Journal of Engineering and Development, Vol. 9, No. 3, September (2005) ISSN 1813-7822

Analysis of Inflatable Dams under Hydrostatic Conditions

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Abstract

Inflatable dams are flexible cylindrical inflatable and deflatable structures made of rubberized material attached to a rigid base and inflatable by air, water, or a combination of air/water.

The interest in inflatable dams is increasing because of the ease of placement and construction.

Behavior of air or water inflated dams is physically studied and analyzed under different conditions of internal pressure, upstream and downstream heads of water.

Experimental data obtained on laboratory test facility for with air and/or water inflated dams are presented and compared with the theoretical results based on a developed computer program describing dam height, cross sectional profiles and dam cross sectional area. Good agreement was obtained between theory and experiment results.

الخلاصـــة

السدود المنتفخة عبارة عن منشآت أسطوانية مطاطية مرنه قابلة للانتفاخ والانضغاط يتم تثبيتها على قاعدة صلبة ونفخها من خلال ملئها بالهواء أو الماء آو كليهما. ويزداد في الوقت الحاضر الاهتمام بهذا النوع من السدود نظراً لسهولة إنشائها وتشغيلها وكذلك نقلها من مكان لأخر.

في هذا البحث تم دراسة أسلوب عمل السدود المنتفخة المملوءة بالهواء أو الماء آو بخليط منهما وتحليلها نظريا وحقليا وتحت مختلف الظروف والمتغيرات ، كتغير الضغط الداخلي للنفخ وتغير ضغط ارتفاع الماء في مقدمة ومؤخرة السد.

هذا وتم أيضا تحليل البيانات المختبرية لبرنامج متكامل من التجارب المختبرية لنماذج من السدود المطاطية تم تصنيعها وتشغيلها بعناية من خلال استخدام الهواء آو الماء آو مزيج منهما لملئ ونفخ هذه النماذج ، وتم تحليل ومقارنة النتائج المختبرية والنظرية لأكبر عدد من الحالات وذلك بكتابة وفحص وتشغيل برنامج حاسبة يتعامل مع عدة متغيرات، حيث وجد توافق جيد بين النتائج العملية والنظرية.

1. Introduction

In the modern times the construction of hydraulic structures becomes essential due to the increase in population, consumption rates and numerous uses against a limited resource of water. In the choice of a suitable dam for the site, economic and technical aspects should be carefully considered. With the advantages of the inflatable dams [2,3,7,10,11], behavior and stability of these dams have highlighted the importance of modeling inflated dam profiles for design and planning purposes.

Prediction of the inflatable dam profiles has been studied by many researchers [2,3]. For these purposes, several mathematical models have been developed. Particular attention has been devoted to the effects of internal pressure and external water pressure on the dam stability [2,3,5,8].

This paper explores the effects on the internal pressure and upstream head as well as the inflation medium (air and water) under hydrostatic conditions on the behavior of the inflatable dams **Fig.(1)**.

The analysis of Harrison [5] was applied to develop a computer program. Good agreement was obtained between theory and experiment.

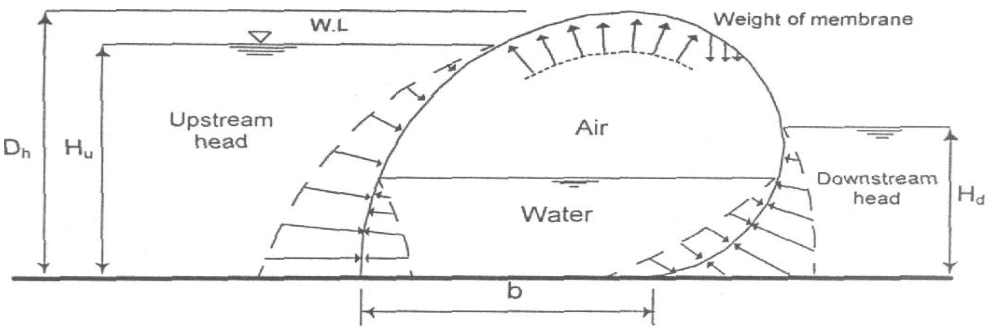

Figure (1) Forces Acting on the Dam (Hydrostatic Condition)

2. Experimental Arrangement

The experiments were performed in rectangular glass walled flume having a length of 20 m, width of 0.9 m and a depth of 0.6 m. the discharge was measured with the help of a rectangular sharp crested weir [4]. Water level and dam profile displacement, horizontally and vertically, were taken with the help of profile gage [1].

Air pressure inside the dam model was measured with a water manometer tube. Air compressor was used to inflate the air dam model. A piezometer connected to a steel column of external diameter of 200mm was used to measure the water pressure inside the water dam model. The steel column was also used to measure inflate the dam model by water.

A rubber material with the properties mention in **Table (1)** is used to build the models. Two models of dams, air inflated dam and water inflated dam, were made from a rectangular rubber sheet with the perimeter length of 0.553m, membrane thickness of 0.001m and base length of 0.15m have been employed [1].

Table (1) Membrane Properties

Material	Thickness (mm)	Weight (kg/m2)	Tensile Strength (kN/m)
Rubber	1	1.3	6.453

3. Analysis and Discussion

3-1 Comparisons between Experimental and Theoretical Results

A compression was carried out between the experimental of height and cross sectional area of the inflated dam with those obtained from the theoretical analysis. A computer program based on the finite elements technique has been developed to analyzed air-inflated dams, and water inflated dams under hydrostatic conditions.

Tables (2) and **(3)** show the difference in dam height and cross sectional area for air-inflated dams, and water-inflated dams respectively. Also the comparison of shape of the dam between the experimental and theoretical results for some cases is included [**Fig.(2)** and **(3)**].

Table (2) shows that the mean percentage of differences for all values of dam height and dam cross sectional area are 2.17%, 3.82% respectively. It can be seen that the higher percentage differences are found when low inflated pressures or higher upstream head is applied; this will increase the distortion of the dam and make the effect of end fixation of the dam model on the central cross sectional profile is clear.

Figure (2), shows a comparison between the experimental and theoretical results of the dam for some cases of low and high internal air pressures. It can be observed from **Fig.(2)** that a good agreement exists for higher internal pressures (4kN/m^2) while a bad agreement exists with low internal pressure (2kN/m^2) **(Fig.2b)**, that is, for the same upstream head (200 mm) and downstream head (0 mm). On the other hand a good agreement is found with low upstream heads (100 mm). Such behavior is attributed to the effect of end fixation of the dam model on its center. In **Table (3)**, the mean percentage of differences for all values of dam height and dam cross-sectional area are 4.53% and 2.63% respectively. The higher percentage differences, same as in air-inflated dam, was found with low inflated pressures (306 mm), **Fig.(3)**.

Table (2) Comparison between Theoretical and Experimental Dam Height and Cross Sectional Area (Air Inflated Dams-Static Conditions)

Test No.	Inside Air Pressure (kN/m^2)	U/S Head (mm)	D/S Head (mm)	Dam Height (mm)		% Abs. Diff. in Height	Cross-section Area (m^2)		% Abs. Diff. In Area
				Exp.	Theo.		Exp.	Theo.	
1	1.5	50	0	205.6	204.09	0.73	0.0380	0.03889	2.34
2		100	0	208	205.67	1.12	0.0381	0.03872	1.63
3		150	0	211.9	207.4	2.12	0.0366	0.03825	4.51
4		200	0	212.6	194.26	8.63	0.038	0.03525	7.24
5			550	213.6	201.01	5.89	0.0378	0.03597	4.84
6			100	216.2	209.5	3.10	0.0379	0.03641	3.93
7			150	221.8	218.92	1.29	0.03746	0.03667	2.10
8	2	50	0	209	206	1.44	0.0396	0.0396	0.00
9		100	0	210.8	207.5	1.57	0.0392	0.0395	0.77
10		150	0	213.4	209.4	1.87	0.0389	0.0392	0.77
11		200	0	214.6	206.1	3.96	0.0391	0.0380	2.81
12	3	50	0	213.6	210.1	1.63	0.0395	0.04114	4.15
13		100	0	214.6	211.38	1.5	0.03895	0.04085	4.87
14		150	0	216.4	212.3	1.89	0.04107	0.04071	0.88
15		200	0	218	212.1	2.71	0.04072	0.04022	1.23
16	4	50	0	216.9	214.33	1.18	0.04033	0.0426	5.63
17		100	0	217.6	215	1.19	0.03902	0.04241	8.68
18		150	0	219.3	216.12	1.45	0.03988	0.04232	6.12
19		200	0	220.2	217	1.45	0.0411	0.042	2.19
20	5	50	0	222.2	219.35	1.28	0.04167	0.04417	6
21		100	0	223	219.72	1.47	0.04241	0.04403	3.82
22		150	0	224.3	220.92	1.5	0.04091	0.0440	7.55
23		200	0	225.7	221.2	1.99	0.04185	0.04364	4.28
24			50	225.8	221.4	1.95	0.04241	0.04374	3.14
25			100	226.8	222.6	1.85	0.0421	0.04363	3.63
26			150	228.2	224.5	1.62	0.04122	0.04377	6.19
Mean						**2.17**			**3.82**

Table (3) Comparison between Theoretical and Experimental Dam Height and Cross Sectional Area (Water Inflated Dams-Static Conditions)

