

# Fast and Multicarrier Frequency Hopping Signals over Frequency Selective Rayleigh Fading Channel

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## Abstract

*In this paper, the performance of frequency hopping spread spectrum systems employing noncoherent reception and transmission diversity is analyzed for frequency selective Rayleigh fading (FSRF) channel.*

*Two different types of frequency diversity systems, fast frequency hopping (FFH) system and a multicarrier frequency hopping (MCFH) system are investigated. In order to combine received signal from transmit diversity channel, the optimum diversity-combining rule based on the maximum-likelihood criterion is described.*

*MCFH systems are found to outperform FFH systems when the channel delay spread ratio is severe at the same diversity order at low error rate. The optimum normalized frequency deviation for MCFH systems is found to increase with delay-spread ratio.*

## الخلاصة

في هذه المقالة تم تحليل أداء منظومات الطيف المنتشر ذو القفز الترددي باستخدام الاستلام غير المتزامن والتباعد الارسالي في قناة (Rayleigh) ذات الخفوت الترددي الاختياري. تم دراسة نوعين مختلفين من أنظمة التباعد الترددي، القفز الترددي السريع (FFH) والقفز الترددي ذو الحامل التعددي (MCFH). لغرض تجميع الإشارة المستلمة من قناة التباعد الارسالي، تم استخدام قانون التجميع التباعدي المبني على مقياس الاحتمالية العظمى. وجد أن منظومة (MCFH) أكثر فاعلية من منظومة (FFH) في حالة كون نسبة قناة التأخير الانتشاري عالية عند الرتبة التباعدية نفسها. وجد أن الانحراف الترددي الأمثل لمنظومة (MCFH) يتزايد مع نسبة التأخير الانتشاري.

## 1. Introduction

Spread spectrum systems have been found to be applicable in combating multipath fading. Both frequency hopping (FH) and direct sequence (DS) schemes have been proposed for such applications <sup>[1]</sup>.

Frequency diversities presented in this work are Fast frequency hopping and multicarrier frequency hopping. They are better than the time diversity because the algorithms of coding or decoding used are so complex, difficult to implement, the signals arrive to the receiver with delay time, and its error rate in HF channel is worse than that with frequency diversity.

Systems operating with BFSK and noncoherent demodulation are examined under very slow fading. These analyses demonstrated the frequency hopping benefits in selective fading. Miller, ... et. al. <sup>[2]</sup> showed a self-normalizing nonlinear combining receiver to achieve a diversity gain without knowledge of signal or jamming levels. Zimer <sup>[3]</sup> illustrated the types of jammers and showed the effectiveness of a repeater jammer, which depends on the hopping rate and the distance between the transmitter, receiver and jammer. Hassan, Hershey, and Schroeder <sup>[4]</sup> evaluated a countermeasure technique to a follower tone jammer for slow frequency hopped spread spectrum communications with MFSK modulation.

## 2. HF Channel Model and Noncoherent Detection

The Gaussian-Scatter Model (more commonly known as Watterson Model) was proposed and experimentally confirmed by Watterson et. al. <sup>[5,6]</sup>. This model was, and still up to date, very widely used in HF channel simulation work.

In FSRF channel, the multipath time delay spread  $T_m$  is assumed to be less than the bit duration  $T_b$  ( $0 < T_m < T_b$ ) <sup>[6]</sup>. A parameter that is commonly used to specify the degree of frequency selectivity is the effective delay spread ratio  $\mu$ :

$$\mu = \frac{T_m}{T_b} \dots\dots\dots (1)$$

Synchronous carrier recovery is a difficult task in a fading multipath environment. FSK with noncoherent detection produces lower error rates than the rectangular pulse DPSK (differentially phase-shift-keying) for channels, which are highly selective <sup>[1]</sup>.

Detection of noncoherent BFSK is accomplished by passing the received signal through the square-law envelope detector <sup>[7,8]</sup>.

