Journal of Engineering and Development, Vol. 10, No. 3, September (2006) **ISSN 1813-7822**

Nonlinear Elastic Analysis and Post-Buckling of Steel Frames with Non-Prismatic Gusseted Plate Members

Prof. Dr. Sabih Z. Al-Sarraf
Building & Construction Engineering Department
University of Technology, Baghdad, Iraq

Asst. Prof. Dr. Jasim Al-Khafaji
Civil Engineering Dept., College of Engineering
Al-Mustansiriya University, Baghdad, Iraq

Dr. Ali Sabah Ahmad
Civil Engineering Dept., College of Engineering
Al-Mustansiriya University, Baghdad, Iraq

Abstract

In this study, a theoretical analysis is presented for estimating the in-plane large displacement elastic stability behavior of steel frames having prismatic and non-prismatic members (Tapered and non-linearly tapered) with end gusseted plates subjected to increasing static loads.

The analysis adopts the beam-column approach and models the structural members as beam-column elements. The formulation of the beam-column element is based on Eulerian approach allowing for the influence of the axial force on bending stiffness. Also, changes in member chord length due to axial deformation and flexural bowing are taken into account.

The effect of gusseted plate is taken and the modified stability and bowing functions are derived for gusset plate with prismatic and non-prismatic members (Tapered and Non-linearly tapered).

The post-buckling analysis is studied, and the incremental load control with different load increment strategies and the modified Newton-Raphson method with different iterative strategies are used to obtain the complete load-displacement curve.

As a result, the beam column approach can be used in the analysis of plane frames with and without gussets and with any varying section. The ultimate load capacity can be increased with gusset-plate members.

الخـلاصـــــة

تتناول هذه الدراسة التحليل النظري للسلوكية المرنة للهياكل الحديدية المستوية مع الأخذ بنظر الاعتبار الإزاحات الكبيرة الخاصة بها. والتي تحتوي على أعضاء موشورية و لا موشورية(خطية و لا خطية التغير) والتي تحتوي نهاياتها على صفائح التقوية والمعرضة لأحمال ساكنة مسلطة على المفاصل.

تبنت هذه الدراسة طريقة العمود-عتبة. إن اشتقاق عنصر العمود-عتبة قد تم بالاعتماد على طريقة اويلر. كما تم الأخذ بنظر الاعتبار تأثير القوة المحورية على صلابة العزم كذلك اخذ بنظر الاعتبار تأثير التغيرات في طول الوتر نتيجة الانفعال المحوري وتقوس الانحناء.

في هذه الدراسة تم اخذ تأثير صفائح التقوية للأعضاء الموشورية واللاموشورية (خطية و لا خطية التغير) وتم اشتقاق دوال الاستقرارية والتقوس المعدلة لها لغرض الحصول على سلوك الهياكل في مرحلة ما بعد الانبعاج فقد تم استخدام طريقة الحمل المتزايد مع استراتيجيات مختلفة لزيادة الأحمال وكذلك تم استخدام طريقة نيوتن-رافسون المحورة مع استراتيجيات مختلفة للحل المتكرر.

ونتيجة لهذه الدراسة تم الحصول على بعض الاستنتاجات المهمة ومنها إمكانية استخدام طريقه العمود-العتبة في تحليل الهياكل لعدة أنواع من المقاطع والأعضاء الموشورية واللاموشورية (خطية ولا خطية التغير) مع اخذ تأثير صفائح التقوية، كما وجد بان سعة التحمل القصوى يمكن أن تزداد مع صفائح التقوية للأعضاء.

1. Introduction

It is usually convenient to work to frame centerlines, so that the ends of members dealt within a structural analysis actually lie with the boundaries of the joints. Although the joints cannot be absolutely rigid, it is more accurate to assume complete rigidity than to assume an effective rigidity equal to that of the rest of the member. Complete flexural rigidity over given lengths at the ends of members may be allowed for in the calculations by introducing modified values of the various stability functions.

The present study allows to analyze structures consisting of members (prismatic and non-prismatic members of varying sections) with gusset plates by treating it as a single member, then the results of analysis are compared with exact solution by considering each member to consist of three elements, one of inner part with classical properties of the member and two terminal parts with infinite rigidity.

The effect of gusseted plate with prismatic and non-prismatic member is taken and the modified stability and bowing functions are derived in this study, and it is presented in this paper.

2. Modified Force-Displacement Relationships for Members with End Gusset Plates (Non-Prismatic Member)

The member ($\mathbf{A'B'}$) of length (L) shown in **Fig.(1)** is completely rigid over the length ends ($\mathbf{A'A = g1 \, and \, BB' = g2}$). The central length AB=L_o has uniform or non-uniform flexural rigidity (E. I).

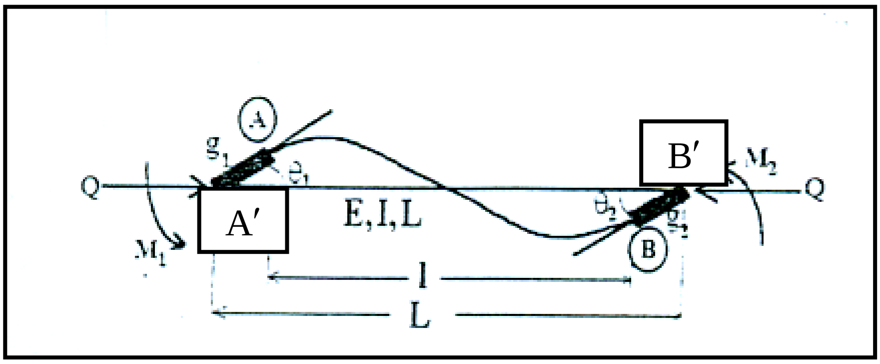

Figure (1) Relative forces in local coordinates for a member with gusset plates

In this study, the non-prismatic member force-deformation relations, obtained from an application of the conventional beam- column theory by expressing the terminal bending moments (M1) and (M2) by rotations of θ_A **and** θ_B **at** A' **and** B' respectively [1]:

$$M1 = \frac{EI\,1}{L_O}(\bar{\gamma}_1 \theta_A + \bar{\gamma}_2 \theta_B) \quad \text{......(1)}$$

$$M2 = \frac{EI\,1}{L_O}(\bar{\gamma}_2 \theta_A + \bar{\gamma}_3 \theta_B) \quad \text{......(2)}$$

$$Q = EA_O(\frac{\bar{u}}{L_O} - \bar{C}b) \quad \text{......(3)}$$

in which ($\bar{\gamma}_1$ and $\bar{\gamma}_2$) are stability functions for non-prismatic members (tapered or non-linearly tapered) with end gusset plates expressed in terms of (S1,Sc and S2) as shown in the following, and A_o: is equivalent area of non-prismatic member.

$$\bar{\gamma}_1 = \frac{S2}{U^m} + \frac{2g1}{L_0}(1 + \frac{g1}{L_O})AG^* \quad \text{......(4)}$$

$$\bar{\gamma}_2 = \frac{Sc}{U^m} + (\frac{S1 + Sc}{U^m})(\frac{g1 + g2}{L_O}) + \frac{2g1.g2}{L_O^2}.AG^* \quad \text{......(5)}$$

$$\bar{\gamma}_3 = \frac{S1}{U^m} + \frac{2g1}{L_O}(1 + \frac{g1}{L_O}).AG^* \quad \text{......(6)}$$

where:

$$AG^* = \frac{(S1 + Sc)}{U^m} - \frac{\pi^2}{2}q \quad \text{......(7)}$$

while U is the depth factor (U=d1/d2) and m is the shape factor.

where S1, Sc and S2 are modified stability function for non-prismatic member, which are shown as following:

3. Modified Stability Function-Approximate Formula (Tapered Member)

AL-Sarraf [2] proposed an approximate formula for the modified stability functions. These functions are:

$$S_1 = U^{\phi m/4}.C_1 \quad \text{.......} \quad (8)$$

$$\overline{SC} = U^{(1+\phi)m/4}.C_2 \quad \text{.......} \quad (9)$$

$$S_2 = U^{(1+\phi/2).m/2}C_1 \quad \text{.......} \quad (10)$$

where: C_1 and C_2 are stability functions of prismatic members having load parameter (q):

$$q = \frac{q_1}{U^{\phi m/2}} \quad , \quad q_1 = \frac{QL^2}{\pi^2 EI_1} \quad \text{.......} \quad (11)$$

The value of (Φ) depends on the shape factor as follows:

For **m = 4 Φ = 1 .... (12)**

For **m ≤ 3 Φ = 1.04 + 0.08 (3-m) .... (13)**

4. Estimation of Stability Functions using Approximate Method (Non-Linearly Tapered Member)

To facilitate the estimation of the elastic critical load of structures with non-prismatic members, it is necessary to tabulate the stability functions. These functions are dependent on three parameters q_1, U, and m [1] .

It is noticed that the modified stability functions for members with non-linear variation of sections can be estimated using the relations [3]:

The value of ϕ depends on the shape factor (m):

A- For members having parabolic distribution of cross section

For **m=4 ϕ=0.825 .... (14)**

For **m ≤ 3 ϕ=0.88-0.284 (3-m) .... (15)**

B- For members having cubic distribution of cross section

For **m=4** **ϕ=0.8429** .... **(16)**

For **$m \leq 3$** **ϕ=0.8365-0.036 (m-3)** .... **(17)**

Figure (2) shows the modified axial force-deformation interaction for a member with gusset plates. The derivation of the modified axial force-deformation relations for members with gusset plates gives:

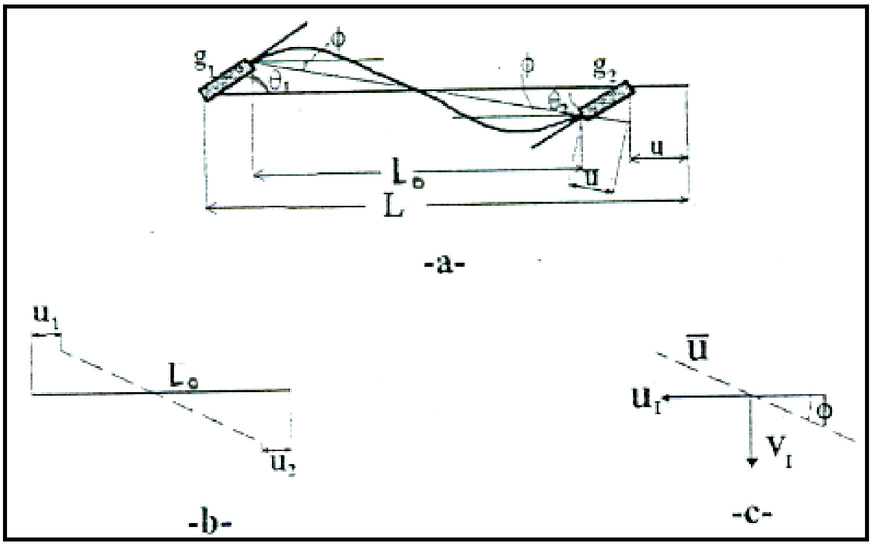

Figure (2) Modified axial force deformation for members with gusset plates

$$L_O = L - g1.\cos\theta_1 - g2.\cos\theta_2 \quad (18)$$

$$\phi = \frac{g1.\theta_1 + g2.\theta_2}{L_O} \quad (19)$$

$$\Delta u = u1 - u2 \quad (20)$$

$$\begin{aligned}\Delta u &= (g1 - g1.\cos\theta_1) - (g2 - g2.\cos\theta_2) \\ &= g1.(1-\cos\theta_1) - g2.(1-\cos\theta_2)\end{aligned} \quad (21)$$

