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Design Study of the Crossed Field Antenna

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Abstract

Besides the fast development in technology of electronics world, there is anther development in the concepts and principles of antennas work.

This paper presents mathematical expressions for the main features of the cross field antenna CFA, like electric field E, magnetic field H, and the poynting vector S.

These expressions are basically derived by considering the principle of two point charges in the space throw measuring the potential at far point, leading to the electric field and magnetic field forms of CFA.

الخلاصــــــة

بجانب التطور السريع في تكنولوجيا عالم الالكترونيات هناك تطور أخر في مبادئ عمل الهوائيات.

*يقدم هذا البحث تعابير رياضية للمعالم الأساسية لهوائي تقاطع المجال، مثل المجال الكهربائي **E**، المجال المغناطيسي **H**، و متجه القدرة **S**.*

تم اشتقاق هذه التعابير باعتبار مبدأ شحنتين نقطيتين في الفضاء من خلال قياس الجهد عند نقطة بعيده، مؤدياً إلى التعبير الرياضي للمجال الكهربائي و المغناطيسي لهوائي تقاطع المجال.

1. Introduction

The first CFA was described by F. M. Kabbary et. al. [1]. It intended to synthesize directly the pointing vector S from separately stimulated electric field intensity E and magnetic field intensity H, according to the formula S= $E \times H$ and one important result of development which is useful in engineering design of antennas in an extremely small size and compact design which is independent of the radiated wavelength.

After the first description which appeared in 1989, as any new design there have been criticisms of design, some correspondents saying that it can not possibly be practicable, but Bryan Well has made models which appear to work well. He described some initial experiments in the November 1989 issue [1].

2. Theory of the CFA

All electrical and communications engineers are in some way acquainted with Heaviside's differential form of the Maxwell equations, which are [2]:

$$\nabla . \mathbf{D} = \rho \quad \text{......} \quad (1)$$

$$\nabla . \mathbf{B} = 0 \quad \text{......} \quad (2)$$

$$\nabla \times \mathbf{E} = \frac{-\partial \mathbf{B}}{\partial t} \quad \text{......} \quad (3)$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \quad \text{......} \quad (4)$$

where:

E: Electric field strength.
H: Magnetic field strength.
J: Current density.
B=μ H: Magnetic flux density.
D=ε E: Electric Displacement.

It's important to realize that the above equations contain the following extremely valuable information:

a. A time-varying magnetic field creates an electric field (or back e.m.f.).
b. A current or a time-varying electric field or both will create a magnetic field.

The essence of Maxwell's equations conveyed through (a) and (b), is that fundamentally they are reaction or field production equations. The physical, mathematical, and engineering importance of the field production nature may be more readily relayed and understood if the forms of eqs.(3) & (4) are reversed:

$$\mathbf{B}' = -\nabla \times \mathbf{E} \quad \text{......} \quad (5)$$

$$\mathbf{J} + \mathbf{D}' = \nabla \times \mathbf{H} \quad \text{...} \quad (6)$$

eq.(3) interpreted as "An electric field can be related to the rate of change of a magnetic field". But eq.(5) is interpreted as a time varying magnetic flux B, creating an electric field E such that the negative of the curl of the induced E field distribution is equal to the source B'. eq.(3) is fully deployed in transformer theory.

Consider now eq.(4) in magnetostatics, it has always been accepted that current produces a magnetic field through the phenomenon called Ampere's Law. But eq.(6) is interpreted as J creates a magnetic field H, such that the curl of H is equal to the source J ($J = \nabla \times H$) or a time-varying D field creates a magnetic field, such that the curl of the H field distribution is equal to the source D' ($D' = \nabla \times H$).

We see now the importance of reversing eq.(4) to eq.(6) which should now be interpreted as J or D' or both can create a magnetic field H such that the curl of the H field distribution is equal to the source ($J + D'$). The plus sign can, and should be interpreted as analogous to the digital-logic OR symbol.

Finally we must know that an H field may at any time be the combination of two separately induced fields from independent types of source i.e. charges motion and displacement current.

2-1 The Magnetic Field Associated with Simple Capacitor

A capacitor essentially consists of two conducting surfaces separated by a layer of an insulating medium called dielectric as shown in **Fig.(1)**. To illustrate the importance of the reversed from eq.(4), and in particular the feature of D' creating an independent magnetic field from **J**. Consider circular capacitor plates with an applied sinusoidal voltage **V,** free charge flowing into and out of the capacitor and also within the capacitor plates themselves, are a source of **J**. Also, due to the build up of free charge in the capacitor E-lines and therefore **D** lines exist between the capacitor plates [1].