Test No.	Inside Air Pressure (kN/m^2)	U/S Head (mm)	D/S Head (mm)	Dam Height (mm)		% Abs. Diff. in Height	Cross-section area (m^2)		% Abs. Diff. in Area
				Exp.	Theo.		Exp.	Theo.	
1	306 (3 kN/m^2)	50	0	186.9	177.9	4.82	0.03704	0.03814	2.97
2		100	0	188	177.5	5.58	0.03721	0.03753	0.86
3		150	0	190.1	175.8	7.52	0.03826	0.03705	3.16
4		170	0	191.8	172.91	9.85	0.0398	0.03627	8.87
5			50	192	173.1	9.84	0.03918	0.0363	7.35
6			100	194	184.8	4.74	0.03928	0.03765	4.15
7			150	198.4	194.2	2.12	0.03868	0.03882	0.36
8	408 (4 N/m^2)	50	0	198.9	191	3.97	0.03932	0.03993	1.55
9		100	0	199.7	192.1	3.81	0.03944	0.03986	1.06
10		150	0	201.6	192	4.76	0.04079	0.03983	2.35
11		170	0	202.3	189.6	6.28	0.03933	0.03915	0.46
12	510 (5 kN/m^2)	50	0	206.9	199.1	3.77	0.04254	0.04165	2.09
13		100	0	207.7	199.8	3.80	0.04273	0.04157	2.71
14		150	0	209.3	200.9	4.01	0.04197	0.04134	1.50
15		170	0	209.4	200.5	4.25	0.04331	0.04127	4.71
16			50	209.7	201.3	4.01	0.04331	0.04114	5.01
17			100	211	203.7	3.46	0.04045	0.04136	2.25
18			150	213.5	206.1	3.47	0.04228	0.0414	2.08
19			170	214.6	206.5	3.77	0.0419	0.04148	1.00
20	612 (6 N/m^2)	50	0	213.5	205.7	3.65	0.042	0.04315	2.74
21		100	0	214.1	206.3	3.64	0.04207	0.04296	2.12
22		150	0	215.4	207.6	3.62	0.04321	0.04304	0.39
23		170	0	215.9	207.1	4.08	0.04446	0.04292	3.46
24	714 (7 N/m2)	50	0	219.1	211.7	3.38	0.04472	0.04494	0.49
25		100	0	219.8	212.9	3.14	0.04466	0.04493	0.60
26		150	0	221.1	213.2	3.57	0.04337	0.04493	3.60
27		170	0	221.5	213.7	3.52	0.04337	0.04467	3.00
Mean						**4.53**			**2.63**

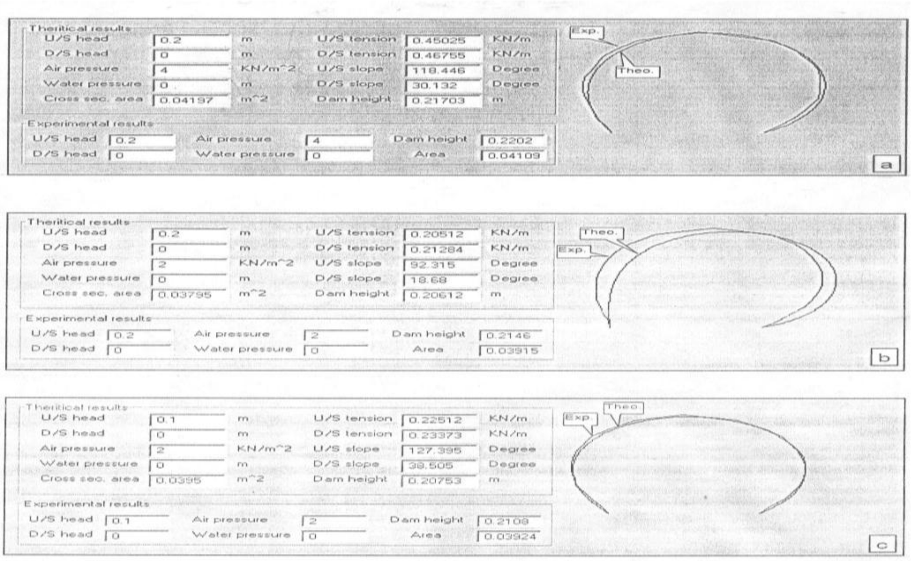

Figure (2) Experimental and Theoretical Shape of Air-Inflated Dam

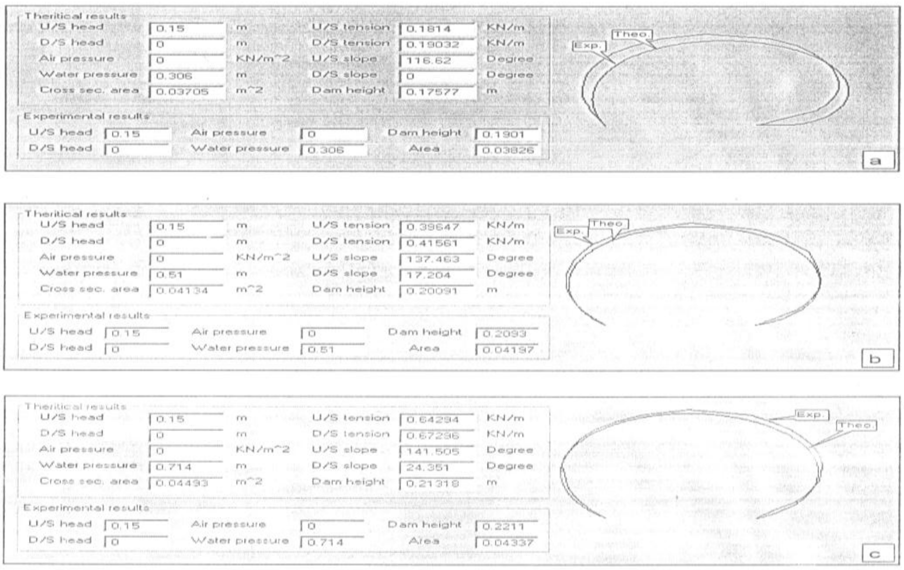

Figure (3) Experimental and Theoretical Shape of Water-Inflated Dam

3-2 Behaviors of Inflatable Dams

Analysis for the results obtained theoretically from the computer program was carried out to investigate the behavior of inflatable dams under different conditions of internal pressure and upstream head for both air and water inflated dams under hydrostatic conditions. The effect of base length, membrane thickness, membrane perimeter length on the behavior of inflatable dams is also included in this work.

3-2-1 Effect of Increasing Internal Pressure for Constant Upstream and Downstream Head

Figures (4) and **(5)** show the effect of rising internal pressure on the cross-sectional profile of air-inflated dam and water-inflated dam respectively. **Figure (4)** shows the change in cross-sectional profile for air-inflated dam when increasing the internal air pressure from 1.5kn/m^2 to 5kn/m^2 with constant upstream head of 200mm and downstream head equal to zero. Large deformation occurred with low internal pressure, the change in cross-sectional profile of dam is due to change in forces on element and therefore causing a change in tension and slope of the element [1].

Figure (5) shows the behavior of water-inflated dam when increasing the internal water pressure from 306mm to 714mm with constant upstream head of 150mm and downstream head equal zero. Again as in air-inflated dam large deformation occurs with low internal pressures and the dam laying flat on the downstream side with low internal pressure (306mm). The curve in **Fig.(6)** shows a steep rise in dam height for air-inflated dam when increasing inflated pressure from 1.5kn/m^2 to 2kn/m^2, however, more increasing in inflating pressure makes the curve flatter reducing the rate of increasing in dam height. In **Fig.(7)** the dam height for water-inflated dam has a steady increased when the inflating pressure increased from 306mm to 714mm.

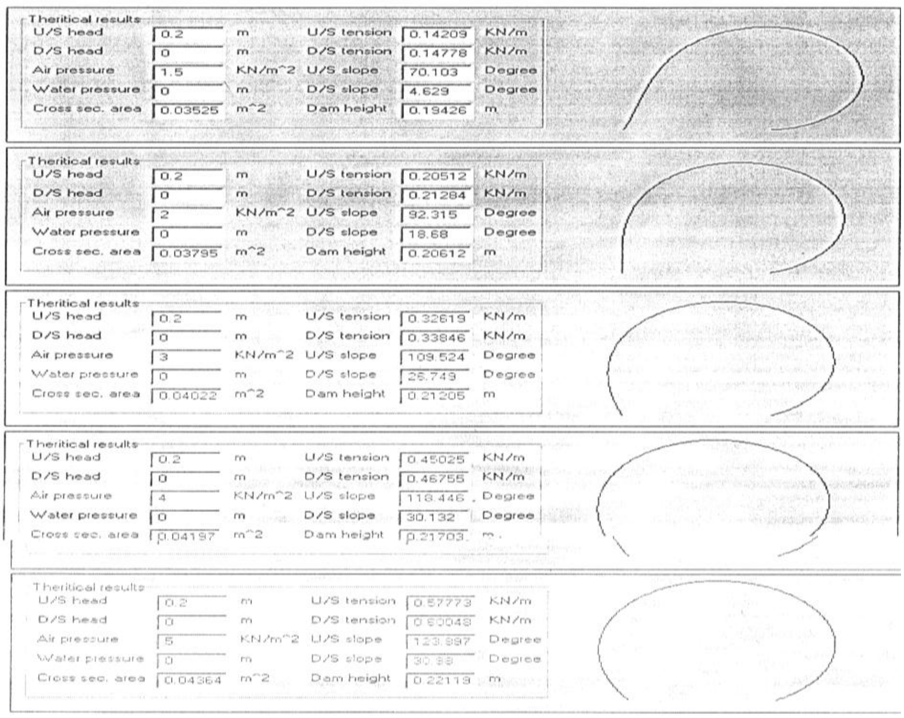

Figure (4) Effect of Increasing Internal Pressure on the Behavior of Air-Inflated Dams for Constant Upstream Head and Downstream Head

