9) is also unique, being with a steep-bed slope. However, the latter is the only one for which the estimated bedload based on (d50) and that based on (is di) are practically equal.

Table (3) Summary of bedload estimates

Channel No.	*General Description*		R' (m)	Q_{SY} (m^3 / year)		PR=(A/B) × 100 (%)
	Texture of bed material	**Bed-slope**		**[A] Based on d50**	**[B] Based on is d_i**	
1	**2**	**3**	**4**	**5**	**6**	**7**
1	Very fine	Mild	0.373	25	2 448	1
2	Moderately fine	Mild	1.005	9 536	555	1 718
3	Moderately fine	Mild	1.505	8 527	1 736	491
4	Very fine	Moderate	0.434	0	2 133	0
5	Very fine	Mild	1.097	0	2 295	0
6	Very fine	Mild	0.981	5 029	76 560	7
7	Moderately fine	Moderate	1.525	1 278 467	913 861	140
8	Coarse	Moderately steep	1.560	1 962 073	1 017 789	193
9	Moderately fine	Steep	1.832	1 900 169	1 863 913	102

3. As mentioned earlier, it is customary to represent the bed-material bulk by its (d50) in estimating the sediment load (considered as bedload only in this research). This practically-common tradition has been considered to give quantitative estimates with satisfactory accuracy.

However, it is obvious that estimates of bedload for a classified bed-material would be more accurate than those for the bulk; despite the increase in calculations effort, the more the number of classes the more accurate the results would be.

The research indicates that the difference in the results, being tens folds for the investigated examples, is practically significant.

4. The annual volumetric bedload (Qsy) could be set in the form:

$$Qsy = C1\ \Phi_* \quad \text{.. (4)}$$

where C1 = collective constant; Φ_* = Einstein's intensity of transport for individual grain size.

(Φ_*) is obtained by an inverse functional relationship with Einstein's intensity of shear on individual grain size (Ψ_*). The involved calculations according to Einstein [1950] yield the value of (Ψ_*). Values of (Ψ_*) calculated for the case study are given in **Table (4)**.

Without being of certain practical significance but just for illustration purposes, a weighted-average value ($\overline{\Psi_*}$) has been computed for each example channel and given in **Table (4)**, where:

$$\overline{\Psi_*} = \sum (\Psi_{*i} \,.\, ib) / \sum ib \quad \text{.......} \quad (5)$$

Table (4) Values of Einstein`s intensity of shear on individual grain size (ψ*)

Channel No.	*Based on d50*	*Based on the Respective Class Size*						
		Class (1)	Class (2)	Class (3)	Class (4)	Class (5)	Class (6)	$\overline{\psi_*}$
1	2.49	3.61	6.19	24.09	10.51	4.69	1.38	4.35
2	6.75	6.45	36.93	74.27	44.42	20.39	6.55	31.66
3	2.79	3.09	14.16	34.16	28.24	12.91	4.84	13.13
4	8.56	5.03	30.02	20.53	11.71	7.13	2.54	9.29
5	11.42	31.79	13.05	13.27	35.04	13.37	4.61	13.47
6	6.86	5.65	4.21	15.79	21.91	9.53	2.62	8.53
7	2.32	2.60	2.19	1.79	1.60	2.21	6.82	2.48
8	2.53	10.38	7.27	3.99	8.49	22.40	42.06	13.74
9	0.23	0.51	0.42	0.30	0.23	0.23	0.28	0.30

5. With all other constituent parameters of (Ψ_*) being the same, (Ψ_*) would then varies directly with Einstein's hiding factor (ξ). This factor is based on the assumption that small particles will hide behind larger ones.