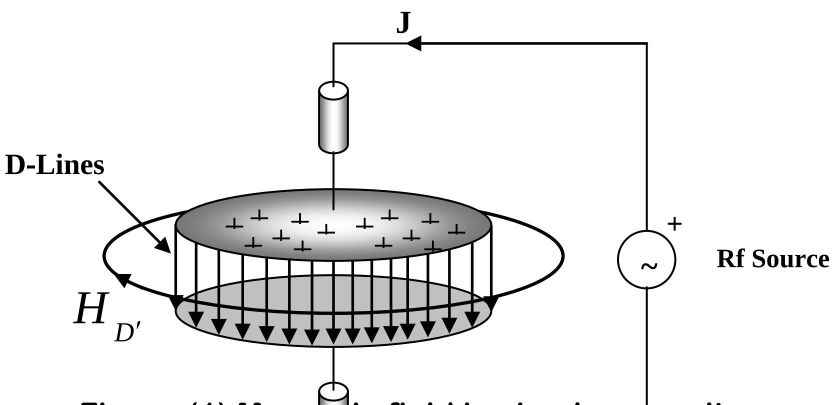

Figure (1) Magnetic field in simple capacitor

The waveforms of **V**, **J** and **D** are shown below, note that **D** follows **V** because D = ε E = ε V/I therefore D and V in phase as shown in **Fig.(2)**.

While J is 90^o phase advanced from V because $I_c = c\frac{dv}{dt}$, if V = V_m sin(wt), then $I_c = \frac{d(V_m \sin(wt))}{dt}$, I_c = cw V_m cos(wt) = cw V_m sin(90^o-α).

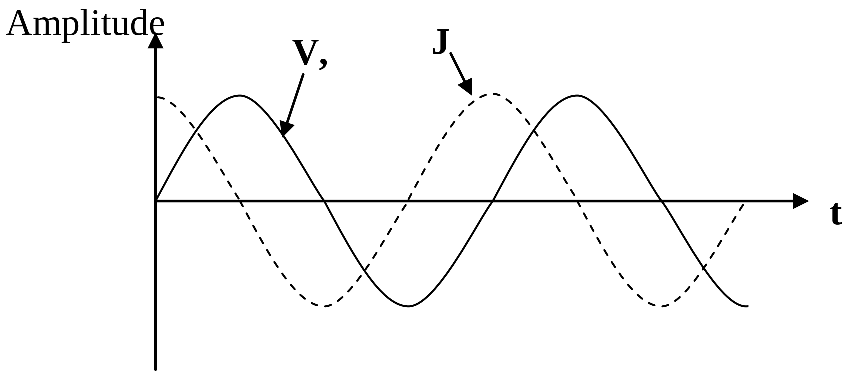

Figure (2) Waveforms of V, D, and J

As the D lines vary in strength due to sinusoidal charge variation on the plates, D will create a sinusoidal magnetic field.

$$\mathbf{D}' = \nabla \times \mathbf{H}_{\mathbf{D}'} \quad \text{......} \quad (7)$$

Since H_D is in time phase with D because $D' = \nabla \times H_{D'}$, then H_D is 90^o phase advanced from D as shown in **Fig.(3)**.

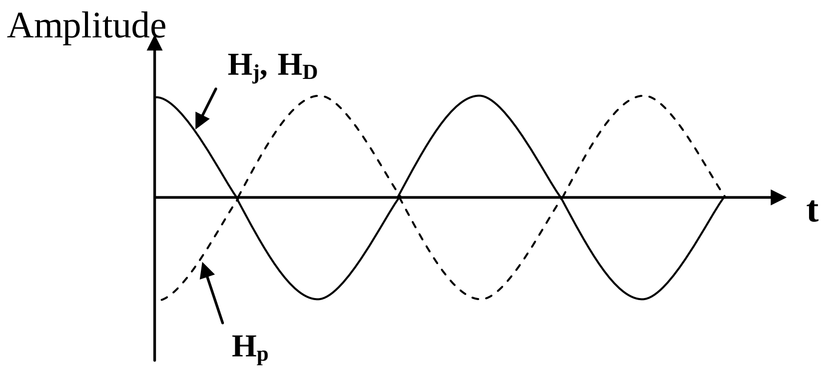

Figure (3) Waveforms Hj, HD, and Hp

Also, since J flowing into and out of the plates is sinusoidal then $J = \nabla \times H_J$, produced a sinusoidal magnetic field H_J (in phase with J). In the vicinity surrounding the capacitor gap, the magnetic field lines from J into and out of the plates and the magnetic field lines from D will be concentric circles surrounding the gap and in phase.

