Shear Behavior of Hybrid Reinforced Concrete I-Beams Containing Steel Fiber Reinforced Concrete (SFRC) and High Strength Concrete (HSC)

Prof. Dr. Husain M. Husain
Civil Engineering Dept., College of Engineering
University of Tikrit, Salahaddin, Iraq

Asst. Prof. Dr. Bayan S. Al-Nu'man
Civil Engineering Dept., College of Engineering
Al-Mustansiriya University, Baghdad, Iraq

Dr. Ali H. Aziz
Civil Engineering Dept., College of Engineering
Al-Mustansiriya University, Baghdad, Iraq

Abstract

This study presents experimental and theoretical investigation of shear behavior of hybrid reinforced concrete I-beams cast monolithically, but, the web is made with concrete differing from concretes of compression and tension flanges.

A rational way has been used by replacing (or strengthening) a certain part(s) or layer(s) of I-shaped reinforced concrete beams by steel fiber reinforced concrete (SFRC) or high strength concrete (HSC). Three beams were tested in this investigation and only the concrete type of the web was varied.

Experimental results showed that the ultimate shear strengths are increased (15 or 25%), while the reserve shear strength (after diagonal cracking) is increased (6 or 10%) for the tested beams containing SFRC or HSC in the web respectively. Crack arrest mechanism of steel fibers, and high compressive strength of concrete improved the tensile response, limited crack propagation, improved reserve strength and altered the failure mode. So, more safety was obtained.

ANSYS finite Element models are used to simulate all tested beams. Three dimensional nonlinear brick element (solid element) was used to model the concrete, while, the steel reinforcement was modeled by discrete bar element.

It was found that the general behaviors of the finite element models showed good agreement with observations and data from the experimental tests.

Key Words: ***Shear, Concrete, I-Beam, Web, Hybrid, Strengthening, HSC, SFRC, ANSYS***

الخلاصـــــــة

*تقدم هذه الدراسة بحثا عمليا ونظريا لتقصي سلوك القص للعتبات الخرسانية المسلحة الهجينية ذات المقطع -**I**- المصبوبة بشكل كامل والتي تكون فيها خرسانة الوترة **(Web)** مختلفة عن خرسانة شفة الضغط و خرسانة شفة الشد **(Tension and Compression Flanges)**.*

*تم استخدام طريقة منطقية وذلك بإبدال (تقوية) جزء معين من المقطع -**I**-، (الوترة)، بخرسانة مسلحة بألياف الحديد**(SFRC)** أو خرسانة عالية الانضغاط **(HSC)**. في هذه الدراسة، تم فحص ثلاثة عتبات كانت فيها خرسانة الوترة متغيرة.*

*أظهرت النتائج المختبرية زيادة في مقاومة القص (١٥ أو ٢٥%) وزيادة في مقاومة القص المؤمن **(Reserve Shear Strength)** (٦ أو ١٠%) للعتبات ذات الوترة **(Web)** المقواة بالخرسانة المسلحة بألياف الحديد أو المقواة بالخرسانة عالية الانضغاط على التوالي.*

*لتقصي سلوك العتبات المفحوصة تحليليا، تم استخدام برنامج العناصر المحددة **(ANSYS)** لإنشاء نماذج رياضية معتمدة على العناصر المحددة ثلاثية الأبعاد اللاخطية لتمثيل الأجزاء الخرسانية وعناصر منفصلة **(Discrete Elements)** لتمثيل قضبان التسليح.*

أظهرت النتائج بشكل عام حصول توافق جيد بين نتائج العناصر المحدودة مع النتائج المختبرية.

1. Introduction

Beams with I-shaped cross sections are used extensively in long span and pre-stressed concrete structures. The wide flange does not only permit a large compressive force to develop but it also maximizes the arm of the internal couple by positioning the resultant of the compressive stress near the compression surface [1].

The minimization of concrete from the tension zone, where only the steel reinforcement is effective in carrying tension, reduces the dead weight and permits the design of smaller and lighter members. Furthermore, the narrow web of I-shaped cross is more effective to carry shear stress [2].

Experimental studies indicate that the addition of steel fibers to the concrete in beams without shear reinforcement (stirrups) may significantly increase the ultimate shear strength [3]. The increase in shear strength attributable to the fibers depends on the amount of fibers (fiber volume fraction V_f), fiber aspect ratio (L_f / D_f) and anchorage conditions for the steel fibers. Fibers are effective after the formation of cracks and continue to resist significant tension until the fibers yield or pull out.

The use of high strength concrete leads to the design of smaller sections, thereby reducing the dead weight, allowing longer spans and more usable area of buildings [4].

