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Investigation of Thermal Anisotropy Influence on Transient Temperature Distribution in an Orthotropic Bar

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Abstract

A study is made of the influence of thermal anisotropy on the transient temperatures distribution after a short time. Such problem simulate the general behavior of the composite long metal bar of rectangular cross-section where it is initially at a temperature T_o and it is then suddenly quenched in a large volume of fluid at temperature T_f, thermal conductivity is assumed orthotropic, having different x, and y direction properties.

The heat dissipation during solidification process related to the transient temperature distribution, thus the directional metal conductivity ply an important role in the volumetric strains due to orthotropic thermal expansion and chemical shrinkage, as well as the dramatic changes of saluted metal. The presented study results show the anisotropic conductivity need for realable simulation procedure.

الخلاصـــــة

تم في هذا البحث تقديم دراسة لتأثير عدم التجانس الحراري على التوزيع العابر لدرجات الحرارة ولفترة زمنية قصيرة. هذا النوع من المسائل يمكن تمثيلها بالتصرف العام للقضيب المعدني بمقطع مستطيل ذو المواصفات غير المتجانسة حيث يكون ابتدائيا بدرجة حرارة معينة ثم يبرد فجائيا في مائع ذو حجم كبير يملك درجة حرارة المائع. الموصلية الحرارية اعتبرت غير متجانسة هندسيا وتملك قيم مختلفة في المحاور الثلاثية المتعامدة.

الحرارة المتشتتة خلال عملية التصلب ترتبط بالتوزيع العابر وفي ضوء هذا فان عدم التجانس في ثوابت التوصيلية الحرارية يلعب دورا مهما في مستويات الانفعال الحجمية نسبة إلى معدلات التمدد الحراري غير المتجانسة والانكماش الكيماوي وبالتالي سينتج تغييرات مهمة للمعدن المتصلب .نتائج الدراسة تميل إلى الصيغة التمثيلية لدور التوصيلية الحرارية غير المتجانسة والحاجة إلى سياق تمثيلي امن.

1. Introduction

During manufacturing of a structural component made of composite material, shrinkage and shape distortions occurring, because of the anisotropic nature of the material where there are different thermal expansion coefficients with directions and location, shrinkage does not only mean that the dimensions of component are scaled down , the shape will also change. Even if the laminate is through-thickness homogenous and isothermally cured, shrinkage leads to an angular decrease.

A number of high-speed flight vehicles, will be exposed to intense thermal, acoustic and aerodynamic environments. The skin panels will become thermally buckled and undergo large random excitation. It is known that the mean-square value of the plate's transverse displacement under stationary random load is inversely proportional to the cubic of transient dynamic behavior [1]. Therefore, the understanding and control of thermal natural characteristics of skin panel using smart or composites under an extreme thermal environment are important for the design of skin panels of high-speed vehicles or any particular structure. The transient thermal behavior of a thermally buckled plate was studied by Bisplinghoff and Pian [2]. They used an extension Marguerre's theory for plates in longitudinal compression and obtained the stress function. The modal equations of motion were derived from Lagrange's equation with the strain energy and the kinetic energy of a thermally buckled plate. Thermally dynamic behavior for symmetric modes versus temperature ratio $\Delta T/\Delta Tcr$ was obtained. Noor and Burton [3] studied the thermal stresses of multilayer composite plates using three-dimensional solutions. A Duhamel-Neumann type constitutive model was used. The plate response variables due to thermal load were assumed to be harmonically time-varying perturbation displacements, strains and stresses. Each of the plate variables was decomposed into symmetric and antisymmetric components in the thickness direction, Yang and Han [4] studied the linear thermally influence of a buckled rectangular plate and the large amplitude free vibration of a square plate with in plane stresses using the finite element method. A 54 DOF triangular shell element was used in the study. A nonlinear eigenvalue equation was formed and an iteration method was adopted to solve the vibration frequencies. Zhou et. al. [5] also studied the free vibration of thermally buckled composite plates using finite element method. DKT (Discrete Kirchhoff theory) element was employed. The solution of the system equation was separated into particular part and homogenous part, which stand for thermal post buckling deflection .Therefore, it is ideal to use SMA for thermal post buckling reduction, vibration control and frequency turning. Birman [6] gave an excellent survey on smart material mechanics, constitutive models and applications in structure control area. Zhong [7] conducted the researches on thermal post buckling, free vibration and random vibration control results showed the great enhancement of static and dynamic characteristics of composite plates under thermal environment if embedded with Shape Memory Alloy (SMA). Duan et. al. [8] proposed a temperature increment procedure for the analysis of thermal analysis of composite plate. The procedure gave accurate prediction on thermal deflection problem with temperature-dependent material properties. In this paper, a finite element formulation is presented for the transient temperature distribution analysis of orthotropic rectangular bar, thermally treated, with temperature-dependent material properties. The transient solution is separated into thermal displacement constitutive relations. Considering temperature-dependent material properties based on orthotropic thermal conductivity in both rectangular coordinates. The approach is to divide the whole deflection procedure under large temperature change into many small increments. In each increment, the temperature step is relatively small and the material properties can be regarded as constants. The incremental thermal displacement, then the displacement and the stresses are summed and updated. The updated total displacement and stresses are considered as initial displacement and stresses in the next temperature increment. The present study tries to exploring the isotropic viscoelastic material temperature dependent which is difficult and expensive to characterize during cure.