Now J flowing within the plates themselves will create a magnetic field H_p. Many components of magnetic field produced from individual J contributions within the plates will cancel, resulting in reduced strength circular field lines surrounding the plates [1].

We should expect the created field H_p to be in phase with H_J, but taking into account the geometry and the current motion within the plates. Then H_p is directed in the opposite direction to H_J, this is equivalent to 180^o phase change between H_p and H_J.

2-2 Barrel Shaped CFA

From the experimental verification of D within large circular capacitor plates, producing a surrounding magnetic field distribution, a revolutionary engineering design of antennas has now been developed in which the poynting vector $S = E \times H$ is directly synthesized by separated E & H field stimulus within a very small volume, these antennas are called Crossed-Field Antennas (CFAs) which were described by F. M. Kabbary, M. C. Hately and B. G. Stewart, one particular CFA design is the “barrel-shaped CFA” as shown in **Fig.(4)** [1].

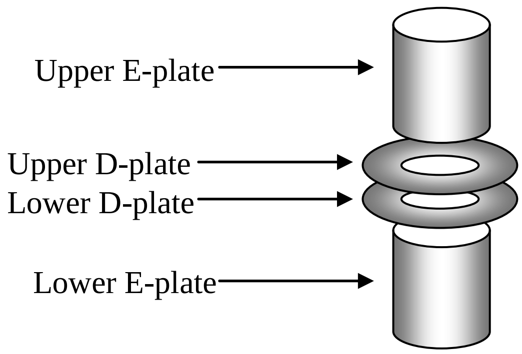

Figure (4) Barrel shaped CFA

D-plates

Large circular capacitor plates when supplied with high voltage will produce strong circular magnetic field around the plates through $D' = \nabla \times H_{D'}$.

E-plates

Two large cylindrical plates of short length but the same radius as the capacitor plates are positioned one above and one below the D-plates.

When the cylindrical plates are driven by an RF power source they produce a high-frequency, E lines “due to voltage difference” between the plates. The power from the transmitter is split roughly in half between the E-plates through suitable design consideration and delay arrangements between the E and D plate voltage. Radiation is then produced through $S = E \times H$ and power flows out to space as vertically polarized radio waves of intense power density.

2-3 The First Model Dimension of CFA

The first model dimension of CFA as appear in the November, 1989, issue is as shown in **Fig.(5)**. The E-plates has a cylindrical shape of 100mm. in radius and 230mm. high. 240mm. separated between the upper and lower plates. The D-plates are two discs of 100mm. radius separated by 80mm; each D-plate has a center hole of 60mm. diameter. These holes are only used for incoming the feed wires.

The distance between the upper E-plate and the upper D-plate is 80mm. same as lowers E-plate and lower D-plate [1].

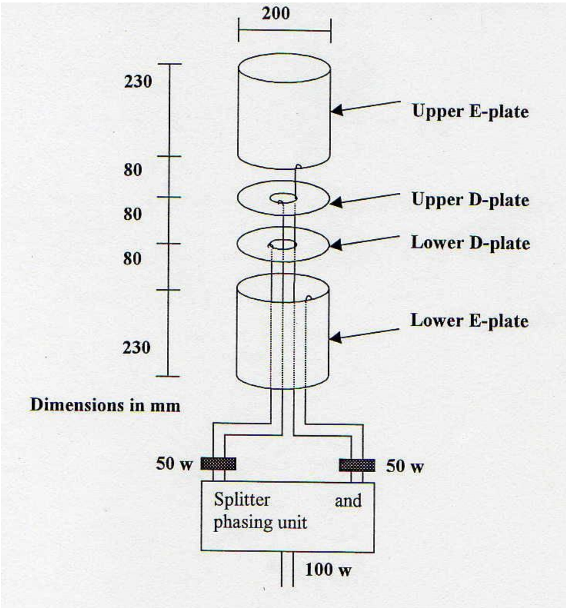

Figure (5) First model dimension of CFA

2-4 Wiring Polarity of the CFA

The feeders of the E-plates is connected to the lower end of the upper E-plate and the upper end of the lower E-plate to be the current direction flow in the surface of the two cylinders as shown in **Fig.(6)**.

