Non-Linear Analysis of Simply Supported Composite Beam by Finite Element Method

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Abstract

In this paper a composite beam element has been developed in this study, the composite beam element can be used to model the nonlinear behavior of composite beams. The element is implemented in a nonlinear finite element program (written by the researchers) and its implementation is verified by the analysis of simply supported composite beam tested by others. The good comparison between the computed results and the experimental data demonstrates the accuracy of the used element.

It was found that the increase in cover plate thickness gives an increase in the ultimate load and decrease in maximum slip at the same load level.

الخلاصـــة

في هذا البحث تم تطوير عنصر العتب المركب، الذي يمكن استخدامه في تمثيل التصرف اللاخطي للعتب المركب. العنصر ينفذ في برنامج عناصر محدده لاخطي(كتب من قبل الباحثين) وهذا التنفيذ قد دقق بواسطة تحليل عتب بسيط مركب مفحوص من قبل باحثين آخرين. المقارنة الجيدة بين القيم المحسوبة والمعلومات العملية بينت دقه العنصر المستخدم.

*لقد وجد إن الزيادة في سمك (**cover plate**) يعطي زيادة في قابليه تحمل العتب القصوى و نقصان في قيمه الانزلاق الاعظم عند نفس الحمل.*

1. Introduction

When two elements which are capable of resisting bending moments are elastically connected together at the interface, interaction, partial or complete, between the two elements takes place. Where the elastic connection is flexible, differential direct strains at the common interface exist resulting in slip, and differential deflections may also result giving rise to uplift between the two elements.

Many studies have been conducted concerning the analysis of composite structures in the past. However, generally either full interaction has been assumed [1] or the shear connectors have been treated as rigid or elastic springs [2,3]. Some of studies assumed that the shear connector is continuous along the length, i.e. discrete connectors are assumed to be replaced by a medium of negligible thickness having normal and tangential modulus [4-7]. Yam and Chapman [4] developed an approach to incorporate nonlinear material and shear connector behavior, and the resulting nonlinear differential equations had bee solved iteratively.

2. Finite Element Idealization

The composite beam has two coordinate system, $\overline{X}$, $\overline{Z}$ for the concrete part and X, and Z for the steel part. Each part of the element has its pertaining end nodes 1 and 2 with three degrees of freedom per node, as shown in **Fig.(1)**, consequently, there are six degrees of freedom (four transitional and two rotational displacements) for each node of the element.

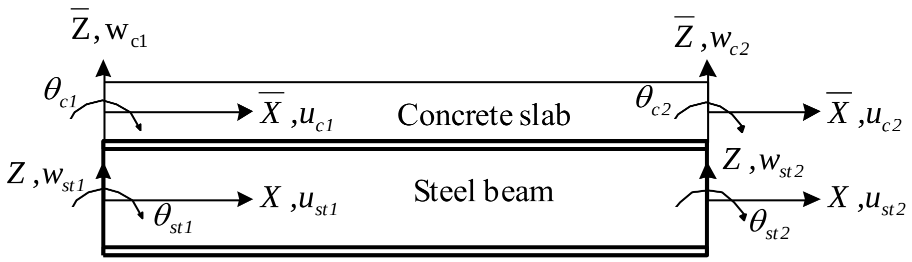

Figure (1) Displacement Components of an Element of a Composite Beam

Assuming that the plane section within each material remains plane, the axial displacement and strain can be expressed in the terms of displacements u and w relative to the local x and z axes. According to **Fig.(2)** the horizontal displacement and strain in each component of the horizontal beam are:

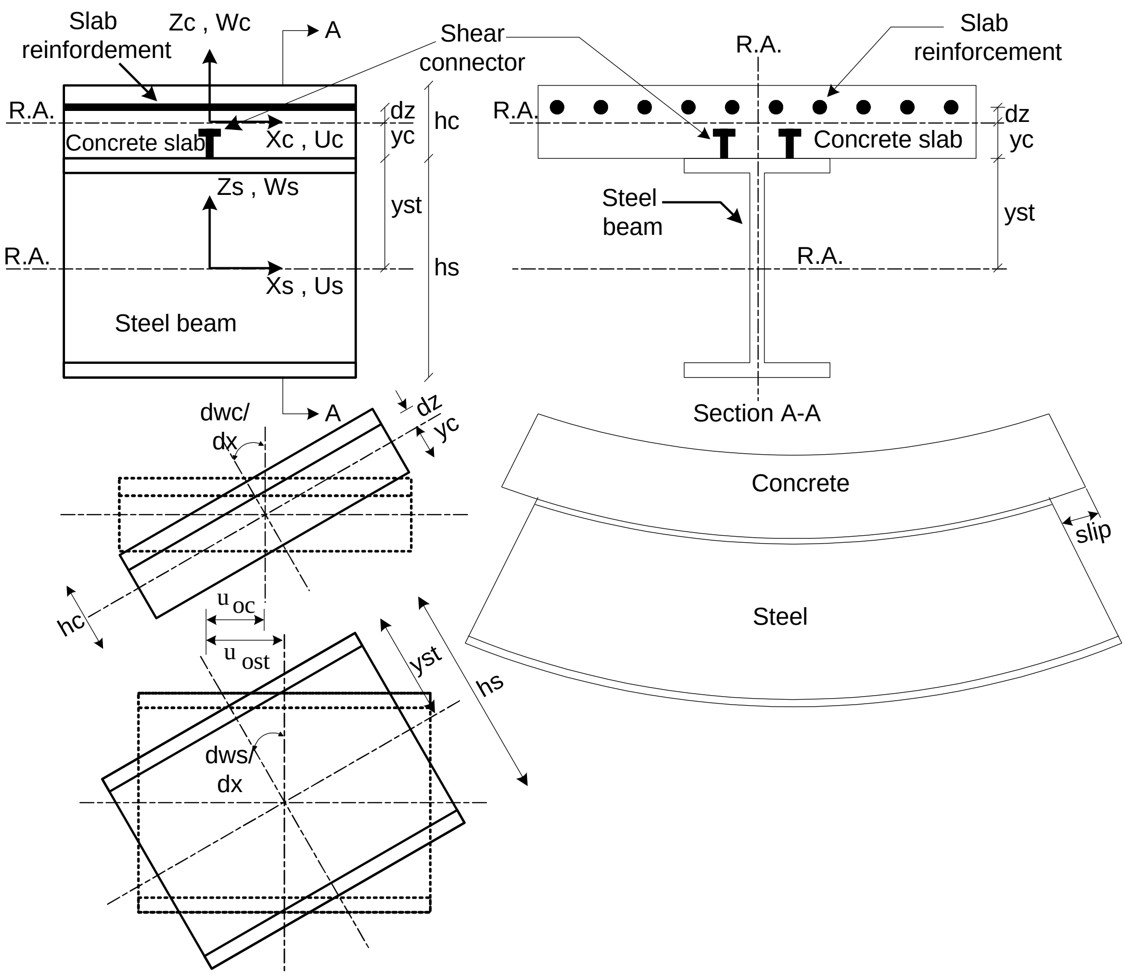

Figure (2) Deformations of Composite Beam Segment

I. Steel Beam:

$$u_{st} = u_{ost} - z\frac{dw_{st}}{dx} \quad \text{.................... (1)}$$

$$\varepsilon_{st} = \frac{du_{ost}}{dx} - z\frac{d^2w_{st}}{dx^2} = \varepsilon_{ost} - z\frac{d^2w_{st}}{dx^2} \quad \text{.................... (2)}$$

II. Concrete Slab:

$$u_c = u_{oc} - \bar{z}\frac{dw_c}{dx} \quad \text{.................... (3)}$$

$$\varepsilon_c = \frac{du_{oc}}{dx} - \bar{z}\frac{d^2w_c}{dx^2} = \varepsilon_{oc} - \bar{z}\frac{d^2w_c}{dx^2} \quad \text{.................... (4)}$$

III. Slab Reinforcement:

$$u_{sr} = u_{oc} - d_z \frac{dw_c}{dx} \quad \textbf{......(5)}$$

$$\varepsilon_{sr} = \frac{du_{oc}}{dx} - d_z \frac{d^2 w_c}{dx^2} = \varepsilon_{oc} - d_z \frac{d^2 w_c}{dx^2} \quad \textbf{......(6)}$$

Where: u_{oc} and u_{ost} are axial displacements in the concrete slab and steel beam, respectively, and $\frac{dw_c}{dx}$ and $\frac{dw_{st}}{dx}$ are the slopes of concrete slab and steel beam with respect to z direction, respectively.

The slip, S, between the concrete slab and steel beam is given as the difference in the displacement between bottom surface of the concrete slab and the top surface of the steel beam at the centerline of the interface, i.e.

$$S = u_{c(\bar{z}=-y_c)} - u_{st(z=y_s)} \quad \textbf{......(7)}$$

$$S = u_{oc} - u_{ost} + y_s \frac{dw_{st}}{dx} + y_c \frac{dw_c}{dx} \quad \textbf{......(8)}$$

The separation (uplift), f_s, between the concrete slab and steel beam in the vertical direction is the difference in deflection (in z-direction) between the steel beam and concrete slab at the node under consideration. It may be expressed as:

$$f_s = w_{st} - w_c \quad \textbf{......(9)}$$

Vectors represent the axial displacement and the bending displacements are {u} and {v}, respectively, are:

$$\left.\begin{aligned} \{u\} &= [u_1 \quad u_2]^T \\ \{v\} &= [w_1 \quad \theta_{1x} \quad w_2 \quad \theta_{2x}]^T \end{aligned}\right\} \quad \textbf{......(10)}$$