Also, the previous researches show that increased bending moment capacity and shear strength are attainable by using steel fiber concrete and high strength concrete. Most experimental investigations and researches have suggested replacing overall depth of reinforced concrete beams by steel fiber reinforced concrete or by high strength concrete with special concentration on rectangular cross-section beams.

In the present work, a rational way has been used by replacing (or strengthening) the web of I-shaped reinforced concrete beams by steel fiber reinforced concrete or high strength concrete.

2. Experimental Study

To accomplish the objectives drawn from this study, an experimental program has been performed and an attempt has been made in order to study the effect of high strength concrete and steel fiber reinforced concrete in the web on shear behavior of these hybrid beams.

2-1 Experimental Program

Three beams were tested in this investigation and only the concrete type of the web was varied, while, the concrete type of compression and tension flanges and the longitudinal reinforcement were kept unchanged.

2-2 Test Specimens

The nominal dimensions of the tested beams are shown in **Fig.(1)**. To avoid mold stripping difficulties, the lower face of the compression flange and the upper face of the tension flange were made with (1/15) and (2/5) slopes respectively. Therefore, the flange thicknesses increased uniformly from the edges towards the web.

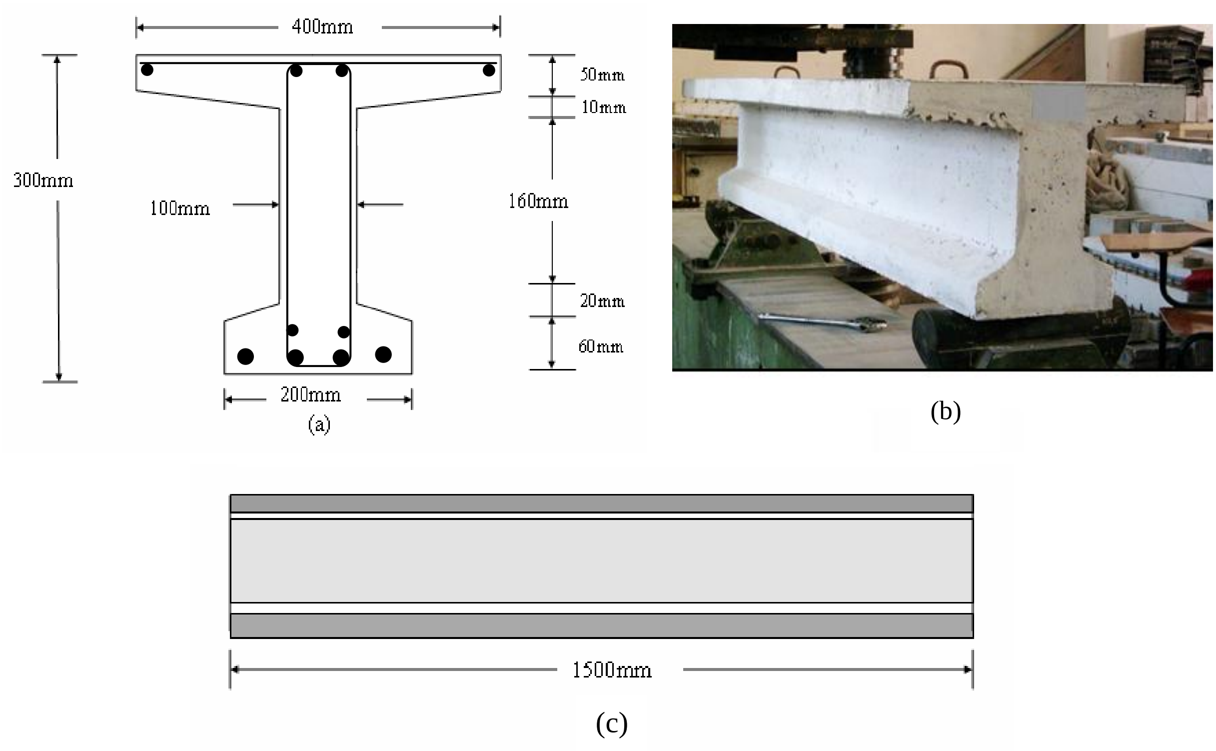

Figure (1) Details of test beams
(a) beam cross-section (b) isometric view (c) side view

All beams were singly reinforced, the longitudinal reinforcement was designed to ensure the upper flange would be in compression and the section would fail in shear mode of failures. A design equation adopted by ACI-318 Building Code [5] was used to estimate the longitudinal tensile reinforcement.

The main reinforcement consisted of (4ϕ16mm) and (2ϕ10mm) with steel ratio of (ρ=0.92%). Mild, hot rolled, deformed steel bars were employed as tension reinforcement. Also, (4ϕ5mm) additional longitudinal smooth steel was used as tie bars in the upper flange to hold the transverse steel in position when concrete was poured and to prevent concrete fragments of compression flange.