$$V_I = g1.\theta_1 + g2.\theta_2 \quad (22)$$

$$u_I = u - \Delta u = u - \left[g1.(1-\cos\theta_1) - g2.(1-\cos\theta_2)\right] \quad (23)$$

$$\bar{u} = u_I.\cos\phi - V_I.\sin\phi \quad (24)$$

$$q = \frac{Q}{QE} = \frac{Q.L_O^2}{\pi^2.E.I1} \quad (25)$$

Equation (25) can be rewritten as:

$$\mathbf{q} = \frac{\overline{\lambda}^2}{\pi^2}\left(\frac{\overline{u}}{L_O} - \overline{C}b\right) \quad \text{.......... (26)}$$

while $$\overline{\lambda} = \frac{L_O}{\sqrt{I1/A_O}} = \frac{L_O}{r} \quad \text{.......... (27)}$$

and L_O =L-g1-g2

For non-prismatic member with gusset plates, assuming $\theta_A = \theta_1$ **and** $\theta_B = \theta_2$, then the derivation of the length correction factor due to bowing actions is shown below:

$$\overline{C}b = (\overline{\beta}_1.\theta_1^2 + 2\overline{\beta}_2.\theta_1.\theta_2 + \overline{\beta}_3.\theta_2^2) \quad \text{.......... (28)}$$

where: $\overline{u}$, $\overline{C}b$: are the modified axial deformation in a member with gusset plate due to axial force, and the length correction factor due to bowing action which have been derived with the modified stability functions, respectively.

While $\overline{\beta}_i$ is a modified bowing function for non-prismatic member with gusset plate, which is derived in the next paragraph in this paper.

A_0 : is equivalent area depending on the shape factor (n) as follows:

I. For n=1, U > 1, $$A_0 = \frac{(U-1)}{\ln(U)} * A_2 \quad \text{.......... (29)}$$

II. For $n \neq 1$, U > 1, $$A_0 = \frac{(U-1)(1-n)}{U^{(1-n)} - 1} * A_2 \quad \text{.......... (30)}$$

III. For U= 1, $A_0 = A_1$ or $A_0 = A_2$ .... **(31)**

$$n = \frac{\log \frac{A_1}{A_2}}{\log U} \quad \text{.......... (32)}$$

where:

A1 and A2: are the areas of larger and smaller depth for non-prismatic member respectively. The non-prismatic member is shown in **Fig.(3)**.

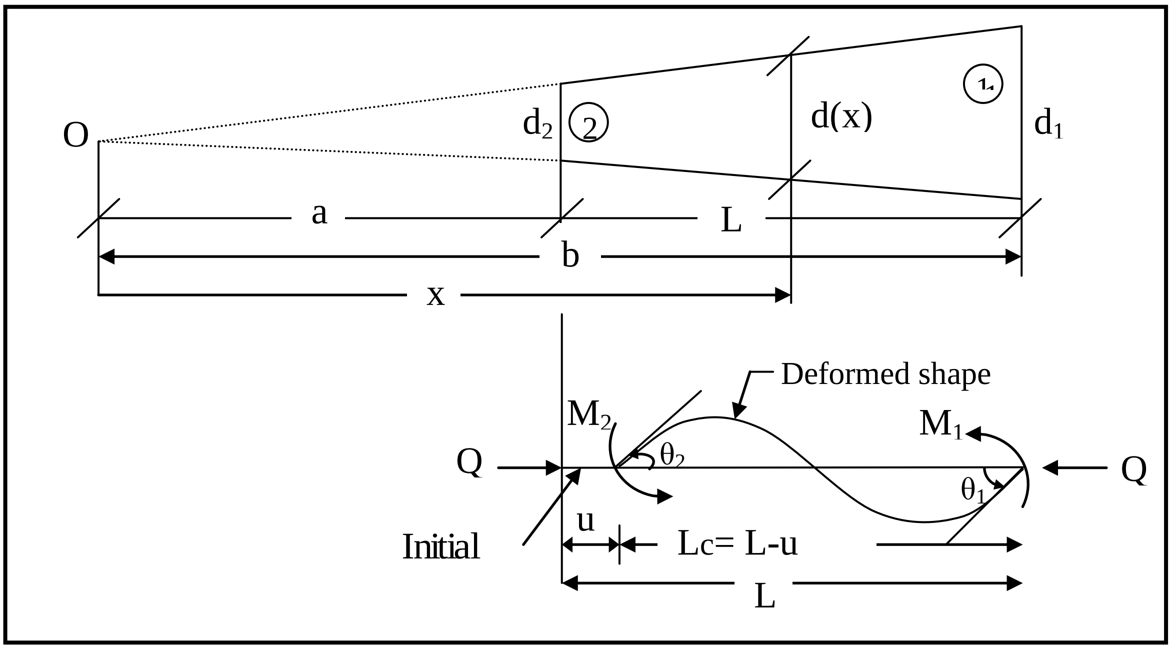

Figure (3) Non-prismatic beam-column element

5. Modified Bowing Functions for Non-Prismatic Member with End Gusset Plates

In this study, the derivation of modified bowing functions for a non-prismatic member which takes into account the effect of the gusset plates are shown below:

$$\bar{\beta}_i = \frac{-\bar{\gamma}'_i}{2\pi^2} \qquad i = 1,2,3 \quad \text{.......... (33)}$$

while $\bar{\gamma}'_i$ is the first derivative of the stability function with gusset plates.

$$\bar{\gamma}'_1 = U^{(2-\phi).m/4}.\bar{C}'_1 \quad \text{.......... (34)}$$

$$\bar{\gamma}_2 = U^{(1-\phi).m/4}.\bar{C}'_2 \quad \text{.......... (35)}$$

$$\bar{\gamma}_3 = U^{-\phi.m/4}.\bar{C}'_1 \quad \text{.......... (36)}$$

where:

ϕ: is a factor depending on the shape factor and the degree of variation of the section (i.e.tapered or non-linearly tapered).

