The Ratio between Static and Dynamic Modulus of Elasticity in Normal and High Strength Concrete

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Abstract

There is a simple linear relationship between the static and dynamic elastic module, this lead to hope that using dynamic elastic modulus may remove the problems in measuring the static elastic modulus of concrete. Normal strength concrete (mix (1)) exhibited lower dynamic modulus of elasticity than that of high strength concrete (mix (2)). The reason for that was the difference in the quality of past, due to the difference in the water content of these mixes. Type (HB) the quartz is higher in static modulus of elasticity of concrete for cylinder specimens than types HA (dolomite), HD (dolomite content organic), HF (round gravel), HE (rough gravel) and HC (calcite) by (9%), (19%), (20%), (21%) and (25%) respectively in 91 days.

Type (HB) the quartz is higher in dynamic modulus of elasticity of concrete for cylinder specimens than types HA (dolomite), HC (calcite), HD (dolomite content organic), HF (round gravel), and HE (rough gravel) by (4%), (22%), (23%), (23%) and (25%) respectively in 91 days. The ratio of static modulus of elasticity of concrete to the dynamic modulus of elasticity of concrete (Ec/Ed) for the normal strength concrete (mix (1)) is between (36 to 55%). While for high strength concrete (mix (2)) the ratio is between (15 to 27%).

الخـلاصــــــة

العلاقة مابين معامل المرونة الستاتيكي و الديناميكي علاقة خطية بسيطة وهذا يقود إلى استخدام معامل المرونة الديناميكي للتخلص من مشاكل قياس معامل المرونة الستاتيكي. الخلطة رقم (١) مقاومة الانضغاط الاعتيادية أظهرت معامل مرونة ديناميكي اقل مقارنته بالخلطة رقم(٢) مقاومة الانضغاط العالية والسبب يعود إلى اختلاف نوعية عجينة الإسمنت بسبب اختلاف محتوى الماء في هذه الخلطات.

النوع (ب) الكوارتز اعلي معامل مرونة ستاتيكي للخرسانة لنماذج الاسطوانة من الأنواع أ(الدولمايتك)، د (دوامايتك يحتوي على مواد عضوية)، ف (ركام اعتيادي غير مكسر)، ي (ركام اعتيادي مكسر)، ج (الكلسايت) بمقدار (٩%)، (١٩%) ،(٢٠%)،(٢١%) و(٢٥%) بالتتابع عند عمر (٩١) يوم. النسبة ما بين معامل المرونة الستاتيكي و معامل المرونة الديناميكي للخلطة الخرسانية ذات مقاومة الانضغاط الاعتيادية خلطة (١) تتراوح مابين (٣٦ إلى ٥٥%) بينما للخلطة الخرسانية عالية الأداء خلطة(٢) تتراوح ما بين (١٥ ألي ٢٧%).

1. Introduction

The purpose of nondestructive testing is to determine the various properties of concrete such as strength, modulus of elasticity, homogeneity, integrity as well as conditions of strain and stress with out damaging the structure. Selection of the most applicable method or methods of testing requires based on information needed, size and nature of the project and the seriousness of observed conditions. The dynamic elastic modulus is used primarily to evaluate soundness of concrete in durability tests ; it is more appropriate value to use when concrete is to be used in structures subjected to dynamic loading (i.e. impact or earthquake), and the equation in the manual can be used to calculate the dynamic modulus of elasticity of concrete [1,2,3].

Mehta [4] reported that, the dynamic modulus of elasticity is generally 20, 30, and 40% higher than the static modulus of elasticity for high, medium, and low strength concrete respectively.

Neville [5] showed that, the ratio of the static to dynamic moduli is higher with the higher strength of concrete and increases with age. The British Code of concrete structures CP110:1972 gives the following equation between static and dynamic modulus of elasticity:

$$\mathbf{E_c = 1.25E_d - 19} \quad \textbf{(1)}$$

A transverse vibration of cylinders and their compressive strength determines atypical relation between the dynamic and static modulus. This relation is unaffected by air entrainment, method of curing, condition at test, or the type of cement. The British Code of Practice for the structural use of concrete CP110: 1972 relates the dynamic modulus of elasticity of concrete E_d to its strength f_c.