In the present work the important role of orthotropic thermal conductivity on the transient temperature distribution is studied. Thus, a finite element formulation is presented for the transient temperature distribution of a thick thermally composite orthotropic laminated bar with temperature-dependent material properties. The temperature-increment solution procedure [9] is adopted.

2. Theory

The thermo-mechanical behavior of thermo set composites during curing process is however viscoelastic and rather than complicated to model accurately, which combined cure kinetics, through-thickness heat conduction and elastic laminated theory, as well as is a fairly complex procedure including thermal , chemical ,and mechanical phenomena. Accordingly, a comprehensive simulation tool for transient temperature distribution has to contain heat conduction.

The heat conduction in linear theory is described by the thermal conductivity matrix $[K_{ij}]^{T^*}$, specifically, the heat flux q_i^T per unit area in the x_i direction is related to the temperature gradient $T_{,i}$ in the x_i direction by [10]:

$$q_i^T = -K^T_{ij}\, T_{,j} \qquad \text{……… (1)}$$

where: T is temperature and $T_{,i}$ is its partial derivative with respect to x_i.

In the material symmetry axes of unidirectional composites, the only heat flux possible due to the temperature gradient $T_{,i}$ is q_i^T therefore, equation (1), for orthotropic material reduces:

$$\begin{aligned} q_x^T &= -K^T_x\, T_{,x} \\ q_y^T &= -K^T_y\, T_{,y} \qquad \text{……… (2)} \\ q_z^T &= -K^T_z\, T_{,z} \end{aligned}$$

The transformation of heat flux however follows from the balance of energy, and the total heat influx must be equal to the total heat efflux, that is:

$$q_x^T\, dy + q_y^T\, dx = q_1^T\, dx \quad \ldots\ldots (3)$$

With the transformation equation known for both heat flux and temperature gradient, it can now express the heat conductivities in any coordinate system in terms of their material symmetry axes. To this end we obtain:

$$q_1^T = -K_x^T\, T_{,x}\, m + K_y^T\, T_{,y}\, n \quad \ldots\ldots (4)$$

where: m, n are the stacking angle direction index (cosθ, sinθ), then for obtaining the governing heat conduction equations for curing, the thermal conductivity in orthotropic axes may expressed after some manipulation.

The thermo-mechanical stress analysis based on the heat conduction during curing treated from the design operating temperature. In such cases, thermal stresses must be accounted for. Residual stresses are undesirable, as they can lower the strength of the structure and cause geometric distortions. The three-dimensional thermo-elastic strain-stress relations are [11]:

$$\varepsilon_i = S_{ij}\, \sigma_j + \alpha_i\, \Delta T \qquad i,j=1,2,\ldots,6 \quad \ldots\ldots (5)$$

where: in the total strain, ε_i is the sum of the mechanical strain, $S_{ij}\, \sigma_j$, and the free thermal, $\alpha_i\, \Delta T$.