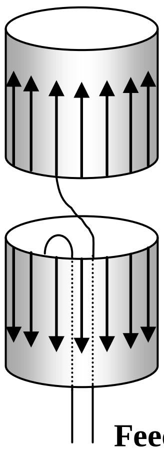

Figure (6) Wiring polarity for E-plates

Similarly for the two D-plates the feeding will be from the internal hole, as shown in **Fig.(7)**.

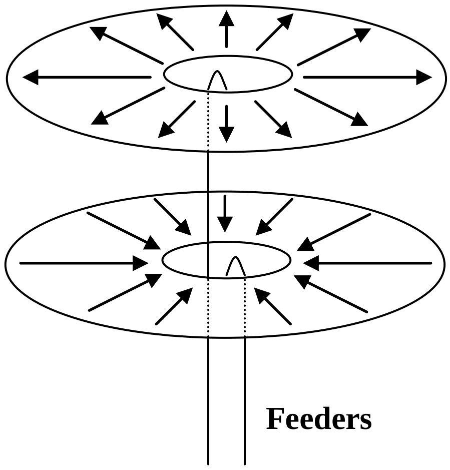

Figure (7) Wiring polarity for D-plates

2-5 Matching Feed System

For efficient power transmission the antenna characteristic should matched with the source and the space also between space and the receiver, this is achieved by matching feed system [3,4].

A matched feed system for the CFA is important because:

1. A matched system with more or less flat lines might avoid the criticism radiation from the feed lines.
2. To obtain correct phasing; Hately confirmed that the required phasing between the plates and cylinders of the CFA is the 90^o, CQ magazine, August 1989, suggested a such phasing unit to give 90^o as shown in **Fig.(8)**.

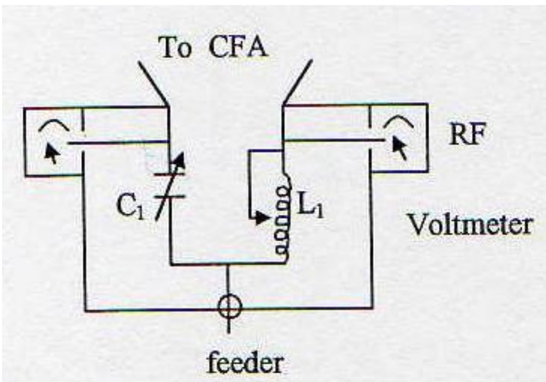

Figure (8) Phasing unit RF voltmeter for checking 50:50 powers split

The complete matched feed system is shown in **Fig.(9)**.

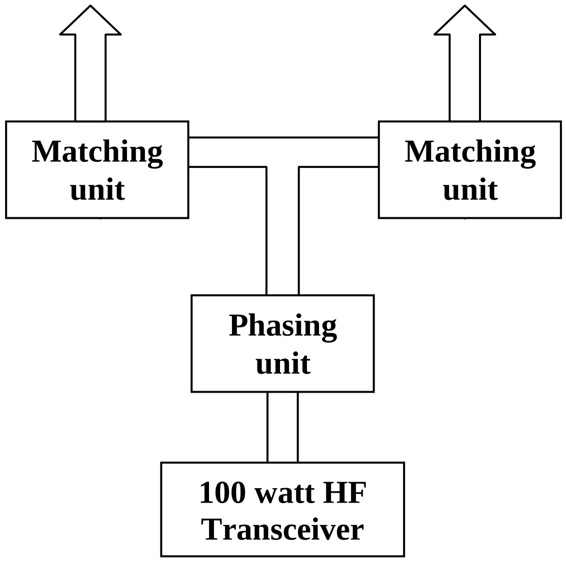

Figure (9) Matched feed system

3. CFA Analysis

To analyze the electric and magnetic field distribution we refer to the principle of dipole assumption which states that we have two point charges of equal magnitude and opposite sign, separated by a distance *d* in free space as shown in **Fig.(10)** [5,6,7].