$$\mathbf{E_d = 22 + 2.8\, f_c^{\,0.5}} \quad \textbf{(2)}$$

where:

E_d =The dynamic modulus of elasticity of concrete (GN/m^2)

f_c = The compressive strength of concrete of cylinder (MN/m^2)

Lydon and Balendran [6] reported that, the simplest empirical relation has been developed between the static modulus of elasticity of concrete and the dynamic modulus of elasticity of concrete. The relation is of limited range to use and it does not apply to concrete containing more than 500 kg/m3 of cement or lightweight aggregate concrete.

$$\mathbf{Ec = 0.83\, Ed} \quad \textbf{(3)}$$

Neville [7] showed that, in the great majority of cases, the modulus calculated from resonant frequency tests exceeds that calculated from static loading tests.

Khalil and Gilles [8] found that, the difference between the dynamic modulus of elasticity of concrete and the static modulus of elasticity of concrete is because the heterogeneity of concrete affects the two moduli in different ways.

Dias et. al. [9] showed that, the dynamic modulus of elasticity determined by a pulse-velocity technique using a pundit instrument seems to be more sensitive to moisture loss, which indicates that the change in the dynamic modulus of elasticity with temperature is greater than that in the static modulus.

Aiticn [10] a recent study has shown that there is a simple linear relationship between the static and dynamic elastic moduli. This leads to expect that using dynamic elastic modulus may remove the problems in measuring the static elastic modulus.

Hansen [11] showed that, the relationship between the dynamic modulus of elasticity and the compressive strength of cube increases with increase in concrete strength and the equation is found to fit the observed result.

$$E_d = 5.31 f_{cu}^{0.5} + 5.83 \quad \text{.......... (4)}$$

where:

E_d= The dynamic modulus of elasticity of concrete GPa

f_{cu}= The compressive strength of cube MPa.

2. Research Significant

This work is undertaken to examine the influence of six aggregate types on the hardened properties for two groups, Mixes of concrete (normal strength and high strength). The differences in the performance are discussed between the ratio of the static and dynamic modulus of elasticity of concrete.

3. Materials and Experimental Work

3-1 Materials

The materials used in this study are locally available and widely distributed over large areas in the mountainous region of Sulaimaniyah and Arbil. These materials include crushed limestone, quartz, dolomite, organic limestone, crushed gravel, rounded gravel and natural silica sand, in addition to the drinking water and Lebanon cement.

3-1-1 Aggregate

Aggregate were used in this study include four types of crushed dolamitic limestone rock [Dolamitic, Marble(quartz), limestone (pure), limestone (contained organic materials)] from north of Iraq and two types of natural crushed and uncrushed gravel in addition to natural fine Al-Ukhaider silica sand. The maximum aggregate size is used (20 mm).

3-2 Mix Proportion

In this study, two mix design combinations are adopted. In the first four types of crushed rocks are used as coarse aggregate with natural silica sand as fine aggregate, and two natural gravel crushed and uncrushed are used as coarse aggregate with silica sand (Al-Ukhaider) as fine aggregate. The mix design is done according to the American method of mix design ACI 211.1.91 (7) for normal concrete. The mix proportions are shown in **Table (1)**. For high strength, the same materials (four types crushed rock, crushed gravel, uncrushed gravel as coarse aggregate and silica sand (Al-Ukhaider) as fine aggregate and superplasticizer (glenum 51) is added to this mix). The proportions of this mix are shown in **Table (1)**.

Table (1) Mix Proportions of Concrete

Mix Type	W/c ratio	Water kg/m^3	Cement kg/m^3	Sand kg/m^3	Gravel kg/m^3	SP/Cement %	Density (kg/m^3)
Normal strength	0.60	200	334	750	1020	------	2304
High strength	0.22	117	530	650	1200	1.5	2497