$\bar{C}'_i$: is the first derivative of the modified stability function for prismatic member with gusset plates.

The modified bowing functions for a non-prismatic member with gusset plates become:

$$\overline{\beta}_1 = \frac{-U^{(2-\phi).m/4}}{2\pi^2}.\overline{C}_1' \quad \text{.......... (37)}$$

$$\overline{\beta}_2 = \frac{-U^{(1-\phi).m/4}}{2\pi^2}.\overline{C}_2' \quad \text{.......... (38)}$$

$$\overline{\beta}_3 = \frac{-U^{(-\phi.m/4)}}{2\pi^2}.\overline{C}_1' \quad \text{.......... (39)}$$

While the modified stability functions for a prismatic member with gusset plates are [4]:

$$\overline{C}_1 = C_1 + \frac{2g1}{L_O}(1+\frac{g1}{L_O}).Ag \quad \text{.......... (40)}$$

$$\overline{C}_2 = C_2 + (C_1 + C_2)(\frac{g1+g2}{L_O}) + \frac{2g1.g2}{L_O{}^2}.Ag \quad \text{.......... (41)}$$

where: C_1 and $C_2$are stability functions for a prismatic member without gusset plates.

The tangent stiffness matrix for non-prismatic member (tapered and non-linearly tapered) with gusseted plate is derived as below:

$$[t] = \frac{E.I1}{L_O}\begin{vmatrix} \overline{\gamma}_1 + \frac{\overline{G1}}{\pi^2\overline{\overline{H}}} & \overline{\gamma}_2 + \frac{\overline{G1}.\overline{G2}}{\pi^2\overline{\overline{H}}} & \frac{\overline{G1}}{\overline{\overline{H}}} \\ & \overline{\gamma}_3 + \frac{\overline{G2}^2}{\pi^2\overline{\overline{H}}} & \frac{\overline{G2}}{\overline{\overline{H}}} \\ \text{Symetric} & & \frac{\pi^2}{\overline{\overline{H}}} \end{vmatrix} \quad \text{.......... (42)}$$

where:

$$\overline{G}1 = -2\pi^2(\overline{\beta}_1.\theta_1 + \overline{\beta}_2.\theta_2) = \overline{\gamma}_1'.\theta_1 + \overline{\gamma}_2'.\theta_2 \quad \text{.......... (43)}$$

$$\overline{G}2 = -2\pi^2(\overline{\beta}_2.\theta_1 + \overline{\beta}_3.\theta_2) = \overline{\gamma}_2'.\theta_1 + \overline{\gamma}_3'.\theta_2 \quad \text{.......... (44)}$$

$$\overline{H} = \frac{\pi^2}{\lambda^2} + (\overline{\beta}_1'.\theta_1^2 + 2\overline{\beta}_2'.\theta_1.\theta_2 + \overline{\beta}_3'.\theta_2^2) \quad \text{.......... (45)}$$

$$L_O = L - g1 - g2 \quad \text{.......... (46)}$$

where ($\overline{C}_i'$) is the derivation of ($\overline{C}_i$) with respect to (q) and the derivation is shown below:

$$\overline{C}'_1 = C'_1 + (\frac{2g1}{L_O} + \frac{2g1^2}{L_O{}^2}).(C'_1 + C'_2 - \frac{\pi^2}{2}) \quad \text{.......... (47)}$$

$$\overline{C}'_2 = C'_2 + (C'_1 + C'_2).(\frac{g1+g2}{L_O}) + \frac{2g1.g2}{L_O}(C'_1 + C'_2 - \frac{\pi^2}{2}) \quad \text{.......... (48)}$$

$$\overline{C}''_1 = C''_1 + (\frac{2g1}{L_O} + \frac{2g1^2}{L_O{}^2}).(C''_1 + C''_2) \quad \text{.......... (49)}$$

$$\overline{C}''_2 = C''_1 + (C''_1 + C''_2).(\frac{g1+g2}{L_O}) + \frac{2g1.g2}{L_O}(C''_1 + C''_2) \quad \text{.......... (50)}$$

where: C'_i and C''_i are shown in reference [2].

6. Modified Tangent Stiffness Matrices for Non-Prismatic Member with Gusset Plates

Introducing the notation:

$$u_1 = \theta_1 \quad ,u_2 = \theta_2 \quad ,u_3 = \frac{\overline{u}}{L_O} \quad \text{and} \quad S1 = M1\ ,S2 = M2\ ,S3 = Q.L_O$$

Then making use of equation $t_{ij} = \frac{\partial S_i}{\partial \overline{u}_j} + \frac{\partial S_i}{\partial q}.\frac{\partial q}{\partial \overline{u}_j}$ .... (51)