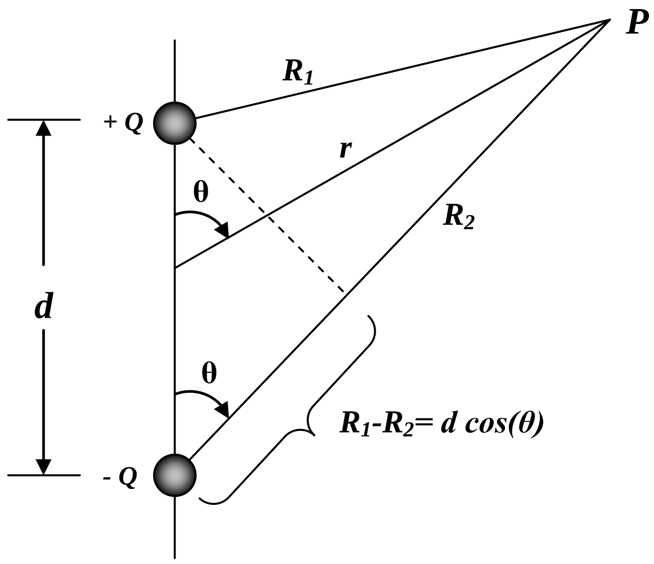

Figure (10) CFA fields analysis

The above figure will be considered for each E-plates and D-plates, first we must find the electric field distribution produced by these two charges by obtaining the potential field which equal for this system:

$$V = \frac{Q}{4\pi\varepsilon_\circ}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{Q}{4\pi\varepsilon_\circ}\cdot\frac{R_2 - R_1}{R_1 R_2}$$, but *d* is very small with respect to R_1 and R_2

$R_1 \cong R_2$ and $R_2 - R_1 = d\cos(\theta)$

$$V = \frac{Q\,d\cos(\theta)}{4\pi\varepsilon_o r^2} \quad \text{(8)}$$

For spherical coordinates system we obtain:

$$E = -\left(-\frac{Q\,d\cos(\theta)}{2\pi\varepsilon_o r^3}\bar{a}_r - \frac{Q\,d\sin(\theta)}{4\pi\varepsilon_o r^3}\bar{a}_\theta\right)$$

or

$$E = \frac{Q\,d}{4\pi\varepsilon_o r^3}\left(2\cos(\theta)\,\bar{a}_r + \sin(\theta)\bar{a}_\theta\right) \quad \text{(9)}$$

3-1 The Magnetic Field Expression

As in section (2) the D-plates will create the magnetic field (H), the expression for the magnetic field (H) coming from the two D-plates measured at a far point.

To find H, we can use eq.(9) and integrate over the whole charge distribution in the D-plates, eq.(9) becomes for incremental element. The charge distribution over the D-plates when supply it by sinusoidal current ($I=I_o \sin(wt)$), each of the surface elements will produce a field toward an external point (*P*) as shown in **Fig.(11)**.

