Optimum Design of Control Devices for Safe Seepage under Hydraulic Structures

Asst. Prof. Dr. Abdulhadi A. Al-Delewy
Civil Eng. Dept., College of Engineering
University of Babylon, Hilla, Iraq

Asst. Prof. Dr. Abdul-Hassan K. Shukur
Civil Eng. Dept., College of Engineering
University of Babylon, Hilla, Iraq

Eng. Waked H. AL-Musawi
M.Sc. in Civil Engineering
University of Babylon, Hilla, Iraq

Abstract

The research aimed at determining the optimal control device in order to decrease seepage under hydraulic structures. The finite-element method is used to analyze seepage through porous media below hydraulic structures with blanket, cut-off, or filter trench as seepage control devices. The effect of length and location of the control devices is investigated.

The formulated optimization model is applied to a hypothetical case study. The optimization problem is solved by the Lagrange-multiplier method to find the minimum costs of control devices with safe exit gradient and uplift pressure.

For a hydraulic structure with different control devices, the minimum total cost has been achieved when using a filter trench, while the maximum total cost was when a downstream cut-off is used.

الخلاصـــة

يهدف البحث إلى تحديد وسيلة السيطرة المثلى لتقليل الرشح تحت المنشآت الهيدروليكية. جرى استخدام طريقة العناصر المحددة لتحليل الرشح خلال الأوساط المسامية تحت المنشآت الهيدروليكية بوجود بطانة، جدار قاطع، أو خندق مرشِح كوسيلة للسيطرة على الرشح، وتمّ تحرّي تأثير طول وموقع تلك الوسائل لتحديد الطول والموقع الأمثلين.

لقد تمّ تطبيق نموذج الأمثلية المُصاغ على حالة تطبيقية افتراضية، وجرى حلّ المسألة بطريقة معامل لاكرانج لإيجاد الكلفة الأدنى لوسيلة السيطرة مع المحافظة على كل من انحدار المخرج Exit gradient وضغط الرفع Uplift pressure ضمن الحدود الآمنة.

لقد بيّن البحث أن الكلفة الكليّة الأدنى تكون باستعمال خندق مرشِح بينما كانت الأعلى عند استعمال جدار قاطع عند المؤخر.

1. Introduction

Optimum use of water nowadays cannot be overemphasized. Hydraulic structures are a specific type of engineering structures designed and executed in order to utilize it to control water and ensure the aforementioned objective. The hydraulic structures represent the important part of any flow network. Examples of such structures are dams, regulators, weirs, … etc. The basic aim of these structures is to control the flow discharge and water levels.

The foundation of any hydraulic structure should be given the greatest importance in analysis and design as compared with other parts of the structure, because failure in the foundation would destroy the whole structure.

One of the most important problems that causes damage to hydraulic structures is seepage under the foundations, which occurs due to the difference in water level between the upstream, (U/S), and downstream, (D/S), sides of the structures. The water seeping underneath the hydraulic structure endangers the stability of the structure and may cause failure.

Water seeping under the base of a hydraulic structure starts from the U/S side and tries to emerge at the D/S end of the impervious floor. If the exit gradient is greater than the critical value for the foundation, a phenomenon called piping may occur due to the progressive washing and removal of the fines of the subsoil [1]. Moreover, the uplift force which occurs as a result of the water seeping below the structure exerts an uplift pressure on the floor of the structure. If this pressure is not counterbalanced by the weight of the floor, the structure may fail by rupture of a part of the floor.

The problems of piping and uplift are practically tackled through a variety of methods of seepage control, aiming at ensuring the safety of the respective structure and at the same time saving the possibly-seeping water. The common provisions in this respect are:

1. Upstream blanket.
2. Upstream or/and downstream cut-offs.
3. Subsurface drain on the downstream side.
4. Filter trench on the downstream side.
5. Weep holes, or pressure relief wells on the downstream side.

This research aims at determining the optimal control device to decrease seepage under hydraulic structures.

2. Review of Literature

There are different methods to find the piezometric head distribution under hydraulic structures from which the hydraulic gradient is computed. As a result of the difficulties which were met through empirical and analytical methods, it was resorted to numerical methods. Such methods provide obtaining the required results with a good accuracy such that they are well comparable to the results of the analytical solutions.

Conner and Brebbia [2], solved the seepage problem under a hydraulic structure with two piles resting on isotropic soil by using a finite element method in order to find the distribution of pressure under the base of the structure.

Cechi and Mancino [3] and Zienkiewicz [4] have given application examples of simple cases for seepage under hydraulic structures by using triangular grid of finite element to find the head distribution under the structures.

Nassir [5] applied the finite element method in a wider range. He analyzed the seepage problems under hydraulic structures by using different shapes of the finite elements in order to choose the best solution that gives the most exact results.

Hatab [6] and Ijam and Hatab [7] used the finite element method in the analysis of seepage through homogeneous and isotropic porous media below hydraulic structures with horizontal filters. They also studied the stability of the structures with vertical drains.

Ijam and Nassir [8] used the finite element method to study seepage under hydraulic structures for isotropic soil with two cut-offs.

Khsaf [9] used the finite element method to analyze seepage in isotropic, anisotropic, homogeneous and non-homogeneous soil foundations underneath hydraulic structures provided with flow control devices.

Gill [10] used the theoretical ideas presented by Bennett [11] criteria for optimal design of rectangular, triangular, and trapezoidal blankets.

Hamed [12] used Rosenbrock constrained optimization and sequential unconstrained minimization technique to solve the non-linear programming problem to find the optimum design of barrage floor (area of concrete and reinforcement).

3. The Finite-Elements Scheme

Real groundwater flow, in general, is a three-dimensional, unsteady, laminar flow of a practically-incompressible fluid (the water), through a heterogeneous and anisotropic medium. However, most of the problems of seepage analysis beneath hydraulic structures are usually simplified by assuming a two-dimensional, steady flow through a homogeneous and isotropic medium.