For plane thermal residual stress of an orthotropic laminate in principal material coordinates. The full derivation of thermo-mechanical matrices are well documented in ref. (11). The transient thermal analysis determines temperatures and other thermal quantities that vary over time .Engineers commonly use the transient thermal analysis results as input data for thermal stresses analysis evaluations, many heat transfer applications, nozzles, engines blocks , and space shuttles involve transient thermal transient analysis.

However, for a comprehensive analysis there is need for a numerical solution, nowadays typically a Finite Element Method (FEM) solution. Many researchers use commercial finite element codes, with user defined subroutines where needed; others, such as our group, have developed special purpose FEM codes. Typically, there is a thermo-chemical module, that solves the thermal problem, including the complex applied boundary conditions and the exothermic response of the matrix material as it cures; a flow module that allows for resin flow prior to gelation; and a stress/deformation module that allows for the buildup of stress due to thermal chemical, and mechanical interactions between the fiber, matrix, layers, and tooling and inserts uses a two-dimensional finite element model to simulate the various phenomena that take place during processing of composite structures.

3. Finite Element Formulation

In order to model process-induced deformations and stresses in complex three-dimensional structures it is necessary to have efficient computational techniques that do not require the full discretization of the domain of the problem consisting of both the composite material and the process tool (mould). This will not only lead to a dramatic reduction in computational times but also in data preparation and mesh generation [12,13]. In the absence of such techniques, the composite structure will have to be modeled in all three dimensions using brick elements leading to a huge computational effort both in mesh generation and solution of equations.

A super finite element technique is presented here to provide an accurate and efficient prediction of the temperature field at any time during the process cycle. The basic idea is to develop an element that is capable of solving a 3D heat transfer problem in plate structures using an analytical series approximation in the through thickness direction and a regular FE model in the plane of the plate. It can be shown [14] that unless radial changes occur in the boundary conditions and/or excessive internal heat is generated within the solid, the first few terms in the analytical series are sufficient to calculate the through thickness distribution of temperature in the plate. There follows a brief outline of the super element formulation, and corresponding results are presented for the 2D thermo-chemical analysis of a composite part in order to facilitate comparison of its accuracy and efficiency with the full-blown finite element code Similar concepts can be used to mathematically construct a three-dimensional super element.

The governing equations for the thermo-chemical analysis of a 2-D orthotropic body as shown in **Fig.(1)** consist of a partial differential equation (PDE) for heat transfer coupled with a cure kinetics equation that describes the evolution of the degree of cure as follows [14]:

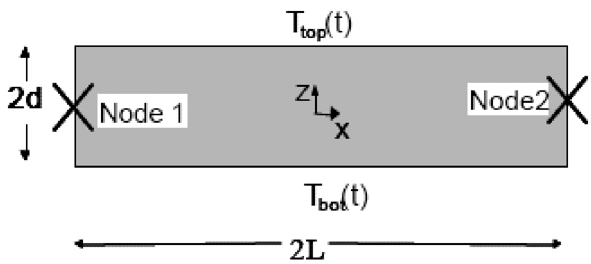

Figure (1) Typical element used in present analysis

$$\begin{aligned}
\rho C_v \frac{\partial T}{\partial t} &= k_x \frac{\partial^2 T}{\partial x^2} + k_z \frac{\partial^2 T}{\partial z^2} + H(x,z,t) \\
H(x,z,t) &= \rho H_R \frac{\partial \alpha}{\partial t} \qquad \text{.......} \quad (6) \\
\frac{\partial \alpha}{\partial t} &= g(T,\alpha)
\end{aligned}$$

where: T denotes the temperature ρ is the density, Cv is the specific heat, kx, kz are the components of thermal conductivity in the *x* and *z* directions, respectively, H_R is the heat of reaction and a is the degree of cure.