As defined previously, the modified member tangent stiffness matrix including the effect of gusset plates would be:

$$[t] = \frac{E.I1}{L_O}\begin{vmatrix} \overline{\gamma}_1 + \frac{\overline{G1}}{\pi^2\overline{H}} & \overline{\gamma}_2 + \frac{\overline{G1}.\overline{G2}}{\pi^2\overline{H}} & \frac{\overline{G1}}{\overline{H}} \\ & \overline{\gamma}_3 + \frac{\overline{G2}^2}{\pi^2\overline{H}} & \frac{\overline{G2}}{\overline{H}} \\ \text{Symetric} & & \frac{\pi^2}{\overline{H}} \end{vmatrix} \quad \text{.......... (52)}$$

where:

$$\overline{G}1 = -2\pi^2(\overline{\beta}_1.\theta_1 + \overline{\beta}_2.\theta_2) = \overline{\gamma}'_1.\theta_1 + \overline{\gamma}'_2.\theta_2 \quad \text{.......... (53)}$$

$$\overline{G}2 = -2\pi^2(\overline{\beta}_2.\theta_1 + \overline{\beta}_3.\theta_2) = \overline{\gamma}'_2.\theta_1 + \overline{\gamma}'_3.\theta_2 \quad \text{.......... (54)}$$

$$\overline{H} = \frac{\pi^2}{\lambda^2} + (\overline{\beta}'_1.\theta_1^2 + 2\overline{\beta}'_2.\theta_1.\theta_2 + \overline{\beta}'_3.\theta_2^2) \quad \text{.......... (55)}$$

$$L_O = L - g1 - g2 \qquad (56)$$

7. Post-Buckling Analysis

It is the consequence of any discrete formulation (e.g. the finite element method), that the deformation of a given structure is described by a set of (N) deformation parameters, which are also called generalized coordinates. In this context, the load-deformation history of a structure presents itself as a curve in a (N+1) dimensional space spanned by the deformation parameters and the magnitude of the applied loads. Such a curve is usually referred to as equilibrium path or deformation path. The problem of elastic stability is intimately connected with singularities that occur somewhere along the path under consideration. These singular points are better known as critical points. Well known is their classification into limit points and bifurcation points. In principle, the elastic stability formulations should be relevant to the problem at hand and should have the capability of:

1. Computing the critical points, i.e. limit or bifurcation points
2. Tracing parts of the path or paths (branches) connected with these points.

From another point of view, the method should have the capability of computing post-buckling.

It is clear now that there are two distinct strategies required for the successful completion of a single load increment in an incremental– iterative method:

1. Selection of a suitable external load increment for the first iterative cycle. The chosen increment is termed as an initial load increment and a particular strategy used to determine it is termed a load incrimentation strategy.
2. Selection of an appropriate iterative strategy for application in subsequent iterative cycles with the aim of restoring equilibrium as rapidly as possible. If iterations are performed on the load parameter as well as the nodal displacement, an additional constraint equation involving the change in the load parameter is required. It is the form of this constraint equation that distinguishes the various iterative strategies.

7-1 Incrimentation of the External Work

The initial load increment is chosen so as to limit the incremental work ΔW_i performed by the applied external loads. The incremental work for the ith load step is computed by [5]:

$$\Delta W_i = \Delta W_{i-1}\left(\frac{Jd}{J_{i-1}}\right)^{\beta} \qquad (57)$$

and $\Delta\lambda_i^1$ is calculated by:

$$\Delta\lambda_i^1 = \frac{\pm\Delta W_i}{\left|\{F_r\}_i^T\{v_t\}_i\right|} \qquad (58)$$

The procedure is initiated by the computation of ΔW_1 from Eq. (58) using specified starting load level $\Delta\lambda_1^1$.

7-2 Iteration at Constant External Work

Iteration at constant external work is an example of the general method described by Powell and Simons [6], and for an increment $\Delta\lambda_i^j\{F_r\}_i$ of external load, the quantity:

$$\Delta W_i = \Delta\lambda_i^j\{F_r\}_i^T\{\Delta v\}_i^j \quad \text{...} \quad (59)$$

is an incremental work term. If the external work is to remain unchanged during equilibrium iterations, then $\Delta W_I = 0$. The expression for the iterative change in the load parameter $\Delta\lambda_I^J$ is:

$$\Delta\lambda_i^j = -\frac{\{F_r\}_i^T\{\Delta v_r\}_i^j}{\{F_r\}_i^T\{v_t\}_i} \quad \text{...} \quad (60)$$

A description of the incremental-iterative method for a single load increment i follows. It is assumed that perfect convergence has been achieved at the conclusion of the (i-1)th load increment, so that the solution $(\lambda_{i-1}, \{v\}_{i-1})$ is known to satisfy total equilibrium. At the first iterative cycle (j=1), the new load increment commences with the computation of the tangent stiffness matrix $[T]_I$, is based on the known displacements and forces at conclusion of the previous load increment. The (tangent displacement), $\{v_t\}_i$ for this load increment are then computed as the solution of [5]

$$[T]_i\{v_t\}_i = \{F_r\}_i \quad \text{...} \quad (61)$$

in which $\{F_r\}_i$ is the reference external load vector, typically as specified in the input data for the problem. Next, the value of the initial load increment $\Delta\lambda_i^1$ is determined according to a particular load incrimentation strategy.

8. Results and Discussions

8-1 Elastic Large Displacement and Post-Buckling Analysis of Plane Structure with Prismatic and Non-prismatic Varying Section (With Gusseted Plate)

EX.1: Gusseted Williams Toggle Frame

This case study aims to examine the reliability of the modified derived tangent stiffness matrix for members with gusset plates and to check the efficiency of the proposed modified elastic bowing functions. It also checks the accuracy of the method with highly nonlinear structure. **Figure (4)** shows the geometry and loading conditions of Williams toggle frame, which is analyzed using the present method with post-buckling behavior. This frame is analyzed in this study using the modified method with 2-elements compared with 6-elements for the whole frame (i.e. each member consists of 3-elements, the medium part with the classical properties of the member and two hug edge members represent the rigid gusset plates). Also, the results are compared with AL-Barazanji [4], who used 2-elements and modified method for prismatic member only. Good agreements are shown for these methods of analysis when (g/L=0.1), as shown in **Fig.(5)**. This figure shows the complete path of load-displacement curve in the present study when using 2 or 6 elements, as compared with AL-Barazanji [4], since the post-buckling is used in this study. **Figure (6)** shows the load-displacement relationship with and without gusset effect with post buckling in this study (i.e. g/L=0.1 and g/L=0). In this modified method (U=1) is taken.