Flow through a saturated porous medium is generally governed by Darcy's law [13]. The governing equation for such a flow is obtained by joining Darcy's law with the equation of continuity. With the aforementioned simplifications, the governing equation for such a flow would be :

Substituting Darcy's Law in the continuity equation for three-dimensional, steady, and incompressible flow results in:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \qquad \text{...... (1)}$$

where: (h) is the piezometric head, (L); (x) and (y) are the horizontal and vertical Cartesian coordinates, respectively.

Thus, the seepage pattern can be completely determined by solving Eq.(1), subject to the boundary conditions of the flow domain.

For the steady state of a confined flow, the boundary conditions are defined as follows:

1. Reservoir Boundaries

The water depth (h_o) above such boundaries is always known; so, the pressure (p) at any point on these boundaries would be:

$$p = \gamma_w \; h_o \quad \text{.......} \quad \textbf{(2)}$$

Therefore, the piezometric head distribution along the reservoir boundaries (S_1) is constant; that is:

$$h = h_o = \frac{p}{\gamma_w} + z \quad \text{.......} \quad \textbf{(3)}$$

For this reason, all the reservoir boundaries are equipotential lines.

2. Impervious Boundaries

At impervious boundaries, (S_2), the water cannot seep through the surface. Thus, the velocity component normal to the boundary, (V_n), must be equal to zero. That is:

$$V_n = k_x \left(\frac{\partial h}{\partial x} \right) L_x + k_y \left(\frac{\partial h}{\partial y} \right) L_y = 0 \quad \text{.......} \quad \textbf{(4)}$$

where: (K_x) and (K_y) are the hydraulic conductivities of the aquifer in the (x) and (y) directions, respectively, [however, for an assumed isotropic aquifer then ($K_x = K_y = K$), where (K) is the average hydraulic conductivity of the aquifer]; (L_x) and (L_y) are the direction cosines of the normal vector on the surface with the directions (x) and (y), respectively. These boundaries represent streamlines of constant stream functions.

Figure (1) illustrates the boundary conditions of a typical problem of seepage under a hydraulic structure.

The basic idea of the finite element method is to discretize the problem domain to sub-domains or finite elements. These elements may be one, two, or three-dimensional and joined to each other by nodes existing on element boundaries; the nodes are regarded as part of the element. After the discretization process, the behavior of the field variable on each element is represented approximately by a continuous function depending on nodal values of the field variable as follows:

$$h^e = \sum_{i=1}^{n} N_i \, h_i \quad \text{.......} \quad \textbf{(5)}$$

where:

h^e = approximate solution for piezometric head distribution in the element (e), (L);
N = shape function of the element (e) ; h_i = nodal values of head of the element (e), (L);
n = number of nodes in the element (e).

Substituting Eq.(5) in Eq.(1) gives:

$$\frac{\partial}{\partial x}\left[k_x \frac{\partial}{\partial x}\sum_{i=1}^{n} N_i h_i\right]+\frac{\partial}{\partial y}\left[k_y \frac{\partial}{\partial y}\sum_{i=1}^{n} N_i h_i\right]=R^e \neq 0 \quad \textbf{(6)}$$

where: R^e = element residual.

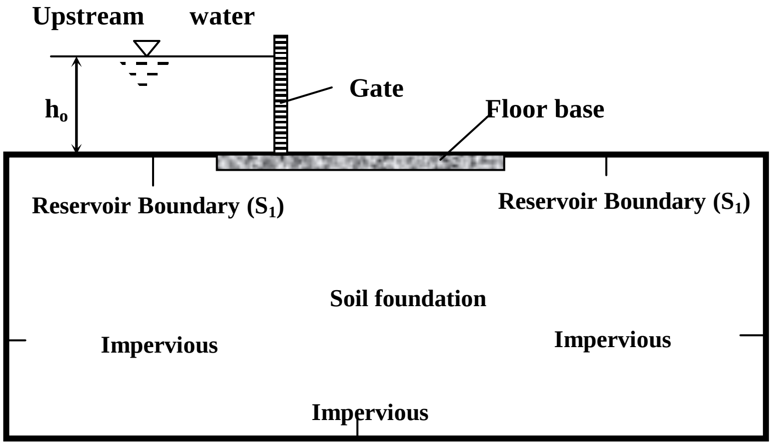


Figure (1) Boundary Conditions of a Typical Problem of Seepage under a Hydraulic Structure

The best solution is the one which makes this residual a minimum or maintains it small at all points of the domain. In order to reach this aim, Eq.(6) should be integrated on the problem domain after weighting by a certain function and should equal zero as follows:

$$\sum_{1}^{n_e}\int_{A^e} W_j R^e \, dA = 0 \quad \textbf{(7)}$$

where: W_j = weighted function.

There are different approaches which can be used, depending on the choice of the weighted function. The best is the one which is known as Galerkin technique where the weighted function is taken equal to the shape function (i.e., $W_j = N_j$) [14].

Then substituting Eq.(6) in Eq.(7) yields:

$$\sum_{1}^{ne}\left[\int_{A^e} N_j^e\left[\frac{\partial}{\partial x}\left(k_x\frac{\partial}{\partial x}\sum_{i=1}^{n}N_i h_i\right)+\frac{\partial}{\partial y}\left(k_y\frac{\partial}{\partial y}\sum_{i=1}^{n}N_i h_i\right)\right]dA\right]=0 \quad \text{……………………. (8)}$$

where: dA = dx .dy; (j=1, 2, …, n); n = number of nodes for each element.

To reduce continuity requirements for the shape function, (N), from (C^1-continuity) to (C^o-continuity), integration by parts with Green's theorem is applied to the second order derivatives terms, where (C^1) and (C^o) are the continuity for the shape function for the first and zero stage, respectively [15].