To ensure shear failure of beams, slightly transverse reinforcement was used. (ϕ3mm) at (200mm) centers, smooth bars were used as stirrups. The stirrups were detailed in form to work as web reinforcement (stirrups) and transverse reinforcement at the top in the flange overhang.

Both longitudinal and transverse bars were arranged and connected together by using (1mm) steel wires to form a reinforcement cage. The reinforcement cage was placed inside the mold with (20mm) concrete cover for tensile reinforcement.

The tested beams were designated as shown in **Table (1)**.

Table (1) Properties of shear beams

	(Designation) Beam No.	Concrete Type		
		Lower Flange	Web	Upper Flange
Shear Beams	SB-I	NSC	NSC	NSC
	SB-II	NSC	SFRC	NSC
	SB-III	NSC	HSC	NSC

2-3 Materials

In the experimental program, (ϕ16mm) and (ϕ10mm) deformed steel bars having (596.8MPa) and (532.78MPa) yield strength were used as flexural reinforcement. Also, (ϕ3mm) and (ϕ5mm) mild smooth (plain) steel bars with (230MPa) yield strength were used as stirrups and tie bar to hold the upper flange reinforcement.

In manufacturing the control and the test specimens, the following materials were used: ordinary Portland cement (Type I); crushed gravel with maximum size of (14mm); natural sand from Al-Ukhaider region with maximum size of (4.75mm) and fineness modulus of (2.84); high-range water-reducer superplasticizer (ASTM C-494; Type A and F) [6]; hooked-ends mild carbon steel fibers with average length of (50mm), nominal diameter of (0.5mm), aspect ratio of (100) and yield strength of (1130MPa).

The mix proportions for NSC, HSC and SFRC are presented in **Table (2)**.

Table (2) Proportions of concrete mixes

Parameter	Concrete Type		
	Normal Strength Concrete	High Strength Concrete	Steel Fiber Reinforced Concrete
Water/cement ratio	0.45	0.3	0.45
Water (kg/m^3)	204	145	204
Cement (kg/m^3)	454	485	454
Fine Aggregate(kg/m^3)	771	800	771
Coarse Aggregate(kg/m^3)	907	1120	907
Steel Fiber volume (%)	-	-	1.0
Superplasticizer (L/ m^3)	-	4.85 *	-
Density (kg/m^3)	2336	2550	2415**

* 1 Liter/ 100 kg cement

** Weight of steel fiber is taken into account.

2-4 Test Measurements and Instrumentation

A hydraulic universal testing machine (MFL system) was used to test the beam specimens as will as control specimens. Central deflections were measured by means of (0.01mm) accuracy dial gauge (ELE type) and (30mm) capacity. The dial gauge was placed underneath the bottom face of the test beam at mid-span.

2-5 Test Results of Specimens

Test results of the mechanical properties of the specimens are summarized in **Table (3)**. Compressive strength was carried out on NSC, HSC and SFRC in accordance with ASTM-C39 [7] for cylinders and BS1881-116 [8] for cubes. Flexural strength (modulus of rupture) tests were carried out in accordance with ASTM-C78 [9]. While, the indirect tensile strength (splitting tensile strength) tests were carried out in accordance with ASTM-C496 [10].

Table (3) Mechanical properties of concrete

Property (MPa)	Concrete Type		
	NSC	HSC	SFRC
Cylinder compressive strength (f_c') ¤	41	71.2	44.2
Cube compressive strength (f_{cu}) ¤¤	51.6	77.3	50.4
Modulus of rupture (f_r) *	4.7	7.3	7.65
Splitting tensile strength (f_{ct}) **	3.58	4.8	7.8

¤ Average of (41), (21) and (21) specimens for NSC, HSC and SFRC respectively; using (152x305mm) cylinders.

¤¤ Average of (41), (21) and (21) specimens for NSC, HSC and SFRC respectively; using (150mm) cubes.

* Average of six specimens for each concrete type; using (100x100x500mm) prisms.

** Average of six specimens for each concrete type; using (102x203mm) cylinders.

2-6 Test Procedure

All beam specimens were tested by using the universal testing machine (MFL system) under monotonic loads to ultimate states. The test beams were simply supported over an effective span of (1400mm) and loaded with two-point loads.

The distance between the two point loads was kept constant at (260mm). The applied loads were distributed across the entire width of the upper flange using (ϕ25mm) steel bars under hydraulic jack. Thin wooden strips were inserted between the concrete and line loads to provide even surface. The loads are applied in successive increments up to failure. At the end of each load increment, observations and measurements were recorded for the mid-span deflection and crack development and propagation on the beam surface.