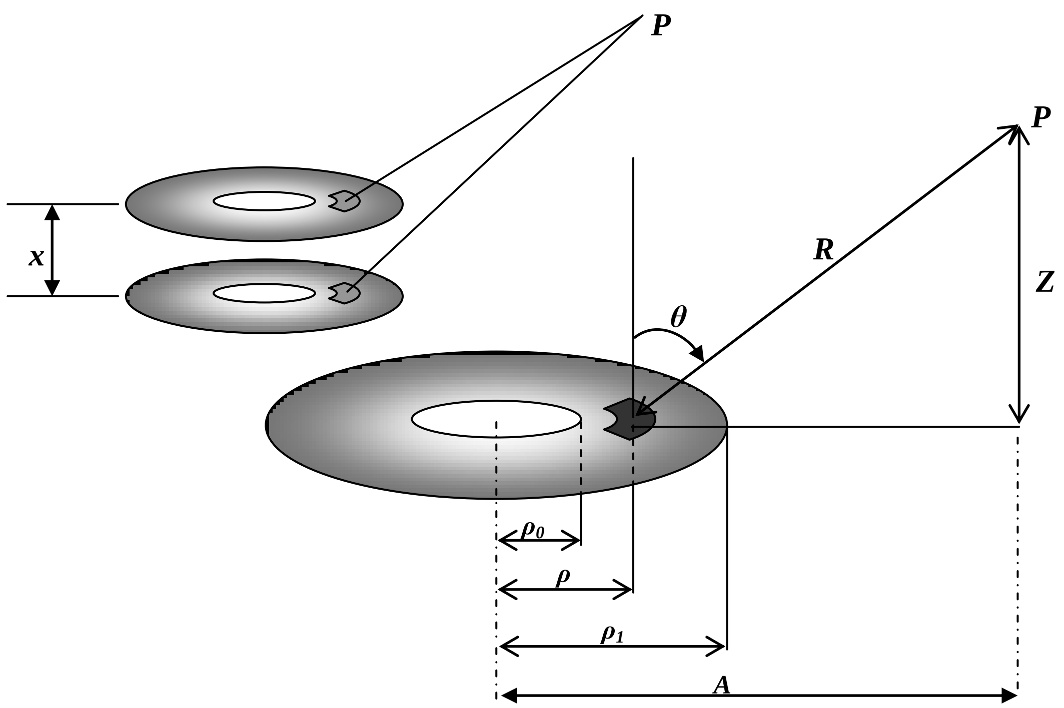

Figure (11) Magnetic field analysis

$$\mathbf{dE} = \frac{x\,dQ}{4\pi\,\varepsilon_o\,R^3}\left(2\cos(\theta)\,\bar{a}_r + \sin(\theta)\,\bar{a}_\theta\right)$$

$$\mathbf{R} = (A-\rho)\,\bar{a}_\rho + Z\,\bar{a}_Z$$

$$dQ = \rho_S\,ds = \rho_S\,\rho\,d\phi\,d\rho$$

here, we can neglect ρ because $A >> \rho$. From the current continuity law:

$$dQ = -\,I\,dt, \qquad \rho_S = \frac{-1}{S}\int I\,dt\,,$$

when $\mathbf{I = I_o \sin(wt)}$

$$\rho_S = \frac{-1}{S}\int I_o\,\sin(wt)\,dt = \frac{I_o}{wS}\cos(wt) \quad \text{.. (10)}$$

where:

S: is the total surface area of the disc

$S = \pi\,(\rho_1^{\,2} - \rho_0^{\,2})$

But to calculate H only the z component is needed, and by transforming to cylindrical coordinates we get:

$$\mathbf{dE} = \frac{x\,dQ}{4\pi\,\varepsilon_o\,R^3}\left(3\cos^2(\theta)-1\right)\bar{a}_Z$$

By integrating for ρ from ρ_o to ρ_1, and for φ from (φ = 0°) to (φ = 2π) then multiplying by ε_o we get:

$$\mathbf{D} = \frac{-x\,I_o\cos(wt)}{2\pi\,w\,R^3}\left(3\cos^2(\theta)-1\right)\bar{a}_Z$$

$$\mathbf{D'} = \frac{\partial D}{\partial t} = \frac{x\,I_o\,\sin(wt)}{2\pi\;R^3}\left(3\cos^2(\theta)-1\right)\bar{a}_Z \quad \text{.. (11)}$$

Let $\mathbf{K} = \frac{x\,I_o\,\sin(wt)}{2\pi}$. Eq.(11) in spherical coordinates becomes:

$$\mathbf{D'} = \frac{\partial D}{\partial t} = \frac{K}{R^3}\left(3\cos^2(\theta)-1\right)[\cos(\theta)\,\bar{a}_r - \sin(\theta)\,\bar{a}_\theta]$$

In CFA, H is obtained from Maxwell equation eq.(4) $D' = \nabla \times H$ and from a_θ component of D'.

$$H_\phi = \frac{5x\,I_o\,\sin(wt)}{\pi\;R}\left(3\cos^2(\theta)-1\right)\sin(\theta) \text{.. (12)}$$

This is the mathematical expression in polar form of the magnetic field in spherical coordinate it is depends on some parameters which can be modified to get optimum value of H_φ. **Figure (12)** shows the polar diagram of the magnetic field intensity.