3.3 Testing Hardened Concrete

3-3-1 Dynamic Modulus of Elasticity

One of the methods used for determining the dynamic modulus of elasticity is the resonant frequent method. This test is carried out on beams subsequently used for flexural strength; the test is done according to ASTM C469-87 [12], using the Erudite resonant frequency tester. The longitudinal resonant frequency is measured for each specimen immediately after removal from water curing tanks. The test is conducted with output voltage of about 1 volt and it is repeated twice for each sample. Two specimens (prisms) are tested for each batch at the specified age and three (cylinders) for each batch. The weight (w) in kilograms of the wet beam in air is measured in addition to the average length, L, and the average width, b, and depth, d, in millimeters. Then, the density, d, in kilograms per cubic meter is calculated, as follows:

$$d = (w/L\ bd) * 10^{9} \quad \text{.......... (5)}$$

Finally, the dynamic modulus of elasticity is calculated using the longitudinal vibration, as follows [13]:

$$E_d = 4 * L^2 * n^2 * d * 10^{-15} \quad \text{.......... (6)}$$

where:

E_d= dynamic modulus of elasticity, GPa

L= length of the specimen in millimeters,
n= natural frequency of the fundamental mode of longitudinal vibration of the specimen in H, Z, and
d = density kg/ m^3

3-3-2 Static Modulus of Elasticity

The static-elastic modulus is calculated from the stress-strain diagram. The chord-modulus method, as recommended by [14] is used. The cord modulus is the slope of the line drawn between two fixed points on the stress-strain diagram. The lower point, established to eliminate the effect of crack from the initial portion of the stress-strain curve, is the point where strain is $5*10^{-5}$ mm, while the up point is the point where stress equals 40 percent of the ultimate stress. The chord modulus is calculated from the relation:

$$E_c = \frac{S_2 - S_1}{\in 2 - 0.00005} \quad \text{............} \quad (7)$$

where:
E_c = The static elastic modulus, MPa
S_2 = stress corresponding to 40 % of ultimate load, MPa
S_1 = stress corresponding to a longitudinal strain (0.00005), MPa
$\in 2$ = longitudinal strain produced by stress S

4. Discussion of Tests Results

Tables **(2)** and **(3)** show the result of the dynamic modulus of elasticity of concrete and the ratio of the static modulus to the dynamic modulus for normal, high strength concrete and the pulse velocity result of this study increase with high strength of concrete.

Figures (1) and **(2)** show the relationship between the dynamic modulus of elasticity of concrete with age. The normal strength of concrete mix (1) and the high strength of concrete mix (2) increase with age, as the age of and this can be expressed by the equation:

$$E_{d1} = 19.0*(t)\ 0.1477 \quad \text{with } R= 0.9659 \quad \text{............} \quad (8)$$

$$E_{d2} = 26.43*(t)\ 0.155 \quad \text{with } R^2=0.9888 \quad \text{............} \quad (9)$$

where:
E_{d1} =The dynamic modulus of elasticity of normal strength concrete (GPa)
E_{d2} = The dynamic modulus of elasticity of high strength concrete (GPa)
t = The age of specimens ((7-91) days)

Table (2) The results of the dynamic and static modulus of elasticity of concrete Mix(1)

Type of Mix(1)	Age (days)	d (kg/m^3)	V (km/sec.)	f_c (MPa)	E_c (GPa)	E_d (GPa)	E_c/E_d %
N A	7	2367	4.2	10	11.5	24.6	46.7
	28	2392	4.25	11.4	14.6	25	53.6
	56	2398	4.3	15.3	15.9	31.2	50.9
	91	2443	4.4	17.8	16.2	33.8	47.9
N B	7	2411	4.27	13.3	11.54	23.7	45.2
	28	2444	4.48	15.3	14.4	24.2	47.6
	56	2466	4.56	21.5	14.54	35.9	40.5
	91	2476	4.62	23.77	14.67	36.4	40.3
N C	7	2406	4.15	10.47	12.7	23.3	54.5
	28	2415	4.2	15.6	17.7	31.1	56.9
	56	2443	4.45	17.2	19.3	35.5	54.3
	91	2476	4.65	22.6	21.4	36.5	58.6
N D	7	2405	4.1	12.5	12.8	28.6	44.7
	28	2410	4.32	14.8	14.85	32.2	46.1
	56	2415	4.34	17.4	15.8	33.1	47.7
	91	2426	4.57	22	17.8	35.2	50.5
N E	7	2369	4.1	9.9	10.5	29.4	35.7
	28	2387	4.34	16	14.8	34.5	44.7
	56	2393	4.5	20.1	17.5	41.1	43.7
	91	2403	4.56	23	21.8	49.8	43.6
N F	7	2390	3.9	9.7	8.7	27	37.0
	28	2396	4.1	12.8	10.2	24	42.5
	56	2411	4.1	13	10.8	29.3	39.7
	91	2462	4.23	15.3	11.6	23.5	39.5