3. Numerical Study

In order to study more thoroughly the performance of hybrid reinforced concrete I-beams under the effect of shear and to simulate all tested beams, ANSYS [11] finite element program is used.

3-1 Finite Element Model

A nonlinear three dimensional brick element (SOLID-65 in ANSYS) is used to model the concrete (NSC, HSC and SFRC). The solid element has eight nodes with three degrees of freedom at each node, translations in the nodal x, y and z-directions. The element is capable of plastic deformation and cracking in three orthogonal directions.

A discrete axial element (LINK-8 in ANSYS) is used to model the steel reinforcement. Two nodes are required for this element; at each node, three degrees of freedom exist identical to those for the brick element.

To avoid stress concentration, (30mm) thick steel plate, modeled by using (SOLID-45 in ANSYS), is added at the support and under the load locations. The element has eight nodes with three degrees of freedom at each node, translations in the nodal x, y and z-directions.

3-2 Materials Properties

3-2-1 Concrete

For the finite element models, compressive uniaxial stress-strain relationship for concrete is described by a multilinear isotropic stress-strain curve. The failure surface is defined by a total of five strength parameters, but it can also be specified by a minimum of two constants (f_t and f_c') with the other three referred as given by **Willam** and **Warnke** criterion [12].

The shear transfer coefficient (β_o) for open cracks and (β_c) for closed cracks, representing conditions of the crack face and determining the amount of shear transferred across the cracks, are used in many studies ranging from (0.0) to (1.0). In this study, (β_o) is assumed to be (0.3) and (β_c) (0.5).

In tension, the stress-strain curve for concrete is assumed to be linearly elastic up to the ultimate tensile strength. The tension stiffening of concrete after cracking is represented by providing a linearly descending branch. Smeared cracking approach is utilized to model the cracking of concrete. Poisson's ratio for concrete is assumed to be (0.2) and it is used for all types of concrete.

The adopted empirical equations of ACI-318, ACI-363 [13] Committee and the two-phase composite material model (Rule of mixture) [14] are used to determine the Moduli of elasticity in the modeling of finite elements, as shown in **Table (4)**.

Table (4) Modulus of elasticity adopted in finite element analysis

Concrete Type	Empirical Equation	f'_c (MPa)	E_c (MPa)	Note
NSC	$E_c = 4700\sqrt{f'_c}$	41	30094.7	ACI-318
HSC	$E_c = 3320\sqrt{f'_c} + 6900$	71.2	34914.2	ACI-363
SFRC	$E_c = E_f V_f + (1-V_f)E_m$	-	31793.7	Rule of Mixture

3-2-2 Steel Reinforcement and Steel Plates

The uniaxial stress-strain relation for steel is idealized as a bilinear curve with Von-Mises yield criterion, representing the elastic-plastic behavior with strain hardening. This relation is assumed to be identical in tension and compression as shown in **Fig.(2)**.

In the present work, the strain hardening modulus (E_T) is assumed to be (0.03 E_s). This value is selected to avoid convergence problems during iteration.

The steel plates are assumed to be linear elastic materials. An elastic modulus equal to (200GPa) and Poisson's ratio of (0.3) are used for the plates and the steel reinforcement.

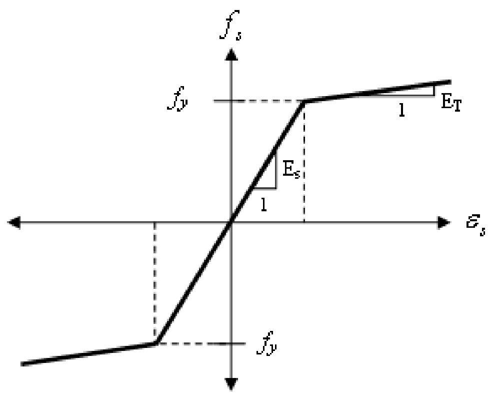

Figure (2) Modeling of reinforcing bars

3-3 Modeling of Beams

The actual dimensions of the tested beams are shown in **Fig.(1)**. By taking advantage of the symmetry of both geometry and loadings, a quarter of the entire model of beam is used for the finite element modeling, **Fig.(3)**.

The origin point of coordinates coincides with the center of the cross-section for the concrete beam. Due to symmetry, only one loading plate and one support plate are needed. The beams, plates and supports are modeled as volumes (solid elements).

The steel reinforcement is simplified in the model by ignoring the shear reinforcement (stirrups). Therefore, only the longitudinal reinforcement (tensile reinforcement) is utilized in the adopted model.

Casting was continuous and accordingly no construction joints occurred between the web and the flanges. Thus, no interface elements are needed in the finite element analysis.

3-3-1 Meshing

To obtain good results, the mesh is set up such that square or rectangular elements are created, **Fig.(3)**.