Accordingly, the first term of Eq.(8) will be:

$$\int_{A^e} N_j^e\frac{\partial}{\partial x}\left(k_x\frac{\partial}{\partial x}\sum_{1}^{n}N_i h_i\right)dA=\int_S N_j^e k_x\frac{\partial}{\partial x}\sum_{1}^{n}N_i h_i\,dy-\int_{A^e}\frac{\partial N_j^e}{\partial x}k_x\frac{\partial}{\partial x}\sum_{1}^{n}N_i h_i\,dA \quad \text{….. (9)}$$

The second term of Eq. (8) will be:

$$\int_{A^e} N_j^e\frac{\partial}{\partial y}\left(k_y\frac{\partial}{\partial y}\sum_{1}^{n}N_i h_i\right)dA=\int_S N_j^e k_y\frac{\partial}{\partial y}\sum_{1}^{n}N_i h_i\,dx-\int_{A^e}\frac{\partial N_j^e}{\partial y}k_y\frac{\partial}{\partial y}\sum_{1}^{n}N_i h_i\,dA \quad \text{… (10)}$$

Substituting Eqs.(9) and (10) in Eq.(8) results in:

$$\sum_{1}^{n_e}\left[\int_{A^e}-\left(\frac{\partial N_j}{\partial x}k_x\frac{\partial}{\partial x}\sum_{1}^{n}N_i h_i+\frac{\partial N_j}{\partial y}k_y\frac{\partial}{\partial y}\sum_{1}^{n}N_i h_i\right)dA\right]+\int_S N_j k_n\frac{\partial}{\partial n}\sum_{1}^{n}N_i h_i\,ds \quad \text{… (11)}$$

where: ($S = S_1^e + S_2^e$) represents the surface boundaries of the element.

Then applying the boundary conditions in Eq. (11) gives:

$$\sum_{1}^{n_e}\left[\int_{A^e}\left(\frac{\partial N_j}{\partial x}k_x\frac{\partial}{\partial x}\sum_{1}^{n}N_i h_i+\frac{\partial N_j}{\partial y}k_y\frac{\partial}{\partial y}\sum_{1}^{n}N_i h_i\right)dx\,dy-\int_{S_1^e}\left(N_j k_x\frac{\partial}{\partial x}\sum_{1}^{n}N_i h_i L_x+N_j k_y\frac{\partial}{\partial y}\sum_{1}^{n}N_i h_i L_y\right)ds\right]=0 \quad \text{…………………….. (12)}$$

and in matrix form:

$$\sum_{1}^{n_e}\left[K^e\right]\{h_i\}=0 \quad \text{...}\ \textbf{(13)}$$

where: $[K^e]$ represents the element matrix:

$$\left[K^e\right]=\int_{A^e}\left[B^e\right]^T\left[D^e\right]\left[B^e\right]dx\,dy=0 \quad \text{..}\ \textbf{(14)}$$

where:

$$\left[B^e\right]=\begin{bmatrix}\dfrac{\partial N_1^e}{\partial x} & \dfrac{\partial N_2^e}{\partial x} & \cdots & \dfrac{\partial N_n^e}{\partial x}\\ \dfrac{\partial N_1^e}{\partial y} & \dfrac{\partial N_2^e}{\partial y} & \cdots & \dfrac{\partial N_n^e}{\partial y}\end{bmatrix}$$

$$\left[D^e\right]=\begin{bmatrix}k_x & 0\\ 0 & k_y\end{bmatrix}$$

and from assemblage:

$$[K]\ \{h\}=0 \quad \text{...}\ \textbf{(15)}$$

where, [K] is the global matrix = $\sum\left[K^e\right]$

The assembled equation, Eq.(15), is solved using a frontal solution because of its efficiency in computer storage requirement [16].

In the present work, two-dimensional quadratic Isoparametric elements are used which have eight nodes, as shown in **Fig.(2)**.

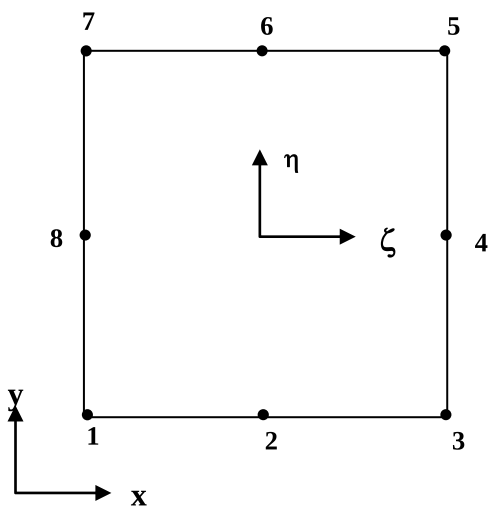


Figure (2) A Quadratic Isoparametric element

From the value of the piezometric head at the nodes, it is possible to compute the hydraulic gradient directly without the need for transformation. That is:

$$h = \sum N_i h_i \qquad i = 1,2,\ldots,8$$

$$h = N_1 h_1 + N_2 h_2 + \ldots + N_8 h_8 \quad \textbf{(16)}$$

Therefore, $\frac{\partial h}{\partial x}$ and $\frac{\partial h}{\partial y}$ can be written as follows:

$$\left.\begin{aligned} \frac{\partial h}{\partial x} &= \frac{\partial N_1}{\partial x} h_1 + \frac{\partial N_2}{\partial x} h_2 + \ldots + \frac{\partial N_8}{\partial x} h_8 \\ \frac{\partial h}{\partial y} &= \frac{\partial N_1}{\partial y} h_1 + \frac{\partial N_2}{\partial y} h_2 + \ldots + \frac{\partial N_8}{\partial y} h_8 \end{aligned}\right\} \quad \textbf{(17)}$$

In matrix form, the above equation becomes:

$$\begin{Bmatrix} \frac{\partial h}{\partial x} \\ \frac{\partial h}{\partial y} \end{Bmatrix} = \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \cdots & \frac{\partial N_8}{\partial x} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_2}{\partial y} & \cdots & \frac{\partial N_8}{\partial y} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_8 \end{bmatrix} \quad \textbf{(18)}$$

or:

$$\begin{Bmatrix} \frac{\partial h}{\partial x} \\ \frac{\partial h}{\partial y} \end{Bmatrix} = [J]^{-1} \begin{bmatrix} \frac{\partial N_1}{\partial \zeta} & \frac{\partial N_2}{\partial \zeta} & \cdots & \frac{\partial N_8}{\partial \zeta} \\ \frac{\partial N_1}{\partial \eta} & \frac{\partial N_2}{\partial \eta} & \cdots & \frac{\partial N_8}{\partial \eta} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_8 \end{bmatrix} \quad \textbf{(19)}$$

where: [J] is the Jacobian matrix.