These displacement components can be assembled in one column vector {d}

$$\{d\} = \begin{Bmatrix} u \\ v \end{Bmatrix} \quad \textbf{......(11)}$$

From Equations (2), (4) and (6), it can be concluded that [8], a C_o-continuity shape function (linear) and C_1-continuity shape function (cubic Hermitten) are required for representing the axial and flexural displacements, respectively. Then let $u_o(x)$ and $v_o(x)$ be the axial and bending displacements at any point along x-axis, respectively

$$u_o = Na\{u\} \quad \& \quad v_o = Nb\{v\} \qquad \textbf{(12)}$$

where: Na is the shape function defining a linear interpolation of $u_o(x)$ between the nodes, and Nb comprises the cubic beam function interpolation polynomial [8]

$$Na = [N1 \quad N2]^T \quad \& \quad Nb = [N3 \quad N4 \quad N5 \quad N6]^T \qquad \textbf{(13)}$$

where:

$$\left.\begin{array}{ll} N1 = 1 - \dfrac{x}{L} & N2 = \dfrac{x}{L} \\ N3 = 1 - \dfrac{3x^2}{L^2} + \dfrac{2x^3}{L^3} & N4 = x - \dfrac{2x^2}{L} + \dfrac{x^3}{L^2} \\ N5 = \dfrac{3x^2}{L^2} - \dfrac{2x^3}{L^3} & N6 = -\dfrac{x^2}{L} + \dfrac{x^3}{L^2} \end{array}\right\} \qquad \textbf{(14)}$$

thus, the displacements field, {d}, is

$$\begin{array}{l}\{d\} = [u_{st1} \quad u_{c1} \quad w_{st1} \quad w_{c1} \quad \theta_{st1} \quad \theta_{c1} \\ \quad u_{st2} \quad u_{c2} \quad w_{st2} \quad w_{c2} \quad \theta_{st2} \quad \theta_{c2}]^T\end{array} \qquad \textbf{(15)}$$

As stated before, six degrees of freedom are needed at each node in the finite element discretization. The nodal displacements at each node will be,

$$\{di\} = [u_{sti} \quad u_{ci} \quad w_{sti} \quad w_{ci} \quad \theta_{sti} \quad \theta_{ci}]^T \qquad \textbf{(16)}$$

For beam element under external load, using the virtual work principles [8]

$$\text{External virtual work} = \int_0^L R_i \, U_i \, dx \qquad \textbf{(17)}$$

where: R_i is the applied load and U_i is the virtual displacement. For nodal displacements, {d}, and equivalent external load $\{R_j\}$

$$\text{External virtual work} = [R_j]\delta\{d\}^T \qquad \textbf{(18)}$$

But the internal work = internal work in steel beam + internal work in concrete slab + internal work in shear connector + internal work in slab reinforcements.

Then:

$$[R_j]\{d\} = \int_{vol.steel} \delta\varepsilon_{st}\,\sigma_{st}\,dvol + \int_{vol.concrete} \delta\varepsilon_c\,\sigma_c\,dvol + \sum_{i=1}^{n}\int_0^L \delta\varepsilon_{sr}\,\sigma_{sr}\,Asr\,dx$$

$$+\sum_{m=1}^{ns}\left[q_x\,\delta S\right]_{x=xs} + \sum_{m=1}^{ns}\left[F_a\,\delta f_a\right]_{x=xs} \quad \textbf{... (19)}$$

where: q_x is the shear force (kN) in x-direction, F_a is the normal force (kN), $F_a=f(f_a)$, n is the number of layer reinforcement, ns is the number of shear connectors in each element, and xs is the location of shear connector. The strains in composite beam component are expressed as:

$$\left.\begin{aligned} \delta\varepsilon_{st} &= [B_{st}]\,\delta\{d\} \\ \delta\varepsilon_c &= [B_c]\,\delta\{d\} \\ \delta\varepsilon_{sr} &= [B_{sr}]\,\delta\{d\} \\ \delta S &= [B_S]\,\delta\{d\} \\ \delta F_a &= [B_f]\,\delta\{d\} \end{aligned}\right\} \quad \textbf{... (20)}$$

where: [B]'s are the strain-displacement relationship matrices. Combing Equations (19) and (20) lead to:

$$\{R_j\} = \int_{steel} [B_{st}]^T \sigma_{st}\,dvol + \int_{concrete} [B_c]^T \sigma_c\,dvol + \sum_{i=1}^{n}\int_0^L [B_{sr}]^T \sigma_{sr}\,A_{sr}\,dx +$$

$$\sum_{m=1}^{ns}\left[[B_S]^T q_x\right]_{x=xs} + \sum_{m=1}^{ns}\left[[B_f]^T F_a\right]_{x=xs} \quad \textbf{.. (21)}$$

and the strain vectors may be written in one column vector, {ε}, as:

$$\{\varepsilon\}=\begin{Bmatrix}\varepsilon_{st}\\ \varepsilon_{c}\\ \varepsilon_{sr}\\ S\\ f_a\end{Bmatrix}=[B_{5\times12}]\begin{Bmatrix}u_{st1}\\ u_{c1}\\ w_{st1}\\ w_{c1}\\ \theta_{st1}\\ \theta_{c1}\\ u_{st2}\\ u_{c2}\\ w_{st2}\\ w_{c2}\\ \theta_{st2}\\ \theta_{c2}\end{Bmatrix}\quad \text{where}[B]^T=\begin{bmatrix} N_1' & 0 & 0 & -N_1 & 0\\ 0 & N_1' & N_1' & N_1 & 0\\ -zN_3'' & 0 & 0 & y_sN_3' & N_3\\ 0 & -\bar{z}N_3'' & -dzN_3'' & y_cN_3' & -N_3\\ -zN_4'' & 0 & 0 & y_sN_4' & N_4\\ 0 & -\bar{z}N_4'' & -dzN_4'' & y_cN_4' & -N_4\\ N_2' & 0 & 0 & -N_2 & 0\\ 0 & N_2' & N_2' & N_2 & 0\\ -zN_5'' & 0 & 0 & y_sN_5' & N_5\\ 0 & -\bar{z}N_5'' & -dzN_5'' & y_cN_5' & -N_5\\ -zN_6'' & 0 & 0 & y_sN_6' & N_6\\ 0 & -\bar{z}N_5'' & -dzN_5'' & y_cN_6' & -N_6 \end{bmatrix} \quad \text{..... (22)}$$

Eq. (21) can be written in compact form as:

$$[K]\{d\}=\{R_j\} \quad \text{.. (23)}$$

where: [K] is the stiffness matrix. The stiffness matrix is generated at the mid-length of composite beam element and assumed to be constant along the element for the non-linear behavior. The stiffness matrix of a composite beam element is given by:

$$[K]^e=\int_{vol}[B]^T[D][B]dvol \quad \text{.. (24)}$$

It is composed from the contribution of composite beam components and can be expressed as:

$$[K]^e=[K]^e_{st}+[K]^e_{c}+[K]^e_{sr}+[K]^e_{S}+[K]^e_{f} \quad \text{.. (25)}$$

where:

$[K]^e_{st}$: steel beam element stiffness matrix, $[K]^e_{c}$: concrete slab element stiffness matrix, $[K]^e_{sr}$: slab reinforcement element stiffness matrix, $[K]^e_{S}$: shear connector element stiffness matrix in x-direction, and, $[K]^e_{f}$: shear connector element stiffness matrix in z-direction.

3. Non-Linear Analysis (Cross-Section Properties)

The modulus of elasticity for each material of composite beam is a function of strain value at the point under consideration. But the strain varies across the depth and width of the beam. Steel beam and concrete slab section are divided into a number of layers as shown in **Fig.(3)** so that:

$$EA = \int_A E\,dA = \sum_{ie=1}^{n} E_{ie}\,A_{ie} \quad \text{.......... (26)}$$

$$EI = \int_A E\,z^2\,dA = \sum_{ie=1}^{n} E_{ie}\,z^2\,A_{ie} \quad \text{.......... (27)}$$

where: n is the number of layers in the material under consideration. E_{ie} is the modulus of elasticity of element. z is the distance from layer to the reference axis of concrete slab or steel beam. A_{ie} is the cross-sectional area of the layer.

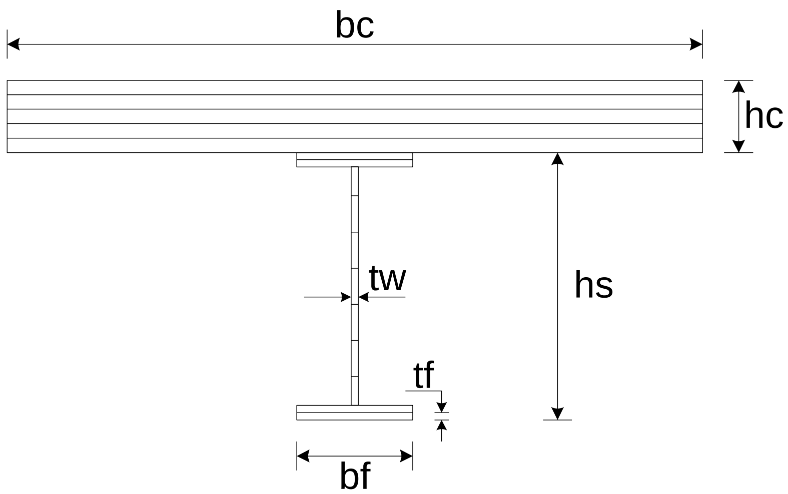

Figure (3) Layered Beam Section

4. Non-Linear Analysis (Materials Constitutive Relationships)

Concrete For concrete in compression the used model for the stress-strain relationship is that proposed in BS 8110 **[9]**, as shown in **Fig.(4-A)**, the ultimate compressive strain, ε_{cu} is limited to 0.0035, the curved portion of stress-strain curve is defined by:

$$\sigma = 5500\sqrt{\sigma_{cu}}\,\varepsilon - 11.3*10^6\,\varepsilon^2 \quad \text{.......... (28)}$$

with $\varepsilon_o = 2.44*10^{-4}\sqrt{\sigma_{cu}}$, and the initial modulus of elasticity is:

$$Ei = 5500\sqrt{\sigma_{cu}} \quad \text{.......... (29)}$$

in which σ_{cu} is the concrete cube strength in MPa.