### 3. Fast and Multicarrier Frequency Hopping Signals over FSRF Channel

#### 3-1 Transmitter Model of FFH and MCFH Signals

The FSK modulator selects one of two-baseband frequency  $f_1$  and  $f_0$  based on the bit rate ( $R_b$ ) and ( $f_1-f_0=\Delta f$  Hz). The  $L$  hop/bit FFH/BFSK system requires each data bit to be broken into  $L$  independent transmission of duration  $T_b/L$  seconds, thus, the hop rate  $R_h = L R_b$ , and  $T_h = T_b / L$

In MCFH systems,  $R_h = R_b$ , hence, one symbol is transmitted during one hop duration and adjacent symbols in time are transmitted in far distant frequency slots such that multipath interference from the previous symbol is negligible.

The synthesizer selects a new hopping frequency every  $T_h$  second from  $N_h$  possible discrete frequencies spaced  $\Delta f$  Hz apart to form hopping cells. Transmitter block diagrams of FFH and MCFH are depicted in **Fig.(1)**, and **Fig.(2)**,<sup>[9]</sup>.

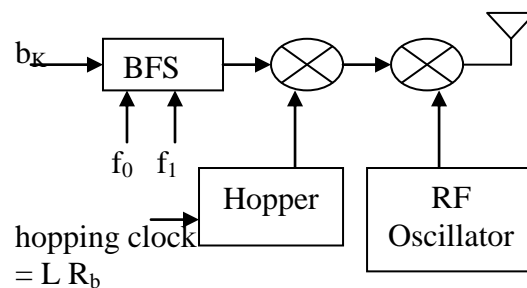


Figure (1) Transmitter block diagram of FFH system

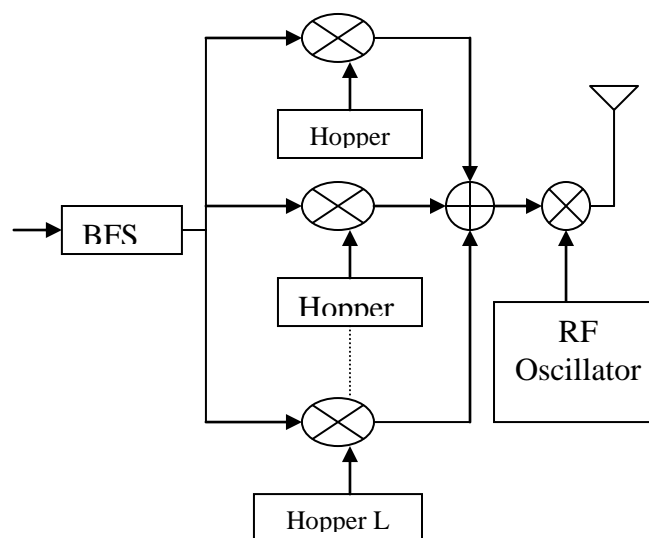


Figure (2) Transmitter block diagram of MCFH system

The optimum  $h$  for noncoherent detection of FFH/BFSK signals is integer values to satisfy the orthogonal condition, if multiple-access interference is not considered<sup>[9]</sup>. Hence:

$$h = 2T_h f_d \dots\dots\dots (2)$$

The frequency deviation of MCFH/BFSK ( $f_d$ ) is a function of the normalized frequency deviation ( $h$ ), which is given by<sup>[9]</sup>:

$$h = 2T_b f_d \dots\dots\dots (3)$$

In fading channels, the optimum  $h$  varies with delay spread and channel variation. Thus, in MCFH the value of  $h$  is not necessary to be integer value as in the FFH system. The optimum  $h$  is found to increase with the delay spread.

**3-2 Receiver Model of FFH and MCFH Signals**

Receiver block diagrams of FFH & MCFH are shown in **Fig.(3)** and **Fig.(4)** respectively. After down converting and dehopping, the complex baseband equivalent of the received signal over the first bit duration may be expressed as<sup>[9]</sup>:

$$r(t) = \sum_{\ell=0}^{L-1} \int_0^{T_m} \sqrt{2S} \alpha_{\ell}(t;\tau) e^{j(2\pi b_0 f_d t + \beta_{\ell}(t;\tau))} d\tau \cdot p_{T_h}(t - \ell T_h) + n_{\ell}(t), t \in [0, T_b) \dots\dots\dots (4)$$