The solution of Eq.(19) will give the hydraulic gradient in the x and y-directions.

4. The Optimization Problem

The purpose of optimization is to find the best possible solution among the many potential solutions satisfying the chosen criteria. Designers often base their designs on the minimum cost as an objective, taking into account mainly the costs of foundation, safety, and serviceability.

A general mathematical model of the optimization problem can be represented in the following form:

A certain function (Z), called the objective function,

$$Z = f\{x_i\} \qquad I = 1, 2, \ldots, n \quad \ldots\ldots \textbf{(20)}$$

which is usually the expected benefit (or the involved cost), involves (n) design variables {x}. Such a function is to be maximized (or minimized) subject to certain equality or inequality constraints in their general forms:

$$g_i\{x_i\} = b_i \qquad ; i = 1, 2, \ldots, I \quad \ldots\ldots \textbf{(21)}$$

$$q_j\{x_j\} \geq b_j \qquad ; j = 1, 2, \ldots, J \quad \ldots\ldots \textbf{(22)}$$

The constraint reflects the design and functional requirements. The vector {x} of the design variables will have optimum values when the objective function reaches its optimum value.

Of the many optimization techniques available, the formulated problem lends itself as a nonlinear model.

4-1 Design Variables

The design variables are taken as follows [see **Fig.(3)**]:

1. Difference head (H): (2, 4, 6, 8, 10, 12 and 14m).
2. Length of floor (b): $3\ m \leq b < 50\ m$.
3. Thickness of the floor base (t): Calculated by Eq.(24).
4. Length of upstream blanket (b_1): Calculated from the floor length value in order to avoid the uplift pressure and critical exit gradient.
5. Depth of the upstream cut-off (d_1): 0 – 12 m; step 0.5 m.
6. Depth of the downstream cut-off (d_2): 0 – 12 m; step 0.5 m.
7. Filter trench: with filter and without filter.

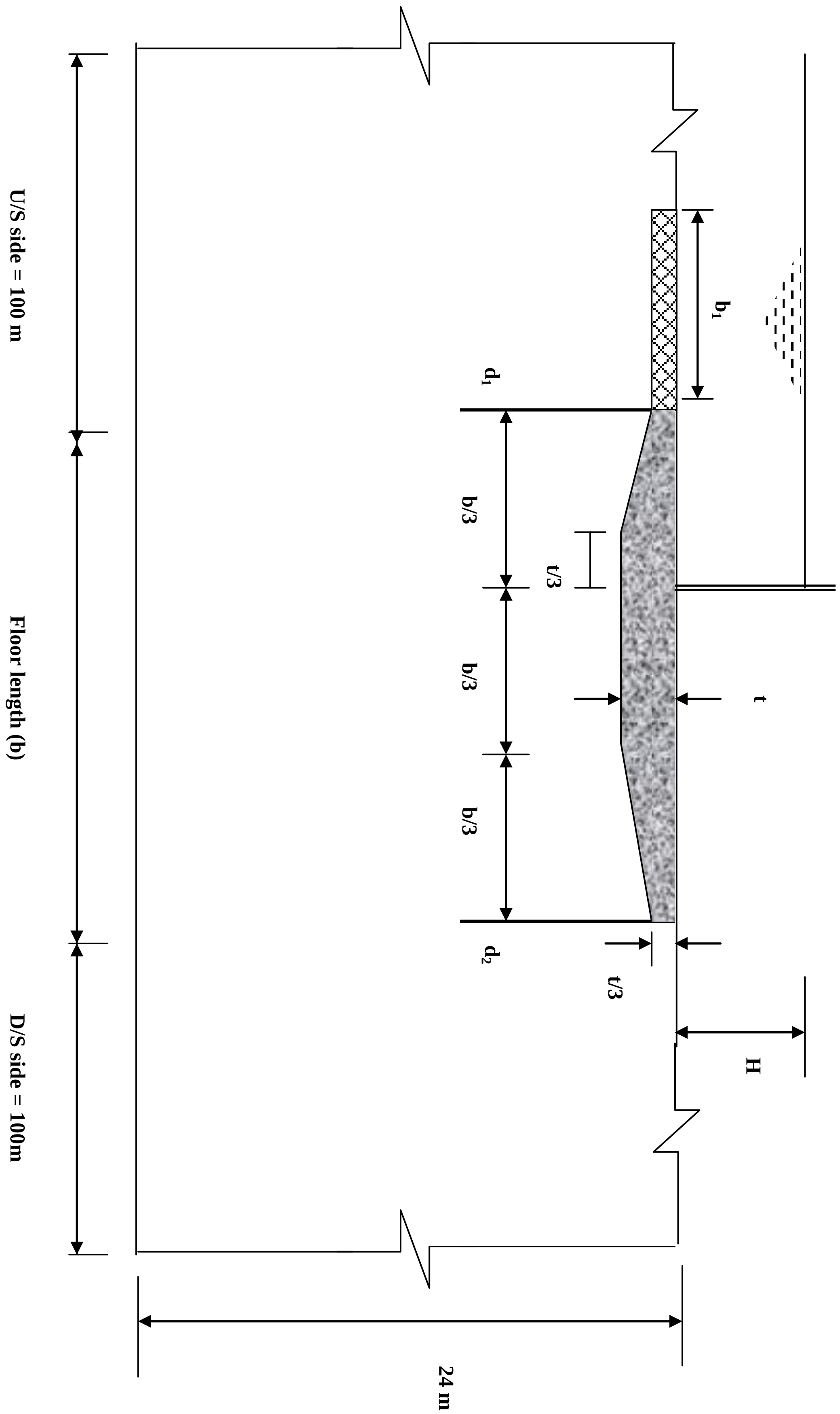


Figure (3) Illustrative Sketch of a Hypothetical Case Study

4-2 The Objective Function

The cost objective function (Z) of the present research involves the cost of both floor and any control device. Such a function is formulated as follows:

$$Z = c_1 a + c_2 d_1 + c_2 d_2 + c_3 b_1 + c_4 (w.z) \quad \text{.. (23)}$$

where:

c_1 = cost of one cubic meter of the floor base material, (10^6 I.D. / m^3);
a = area of floor base, (m^2);
c_2 = cost of one square meter of cut-off, (10^6 I.D. / m^2);
d_1 ,d_2 = depths of upstream and downstream cut-offs, respectively, (m);
c_3 = cost of one square meter of upstream blanket, (10^6 I.D. / m^2);
b_1 = length of upstream blanket, (m);
c_4 = cost of one cubic meter of the filter trench, (10^6 I.D. / m^3);
w = width of filter trench, (m); and
z = depth of filter trench, (m).

4-3 Constraints

The objective function is to be minimized, subject to the following constraints:

1. The ratio of the length of any control device used to the length of floor base for a certain differential head (H) should be such that a safe exit gradient (used as 0.25 in this research) will be ensured.
2. The floor thickness, with any control device used, shall be limited to a certain value to help in counteracting the uplift water pressure. This requirement is stated as follows:

For equilibrium: Uplift pressure = Downward pressure; that is: $\gamma_w(h+t) = \gamma_w.G_c.t$ or:

$$t = \frac{h}{G_c - 1} \quad \text{.. (24)}$$

where:

t = thickness of floor base under the gates , (L);
h = head at any point under the hydraulic structure, (L);
G_c = specific gravity of concrete (the floor material) = 2.4; and
γ_w = unit weight of water, (F/L^3).

3. The cut-off depth should be equal to or less than (12 m).

4-4 Method of Optimization

The method of optimization employed in this research is the Lagrange multiplier method. The Lagrangian would be:

$$L = F(\{x\}) + \sum_{i=1}^{m} \lambda_i \, g_i(\{x\}) + \sum_{j=i+1}^{m} \lambda_j \left\{ q_j(\{x\}) - \theta_j^2 \right\} \quad \text{..................................... (25)}$$

where: (m) denotes the total number of constraint equations, (λ_1, λ_2, …, λ_m) are the Lagrange multipliers, and(θ) is a dummy variable. The Lagrangian involves the original (n) variables plus (m) number of (λ) variables and a similar number of (θ) variables, making a total of (n+2m) unknowns.

Differentiating the Lagrangian with respect to each of the stated unknowns, one at a time, and equating each formulated derivative to zero (for the minimization goal) will yield (n+2m) equations as follows:

$$\frac{\partial L}{\partial x_j} = \frac{\partial F}{\partial X_j} + \sum_{i=1}^{m} \lambda_i \frac{\partial g_i}{\partial x_i} - \sum_{j=1+1}^{m} \lambda_j \frac{\partial q_j}{\partial x_j} = 0 \quad \text{.. (26)}$$

$$\frac{\partial L}{\partial \lambda_i} = g_i = 0 \quad \text{.. (27)}$$

$$\frac{\partial L}{\partial \theta_j} = 2\lambda_j \theta_j = 0 \quad \text{.. (28)}$$

Solving Eqs. (26), (27), and (28) simultaneously gives the required optimal solution.

5. Analysis of the Results

In this research, different cases were analyzed to investigate the effect of length and location of control devices (blanket, cut-off, and filter trench) on uplift pressure and exit gradient by using the finite-element method. **Figure (4)** shows the discretization mesh of the finite-elements of a hydraulic structure with an upstream blanket (b_1), downstream blanket (b_2), upstream cut-off (d_1), downstream cut-off (d_2), and filter trench.

In order to determine the optimum design of control devices, many cases were examined using the Lagrange multiplier method and cost calculation curves with various values of active head (H). For this purpose, the difference ratio of piezometric head is considered, that is:

$$H^* = \frac{h - h_2}{h_1 - h_2} \quad \text{.. (29)}$$

where:

H^* = ratio of head difference;

h = piezometric head at any point in the flow domain, (L);

h_1, h_2= piezometric head (U/S) and (D/S) of the structure, respectively, (L). Consequently, (H^*) equals (one) for the upstream bed and (zero) for the downstream bed.