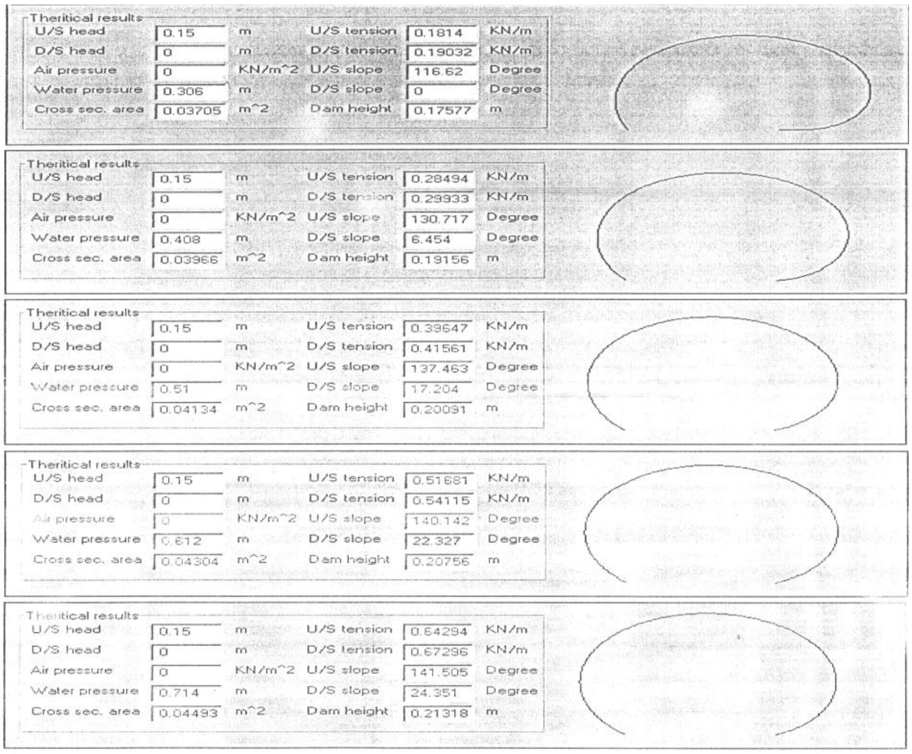

Figure (5) Effect of Increasing Internal Pressure on the Behavior of Water-Inflated Dams for Constant Upstream Head and Downstream Head

3-2-2 Effect of Rising Upstream Head and Internal Pressure for Constant Downstream Head

The effects of rising upstream head and internal pressure on the behavior of inflatable dams are investigated through the following parameters:

- Tension in the membrane.
- Dam height (height of dam crest).
- Upstream membrane slope (at upstream fixture).
- Downstream membrane slope (at downstream fixture).
- Cross-sectional area of the dam membrane elongation in the membrane.

3-2-2-1 Tension in the Membrane

The average tension between the upstream fixture and downstream fixture has been computed. **Figures (8)** and **(9)** show that the tension in the membrane decreases with increasing upstream head also the tension was increased when the internal pressure increased. The decrease in the membrane tension when increasing the upstream head may be due to the increasing in the force f_u on the element and therefore the tension in the membrane decreased. At the same time when decreasing the internal pressure, the components of forces F_a and F_{wa} are decreased causing a decrease in the membrane tension [1].

3-2-2-2 Dam Height

Figure (10) shows the dam height of air-inflated dam increases with increasing the upstream head for all inside pressure. This behavior may not continue depending on the internal pressure. For example, when the internal air pressure increased to 5kn/m^2 and upstream head equal to 180mm the dam height reaches a peak value of 221.98mm and decrease to 221.03mm when increasing the upstream head to 220mm. **Figure (11)** shows the variation of dam height with increasing upstream head for water-inflated dam. Similar as in air-inflated dam but with low water pressures (306mm), the dam height decreases as upstream head increasing. The dam height rises slightly when the upstream head increasing until the dam height reach a peak value but falls slightly as upstream head increasing (water pressure 408mm, to 510mm).

3-2-2-3 Upstream Membrane Slope

Figures (12) and **(13)** show the variation of upstream slope (slope at upstream fixture) with increasing upstream head and internal pressure for air and water inflated dams. The upstream slope decreases when increasing upstream head, this was due to the deformation towards the downstream side. In **Fig.(12)**, the rate of decrease in upstream slope is greater for low internal pressure (3kn/m^2) than high internal pressure (5kn/m^2). The same behavior was found in water-inflated dams **(Fig.13)**, but the upstream slope is higher for low water pressure than upstream slope of high water pressure with low upstream head. But when increasing the upstream head, the upstream slope for high water pressure is greater than those for low water pressure. This may be due to that the dam begins to lie flat at the upstream fixture when the internal pressure and upstream head are decreasing.

3-2-2-4 Downstream Membrane Slope

Figure (14) shows the downstream slope is higher for low air pressure than that of high air pressure with low upstream head. But when increasing the upstream head, the downstream slope for high air pressure is greater than that for low air pressure. In **Fig.(15)**, the downstream slope decreases when the upstream head increases for all water pressures with no convergence observed to the graphs. When the dam is inflated by low water pressure (306mm) and upstream head increases above 100mm the dam is lid at the downstream fixture (downstream slope equal zero).

3-2-2-5 Cross-Sectional Area

From **Fig.(16)**, the cross-sectional area for air-inflated dams is increased when increasing internal air pressure and decreases as the upstream head increases. Also the rate of decreasing in cross-sectional area of the dam for low air pressure (3kn/m) air pressure is higher than when the dam inflated with high air pressure (5kn/m^2).

Same behavior was found in water-inflated dams **(Fig.17)**, the cross-sectional area increased when increasing internal water pressure and decreased when increasing upstream head.

3-2-2-6 Elongation in the Membrane

The stretch in the membrane original length has been usually noticed this, occurs due to the applied loads (upstream head, downstream head and internal pressure) and can be found from the stress-strain relationship [1].

The elongation of membrane material was found by subtracting the original length from the new length (stretch length). **Figures (18)** and **(19)** show that the elongation increases when the internal pressure increases or when the upstream head decreases. It is well known also that the elongation in membrane material is directly affected by the properties of the materials of the membrane [1].

3-2-3 Effect of Variation in Membrane Perimeter Length

The membrane tension and dam height were investigated by increasing membrane perimeter length from 450mm to 650mm for both air and water inflated dams. **Figure (20)** shows that when increasing membrane perimeter length for air-inflated dam results in increasing in the membrane tension. Also the height of the dam increases when increasing the membrane perimeter length **Fig.(21)**.

The same behavior was found in the case of water-inflated dam [**Figs.(22)** and **(23)**] but with low rate of increasing in tension and height compared with air-inflated dam.

3-2-4 Effect of Variation in Membrane Thickness

The effect of variation in membrane thickness on the tension and the dam height was investigated by increasing membrane thickness from0.5mm to 3.5mm. **Figures (24)** and **(25)** show the behavior of air-inflated dam when increasing membrane thickness. Increasing in membrane thickness produce a decrease in tension of the membrane also the height of the dam shall decreases when increasing membrane thickness.

The behavior of water-inflated dam when increasing membrane thickness is the same as air-inflated dam. But the over all rate of decreasing is lower than that in the case of air-inflated dam [**Figs.(26)** and **(27)**].

3-2-5 Effect of Variation in the Base Length of the Dam

The effect of variation in the base length of the dam (distance between upstream and downstream anchor) on the tension in membrane and dam height was investigated for both air and water inflated dams by increasing in base length from 120mm to 250mm as shown in **Figs.(28, 29, 30)** and **(31)** respectively.

For air-inflated dam, as the base length increasing the membrane tension also increases, **Fig.(28)**, the dam height increases as well, reaching its peak value at 219.22mm with internal air pressure equal to 4kN/m2, however, more increasing in base length up to 220mm, the dam height starts to decrease in opposite trend **Fig.(29)**.

In the case of water-inflated dam the same behavior is found as in air-inflated dam that is, increasing in the base length causes an increase in tension in the membrane. However, the peak value of the height of the dam shall reach 197.15mm,occurs at a base length of 220mm, beyond this value the height of the dam starts to decrease in reverse trend(internal water pressure equal 408mm) [**Fig.(30)** and **(31)**].