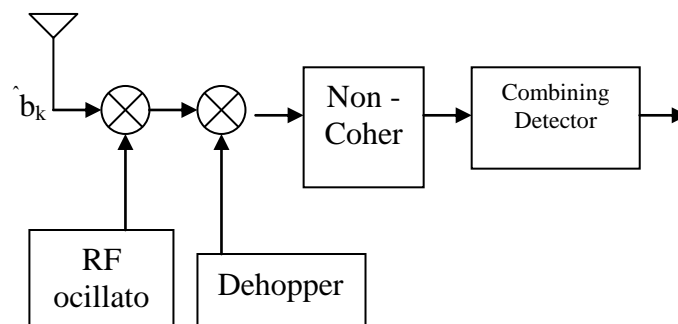
where:

$\alpha_{\ell}(t;\tau)$ 's are independent and identically distributed (i.i.d.) Rayleigh random processes,  $p_{T_h} = 1$  for  $t \in (0, T_h)$ .

$\beta_{\ell}(t;\tau) = \theta_{\ell}(t;\tau) + \phi_{\ell,0}$ ,  $\theta_{\ell}(t;\tau)$ 's are i.i.d. uniform random processes over  $(0, 2\pi)$ .

$n_{\ell}(t)$ : represents a complex-value background noise and modeled as a low pass equivalent additive white Gaussian noise (AWGN) process with power spectral density  $N_{\ell}$ .

$b_0$ : is either +1 or -1 with equal probability, without loss of generality,  $b_0$  is +1.



**Figure (3) Receiver block diagram of FFH system**

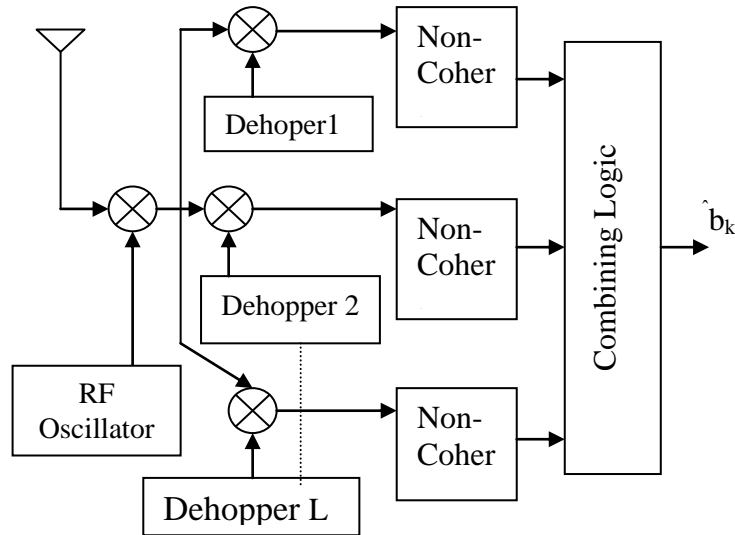


Figure (4) Receiver block diagram of MCFH system

A noncoherent detector demodulates each diversity reception, and a synthesizer assumed to be in perfect synchronism with the transmitter [2,9].

As shown in Fig.(5), a noncoherent detector consists of two branches of correlator followed by an envelope detector. The two-correlator outputs of the  $\ell$ th diversity reception are respectively denoted by  $Z_{\ell,1}$  and  $Z_{\ell,-1}$ , and may be expressed as:

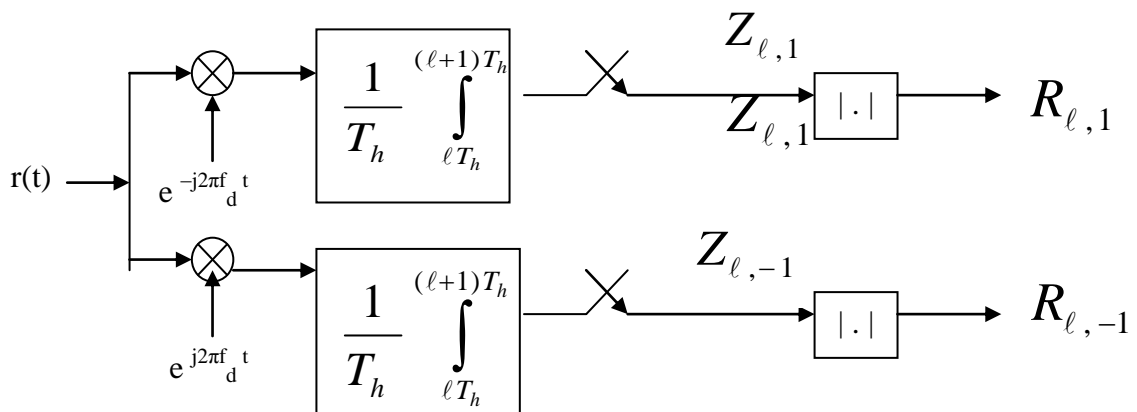


Figure (5) Non-coherent detector for the  $\ell$ th diversity reception

$$Z_{\ell,1} = \frac{1}{T_h} \int_0^{T_h} \int_0^{T_h} \sqrt{2S} \alpha_\ell(t;\tau) e^{j\beta_\ell(t;\tau)} dt d\tau + \frac{1}{T_h} \int_0^{T_h} n_\ell(t) e^{-j2\pi f_d t} dt \dots\dots\dots (5)$$

$$Z_{\ell,-1} = \frac{1}{T_h} \int_0^{T_m} \int_0^{T_h} \sqrt{2S} \alpha_\ell(t;\tau) e^{j\beta_\ell(t;\tau)} e^{j2\pi\Delta f t} dt d\tau + \frac{1}{T_h} \int_0^{T_h} n_\ell(t) e^{j2\pi f_c t} dt \dots\dots\dots (6)$$

Since all the terms in Eq. (5) and Eq. (6) are zero-mean complex Gaussian random variables,  $Z_{\ell,1}$  and  $Z_{\ell,-1}$  are also zero-mean complex Gaussian random variables whose variances and correlation coefficient are given by [9]:

$$\sigma_{\ell,1}^2 = \frac{2S}{T_h} \int_0^{T_m} \int_0^{T_h-\tau} R_c(t;\tau) \left(1 - \frac{t+\tau}{T_h}\right) dt d\tau + \frac{N_\ell}{T_h} \dots\dots\dots (7)$$

$$\sigma_{\ell,-1}^2 = \frac{2S}{T_h} \int_0^{T_m} \int_0^{T_h-\tau} R_c(t;\tau) \cos(2\pi\Delta f t) \left(1 - \frac{t+\tau}{T_h}\right) dt d\tau + \frac{N_\ell}{T_h} \dots\dots\dots (8)$$

As shown in Fig.3, Fig.4, and Fig.5, decisions are made based on L pairs of noncoherent detector outputs,  $R_{\ell,1} = |Z_{\ell,1}|$  and  $R_{\ell,-1} = |Z_{\ell,-1}|$  for  $\ell \in \{0,1,\dots, L-1\}$ . They should be combined in some way to form decision statistics for the receiver.

### 3-3 Optimum Diversity Combining Rule and Probability of Error of FFH and MCFH

The optimum diversity-combining rule [9] is based on the maximum-likelihood criterion, and the conditional joint probability density function (pdf) of noncoherent detector outputs,  $R_{\ell,1}$  and  $R_{\ell,-1}$  should be found. After straightforward algebraic manipulation and extraction of common terms in the log-likelihood, the optimum decision rule is obtained as [9]:

$$\sum_{\ell=0}^{L-1} \frac{\sigma_{\ell,1}^2 - \sigma_{\ell,-1}^2}{\sigma_{\ell,1}^2 \sigma_{\ell,-1}^2 (1 - |\rho_\ell|^2)} \cdot (R_{\ell,1}^2 - R_{\ell,-1}^2) \geq 0 \quad \mathbf{b}_0 = +1$$

$$\sum_{\ell=0}^{L-1} \frac{\sigma_{\ell,1}^2 - \sigma_{\ell,-1}^2}{\sigma_{\ell,1}^2 \sigma_{\ell,-1}^2 (1 - |\rho_\ell|^2)} \cdot (R_{\ell,1}^2 - R_{\ell,-1}^2) < 0 \quad \mathbf{b}_0 = -1$$