The tensile strength of concrete is relatively low so that, concrete is assumed incapable to resist any tension.

Steel Reinforcement A bilinear stress-strain curve is adopted for this type of steel as shown in **Fig.(4-B)**. In this stress-strain curve, the yield stress, fy, in tension and compression is equal.

Shear Connectors Load-slip curves and information concerning shear connectors can be obtained from push-out tests, although they cannot be assumed to represent what really happens because the distribution of longitudinal stress in the concrete flange of a beam is different from that in the slab in push-out test [10].

Many different load-slip relationships for stud connectors have been proposed, an exponential model is the best of these models. An exponential model for the load-slip relationship of shear connectors was used by Al-Amery and Roberts [7]. This is represented by the following function,

$$Q = Qu\left(1 - Exp\left(-\alpha\, u_{ab}\right)\right) \quad \textbf{(30)}$$

in which Qu is the ultimate shear strength of a connector and α is a constant which can be determined from test results, as shown in **Fig.(4-C)**. If, for example, the slip at load $\overline{Q}$ is equal to $\overline{u}_{ab}$, then from Eq. (30)

$$\alpha = \frac{1}{\overline{u}_{ab}} \ln\left(\frac{Qu}{Qu - \overline{Q}}\right) \quad \textbf{(31)}$$

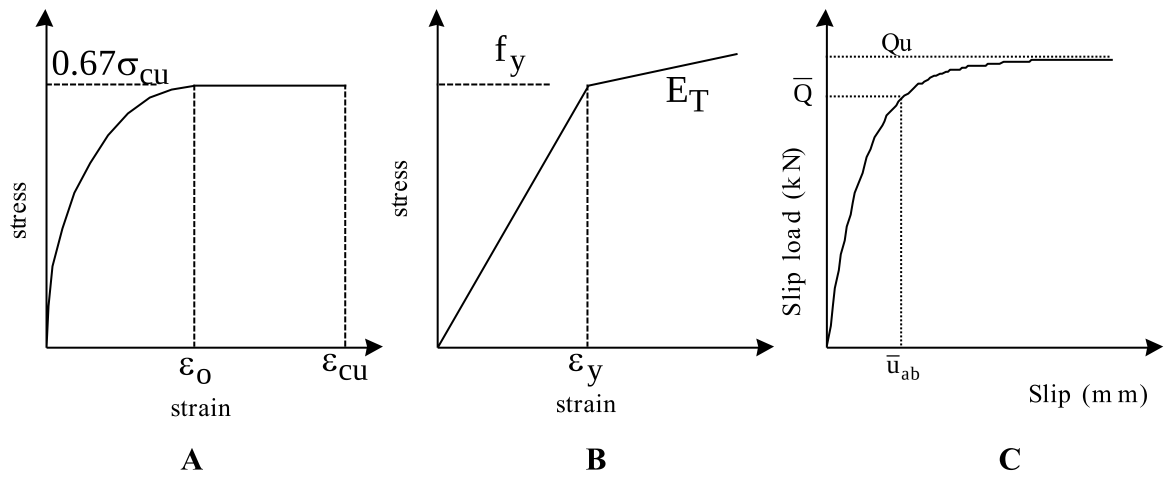

**Figure (4) A-Stress-Strain Relationship for Concrete;
B- Bilinear Stress-Strain Relationship for Steel;
C-Load-Slip Relationship for Shear Connector**

5. Convergence Criteria

The nonlinear algebraic equations can be solved iteratively, as illustrated in **Fig.(5)** in which R and d denote a representative load and displacement, respectively.

For the first stage of solution, the material properties are assumed constant and a set of nodal displacements corresponding to a specified applied loading is determined. From these displacements, strains throughout the beam are determined, which are used to define the secant values of material properties for the second stage of the solution. The process is repeated until the calculated displacements have converged.

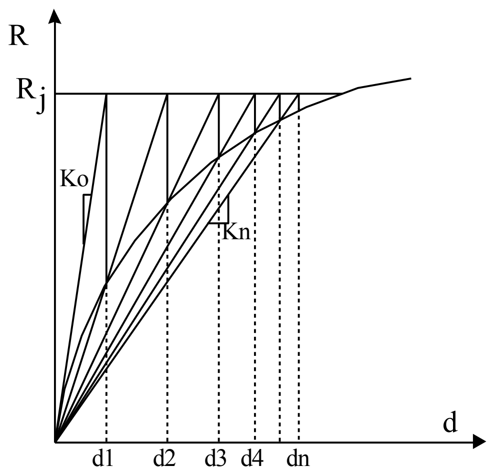

Figure (5) Solution Procedure in a Nonlinear Problem (Secant Method)

6. Results and Discussion of Numerical Example

Chapman and Balakrishnan [11] tested a series of simply supported composite beams, EII is one of simply supported beams, and **Fig.(6)** illustrates the dimensions of this beam. The material properties are listed below.