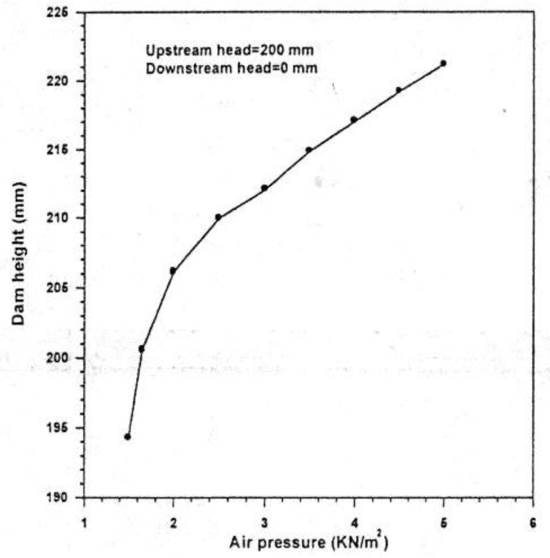

Fig.6 Variation of Dam Height with Increasing Internal Air Pressure for Constant Upstream Head and Downstream Head

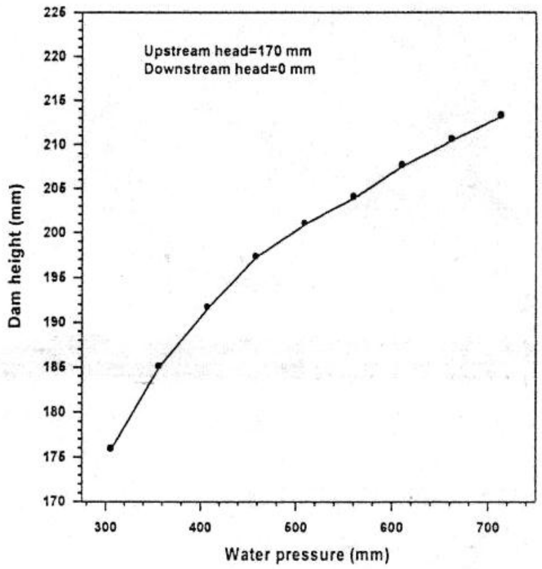

Fig.7 Variation of Dam Height with Increasing Internal Water Pressure for Constant Upstream Head and Downstream Head

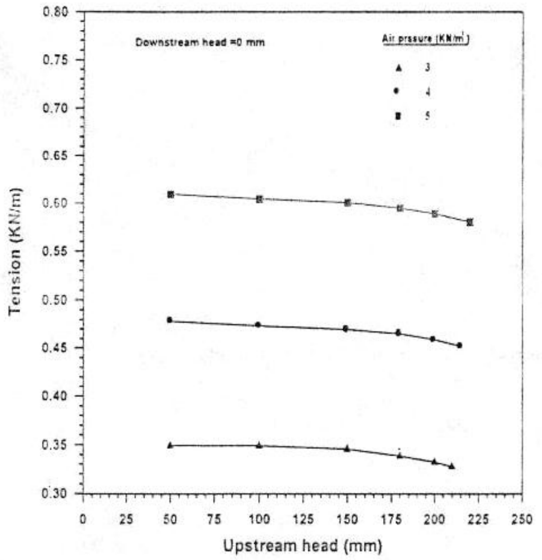

Fig. 8 Variation of Membrane Tension with Rising Upstream Head for Various Air Pressures.

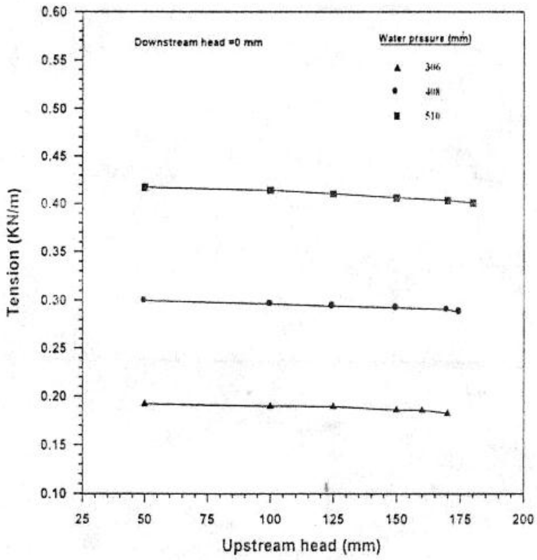

Fig. 9 Variation of Membrane Tension with Rising Upstream Head for Various Water Pressures.

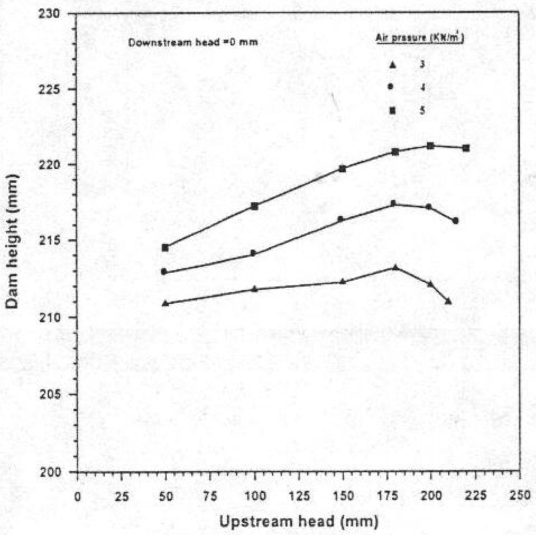

Fig. 10 Variation of Dam Height with Rising Upstream Head for Various Air Pressures.

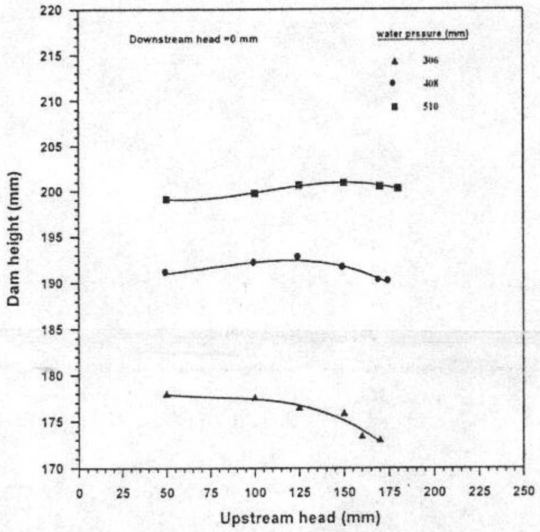

Fig. 11 Variation of Dam Height with Rising Upstream Head for Various Water Pressures.

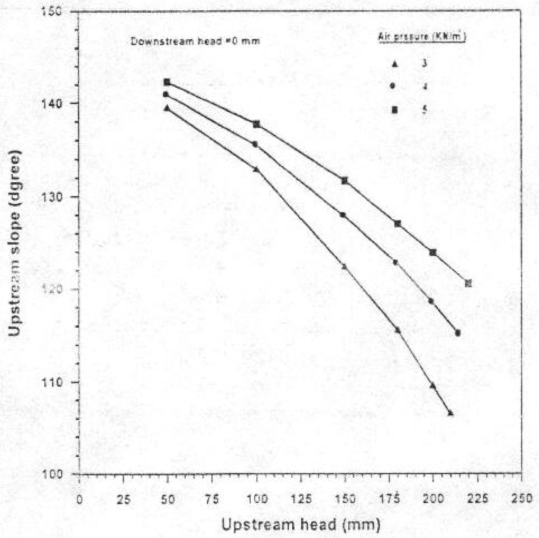

Fig.12 Variation of Membrane Slope at Upstream Fixture for Rising Upstream Head and Various Air Pressures.

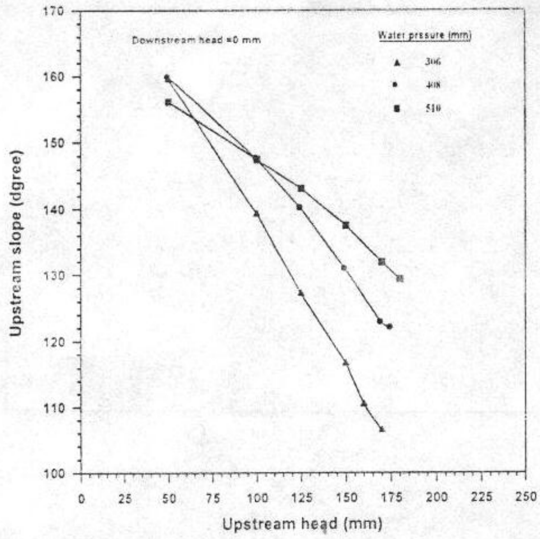

Fig.13 Variation of Membrane Slope at Upstream Fixture for Rising Upstream Head and Various Water Pressures.

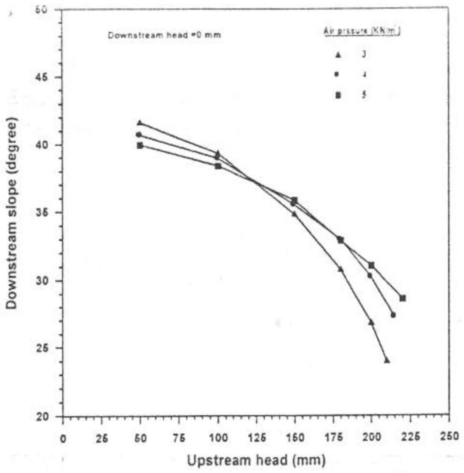

Fig.14 Variation of Membrane Slope at Downstream Fixture for Rising Upstream Head and Various Air Pressures.

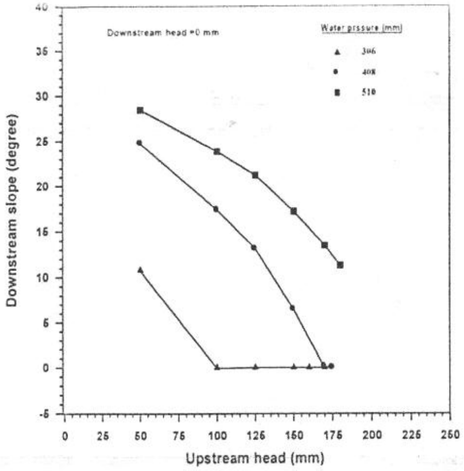

Fig.15 Variation of Membrane Slope at Downstream Fixture for Rising Upstream Head and Various Water Pressures.