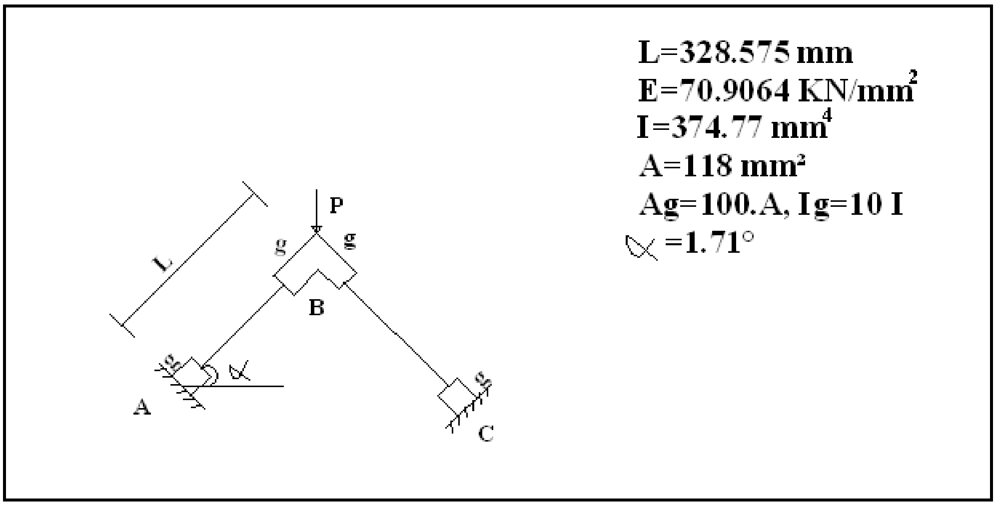

Figure (4) Geometry and loading conditions

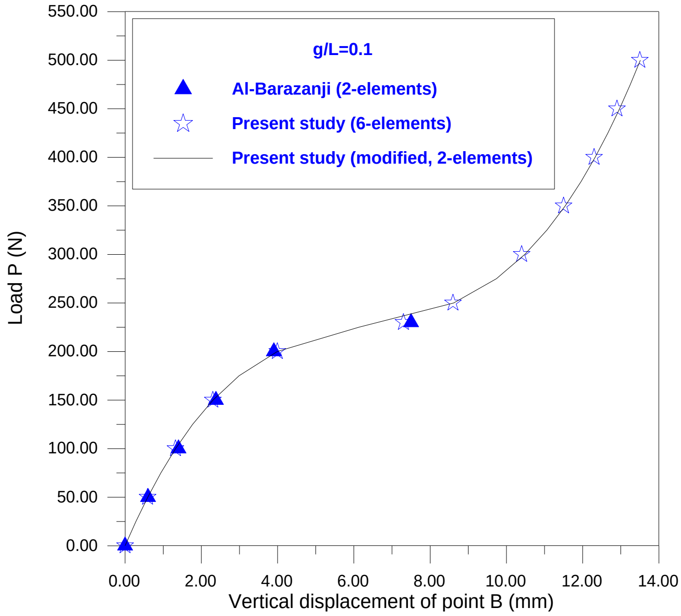

Figure (5) Load-displacement curves

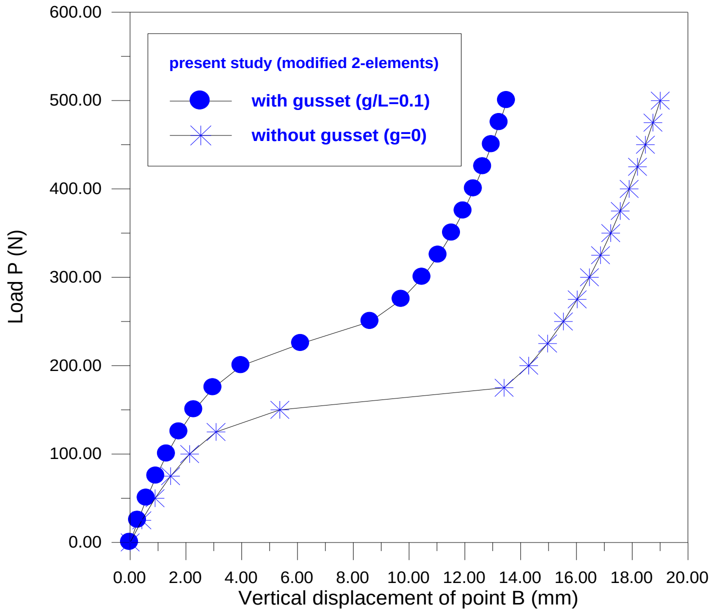

Figure (6) Load-displacement curves with gusseted effect

Ex.2: Effect of Gusseted Plate Length on Member Capacity

Figure (7) shows the geometry and loading conditions of toggle frame with different gusset lengths, g1 and g2. **Figure (8)** shows the load-displacement curves for vertical displacement at point B with different gusset lengths. The post-buckling, which is taken in this study appears in these curves. The load-capacity is increased when gusset length is increased.

Also, it is noticed that when gusset length (g1=0.2L), the structure becomes more stiff, and the load-capacity is increased.