***Steel Beam*:** 305 mm* 153 mm *65.49 kg/m rolled steel joist. Flange 153 mm* 18.2 mm. Web thickness 10.16 mm. Young's Modulus 205000 MPa. Yields stress 265 MPa. Strain –hardening factor 0.022.

Concrete Slab: 1220 mm* 153 mm. The cube strength 50 MPa. Young's Modulus 26700 MPa.

Shear Connector: 12.5 mm diameter 50 mm height Spacing 110 mm Number of rows 2. Load-slip relation $Q = 59\left(1 - \mathrm{Exp}\left(-3.1265\ u_{ab}\right)\right)$.

For numerical integration of Eqs. (26) and (27), the concrete slab was divided into 15 equal layers. Each flange of steel beam was divided into four equal layers and web of steel beam was divided into 10 equal layers. The below results were obtained using 21 nodes along the length of the beam.

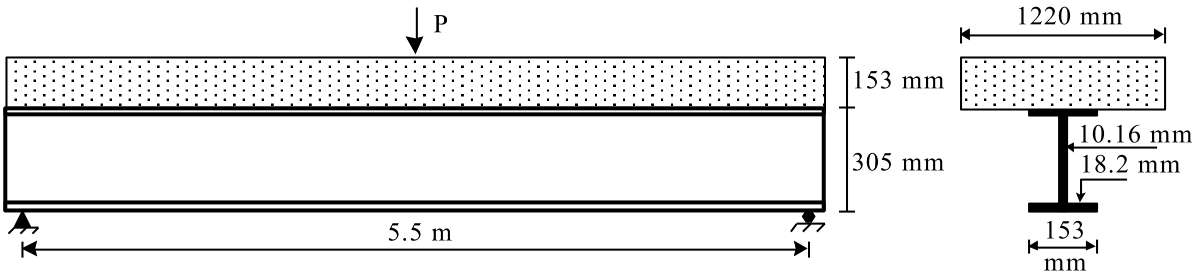

Figure (6) Simply Supported Composite Beam EII [4]

Load-deflection relationships and slip distributions between the concrete slab and steel beam obtained from analysis and tests are shown in **Figs.(7-A)** and **(7-B),** respectively, for a simply supported composite beam with partial interaction. These figures compare the results of the present analysis with the experimental data given by Chapman [11] as well as with the analytical data obtained from Ref. (4), where they show good agreement between analysis and tests. **Fig.(7-A)** represents the load mid-span deflection curve. The same trend of behavior is seen for the analysis and the test results, but comparison with experimental results indicates a close agreement till about 60% of the ultimate load. A stiffer behavior the finite element model was observed during the next load increments. This may be attributed to the selfweight effects on the stresses and strains, which were neglected in the analysis. However, the analytical ultimate load level (550kN) is detected quite well compared with the experimentally observed of 519 kN, with an error of only 5.98 %.

A comparison between the calculated results of slip distribution for beam EII [shown in **Fig.(6)**] with the experimental results and the analytical solution given by Yam and Chapman [4] are shown in **Fig.(7-B)** at load equal to 450 kN. In general, the figures show the same general trend of calculated and experimental results. The discrepancy between the figures is due to the fact that slip is very sensitive to changes of load. It should also be noted that, friction and bond between steel beam and concrete slab are neglected. Both analytical and test result in **Fig.(7-B)** show a characteristic that the maximum slip exists at approximately one-fifth of the half span. The maximum experimental slip was equal to 0.585 mm, while the maximum calculated slip is equal to 0.555 mm with error of 5.13 % only, Yam and Chapman gave maximum slip equals to 0.556 mm and this value is very close to the calculated value. But we must be know that, the minimum slip occurs at the support for all studies (experimental, Ref. (4) and present study but the present study gives minimum slip greater than the experimental value and Ref. (4).