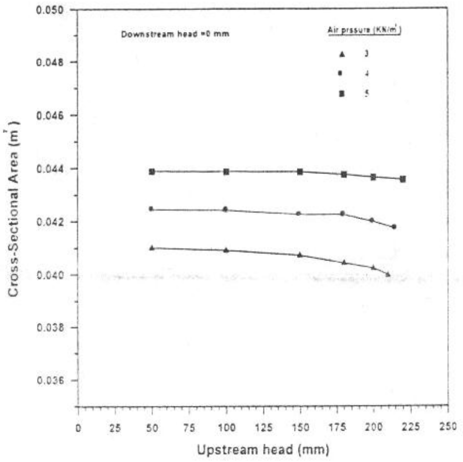

Fig.16 Variation of Dam Cross-sectional Area for Rising Upstream Head and Various Air Pressures.

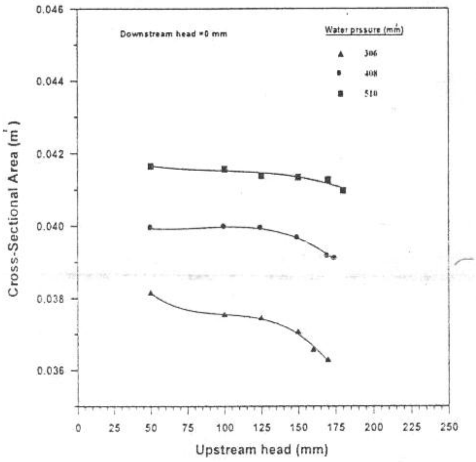

Fig.17 Variation of Dam Cross-sectional Area for Rising Upstream Head and Various Water Pressures.

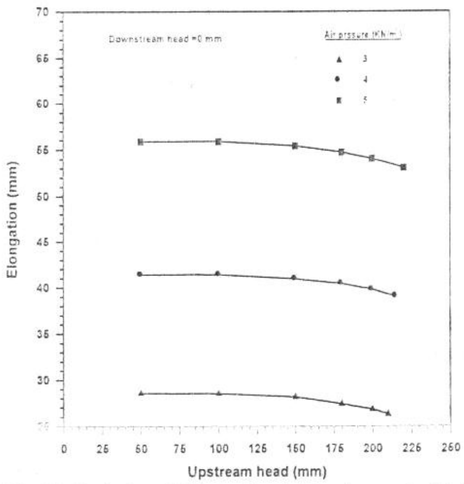

Fig.18 Variation of Elongation in Membrane for Rising Upstream Head and Various Air Pressures.

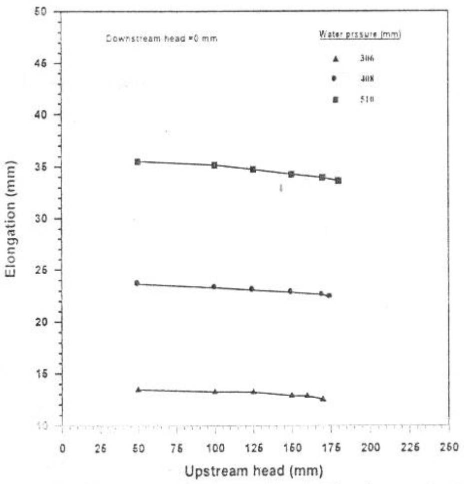

Fig.19 Variation of Elongation in Membrane for Rising Upstream Head and Various Water Pressures.

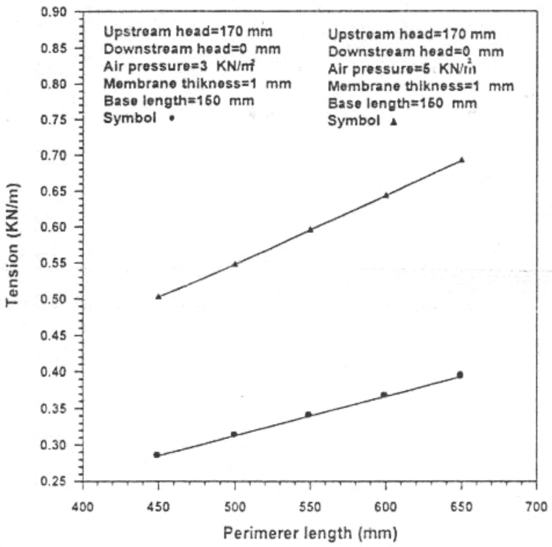

Fig.20 Variation of Membrane Tension with Membrane Perimeter Length (Air-inflated Dam)

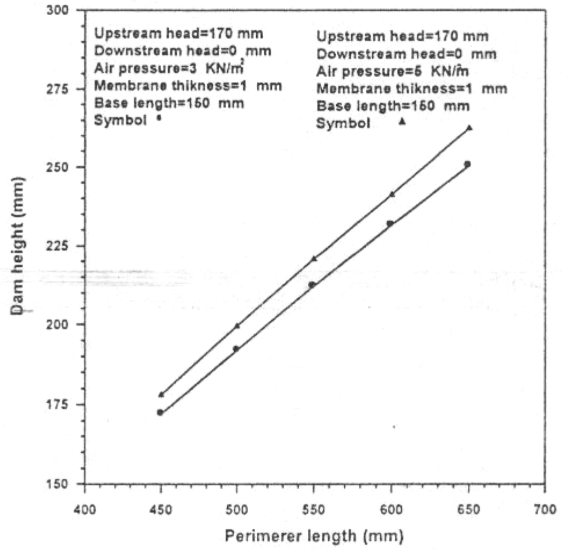

Fig.21 Variation of Dam Height with Membrane Perimeter Length (Air-inflated Dam)

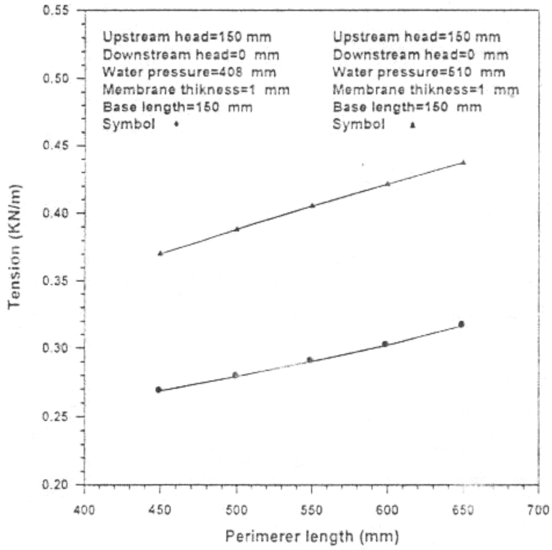

Fig.22 Variation of Membrane Tension with Membrane Perimeter Length (Water-inflated Dam)

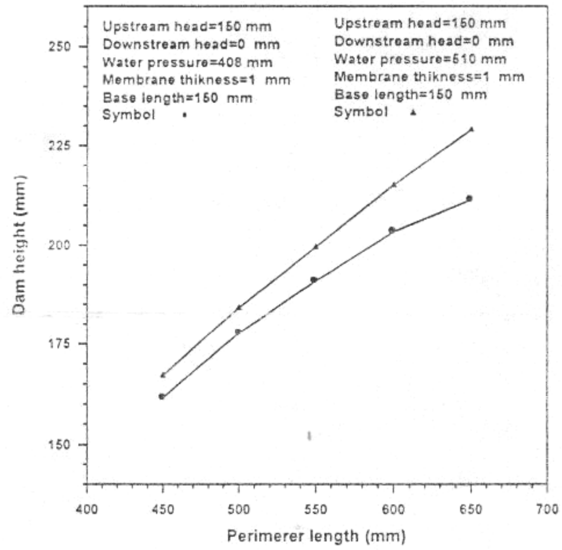

Fig.23 Variation of Dam Height with Membrane Perimeter Length (Water-inflated Dam)

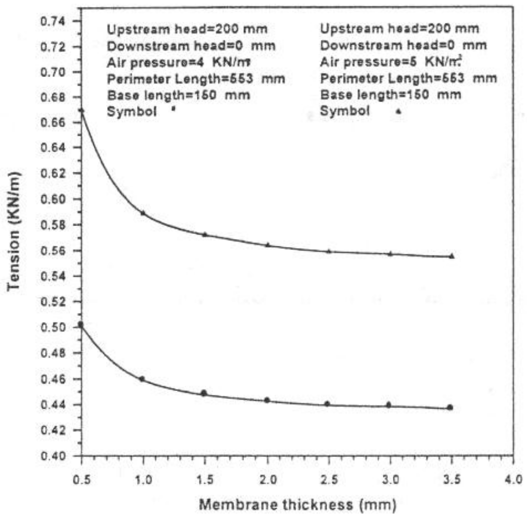

Fig.24 Variation of Membrane Tension with Membrane Thickness (Air-inflated Dam)