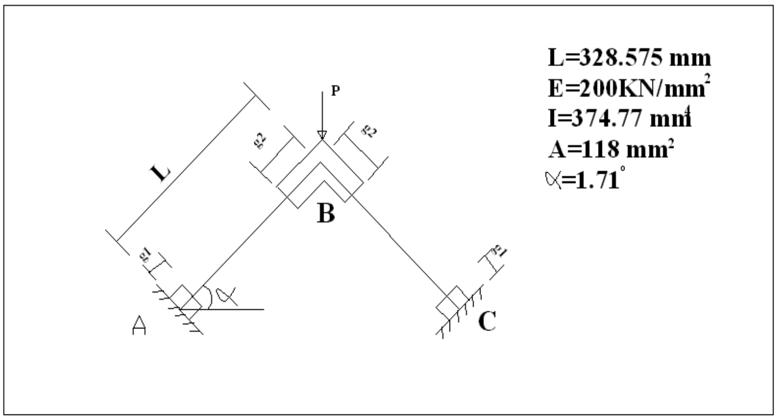

Figure (7) Geometry and loading

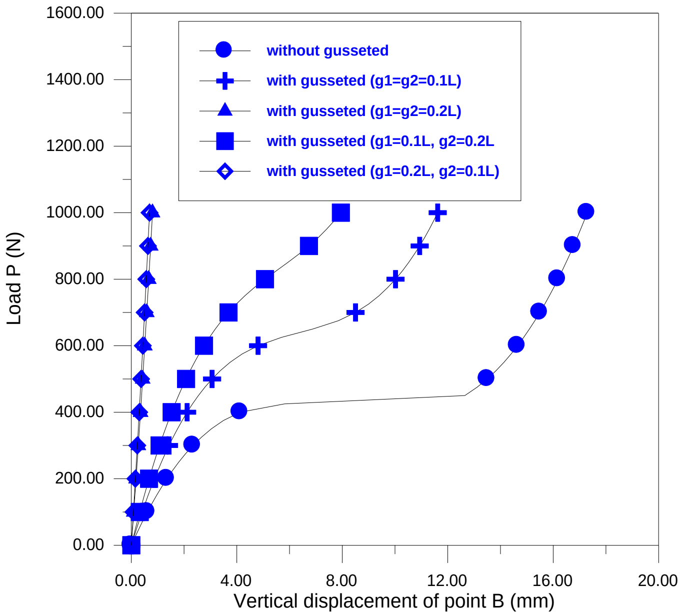

Figure (8) The effect of gusseted plate length on load-displacement curves

Ex.3: Effect of using Different Types of Variation of Cross-Section (Prismatic, Tapered and Nonlinear Tapered) with Gusseted Plate

The effect of prismatic or non-prismatic (linear and non-linear tapered) is very important for load carrying capacity. In this problem, three cases of column are taken, prismatic (U=1), tapered (U=2) and non-linear tapered (U=2) with parabolic distribution. The gusset ratio for length 0.1 is taken as shown in **Fig.(9)**, for geometry and loading conditions and one gusset element for modified stability function is used for representing the columns. The displacement ratio for horizontal, vertical and rotation is shown in **Fig.(10)** respectively. From the results shown in **Fig.(10)**, it is shown that for horizontal displacement ratio u/L= 0.775 for prismatic without gusset and 0.65 for prismatic with gusset and 0.1 for parabolic member with gusset and 0.085 for tapered with gusset for the same value of load which is 20000 kN at point B.

For vertical displacement ratio, it is shown that V/L=0.567 for prismatic member without gusset and about 0.37 for prismatic with gusset and about 0.01 for tapered and parabolic member which gives the results with very small difference.

For rotation ratios (rot./6.28), they are equal to 0.25 for prismatic member without gusset, 0.215 for prismatic with gusset, 0.0265 for parabolic, and about 0.026 for tapered column with gusset.

The member with gusset gives load capacity higher than that without gusset, while the non-prismatic gives more capacity and the tapered member gives nearly, equal stiffness to non-linear tapered.