Due to load concentration directly on concrete elements, in the early attempt (before spreading the load by using steel plates), crushing of the concrete started to develop in elements located directly under loads. Subsequently, adjacent concrete elements crushed within few load steps. As a result, the model showed a large displacement, the solution diverged and finally, the finite element model failed prematurely. Therefore, to prevent this premature failure phenomenon, two techniques are used:

1. Finer mesh is used under applied load and near beam supports.
2. Steel plates are used under load and near beam supports.

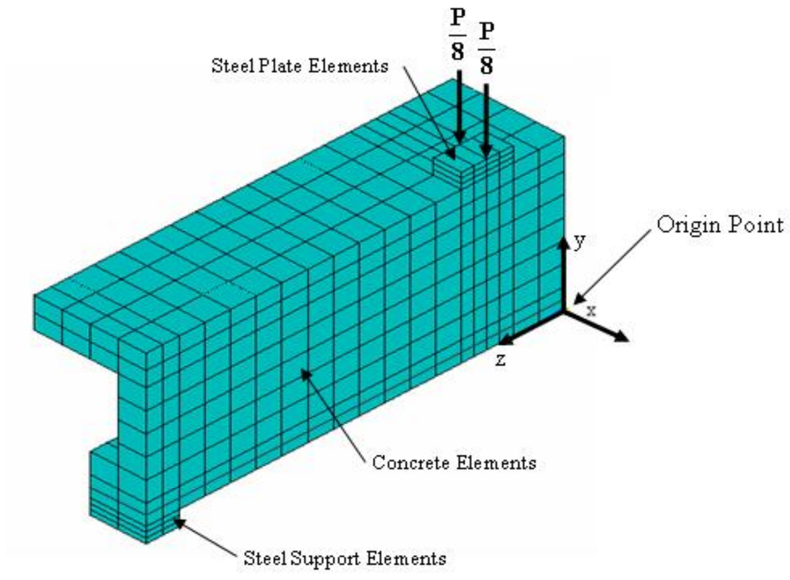

Figure (3) Mesh of the concrete, steel plate, and steel support

3-3-2 Loads and Boundary Conditions

Displacement boundary conditions are needed to constrain the model for obtaining a unique solution. To ensure that the model acts the same way as the experimental beams boundary conditions need to be applied at points of symmetry, and where the supports and loadings exist. This approach reduced computational time and computer disk space requirements.

Planes of symmetry are required at the internal faces. At the plane of symmetry, the displacement in the direction perpendicular to that plane is held zero (hinge), (U_x=0). The displacements in the plane of symmetry constrained by providing rollers along the axes of symmetry ($U_y \neq 0$), **Fig.(4)**.

The external load was applied on the steel plate across the entire centerline of the plate; thus, the external applied load was represented by the equivalent nodal forces on the top nodes of the same plate. Therefore, the equivalent force at each node on the plate was (P/8) of the actual force applied.

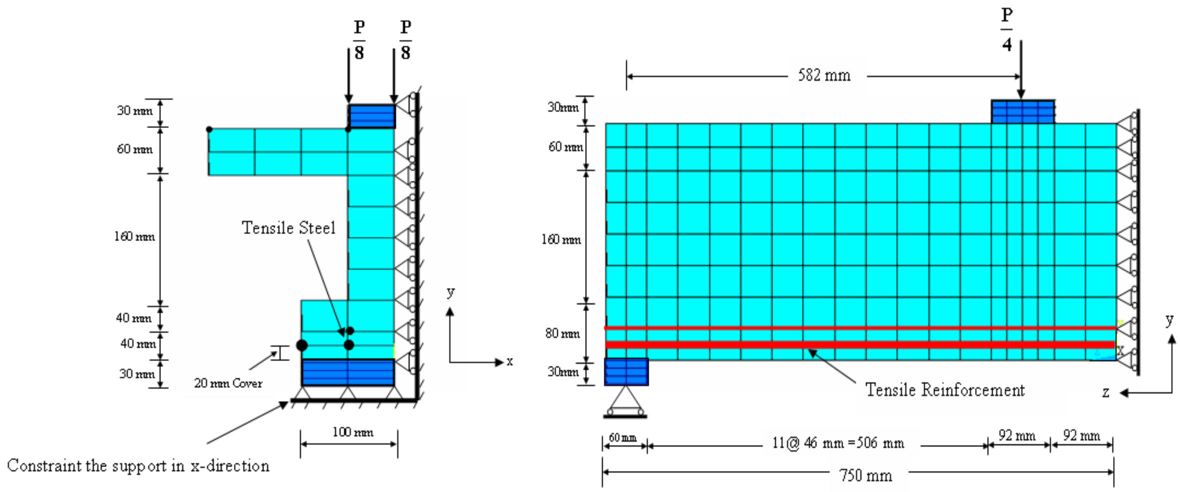

Figure (4) Boundary condition for support in x-direction and plane of symmetry in y-direction (side and front views)

4. Results and Discussion

As mentioned before, the main objectives of this study were to examine or assess the shear behavior of hybrid reinforced concrete I-shaped beams containing different concrete types.