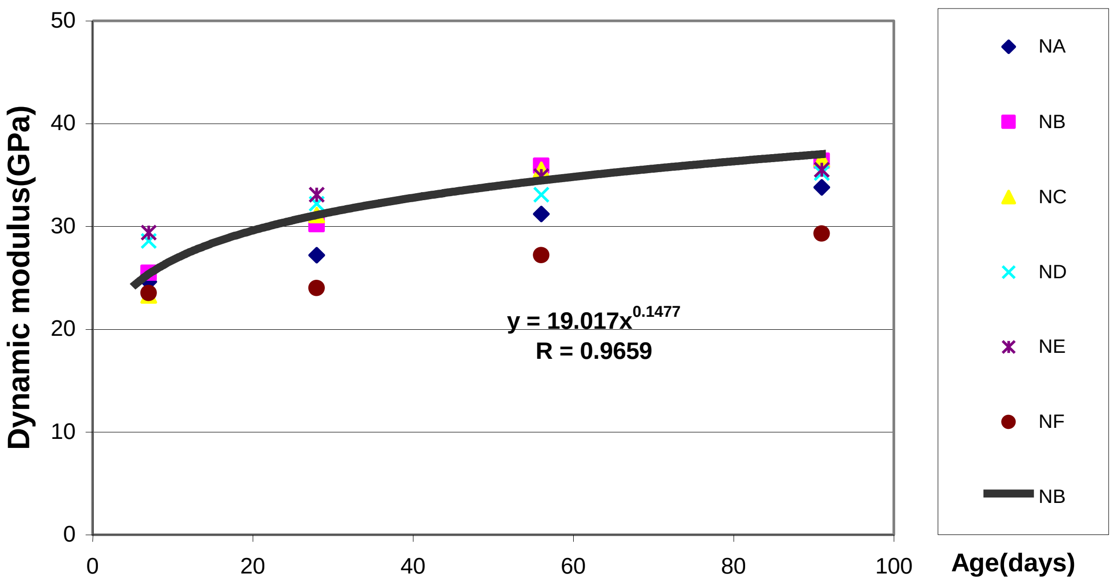

Figure (1) The relationship between the dynamic modulus of elasticity of concrete mix (1) with age

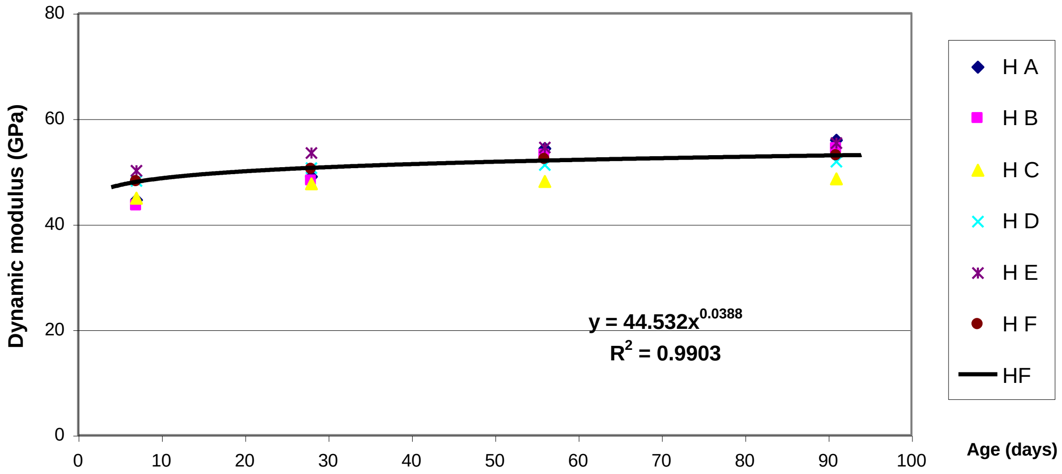

Figure (2) The relationship between the dynamic modulus of elasticity of concrete mix(2) with age