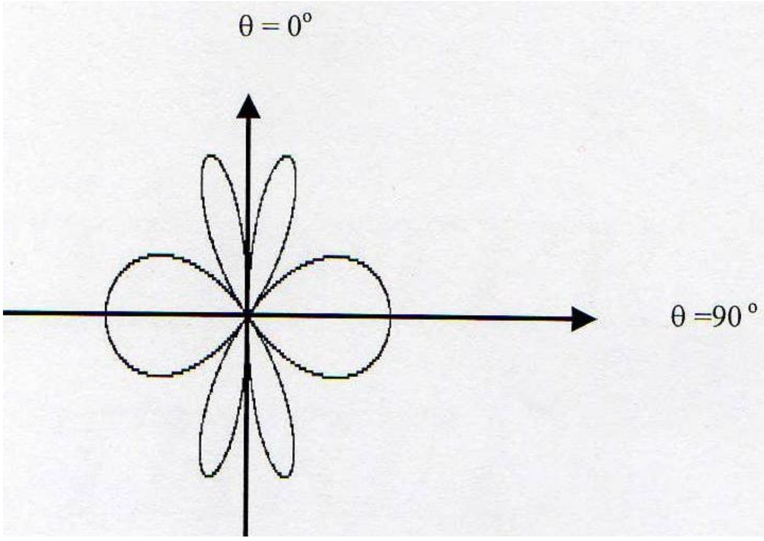

Figure (12) Pattern of the magnetic field of CFA

3-2 The Electric Field Expression

The electric field is generated from the E-plates, to derive the mathematical expression of the electric field coming from the two cylinders eq.(9) must be used as an incremental field coming from only two charges then integrate it over the whole surface of the two cylinders. **Figures (13-a)** and **(13-b)** show the assumption to calculate the electric field at point (P).

$$\mathbf{R} = (\rho - \rho_1)\,\bar{\mathbf{a}}_\rho + (\mathbf{A} - \mathbf{Z})\,\bar{\mathbf{a}}_Z$$

$$|\mathbf{R}| = \sqrt{(\rho - \rho_1)^2 + (\mathbf{A} - \mathbf{Z})^2}$$

$$|\mathbf{R}|^3 = \left(\sqrt{(\rho - \rho_1)^2 + (\mathbf{A} - \mathbf{Z})^2}\right)^3$$

But here $\rho >> \rho_1$ and $A >> Z$

$$|\mathbf{R}|^3 = \left(\sqrt{\rho^2 + \mathbf{A}^2}\right)^3$$

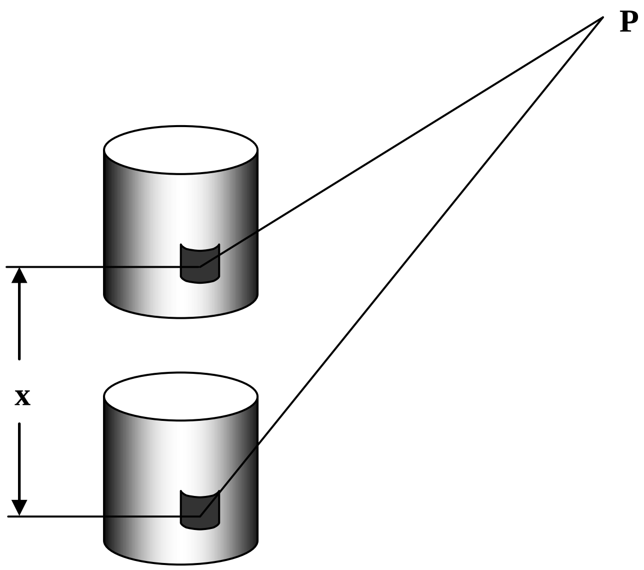

Figure (13-a) Electrical field analysis

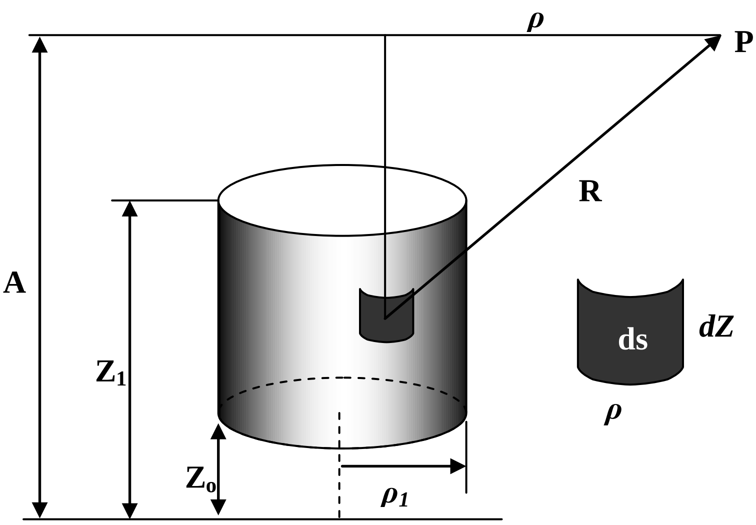

Figure (13-b) Electrical field analysis

$$dE = \frac{x\, dQ}{4\pi \varepsilon_0 R^3}(2\cos(\theta)\,\bar{a}_r + \sin(\theta)\,\bar{a}_\theta)$$

To produce pointing vector in $\bar{a}_r$ direction, therefore E will be mainly in $\bar{a}_\theta$ direction, by making surface integral and considering *x* as (Z_1+Z_o):

$$E_\theta = \frac{\rho_S\, \rho_1\, (Z_1^2 - Z_0^2)}{2\, \varepsilon_0\, R^3}\sin(\theta)$$

$$|E_\theta| = \frac{(Z_1 + Z_0)\, I_0}{4\pi\, w\, \varepsilon_0\, R^3}\cos(wt)\sin(\theta) \quad \text{.......... (13)}$$

Figure (14) shows the polar diagram of electric field intensity.