During the experimental work, ultimate loads, loads versus deflections at mid-span were recorded for each beam specimen, and for each load increment. The recorded data, general behavior and tests observations were reported and discussed as follow:

4-1 General Behavior

All beams of this category were designed to fail in shear. At early stage of loading, flexural cracks were observed first in the maximum moment region. As the load was increased, flexural cracks speared into the shear span. Some of these cracks gradually increased in depth and began to incline towards the applied loads. Since all the beams failed in shear, final failure took place by opening up of one of the diagonal cracks over the entire depth of the web. Also, at this stage, some horizontal cracks appeared at the level of the tension reinforcement. These horizontal cracks extended to the end of the beam. This mode of failure indicated a diagonal tension crack mode (splitting failure), as shown in **Fig.(5)**. A diagonal tension crack is defined as a major inclined crack extending from the level of the longitudinal reinforcement to the region of point load [15].

Figure (5) Crack pattern for shear beams

The primary difference between the observed cracking patterns was the angle at which the primary shear crack formed. The crack inclination observed in beams (SB-I and SB-III) was steeper than that observed in beam (SB-II). Another observed difference was the horizontal cracks, clearly, both beams (SB-I and SB-III) had horizontal portions at the ends of diagonal cracks which may indicate to bond-slip failure of tension reinforcement. This phenomenon did not occur in (SB-II), where presence of SFRC in web reduced the damage of concrete and this led to prevent bond-slip failure to take place.

4-2 Failure Mode

The failure modes for all of the tested beams are reported in **Table (5)**. The mode of failure was typical of shear failure with diagonal tension (splitting failure).

The inclined cracks formed suddenly over the entire depth of the web, and extended along the junction of the web and the tension flange, thus splitting the web from the flange and causing the beam to fail instantaneously (for the beams SB-I and SB-III) along a single shear crack.

Table (5) Failure mode and reserve shear strength

Beam No.	Experimental		% Reserve Shear*	Failure Mode
	V_u (kN)	V_{cr} (kN)		
SB-I	104	68	52.9	Diagonal tension
SB-II	130.5	80	63.13	Diagonal tension
SB-III	119.5	75	59.33	Diagonal tension

* Reserve Shear = $\left(\frac{V_u - V_{cr}}{V_{cr}}\right)$

Although the tested beams failed in diagonal tension, the effect of concrete type of the web greatly affected the observed cracking patterns, **Fig.(5)**.

In beams (SB-I and SB-III), which had no steel fibers, the diagonal tension cracks were wide and tended to branch off, whereas in beam (SB-II), the diagonal crack was narrower. Randomly distributed steel fibers in the web were effective after the formation of cracks and continued to resist significant tension until the fibers yielded or pulled out.

For beam (SB-III), which had HSC in the web, the diagonal tension crack was wider in comparison with (SB-I). This may be due to the brittle behavior of HSC which makes the failure to arise suddenly.

4-3 Ultimate Shear Strength (Vu)

The ultimate shear strength of the tested beams were compared with the reference (control) beam, (SB-I), and reported in **Table (6)** in terms of shear force at failure (V_u).

The shear strength of the beam (SB-II), with SFRC in the web, and the beam (SB-III), with HSC in the web was increased (25% and 15%) respectively. For beam (SB-II), the increase in strength was due to the presence of steel fibers, the fibers spanning the micro-cracks, controlled the crack propagation and the rate of widening of cracks, which ultimately led to a higher load carrying capacity.

Table (6) Ultimate Shear Strength

Beam No.	Ultimate shear strength (kN)		$\frac{(V_u)_{FEM}}{(V_u)_{EXP.}}$	$\frac{V_u}{(V_u)_R}$ **
	$(V_u)_{EXP.}$	$(V_u)_{FEM.}$		
SB-I *	104	107.5	1.013	1.0
SB-II	130.5	144.6	1.108	1.25
SB-III	119.5	123	1.029	1.15

* Reference Beam

** $(V_u)_R$= Shear strength of Reference Beam=104kN

For beam (SB-III), employing HSC instead of NSC in the web had increased compressive strength, thus the beam stiffness and the resistance to tensile cracks in web increased and as a result, the overall strength of the beam was increased.

The numerical model predicted an ultimate shear strength of (107.5kN), (144.6kN) and (123kN) for beams (SB-I), (SB-II) and (SB-III) respectively and captured well the shear mode of failure.