The British Code of Practice for the structural use of concrete CP110: 1972 relates the dynamic modulus of elasticity of concrete Ed to its strength fc. Hansen [11] showed the relationship between the dynamic modulus of elasticity and the compressive strength of cube which increases with the increase in concrete strength. The equation is found to fit the observed results in **Figures (3)** and **(4)** at test Mix (1) which exhibits a lower dynamic modulus of elasticity than that of Mix (2) .This is due to the difference in the quality of paste due to the difference in the water content in these mixes (high compressive strength). Hence, this relation can best be expressed in the equation:

$$E_{d1} = 7.3*(f_c)\ 0.533 \quad \text{with R=0.9958} \quad \textbf{(10)}$$

$$E_{d2} = 29.0*(f_c)\ 0.139 \quad \text{with R=0.991} \quad \textbf{(11)}$$

where:

E_{d1} =The dynamic modulus of elasticity of normal strength concrete(GPa)

E_{d2} = The dynamic modulus of elasticity of high strength concrete (GPa)

f_c= The compressive strength of cylinder specimens (MPa)

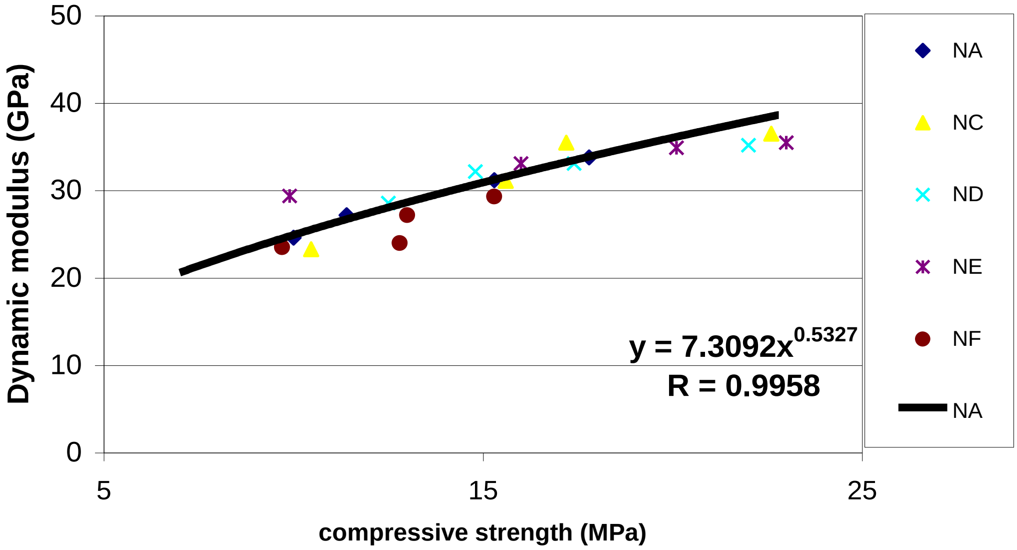

Figure (3) The relationship between the dynamic modulus of elasticity of concrete mix (1) with compressive strength