In the proposed super element formulation, the temperature field is approximated in the following series form [15]:

$$\mathbf{T = a(x,t)z + b(x,t) + \sum_{n-1}^{\infty} W_n(x,t) Z_n(z)} \quad \ldots\ldots \textbf{(7)}$$

where: Z is a trigonometric shape function which is selected based on the operative homogeneous boundary conditions (B.C.'s) at the top and bottom of the composite part; W is approximated using the conventional finite element method; and a and b are dependent on the non-homogeneous B.C.'s. using the Galerkin based weak formulation and FE discretization [15], it can be obtain the modified following system of algebraic equations for the unknown nodal variables Wi (or W in vector form):

$$\boldsymbol{\rho C_v N W + [(k_x / L^2) N^p + (\beta_i k_z / d)^2 N] W + Q(t) = 0} \quad \ldots\ldots \textbf{(8)}$$

where: N and NP are matrices that are calculated from the integration of the in-plane shape functions and spatial derivative of the element shape functions, respectively; βi is the ith eigenvalue of the PDE and is dependent on the B.C., and Q is obtained from the weighted integral of the internal heat generation term.

The time integration in the above equation is carried out using a backward Euler finite difference scheme. The constitutive relations of the thermally orthotropic composite bar then become [15]:

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{Bmatrix} \mathbf{e^o} \\ \mathbf{k} \end{Bmatrix} + \begin{Bmatrix} \mathbf{N_r} \\ \mathbf{M_r} \end{Bmatrix} - \begin{Bmatrix} \mathbf{N_{\Delta T}} \\ \mathbf{M_{\Delta T}} \end{Bmatrix} + \begin{Bmatrix} \mathbf{N_\sigma} \\ \mathbf{M_\sigma} \end{Bmatrix} \quad \ldots\ldots \textbf{(9)}$$

where: [A], [B] and [D] are the laminate stiffness matrices. The stress resultant components {Nr} and {Mr} are due to the incremental recovery stress {σr}, {NΔT} and {MΔT} are due to the incremental temperature ΔT, and {Nσ} and {Mσ} are due to the initial curing stress {σo}.For a bar element, the virtual work of the curing internal and external forces is expressed as:

$$\boldsymbol{\delta W_{int} = \int_A \left(\{\delta e^o\}^T \{N\} + \{\delta k\}^T \{M\}\right) dA,\ \delta W_{ext} = \int_A -\rho h(\ddot{u}\delta u + \ddot{v}\delta v + \ddot{w}\delta w) dA} \quad \ldots \textbf{(10)}$$

Assembling the element equations to system level and applying the boundary conditions, the finite element system equation of heat conduction with respect to temperature quenching increment ΔT, and initial displacement and initial stresses can be expressed as:

$$\rho c[(\delta T/\delta t]+\{v\}^{T} L\{T\}]+\{L\}^{T}\{q\}=\ddot{I} \quad \text{.......... (11)}$$

where: t is the time, ,L 3-D vector operator , v is the heat transfer quantity, q is heat flux vector, and Ï is the heat generated.

And the heat flux vector can be defined as [16]:

$$\{q\}=- [K] \{L\}T \quad \text{.......... (12)}$$

where: [K], is diagonal orthotropic conductivity matrix(k_{xx} ,k_{yy} ,k_{zz}), and the temperature vector defined by:

$$T = \{N\}^{T}\{T_{e}\} \quad \text{.......... (13)}$$

where: {N}, is the element shape function, T_e element temperature,

$$\delta T=\{ \delta T_{e} \}^{T} \{N\} , [B]=\{L\}\{N\}^{T} \quad \text{.......... (14)}$$