..... (9)

This equation indicates that the decision variable associated with  $\hat{\mathbf{b}}_0 = +1$  is constructed as the weighted sum of squares of  $R_{\ell,1}$  for all  $\ell$ , and the decision variable associated with  $\hat{\mathbf{b}}_0 = -1$  is constructed in a similar manner. These two variable values are compared to estimate the transmitted bit. In Eq. (9), it can be seen that the  $\ell$ th weighting factor depends on the variance and the correlation coefficient of correlator outputs for the  $\ell$ th diversity reception. After simplifications, the probability of error may be expressed as [9]:

$$P_e = \frac{1}{(1+\gamma)^{2L-1}} \sum_{\ell=0}^{2L-1} \binom{2L-1}{\ell} \cdot \gamma^\ell \frac{1}{2\pi j} \int_{\Gamma} \frac{1}{u^{L-\ell}(1-u)} du \dots\dots\dots (10)$$

where:  $\Gamma$  is a circular contour of radius less than unity that encloses the origin, and  $\gamma$  is defined as:

$$\gamma = \frac{\sigma_1^2 - \sigma_{-1}^2 + \sqrt{(\sigma_1^2 + \sigma_{-1}^2)^2 - 4|\rho|^2 \sigma_1^2 \sigma_{-1}^2}}{\sigma_{-1}^2 - \sigma_1^2 + \sqrt{(\sigma_1^2 + \sigma_{-1}^2)^2 - 4|\rho|^2 \sigma_1^2 \sigma_{-1}^2}} \dots\dots\dots (11)$$

Thus, the probability of error expression in Eq. (10) may be simplified to <sup>[9]</sup>:

$$P_e = \sum_{\ell=0}^{L-1} \binom{2L-1}{\ell} \frac{\gamma^\ell}{(1+\gamma)^{2L-1}} \dots\dots\dots (12)$$

#### 4. Effects of Partial Band Jammer and Repeater Jammer in FFH and MCFH over FSRF

In partial band jammer, the fraction of the communication bandwidth that is jammed is denoted by  $\eta$ , then the bit error probability is given by <sup>[2,6]</sup>:

$$P_b = \frac{\eta}{2} e^{-\eta(E_b/2N_J)} \dots\dots\dots (13)$$

The repeater jamming waveform is modeled as a wide-sense-stationary Gaussian noise, its symbol error probability is defined as <sup>[4]</sup>:

$$P_J = \frac{N_t}{2N_t + N_J} e^{(-R_s/(2N_t+N_J))} \dots\dots\dots (14)$$

where:

$N_t$ : is the average thermal noise power,

$N_{aj}$ : is the average jamming power, and

$R_s$ : is the average power of the desired signal at the receiver.

#### 5. BER Performance of FFH and MCFH Systems

##### 5-1 Comparisons between FFH and MCFH Systems without Jammer

The Bit error rate performance of FFH over HF channel having different values of effective delay spread ratio ( $\mu=0.1,0.15,0.2$ ) with different diversity order ( $L$ ) and normalized.

Doppler spreads  $B_d T_b = 0.01$  [4] are illustrated in **Figs. (6a)**, and **(6b)**. The performance of FFH system is significantly degraded in frequency selective fading environments with delay spread, because the probability of error from Eq.(12) is a monotonically decreasing function of  $\gamma$ , which can be treated as a performance measure.