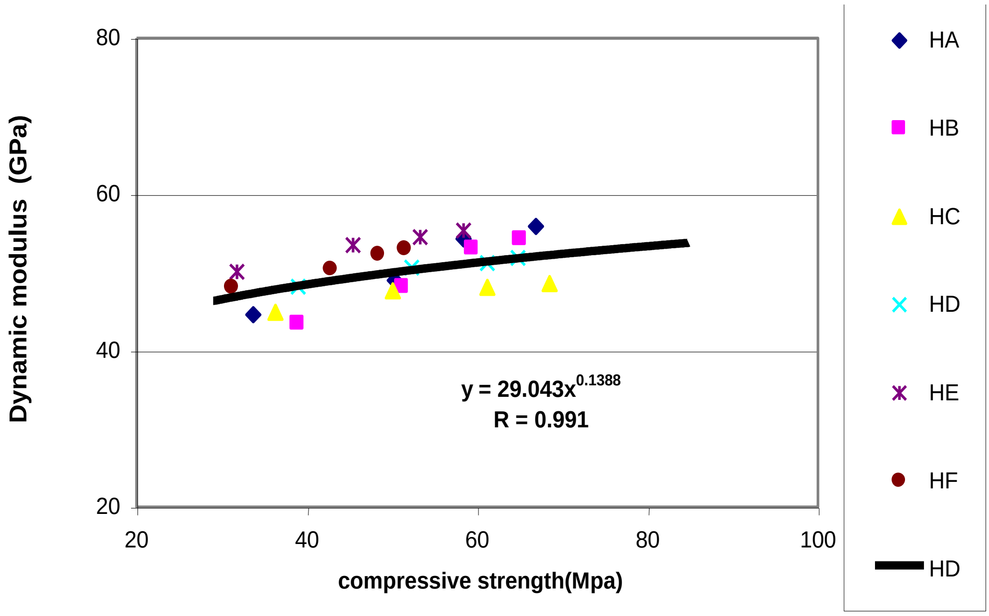

Figure (4) The relationship between the dynamic modulus of elasticity of concrete mix (2) with compressive strength

Figures (5) and **(6)** show the simplest empirical relation which has been developed between the static modulus of elasticity of concrete and the dynamic modulus of elasticity of concrete. For higher static moduli, both dynamic and static moduli of elasticity are in close agreement. Aiticn [10] a recent study has shown that there is a simple linear relationship between the static and dynamic moduli ,which leads to hope that using dynamic modulus may remove the problems in measuring the static modulus. The dynamic modulus of elasticity is generally 20, 30, and 40% higher than the static modulus of elasticity for high, medium and low strength concrete respectively [4]. Hence, the test results shown in **Table (2)** for Mix (1) indicated that the ratio of the static to dynamic modulus of elasticity (E_c/E_d) which ranges (36 to 59%). **Table (3)** shows the static modulus of elasticity, which is lower than dynamic modulus of elasticity by (15 to 27%) for mix (2) .This can be expressed in the equation:

$$E_{d1} = 5.82*(E_c)^{0.63} \quad \text{with } R=0.988 \qquad (12)$$

$$E_{d2} = 26.27*(E_c)^{0.19} \quad \text{with } R=0.9972 \qquad (13)$$

where:

E_{d1} =The dynamic modulus of elasticity of normal strength concrete (GPa)

E_{d2} = The dynamic modulus of elasticity of high strength concrete (GPa)

E_c= The static elastic modulus of elasticity of concrete (GPa).

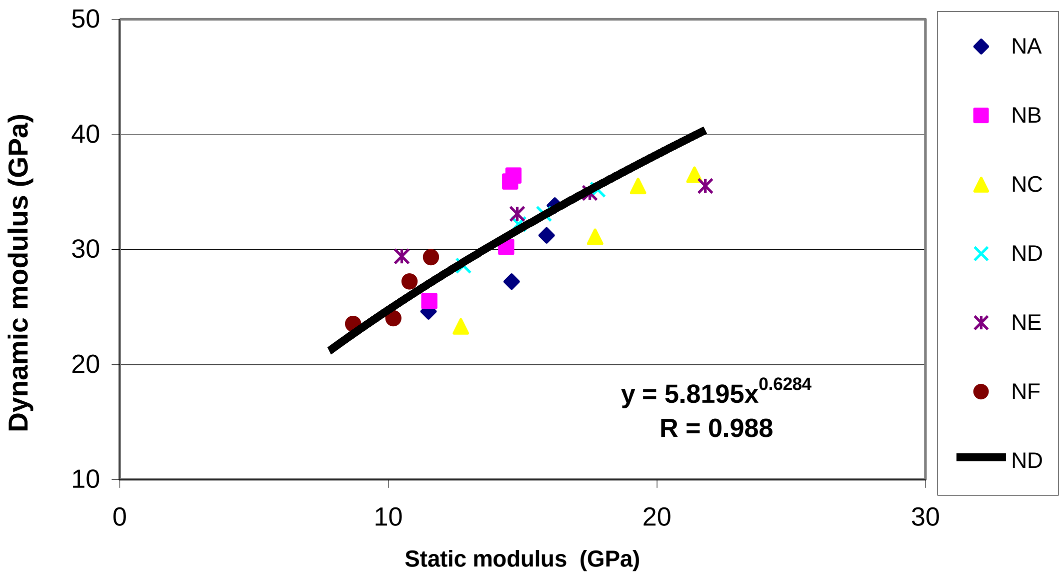

Figure (5) The relationship between the dynamic modulus with the static modulus of elasticity for concrete mix (1)