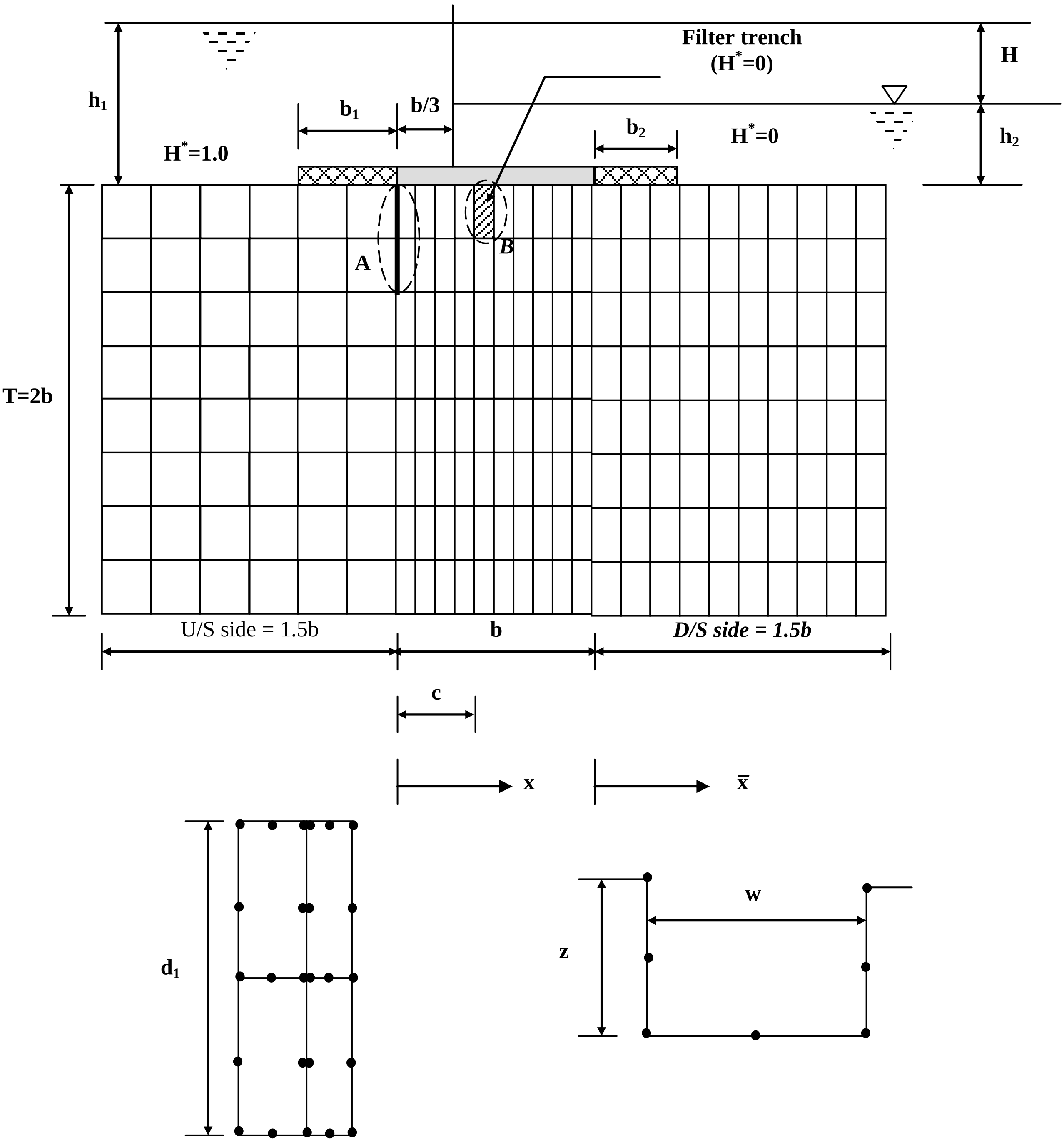


Figure (4) The Finite Element Mesh Discretization of a Hydraulic Structure with U/D Blankets, U/S Cut-Off, and Filter Trench

Figure (5) shows the effect of using an upstream or downstream blanket on the uplift pressure. It has been noted that the uplift pressure is decreased when using an upstream blanket, as compared with the case without blanket, other parameters being unchanged. Also, the uplift pressure increases with using a downstream blanket. The same aforementioned effect can be noticed on the exit gradient and as it is shown in **Fig.(6)**.

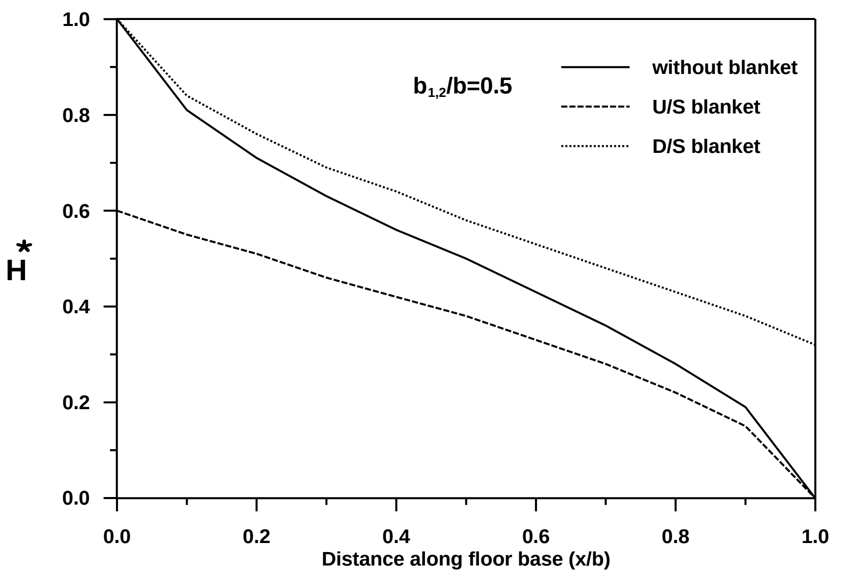


Figure (5) The Effect of using the U/S or D/S Blanket on the Uplift Pressure

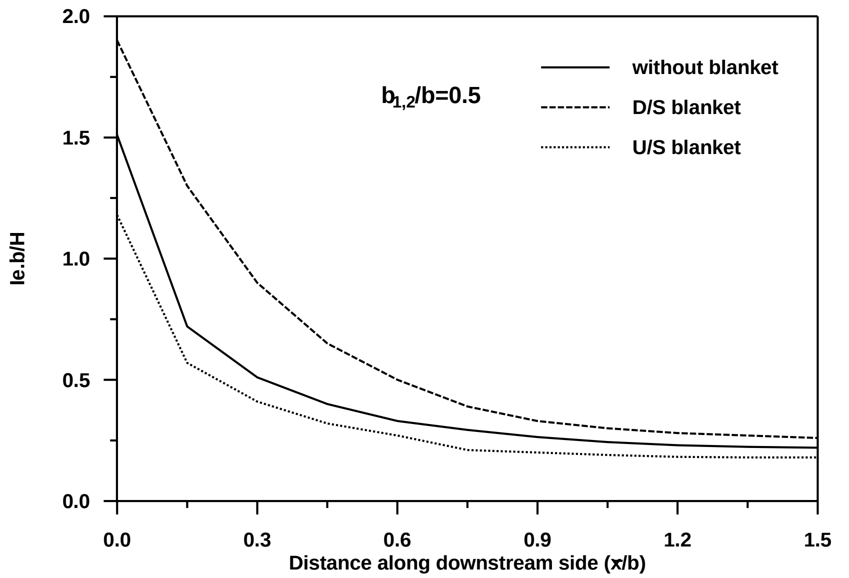


Figure (6) The Effect of using the U/S or D/S Blanket on the Exit Gradient

Figures (7) and **(8)** show a comparison between the use of upstream cut-off, downstream cut-off, or two cut-offs at upstream and downstream (U/D). It has been noted from **Fig.(7)** that the upstream cut-off is more effective in reducing the uplift pressure. However, **Fig.(8)** indicates that the downstream cut-off is more effective than the upstream cut-off in terms of the reduction of the exit gradient. Moreover, using the two cut-offs at the ends is more effective than using the downstream cut-off only.