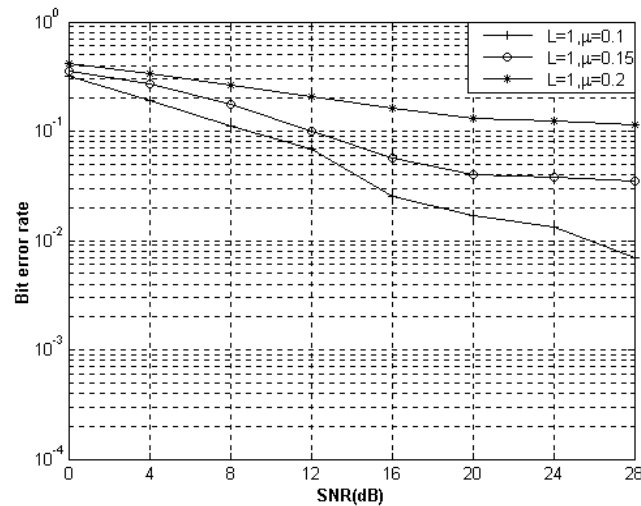


Figure (6-a) Performance of FFH system for L=1

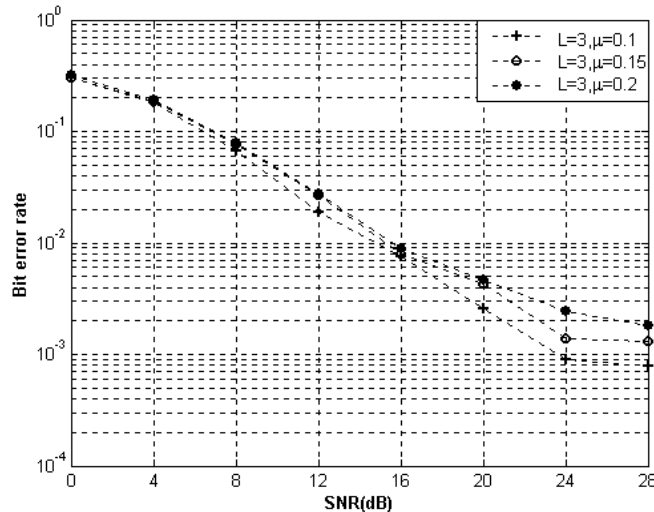


Figure (6-b) Performance of FFH system for L=3

**Figures (7a)**, and **(7b)** give the effects of diversity order and the effective delay spread ratio on the bit error rate (BER) performance of MCFH system over FSRF channel at diversity orders  $L=1$ , and  $L=3$  respectively. The normalized Doppler spread  $B_d T_b = 0.01$  [4] and the effective delay spread ratio  $\mu=(0.1,0.15,0.2)$  at normalized frequency deviation  $h=1.6$ .



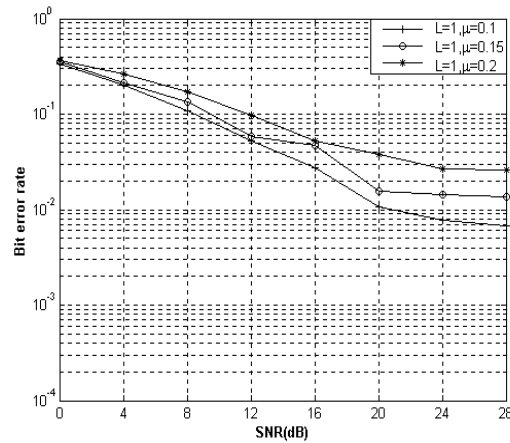


Figure (7-a) Performance of MCFH system for L=1

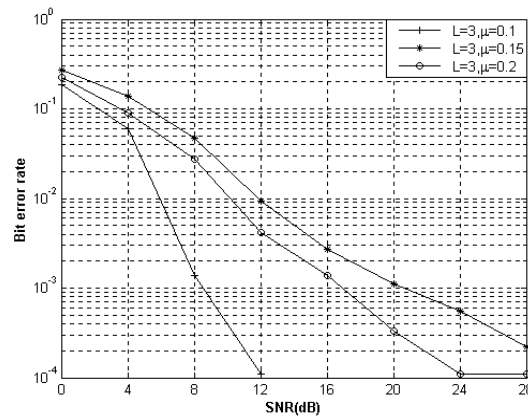


Figure (7-b) Performance of MCFH system for L=3

Figures (8a), and (8b), give the effects of changing  $h$  from 1.4 to 1.6 on the BER performance of MCFH system at diversity order  $L=1$ , and  $L=3$  respectively. The normalized Doppler spread  $B_d T_b = 0.01$  [4], and the effective delay spread ratio  $\mu = (0.1, 0.15, 0.2)$ .