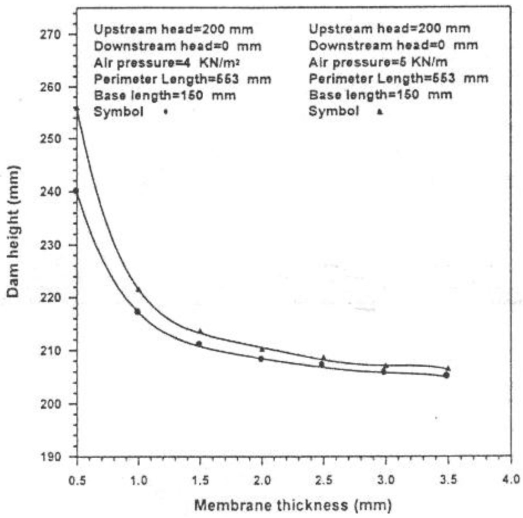

Fig.25 Variation of Dam Height with Membrane Thickness (Air-inflated Dam)

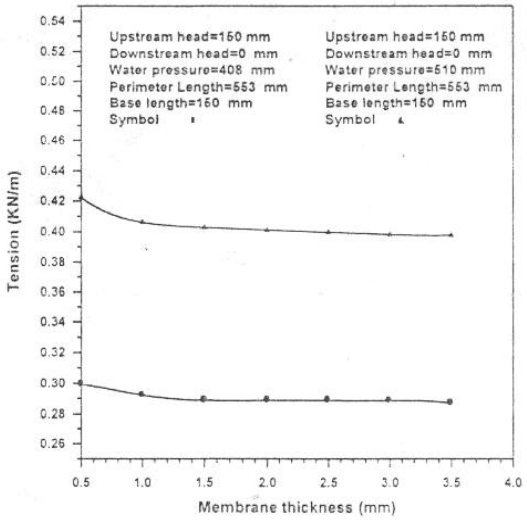

Fig.26 Variation of Membrane Tension with Membrane Thickness (Water-inflated Dam)

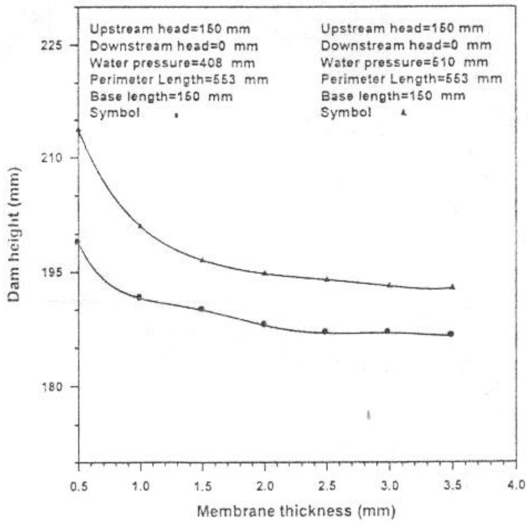

Fig.27 Variation of Dam Height with Membrane Thickness (Water-inflated Dam)

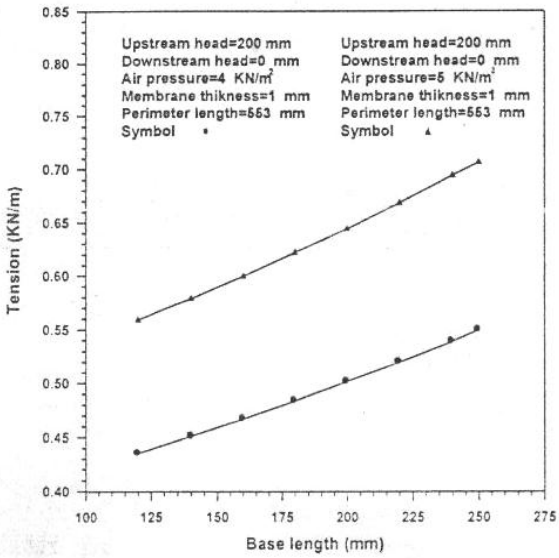

Fig.28 Variation of Membrane Tension with Base Length of the Dam (Air-inflated Dam)

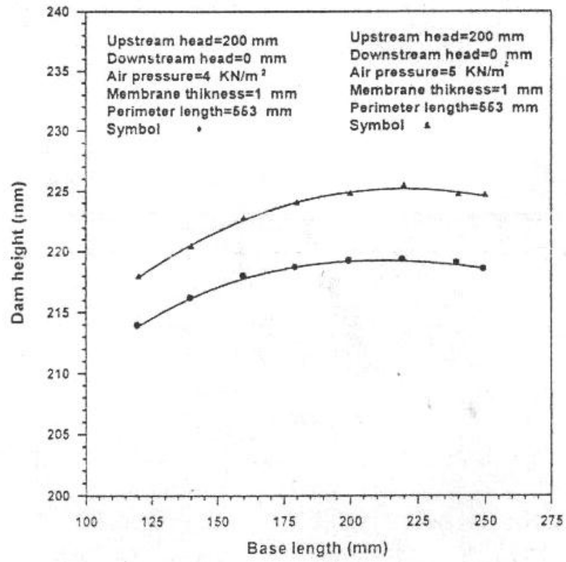

Fig.29 Variation of Dam Height with Base Length of the Dam (Air-inflated Dam)

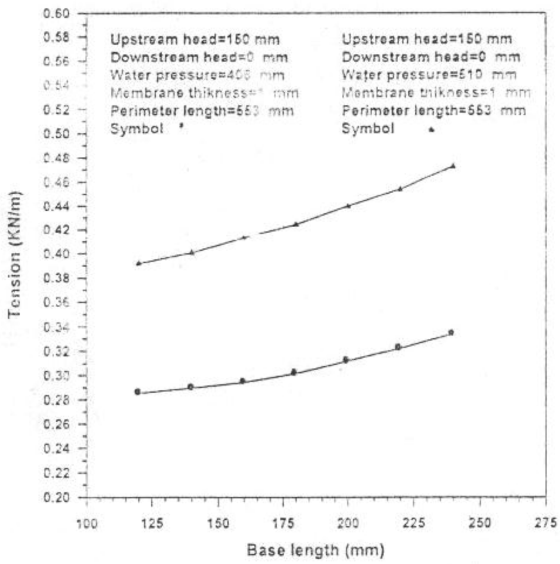

Fig.30 Variation of Membrane Tension with Base Length of the Dam (Water-inflated Dam)

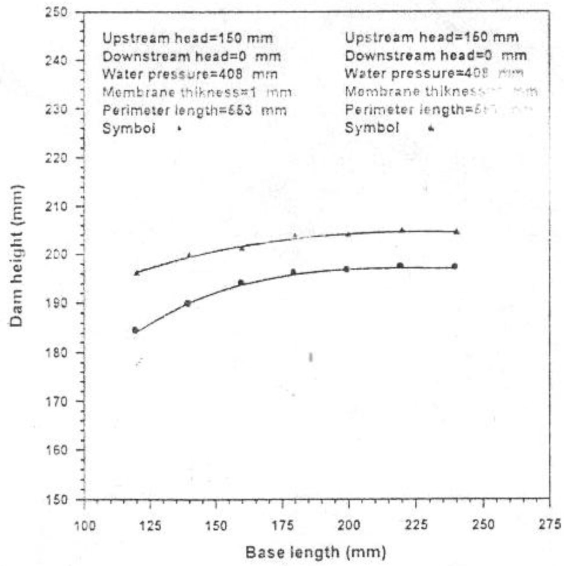

Fig.31 Variation of Dam Height with Base Length of the Dam (Water-inflated Dam)

4. Conclusions

For a laboratory model of rubber inflatable dam, the shape, dam height and cross sectional area, which obtained from the theoretical analysis under static conditions, were compared with those obtained from the experimental work. In general, it was found a good agreement between the experimental measurements and the results of theoretical analysis obtain from the computer program.

From the previous analysis, the tension in the membrane and dam height of air-inflated dam was higher than that for the water-inflated dam for the same conditions of internal pressure, upstream head and downstream head, this means that air-inflated dam support upstream head greater than water inflated dam. Also the stretch in the membrane of air-inflated dam is higher than that of water-inflated dam, this is due to the high tension in an air-inflated dam and this may cause a reduction in the dam life.

As shown in **Fig.(12)** when increasing the upstream head of air-inflated dam from 50mm to 175mm with internal pressure 4kN/m2 the decreasing in upstream slope is 16.9^o. In comparison with water inflated-dam at the same condition (408mm internal pressure) the decreasing in upstream slope is 37.98^o when increasing of upstream head from 50mm to 175mm, **Fig.(13)**. This means that the magnitude of deformation depends on the type of medium of inflation. The inflatable dams become rigid when inflated to high pressure which make the deformation shape to be insignificant, however, the upstream head will change accordingly.

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Notations

F_u = Upstream hydrostatic force on the element.

F_a = Internal air force on the element.

F_{wa} = Force due to internal water pressure.

Appendix A

BASIC EQUATIONS AND THEORETICAL ANALYSIS

A-1 Forces Acting on an Upstream Element

$$F_u=\gamma h_{c1} l \quad \text{(A-1)}$$

where:

F_u = Upstream hydrostatic force on the element per unit length (F/L).
γ = Specific Weight of the water (F/L^3).
h_{c1}= Depth of center of the upstream element below the upstream free water surface (L).
l = Length of the element (L).

$$F_{wa}=\gamma . h_{c3} . l \quad \text{(A-2)}$$

where:

Fwa = Internal water force on the element per unit length (F/L).
γ = Specific Weight of water inside the dam (F/L^3).
H_{c3} = Depth of center of the upstream element or downstream element below the Internal free water surface (L).

$$F_a = p_{ia} . l \quad \text{(A-3)}$$

where:

F_a = Internal air force on the element per unit length (F/L).
P_{ia} = Internal air pressure of the dam (F/L2).

$$F_w = wl \quad \text{(A-4)}$$

where:

F_w = Force due to weight of element per unit length (F/L).
w = Weight of the element per unit area (F/L2).