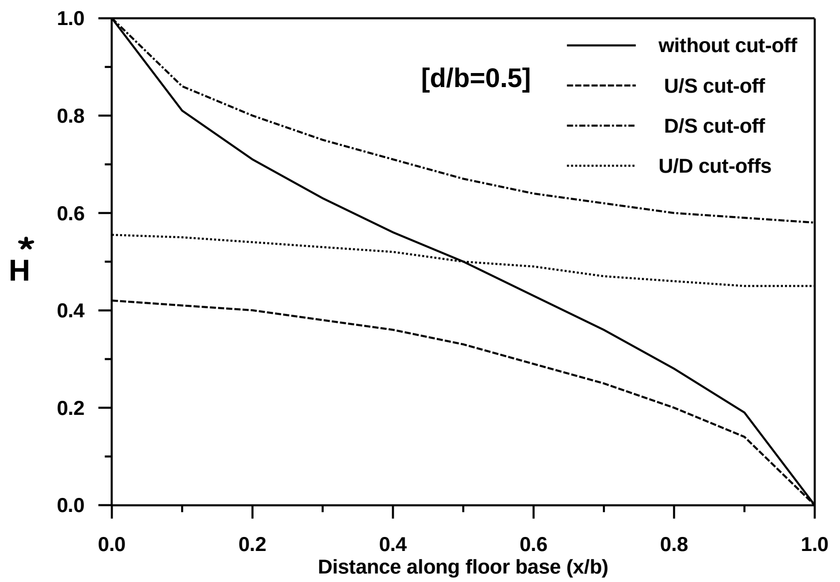


Figure (7) Effect of using U/S, D/S and U/D Cut-Offs on the Uplift Pressure Distribution under a Hydraulic Structure, (d/b=0.5)

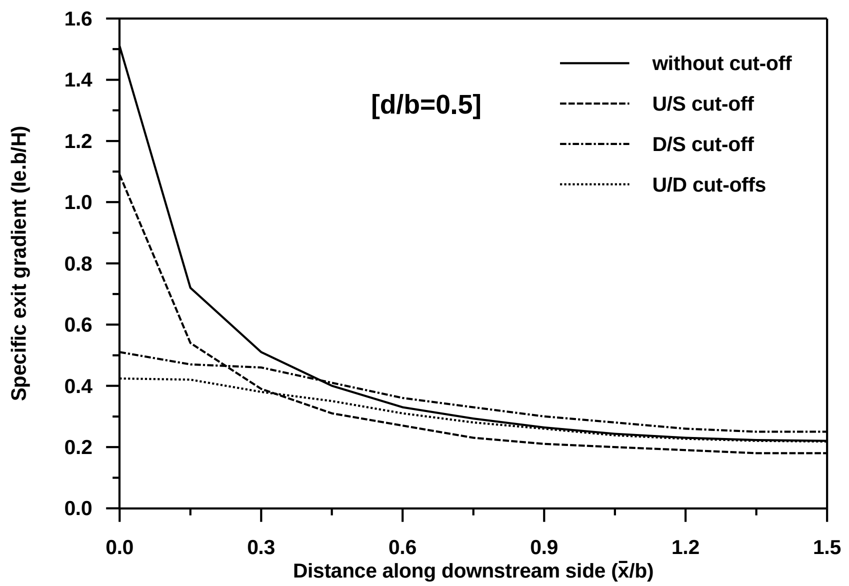


Figure (8) Effect of using U/S, D/S and U/D Cut-Offs on the Exit Gradient Distribution D/S of a Hydraulic Structure, (d/b=0.5)

Figure (9) illustrates the distribution of the uplift pressure along the floor for various (width/depth) ratio of filter trench with the same sectional area of filter. It has been noted that the uplift pressure decreases with increasing (w/z) ratio for the same sectional area of filter. **Figure (10)** shows the effect of different (w/z) ratio of filter on the exit gradient with the same sectional area of filter. This figure indicates that the exit gradient decreases with increasing the (w/z) ratio.

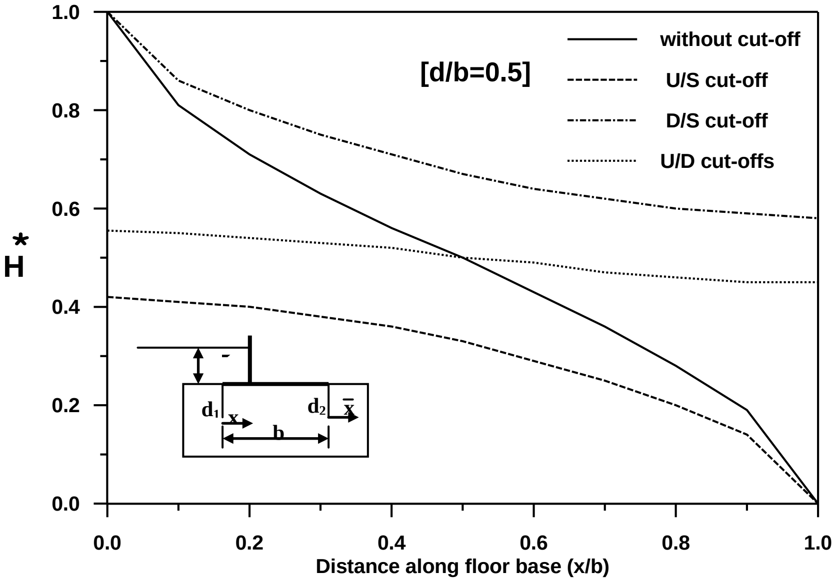


Figure (9) Effect of using U/S, D/S and U/D Cut-Offs on the Uplift Pressure Distribution under a Hydraulic Structure, (d/b=0.5)