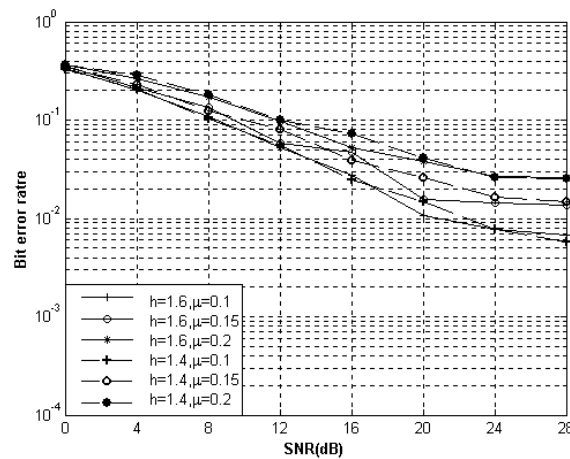


Figure (8-a) Performance of MCFH system for  $h=(1.6, 1.4)$  and  $L=1$

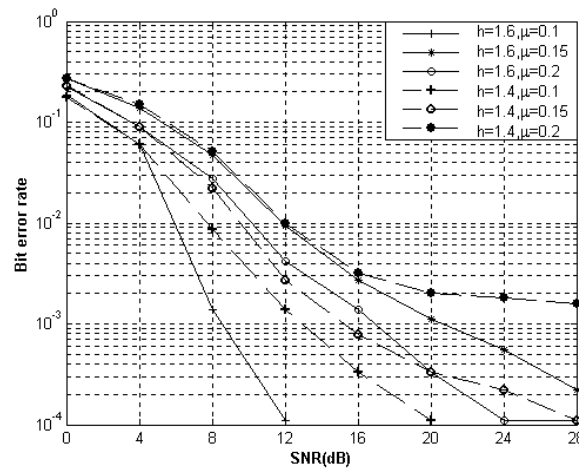


Figure (8-b) Performance of MCFH system for  $h=(1.6,1.4)$ , and  $L=3$

### 5-2 Comparisons between FFH and MCFH Systems with Partial Band Jammer and Follower Jammer

Figure (9) and Fig.(10) give the effect of partial band jamming on the performance of FFH and MCFH systems over FSRF channel for  $L=3$ . The normalized Doppler frequency  $B_d T_b=0.01$ , and two values of effective delay spread ratio  $\mu=(0.1,0.15)$ .

Figure (11) and Fig.(12) give the effect of follower jammer on the performance of FFH and MCFH systems over FSRF channel at  $L=2$  and 3 diversity orders. The normalized Doppler spread  $B_d T_b=0.01$  and effective delay spread ratio  $\mu=0.1$ .

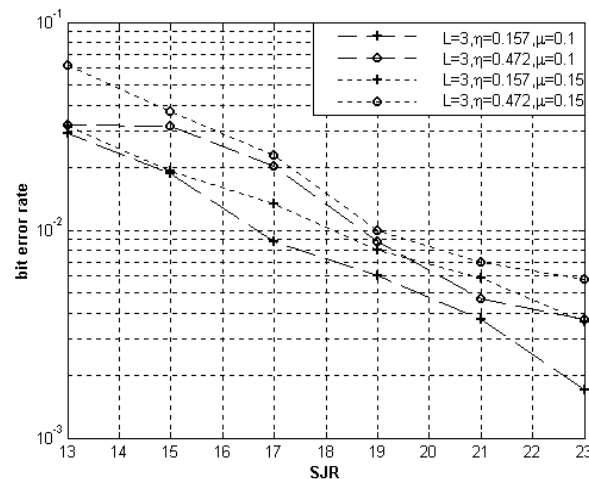


Figure (9) BER performance of FFH system over FSRF Channel in partial band jammer at  $L=3$