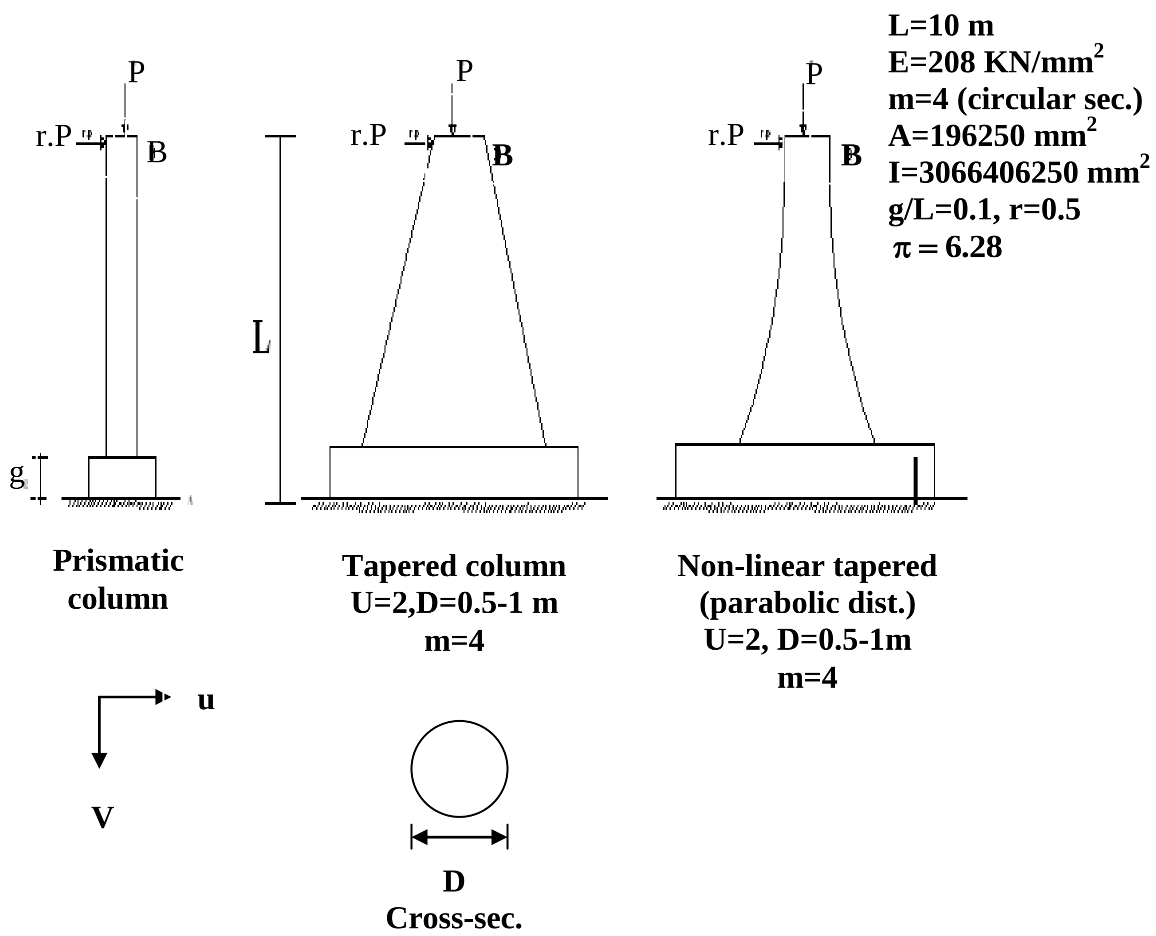

Figure (9) Geometry and loading conditions of prismatic and tapered columns

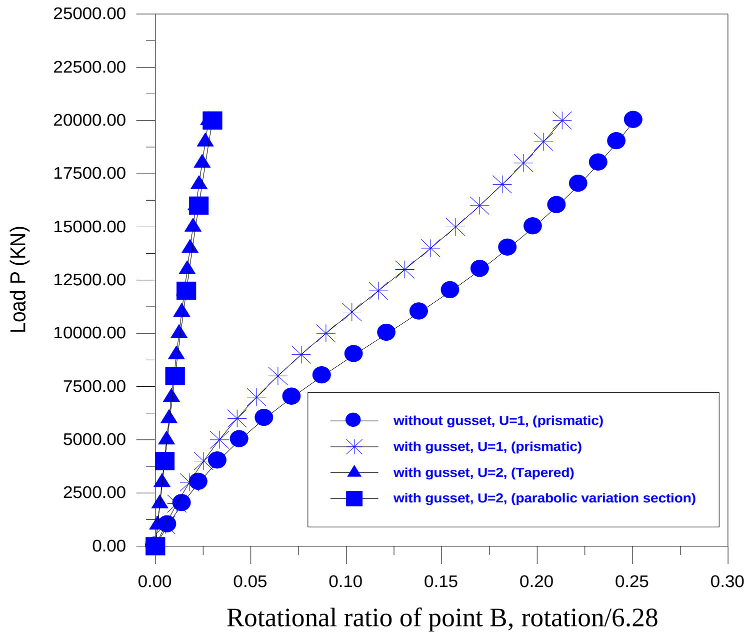

Figure (10) Continued

9. Conclusions

1. In all the examples and case studies, the used load incrimentation strategies are efficient for enabling the program to estimate the suitable size of load increments and the used iterative strategies are efficient for enabling the program to change the load level and rapidly achieve the convergence.
2. Using one element analysis for structures with gusseted members produces higher advantage of the software facilities due to the use of single precision in analysis of gusseted structures which makes full benefit of the software memory storage capacity for matrices dimensions required in the program.
3. The increase of gusset ratio produces high structure stiffness, which leads to higher elastic critical load of the structure, and it produces lower structure deformations due to the opposite increase in the structure stiffness. The increase in elastic critical load and the amount of deformation decrease varies from a structure to another depending on the structure geometry.
4. The gusseted plate increases the load capacity of the frame and this is dependent on the gusset length and the varying sections of the member.

10. References

1. AL-Amili, ***"Large Displacement Elastic and Elastic-Plastic Analysis and Post-Buckling of Steel Frames with Varying Non-Prismatic Members and Gusseted Plates"***, Ph.D. Thesis, University of AL- Mustansiriya, Iraq, 2005.

2. AL-Sarraf, S. Z., ***"Elastic Instability of Frames with Uniformly Tapered Members"***, The Structural Engineering, March 1979, pp. 18-24.

3. Faris, ***"Large Displacement Elastic-Plastic Analysis of Plane Frames with Non-Prismatic Members of Non-linearly Varying Sections"***, Ph.D. Thesis, University of Technology, Iraq, 2002.

4. AL-Barazanji, ***"Large Displacement Elastic Stability Analysis of Gusseted Frames"***, M.Sc. Thesis, University of Technology, Iraq, 1997.

5. Clarck, M. J., and Hancock, G. J., ***"A Study of Incremental-Iterative Strategies for Non-Linear Analysis"***, International Journal for Numerical Methods in Engineering, Vol. 29, 1990, pp. 1365-1391.

6. Powell, G., and Simons, J., ***"Improved Iteration Strategy for Nonlinear Structures"***, International Journal of Numerical Methods in Engineering, Vol. 17, 1981, pp. 1455-1467.