Knowing the properties of inflation fluid, and the dam membrane material, the forces on the element can be calculated using the previous equations.

Considering the horizontal and vertical equilibrium of these forces on element AB see **Fig.(A.1b)**.

For horizontal equilibrium:

$$T_B \cos\theta_B = T_A \cos\theta_A + (F_a + F_{wa} - F_u) . \sin\theta_A \quad \text{(A-5)}$$

For vertical equilibrium:

$$T_B \sin\theta_B = T_A \sin\theta_A + F_{wa} + (F_u - F_a + F_{wa}) . \cos\theta_A \quad \text{(A-6)}$$

where:
T_A = tension at node A per unit length (F/L).
θ_A = slope of element AB at node A.
T_B = tension at node B per unit length (F/L).
θ_B = slope of element AB at node B.

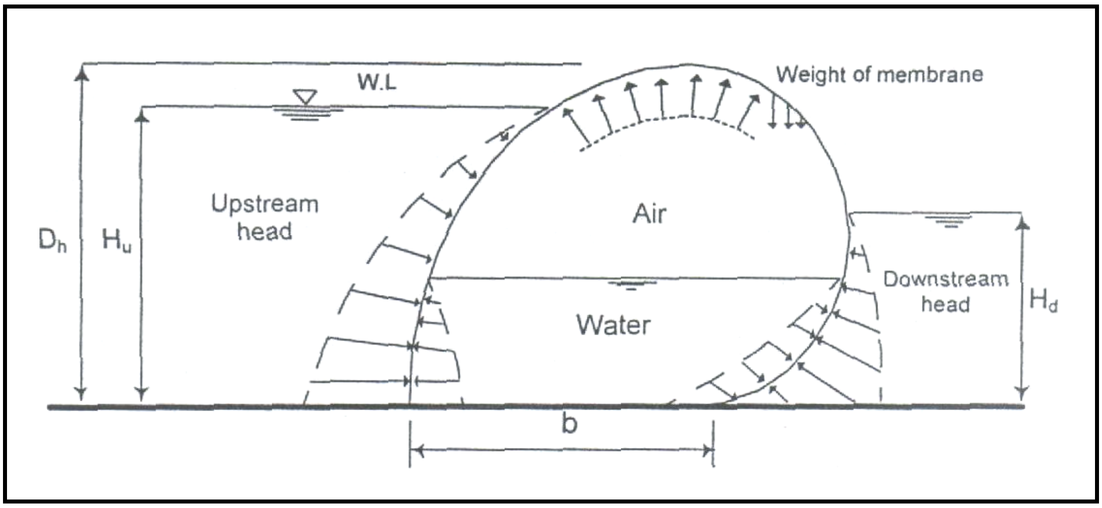
A-2 Forces Acting on the Downstream Element

The analysis for the downstream element is similar to that for the upstream element but the forces F_u equation (A.1) is equal to zero, and it is necessary to take into account the effect of the downstream head F_d.

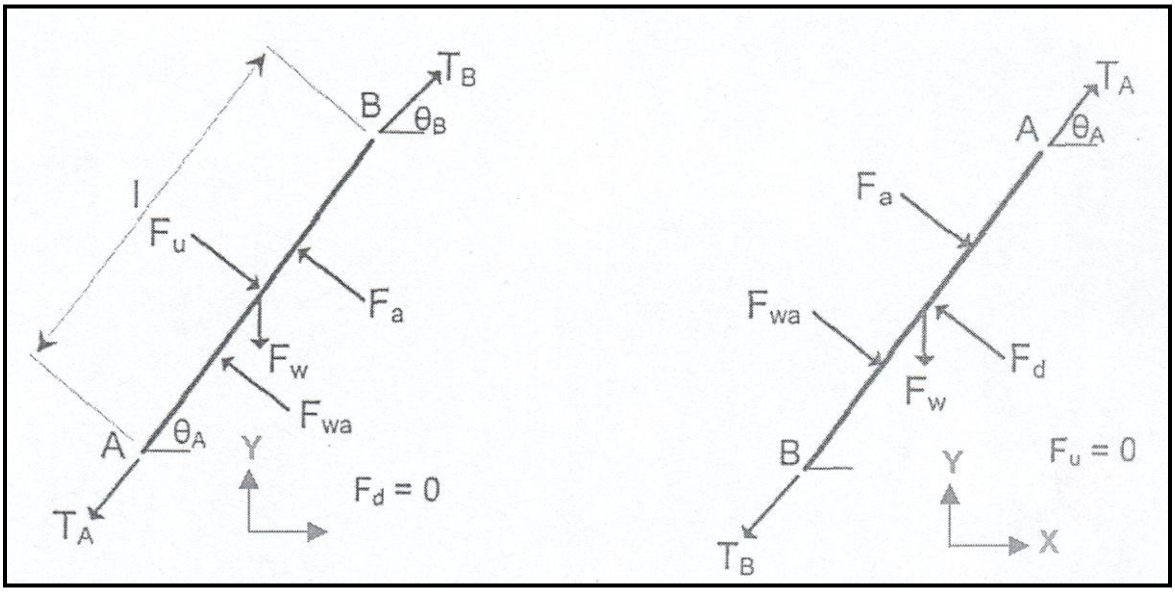
$$F_d=\gamma h_{c2} l \qquad \text{(A-7)}$$

where:
F_d = Downstream hydrostatic force on the element per unit length (F/L).
H_{c2} = Depth of center of the downstream element below the downstream free water surface (L).

It should be noted that the forces F_u, F_d can not both act on the same element at the same time.



(a) Forces acting on the dam



Upstream element *Downstream element*

(b) Forces acting on an element

Figure (A-1) Forces Acting on the Dam (Hydrostatic Condition)

A-3 Initial Values of Tension and Slope

To obtain the initial trial values of tension and slope of the first upstream element assume the shape of the membrane to be circular (Parbery, 1978), the tension in the membrane T can be found from the following expression:

$$T=p_i . r_i \qquad \text{(A-8)}$$

Where:

T = Maximum tension force of the membrane material per unit length (F/L).

p_i = Internal pressure (F/L^2).

r_i = Initial radius of curvature of the dam (circular) (L).

Assuming the downstream slope equal to zero, the horizontal equilibrium of static force acting on the dam **(Fig. A.1a)** is:

$$\tfrac{1}{2}\gamma(H_u)^2=T+T\cos\theta_1+\tfrac{1}{2}\gamma(H_d)^2 \qquad \text{(A-9)}$$

where:

θ_1 = Upstream slop of the dam at upstream fixture.

H_u = Upstream head (L).

H_d = Downstream head (L).

A-4 Co-Ordinate of Nodes of the Dam Profile

Stresses can be calculated from the following expression:

$$\sigma = T/t \quad \text{(A-10)}$$

where:

σ = Stress in the membrane material (F/L^2).
T = Tension of the membrane material per unit length (F/L).
t = Thickness of membrane material (L).

Knowing the initial slope and new length ($l+\Delta l$) of the first element, and the co-ordinates of the first node [assumed (0,0)], the co-ordinates of the second node (x, y) can be found (**Fig. A.2**, step 4) from the following equations:

$$X = (l + \Delta l).\ \text{Cos}\theta \quad \text{(A-11)}$$

$$Y = (l + \Delta l).\ \text{Sin}\theta \quad \text{(A-12)}$$

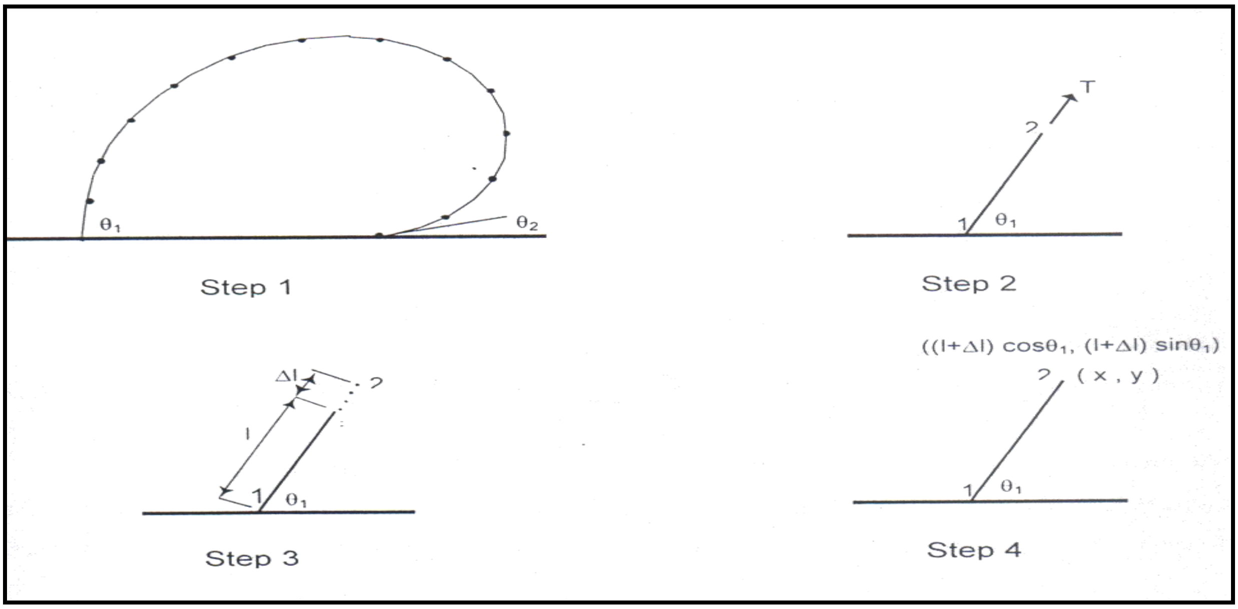


Figure (A-2) Co-ordinates of Nodes of the Dam Profile

A-5 Improving Initial Values of Tension and Slope

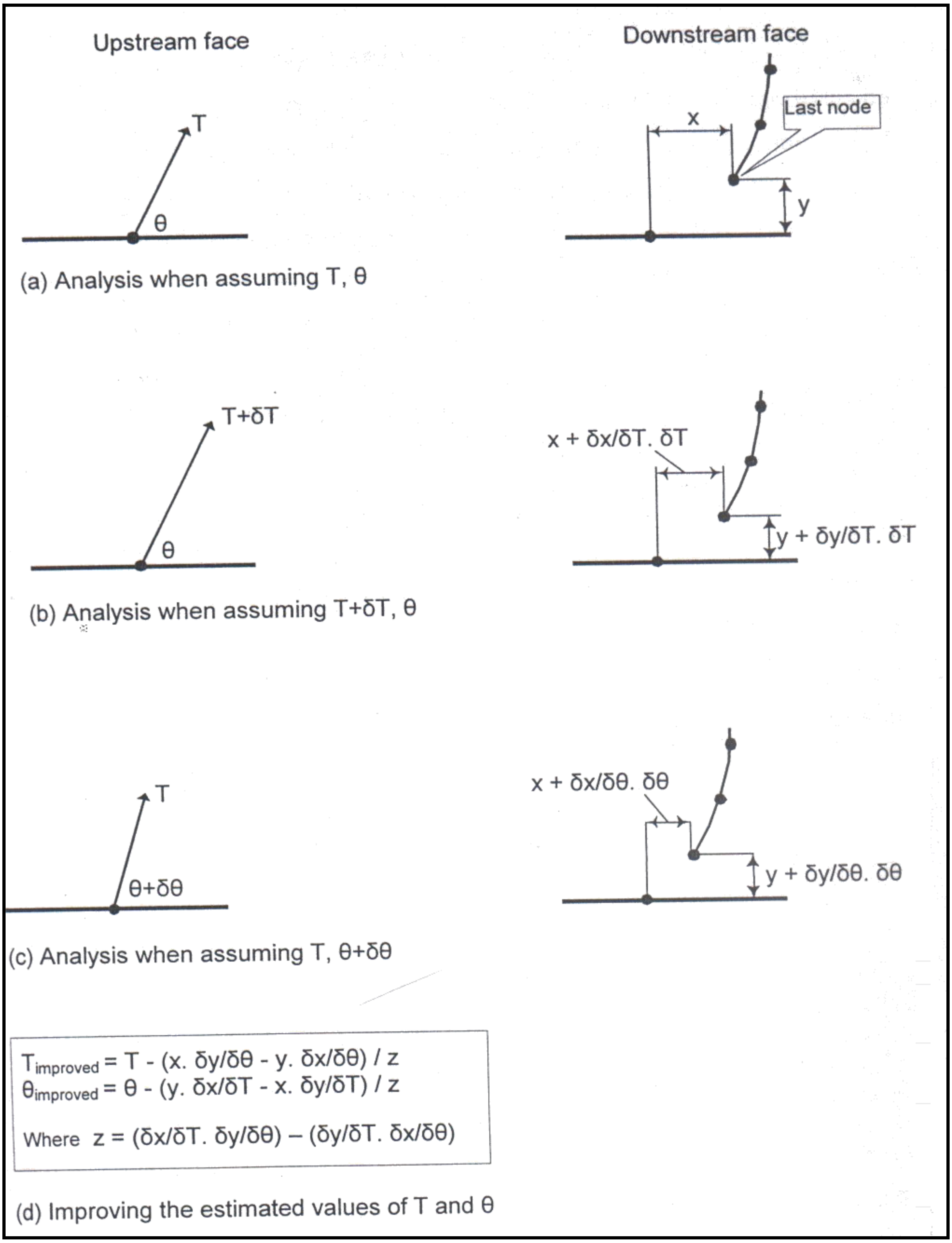
The improved values of T and θ are determined numerically from Newton's expressions:

$$T_{improved} = T - (x.\delta y/\delta\theta - y.\ \delta x/\delta\theta) / z \quad \text{(A-13)}$$

$$\theta_{improved} = \theta - (y.\ \delta x/\delta T - x.\ \delta y/\delta T) / z \quad \text{(A-14)}$$

where:

$$z = (\delta x/\delta T.\ \delta y/\delta\theta) - (\delta y/\delta T.\ \delta x/\delta\theta) \quad \text{(A-15)}$$



(a) Analysis when assuming T, θ

(b) Analysis when assuming T+δT, θ

(c) Analysis when assuming T, θ+δθ

(d) Improving the estimated values of T and θ

Figure (A-3) Improving Initial T and θ to Reduce the Miscloses

Appendix B

COMPUTER PROGRAM AIDED

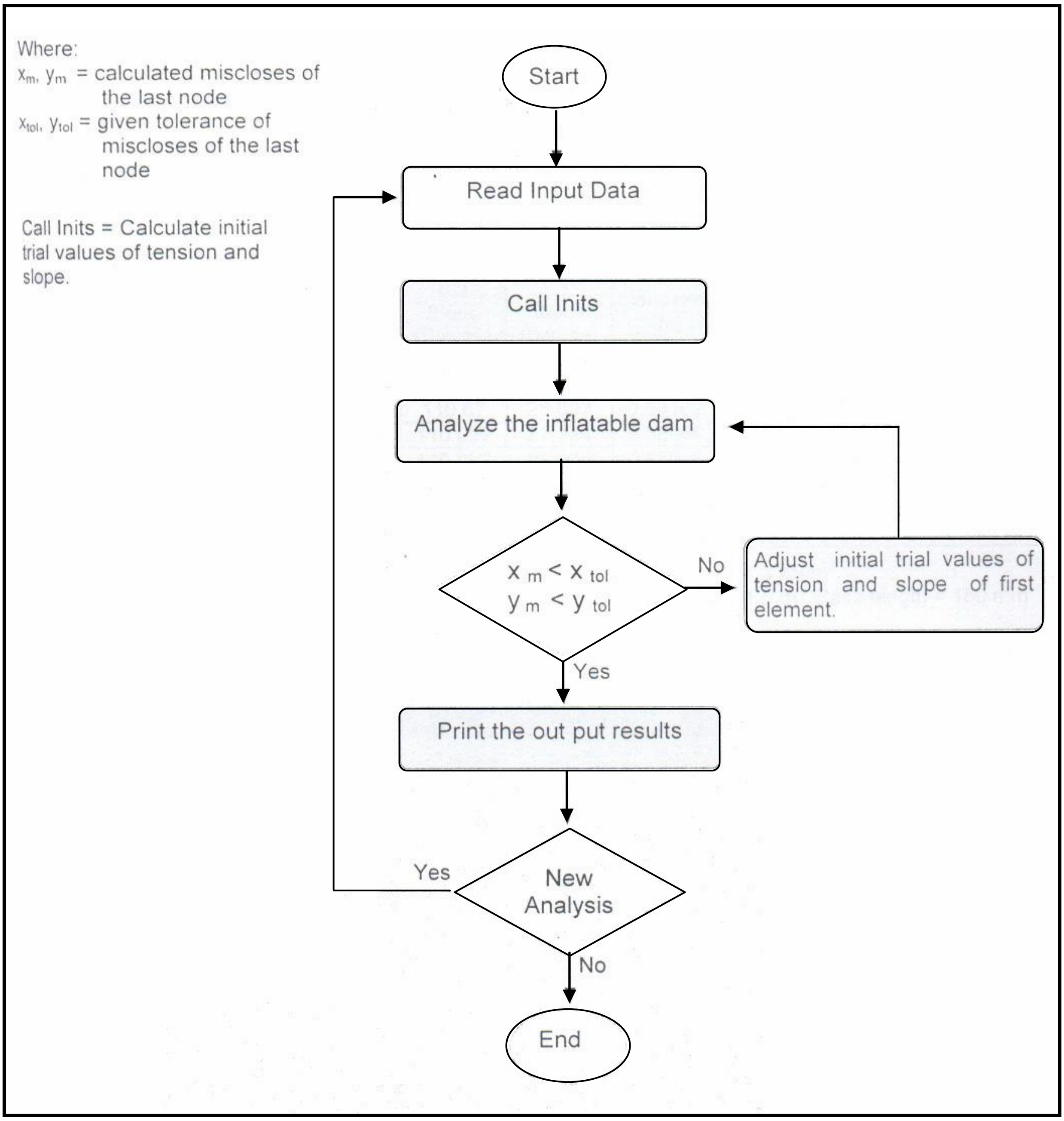


Figure (B-1) Flow Chart of the Computer Program Aided