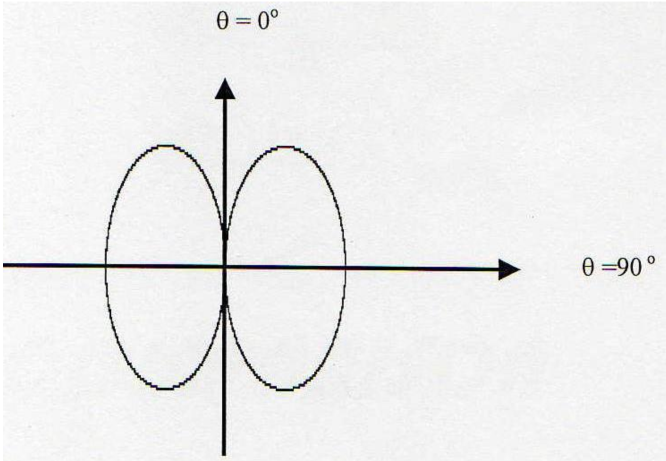

Figure (14) Pattern of the electric field of CFA

3-3 Pointing Vector Equation of CFA

The pointing vector can be obtained directly by crossing E_θ and $H\phi$, which represents the power radiated from the CFA per unit area as follow:

$$\mathbf{S} = \mathbf{E}_\theta \times \mathbf{H}_\phi$$

$$\mathbf{S} = \frac{(Z_1 + Z_0)\, I_0}{4\pi\, w\, \varepsilon_0\, R^3} \sin(\theta) \cdot \frac{5x\, I_o}{\pi\ R} \left(3\cos^2(\theta) - 1\right) \sin(\theta)\, \bar{a}_r$$

$$\mathbf{S} = \frac{5x(Z_1 + Z_0)\, I_0^2}{4\pi^2\, w\, \varepsilon_0\, R^4} \sin^2(\theta) \left[3\cos^2(\theta) - 1\right] \bar{a}_r$$

Figure (15) shows the pattern of the pointing vector *S*.

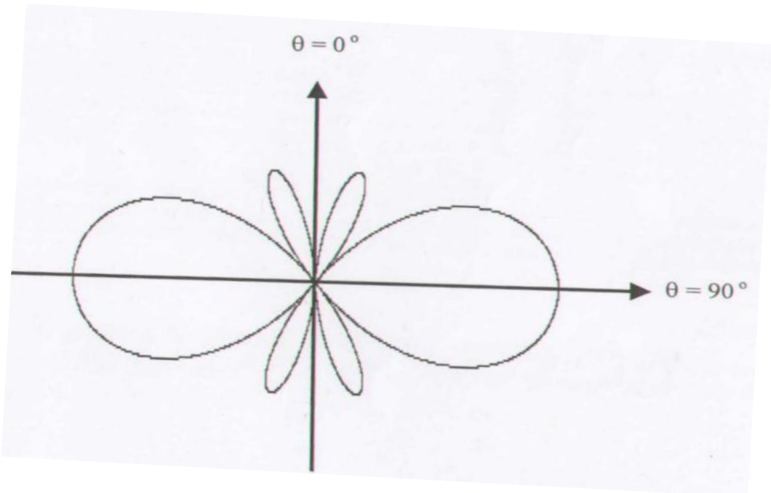

Figure (15) Pattern of the Pointing vector of CFA

4. Conclusions and Recommendation

1. The CFA has special properties among the frequency independent antennas due to the generation of electromagnetic wave by separated electric field and magnetic field from two independent sources, which is not appear in Log-periodic antenna, Spiral antenna, and other types of frequency independent antennas.

2. There are no dimension limitations in design for the CFA comparing with the other frequency independent antennas.

3. It is recommended that the derivations in this paper can be verified with simulation or practical model of CFA.

5. References

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