When comparing with the experimental values, the numerical models showed (1.3%), (10.8%) and (2.9%) increase in ultimate loads for the beams (SB-I), (SB-II) and (SB-III) respectively.

4-4 Load Deflection Response

The load-deflection response for the tested beams is shown in **Fig.(6)**. By observing the load-deflection characteristics of the tested beams, it is seen that the ductility of the beams increased when the concrete of the web was changed from (NSC) to (HSC) and increased further for beam with (SFRC) in the web.

The ductility of beam specimen (SB-II) was better when compared to those of (SB-I) and (SB-III). Presence of SFRC increased both the load carrying capacity and the ultimate deflection, which is defined as the deflection at which the load resistance drops significantly. Here, ductility is measured as the ratio of deflection at ultimate load to the deflection at first cracking or yielding.

4-5 Reserve Shear Strength

Shear forces corresponding to a diagonal tension cracking (V_{cr}) and failure (V_u) were given in **Table (5)** in addition to the percentage reserve shear strength for all tested beams.

The reserve shear strength, which is defined as the ratio of the difference of ultimate load and diagonal cracking load to the diagonal cracking load [16], is a measure of the reserve strength beyond diagonal cracking. The percentage of reserve shear strengths were (52.9, 63.13 and 59.33%) for the tested beams (SB-1, SB-II and SB-III) respectively.

The tested beam (SB-II) had the greater value, this is due to the crack arrest mechanism of steel fibers which improves the tensile response, and limits the crack propagation, thus improved the reserve strength of this beam.

For beam (SB-III), the presence of HSC in the web improved both the resistance of tensile crack and the reserve strength.

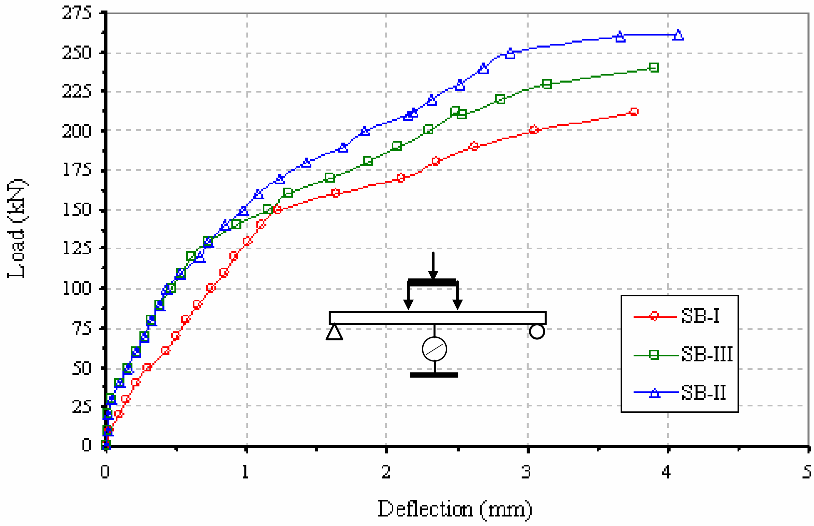

Figure (6) Load-Deflection relationship for shear beams

4-6 Crack Patterns

ANSYS program records a crack pattern at each applied load step. Crack patterns obtained from the finite element analyses and from the failure modes of the experimental beams agree very well, as shown in **Fig.(8)**.

The appearance of the cracks reflects the failure mode of the beams. The finite element model accurately predicts that the beams fail in shear and predicts that diagonal tension cracks are formed diagonally and move up towards the load location.