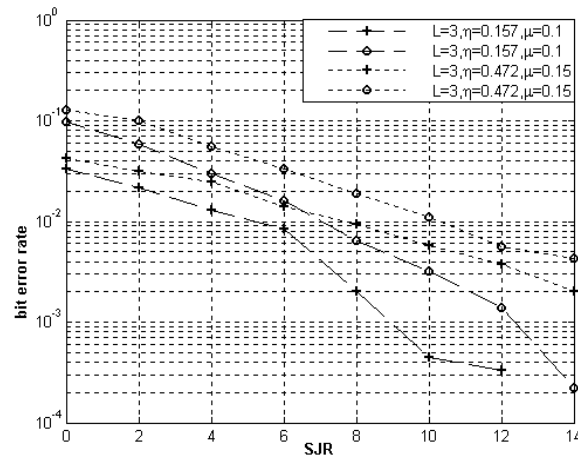


Figure (10) BER performance of MCFH system over FSRF channel in partial band jammer at L=3

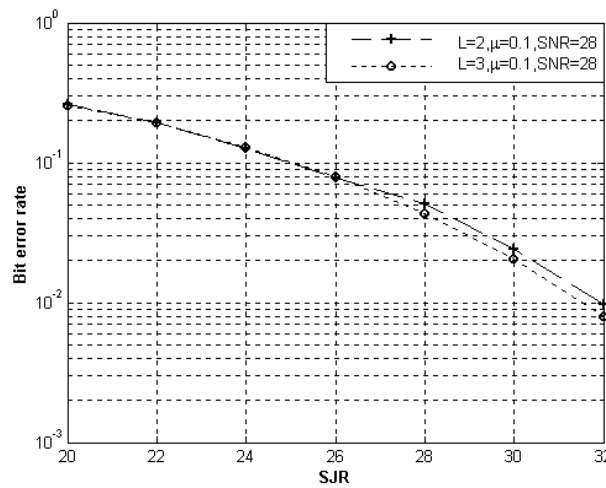


Figure (11) BER performance of FFH system over FSRF channel in follower jammer at L=2 and L=3

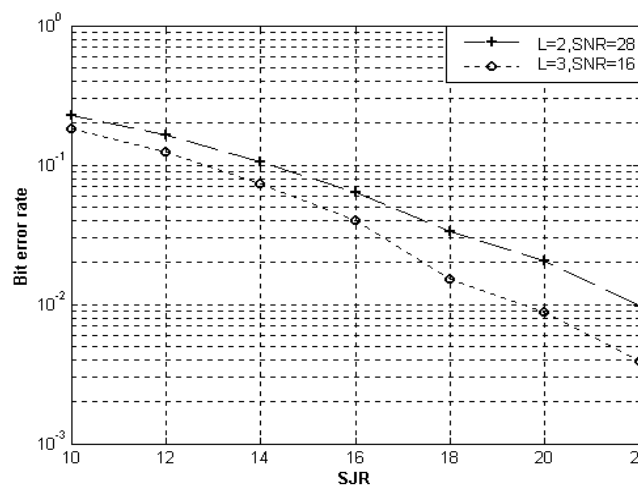


Figure (12) BER performance of MCFH system over FSRF in follower jammer at L=2 and L=3

## 6. Conclusions

Fast frequency hopping over FSRF channel has been analyzed to demonstrate the effect of changing diversity orders in improving the BER performance of FFH system. In this system, the simulation shows that the effective delay spread ratio increases with diversity order. Due to an increase in the effective delay spread ratio, the overall performance enhancement has been reduced with an increase of diversity order.

Multicarrier frequency hopping over FSRF channel has been analyzed to demonstrate the effects of changing diversity orders and normalized frequency deviation in improving the BER performance of MCFH system. It is found that an increase in diversity order improves the BER performance in large extent because the effective delay spread ratio does not change with diversity order. It is also shown that the optimum value of normalized frequency deviation increased as the delay spread increases. Hence, the diversity gain of MCFH system is greater than that of FFH system so that MCFH system is superior to the FFH system in FSRF channel.

It is found that for the same diversity order and the same partial jamming bandwidth, but with less value of SJR, the MCFH system is better than the FFH system around 9dB.

Follower jammer has been analyzed for two systems FFH system and MCFH system in worst case, when the jammer is jammed half of the bit duration. It is found that the MCFH system is better than the FFH system in the same conditions of diversity order and effective delay spread, but in values of SJR less than that of FFH system around 10 dB.

## 7. References

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