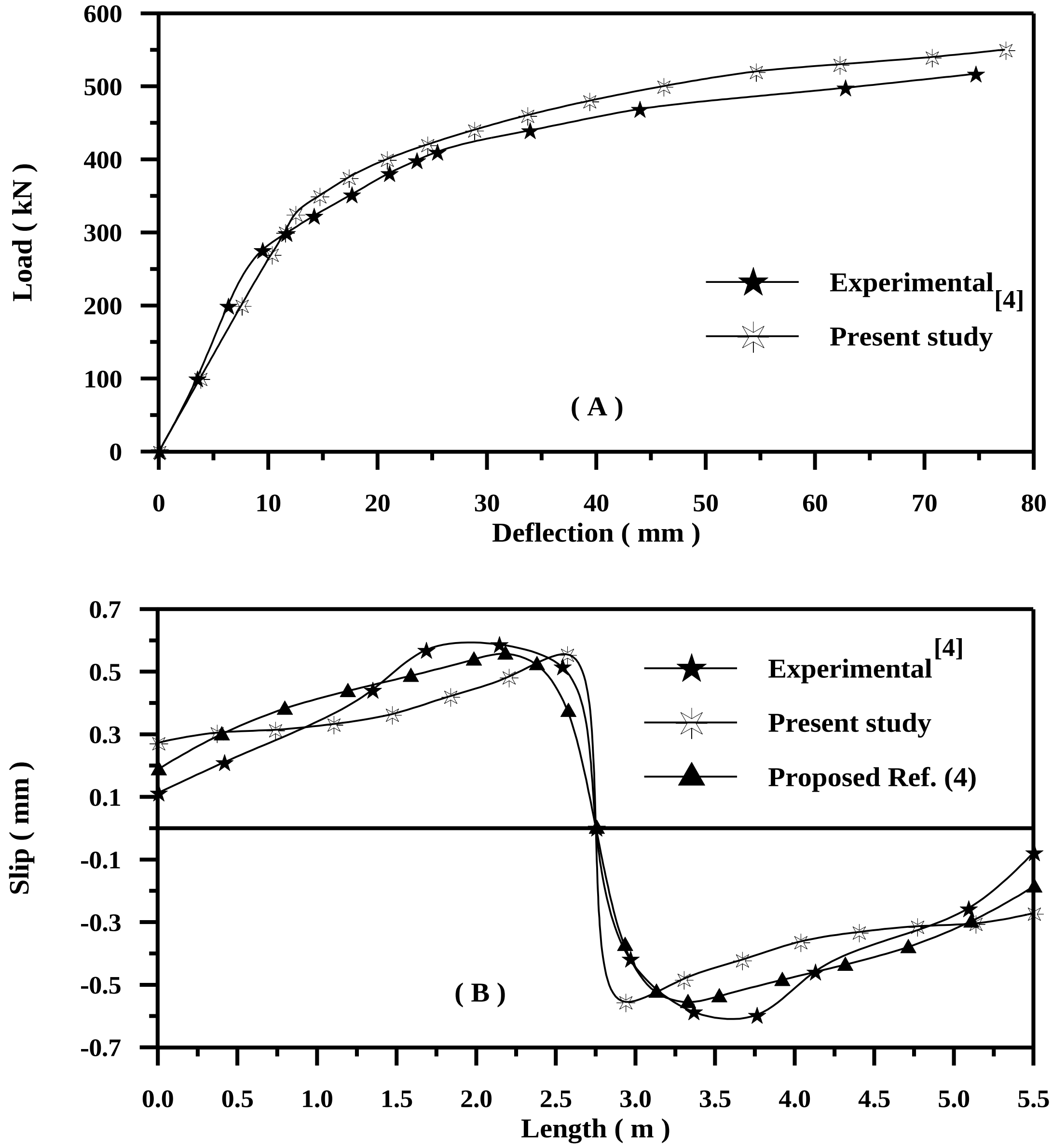

Figure (7) Comparison between theory and Experimental Results of Beam EII

For best economy use tension flange larger than compression flange for welded I-beams in composite construction. For rolled steel beam use cover plate on tension flange as shown in **Fig.(8)** to lower neutral axis and increased the second moment of inertia. Therefore, we study the effect of cover plate on the behavior of simply supported composite beam especially on ultimate load capacity and slip.

To study the effect of cover plate use the same composite beam [EII shown in **Fig.(6)**] with cover plate length equals to 3.3 m, width 100 mm and with various thicknesses. Assuming that the cover plate and lower flange have full interaction occurring between them (the interface deflections and strains are equals so that no slip and no uplift occur in the interface). For simplicity use the same material properties of steel beam and cover plate.

Load-deflection relationships obtained from the present analysis are shown in **Fig.(9)** for simply supported composite beam with various cover plate thicknesses. It is clear when increasing the thickness of cover plate, the ultimate load capacity of the section will increase by 7.27 %, 16 % and 32 % for plate thickness 7.5 mm, 15 mm and 30 mm respectively, in comparison with no cover plate. The deflection values when the beams failed decrease with the increase of the plate thickness.