The final form of variation statement stated yields:

$$\int_{vol} \rho c\{\delta T_{e}\}^{T}\{N\}\{N\}^{T}\{T_{e}\}d(vol) +\int_{vol}(\rho c\{\delta T_{e}\}^{T}\{N\}\{v\}^{T}[B]\{T_{e}\}d(vol))+$$
$$\int_{vol} \rho c\{\delta T_{e}\}^{T}[B][K][B]^{T}\{T_{e}\}d(vol)= \int_{S2}(\rho c\{\delta T_{e}\}^{T}\{N\}^{q^{*}} d(S2))+$$
$$\int_{s,} \{\delta T_{e}\}^{T}\{N\}(T_{f}-T)\{T_{e}\}d(s) \quad \text{.......... (15)}$$

The solution procedure of equation (15), is developed by considering that equation is a set of nonlinear partially differential equations In order to verify the above technique we consider a long orthotropic composite bar of rectangular section is initially at a temperature T_o and is then suddenly quenched in a large volume of fluid at T_f , for determining the transient temperature distribution in the slab after a short time, such case study of composite material undergoing the two-hold autoclave temperature shown in **Fig.(2)**. The problem is analyzed using the current super element method. It is consider the case where the temperature at the top and bottom surfaces of the composite are specified to be distributed uniformly and follow the time histories.

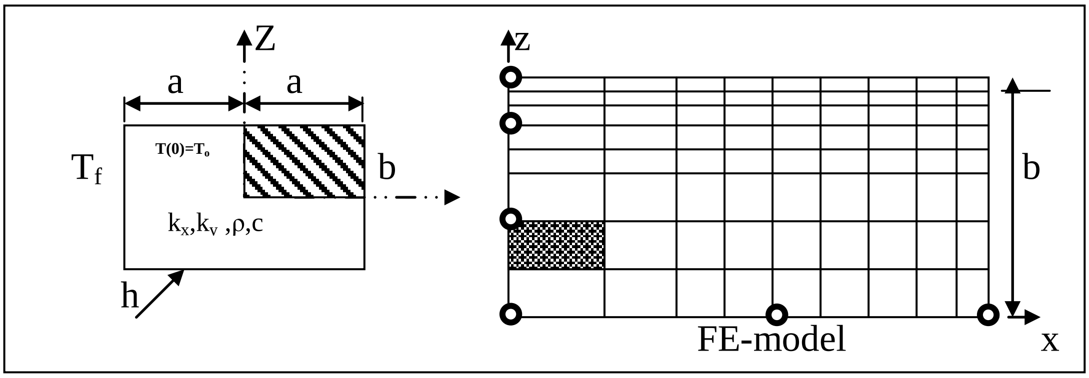

Figure (2) The schematic of case study problem

In this study the cure kinetics model is identical to that used in [17] and the relevant material properties are as follows:

$\rho = 1.581\ kg/m^3$ $\quad$ $C_v = 9.39x10^2\ J/kgK$

$K_x = 3.74\ W/mK$ $\quad$ $K_z = 0.4334\ W/mK$

$H_R = 5.40x10^5\ J/kg$ $\quad$ $V_f = 0.427$ (fiber volume fraction)

And the Appropriate transverse shape functions are selected to satisfy the essential boundary conditions on the upper and lower surfaces of the composite [18].

$$Z_i(z) = \begin{cases} \cos\left(i\dfrac{\pi}{2}\dfrac{z}{d}\right) & i = 2k-1 \\ \sin\left(i\dfrac{\pi}{2}\dfrac{z}{d}\right) & i = 2k \end{cases} \quad k = 1,2,3,\ldots \qquad \text{(16)}$$

4. Results and Discussion

The transient temperature distribution of the composite orthotropic bar during and after a short time based on considering the quenching process as a step function of time at T_f, thus the distribution of temperature calculated at different three main locations firstly at upper surface, and at the mid-thickness, and the third at interior points.

From **Table (1)**, and **Table (2)** it can distinguished that the distribution of orthotropic composite bar trend with the same gradient for all the three different positions, where it is higher logically near the bottom location then tends to slow down when approaches the upper surface with increasing the time of curing, the main note in the present study is the large significant differences in temperatures for different positions with time in a small dramatic manner because of the thermally anisotropic conductivity, as well as the states tends from the interiors point to the surface positions, then it indicates clearly the role of changes in thermal conductivity between the principal directions of the orthotropic composite bar.