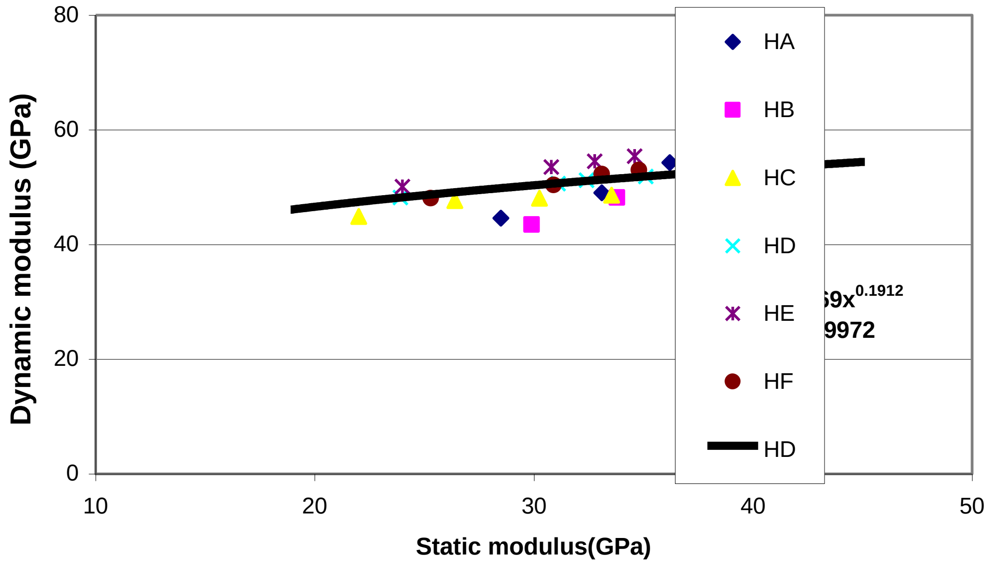

Figure (6) The relationship between the dynamic modulus with the static modulus of elasticity for concrete mix (2)

The ratio of the static modulus of elasticity to the dynamic modulus of elasticity of the concrete is lower with the higher compressive strength of concrete and increases with age. The modulus calculated from resonant frequency tests exceeds that calculated from static loading tests. The difference between the dynamic modulus of elasticity of concrete and the static modulus of elasticity of concrete is due to the fact that the heterogeneity of concrete affects the two moduli in different ways (5).

5. Conclusion

The mineralogical characteristic of coarse aggregate appears to be an important factor, which influences the mechanical properties of concrete. Based on the results of the experimental work carried out in this study to verify the types of aggregate and mix proportion, the following conclusions are deduced:

1. The normal strength of concrete Mix (1) has lower compressive strength, the static elastic modulus, the dynamic modulus of elasticity of concrete and flexural strength in comparison with the high strength of concrete Mix. As a result, the role of the crushed rock aggregate does not appear in Mix (1) consequently, the influence of coarse aggregate on all properties of concrete is obvious in Mix (2).

2. The compressive strength and the elastic modulus of high strength concrete are significantly influenced by the mineralogy of coarse aggregate.

3. Dolomitic concrete type (HA) shows more linear behavior than crushed and uncrushed aggregate. So, gives higher modulus in Mix (2) compared with types (HE) by (7%, 9.7%) and HF by (6.6%, 9.1%) respectively at 28 days and 91 days. Type (NA) in Mix (1) is of lower value by (58%) compared with type (HA) in Mix(2) because of the difference in Mixes.

4. There is a simple linear relationship between the static and dynamic elastic moduli, which leads to predict that using dynamic elastic modulus may remove the problems in measuring the static elastic modulus of concrete.

5. The ratio of the static to dynamic modulus of elasticity (E_c/E_d) ranges between (46% for Mix (1)) and (21% for Mix (2)) for the total results.

6. The normal strength of concrete Mix (1) exhibited lower dynamic modulus of elasticity than that of the high strength of concrete Mix (2) because of the difference in the quality of paste due to the difference in the water content in these mixes.

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