On following the sequence of steps for obtaining the respective value of (ξ), it will appear that the grain size (d) plays the major role in this respect. Values of (ξ) calculated for the case study are given in **Table (5)**. The values ranged from its minimum value of (1.00) for coarse grains to as large as (135.30) for fine grains. Similar to ($\overline{\Psi_*}$), a parameter ($\bar{\xi}$) is computed and given in **Table (5)** too, where:

$$\bar{\xi} = \sum (\xi_i \,.\, ib) / \sum ib \quad \text{.......} \quad (6)$$

Table (5) Values of Einstein`s hiding factor (ξ)

Channel No.	*Based on d50*	*Based on the Respective Class Size*						
		Class (1)	Class (2)	Class (3)	Class (4)	Class (5)	Class (6)	$\bar{\xi}$
1	135.30	1.00	10.64	135.30	135.30	135.30	135.30	128.85
2	1.75	1.04	32.99	135.30	135.30	135.30	135.30	106.35
3	2.46	1.16	18.70	98.76	135.30	135.30	135.30	92.75
4	135.30	1.00	85.50	135.30	135.30	135.30	135.30	124.51
5	45.60	1.00	1.00	10.64	135.30	135.30	135.30	95.41
6	17.16	1.00	3.88	36.62	135.30	135.30	135.30	42.31
7	1.03	1.00	1.01	1.03	1.23	2.56	15.77	2.84
8	1.51	1.00	1.00	1.28	5.12	27.02	135.30	20.72
9	1.01	1.00	1.00	1.01	1.08	1.54	2.19	1.08

6. For specified (B), (Z), and (QX) for a channel in alluvium with non-cohesive bed-material of almost rounded particles, the process of bedload estimation by Einstein [1950] approach is basically characterized by the grain-size distribution and the parameter $(R' \times S)$ [8]. However, the way in which the governing parameters, namely, (d50, is di and ib, R, R', and S) interact in the problem is too complex and not amenable to direct analysis and interpretation.

In an attempt to go through this dilemma, a parameter, called the ***texture coefficient*** (TC), defined as:

$$TC\ (\%) = [\sum\ (is\ di \times ib) / d50] \times 100 \quad \text{.......................................} \quad \textbf{(7)}$$

has been calculated. Then, a variety of combinations of the governing parameters were calculated, namely, (R'/R), $(R' \times S)$, $(R' \times S/R)$, $(d50 \times S)$, $(S/d50)$, $(TC \times S)$, $(TC \times R')$, $(TC \times R' \times S)$, $(TC \times R' \times S/R)$, (TC/S), (TC/R'), $(TC/R' \times S)$, and $((TC \times R)/(R' \times S))$.

Whereas some of the aforementioned combinations of the involved parameters may serve as indicators for the quantity of bedload (which is not the direct goal of this research), the ones of indicative sense for the aimed comparison are given in **Table (6)**.

The results in **Table (6)** indicate that, for the considered case study and mild or moderate bed-slopes [thus, discarding channel (9)], the parameter $(R' \times S/TC)$ is the most indicative one. It seems that the ratio (PR) varies successfully with this parameter; a value of this parameter of around (200) (in the units used in the research) would make (d50) to give comparatively reasonable bedload estimates.

6. Conclusions

Realizing that no specific rigid conclusion could be abstracted from relatively limited examples, nevertheless, based on the results outlined in the preceding section, the following conclusions are evident:

(1) Estimates of bedload based on (d50) may be reasonably acceptable for channels on medium-textured beds and moderate bed-slopes (like example channel No. 7).

(2) Bedload estimates based on (d50) for channels on beds whether fine-textured or coarse-textured would be only indicative and may serve as good rough-estimates, particularly with mild bed-slopes. Consequently, the reference to a classified bed material would be indispensable when more accurate estimates are aimed at.

(3) The texture of the bed-material and, to a lesser extent the channel bed-slope, plays a significant role in estimating the bedload. This is clearly noticeable through Einstein's hiding factor (ξ).

(4) With the texture coefficient (TC) proposed in this research, the comprehensive parameter $(R' \times S / TC)$ as defined in the text may serve as a reasonable indicator to verify the use of (d50) in bedload estimation. A value around (200) was found to be appropriate for channels on moderately textured beds and moderate bed-slopes.

(5) Due to its recognized role in estimating the sediment load, the parameter (ξ) worth to be the subject of a separate research (which is lacking for the time being).

(6) A more rigid conclusion concerning the basic subject of this research could be set when more extensive and dependable data are available.

Acknowledgement

The researcher is grateful to Miss Hamida Hasan Nuwayss for her assistance in performing the laboratory work at the Soil Lab of the Civil Engineering Department, and to his daughters Shatha and Zainab for typing the manuscript.

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