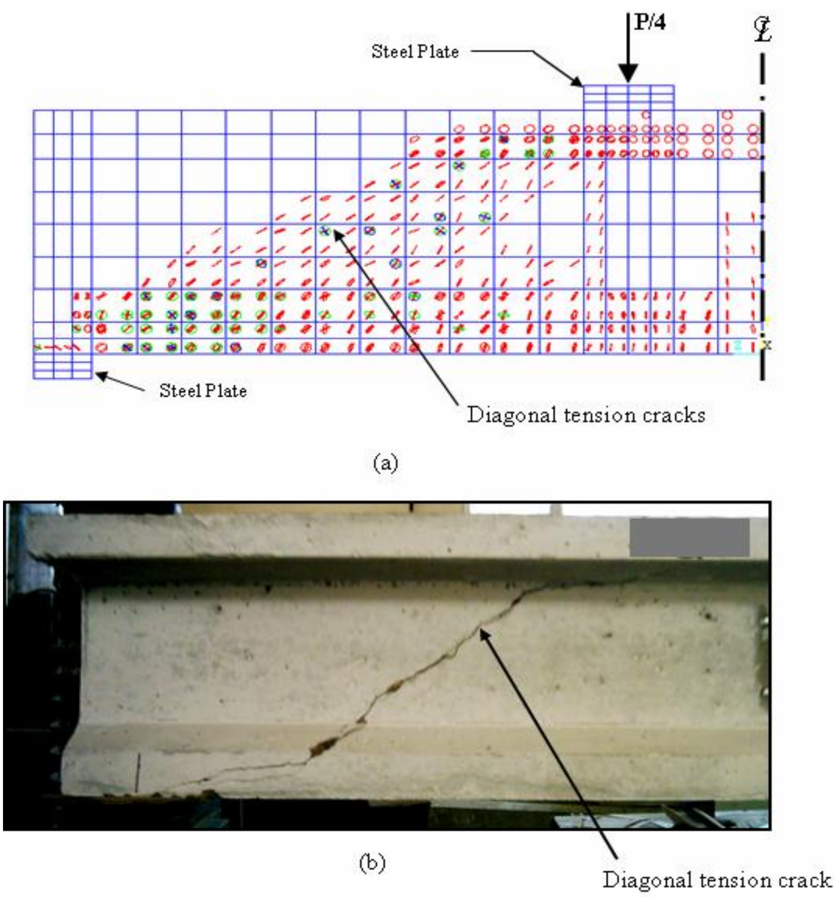

Figure (8) Crack patterns for shear beams, (a) crack pattern from FE model (b) crack pattern of experimentally tested beams

5. Conclusions

Based on the results obtained by the experimental work and the finite element analysis for hybrid I-shaped beams, the following conclusions are presented:

1. The ultimate shear strengths are increased (15%) and (25%) for tested beams with SFRC and HSC in the web respectively. The increase in strength was due to the presence of steel fibers, the fibers are spanning the micro-cracks, and controlling crack propagation and the rate of widening of cracks, which ultimately leads to higher, load carrying capacity.

2. The reserve shear strength is increased (6%) and (10%) for the tested beams with SFRC and HSC in the web respectively. Crack arrest mechanism of steel fibers, and high compressive strength of concrete improve the tensile response, limit the crack propagation, improve the reserve strength and alter the failure mode. So, more safety would be obtained.

3. Presence of SFRC in web reduces the damage of concrete and this leads to prevent or limit the occurrence of horizontal cracks at ends and prevent bond slip in longitudinal reinforcement.

4. The cracks inclination observed in beams that had NSC or HSC in the webs was steeper than that observed in beam with SFRC in the web. This means, the diagonal cracks in SFRC in web will extend to a longer distance than in other webs. Therefore, the diagonal cracks will be passing through two adjacent stirrups in SFRC beams and will not split a beam into two parts between stirrups. As a result, the section becomes more effective to resist shear stresses.

5. The failure mechanism of the reinforced concrete beam was modeled quite well by the finite elements, and the failure loads predicted were very close to the failure loads obtained from the experimental testing. The general behavior by the finite element models represented by the load-deflection plots at mid-span showed good agreement with the test data from the experimentally tested beams. The crack patterns at the final loads from the finite element models corresponded well with the observed failure modes of the experimental beams.

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Notation and Abbreviations

D_f =	steel fiber diameter;
E_c =	modulus of elasticity of concrete;
E_f =	modulus of elasticity of steel fiber;
E_m=	modulus of elasticity of plain concrete (matrix);
E_s=	modulus of elasticity of steel;
E_T =	tangent modulus of elasticity of steel;
f_c' =	cylinder compressive strength of concrete;
f_{cu} =	cube compressive strength of concrete;
f_{ct} =	indirect tensile strength (splitting tensile strength);
f_r =	flexural strength of concrete (modulus of rupture);
f_t =	tensile strength of concrete;
f_s =	steel stress;

f_y=	yield strength of steel;
L_f =	steel fiber length;
V_f =	steel fiber volume fraction;
V_{cr} =	diagonal cracking load;
V_u=	ultimate shear strength;
$(V_u)_{EXP.}$=	ultimate shear strength obtained from experimental tests;
$(V_u)_{FEM.}$=	ultimate shear strength obtained from finite element analysis.
β_o =	shear transfer coefficient for open cracks;
β_c =	shear transfer coefficient for closed cracks;
ε_s =	steel strain;
ϕ =	diameter of reinforcement bar;
ACI =	American Concrete Institute
ASTM =	American Society for Testing and Materials
ASCE =	American Society of Civil Engineering
ANSYS =	Analysis System Program (package)
BS =	British Standards
F.E.M =	Finite Element Method
HSC =	High Strength Concrete
NSC =	Normal Strength Concrete
SFRC =	Steel Fiber Reinforced Concrete