Table (1) Temperature distribution of orthotropic bar after short time with T_0=500K, T_f=100K, at three different positions($k_{xx}=k_{zz}$)

Time (sec)	Interior position (K)	Intermediate position (K)	Upper-surface position (K)
0.00	500.0	499.97	499.95
0.05	499.585	499.051	499.00
0.01	497.466	497.234	496.105
0.05	497.0231	496.201	496.009
0.09	495.057	493.161	491.566
0.1	495.002	492.837	491.035
0.5	494.883	492.053	488.465
1.0	493.965	491.954	473.889
2.5	493.831	491.021	465.902
3.00	492.941	488.291	422.218
4.00	492.523	487.433	412.746

Table (2) Temperature distribution of orthotropic bar after short time with T_0=500K, T_f=100K, at three different positions($k_{xx}=2k_{zz}$)

Time (sec)	Interior position (K)	Intermediate position (K)	Upper-surface position (K)
0.00	500	499.97	499.95
0.05	499.58	496.05	495.00
0.01	497.466	495.23	493.105
0.05	497.023	491.201	490.609
0.09	495.151	489.654	487.566
0.1	495.503	489.087	483.035
0.5	494.086	488.603	474.545
1.0	493.644	464.954	442.089
2.5	492.183	421.021	405.92
3.00	490.494	398.291	332.28
4.00	490.005	327.433	302.176

The clear jump in temperature distribution starts after a short time with respect to first conditions of equally directional thermal conductivity, the this jumps correlated with the positive increment in the k_{xx}, and the assumed changes is given for simulating the influence of such anisotropy in thermal properties in the transient distribution of temperatures, other parameter of transient behavior may be extensively studied in future work. When the thermal anisotropy increased in significant changes the wide and fast changes can be noticed clearly, as shown in **Table (3)**, for the k_{xx}=5k_{zz}, and the inverse relation is concluded very well for the new ratio (k_{xx}=0.5k_{zz}), in **Table (4)**, thus these changes introduces the thermal anisotropic degree across the laminate thickness.

Table (3) Temperature distribution of orthotropic bar after short time with T_0=500K, T_f=100K, at three different positions(k_{xx}=5k_{zz})

Time (sec)	Interior position (K)	Intermediate position (K)	Upper-surface position (K)
0.00	500.0	495.97	495.95
0.05	495.85	495.051	494.00
0.01	493.606	493.401	492.512
0.05	491.321	491.190	489.309
0.09	488.567	488.1	485.016
0.1	486.002	485.705	483.315
0.5	484.893	482.903	480.655
1.0	483.511	481.974	476.901
2.5	482.381	480.218	475.211
3.00	482.001	479.961	466.198
4.00	481.823	476.013	465.006

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Table (4) Temperature distribution of orthotropic bar after short time with T_0=500K, T_f=100K, at three different positions(k_{xx}=0.5k_{zz})

Time (sec)	Interior position (K)	Intermediate position (K)	Upper-surface position (K)
0.00	500	499.97	499.95
0.005	499.88	498.75	498.00
0.01	499.606	498.123	498.05
0.05	498.883	497.801	496.309
0.09	497.561	497.004	496.029
0.1	497.003	496.01	496.395
0.5	496.896	495.993	494.458
1.0	495.040	494.154	493.09
2.5	493.803	493.199	491.92
3.00	492.974	490.991	489.899
4.00	492.005	488.023	489.107

5. Conclusions

The present work has shown the simulation tool under estimate the orthotropic thermal conductivity influence on the transient temperature distribution of composite orthotropic bar subjected to a common case of quenching at a sequence fabrication process step. A finite element formulation and solution is presented. A temperature distribution is evaluated under two different cases with and without changes in directional material thermal conductivity. The results show the clear and significant role of jumps in temperatures distribution due anisotropic material properties.

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