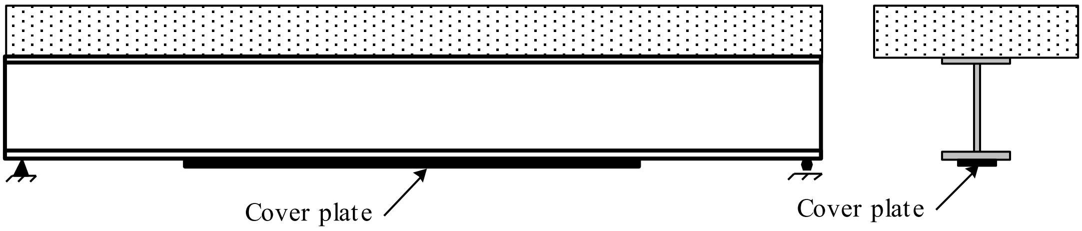

Figure (8) Details of Composite Beam with Cover Plate

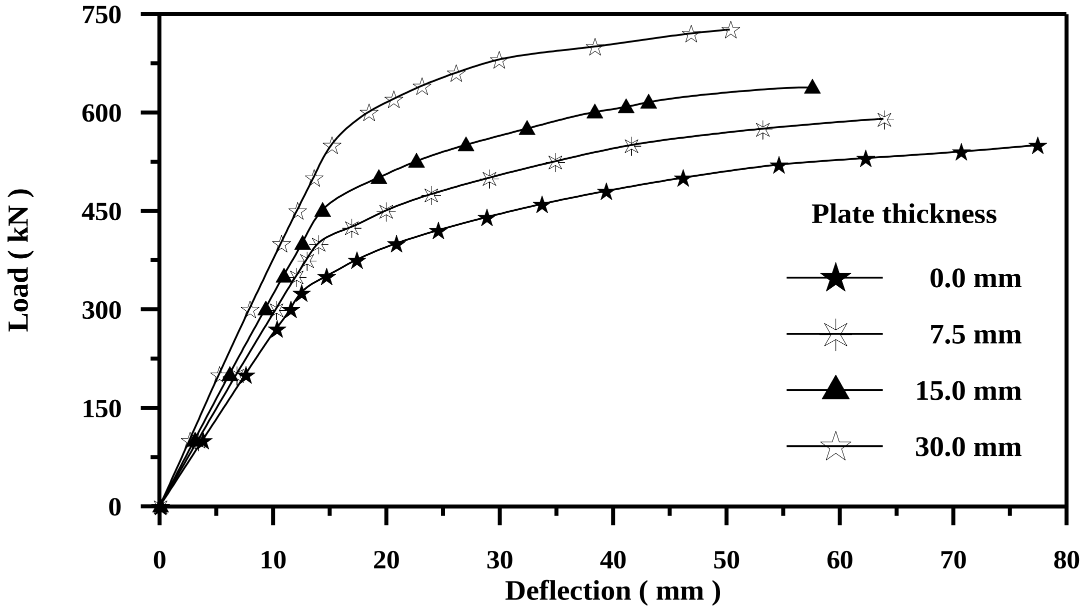

Figure (9) Load-Deflection Relationships for Beams with Various Plate Thicknesses

Figure (10) show the slip distribution at a load level equals to 550 kN. When increasing plate thickness, the maximum slip will decrease, the maximum slip will occur at different locations for each plate thickness. For plate thickness equals to 30 mm the maximum slip occurs near the support, but for plate thickness equals to 15 mm and 7.5 mm the maximum slip occurs near the point load. For the original beam (plate thickness equals zero), the maximum slip exists at approximately one-fifth of the half span and this shape is similar to the experimental shape at load 450 kN [**Fig.(7-B)**].

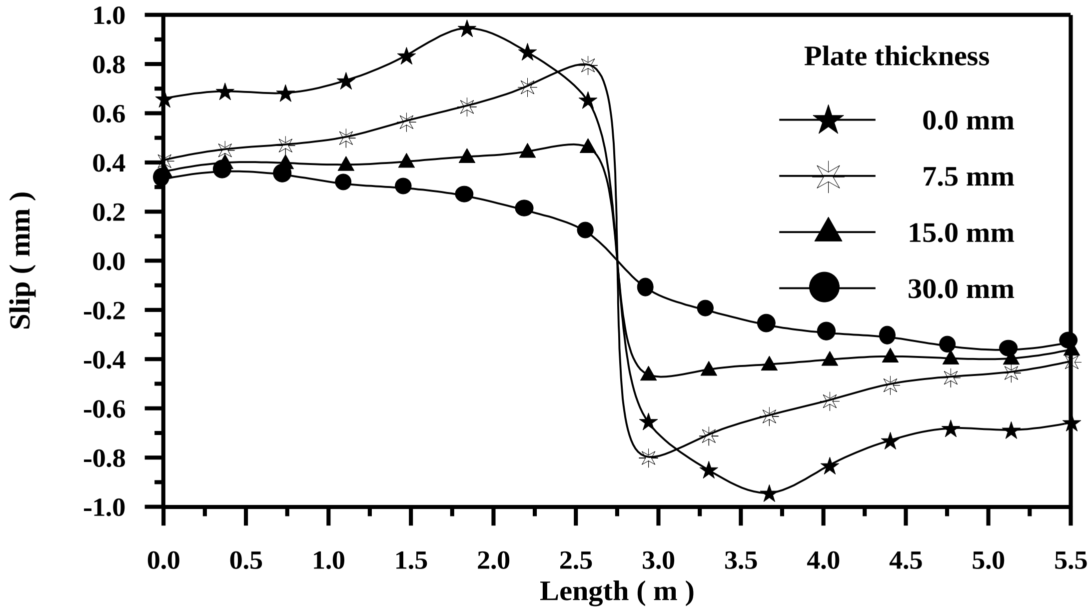

Figure (10) Slip Distribution for Beams with Various Plate Thicknesses at Load 550 kN

7. Conclusions

The following points are the results concluded from the above discussion:

1. The developed composite beam element gives good results of deflection and slip of a composite simply supported beam when compared with published experimental data.

2. The increase in thickness of cover plate will give increase in ultimate load value of composite beam by 7.27 %, 16 % and 32 % for plate thickness 7.5 mm, 15 mm and 30 mm respectively.

3. The increase in thickness of cover plate will give decrease in slip value at the same load.

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