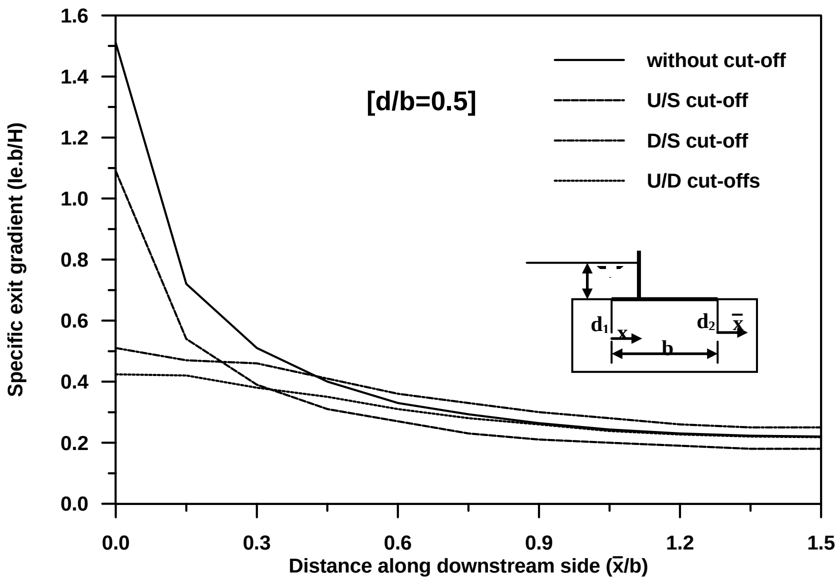


Figure (10) Effect of using U/S, D/S and U/D Cut-Offs on the Exit Gradient Distribution D/S of a Hydraulic Structure, (d/b=0.5)

6. The Optimum Design

In this research, the optimization process considers the hypothetical case shown in **Fig.(3)**. The aim is to calculate the optimum design to decease the seepage under the shown structure by using different control devices.

The hydraulic structure would be safe when the existing exit gradient (I_e) is less than the critical gradient (I_c).

The existing exit gradient is defined as in Eq.(19) whereas the critical exit gradient is defined as:

$$I_c = \frac{G_s - 1}{1 + e} = \frac{\bar{\gamma}}{\gamma_w} \quad \textbf{(30)}$$

where:

I_e = exit gradient;
I_c = critical exit gradient;
G_s = specific gravity of the soil;
e = voids ratio of the soil;
$\bar{\gamma}$ = Submerged unit-weight of the soil؛
γ_w = unit weight of water.

Practical safety is ensured when:

$$Fs = \frac{I_c}{I_e} > 1 \quad \textbf{(31)}$$

where: (Fs) = factor of safety against piping.

Harza (1935) [quoted in Janice, 1976] mentioned that a factor of safety between (3) and (4) is enough for the safety of the structure. A factor of safety of (Fs = 4) has been chosen for use in this research.

The effect of floor length (b) on the exit gradient with different values of the head difference (H) is shown in **Fig.(11)**. The figure indicates that the exit gradient decreases as the length of the floor increases. This figure could be used to find the safe length of floor with any head difference applied.

The relationship between the cost of floor and its length (b) for each head difference is direct, i.e., cost increases with the increase of the floor length, as shown in **Fig.(12)**.

Figure (13) shows the total costs (Z) for upstream blanket length (b_1) and floor length (b) with safe exit gradient. This figure could be used to find the necessary upstream blanket length (b_1) for any floor length (b). Moreover, the total cost is decreased with increased upstream blanket length (b_1), but it increases with increased floor length (b) for all the considered values of head difference (H).

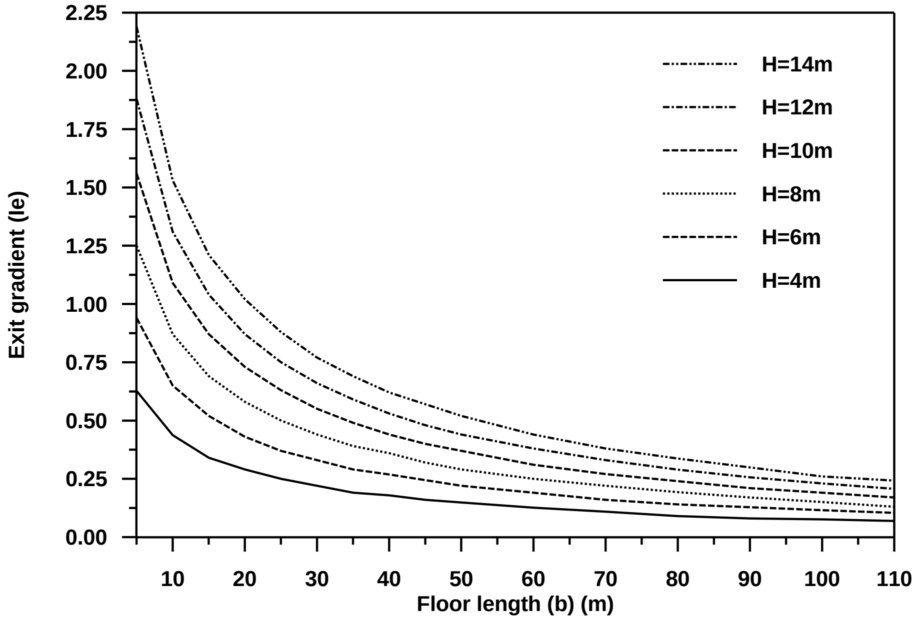


Figure (11) Effect of Floor Length (b) on the Exit Gradient for Various Head Difference (H)

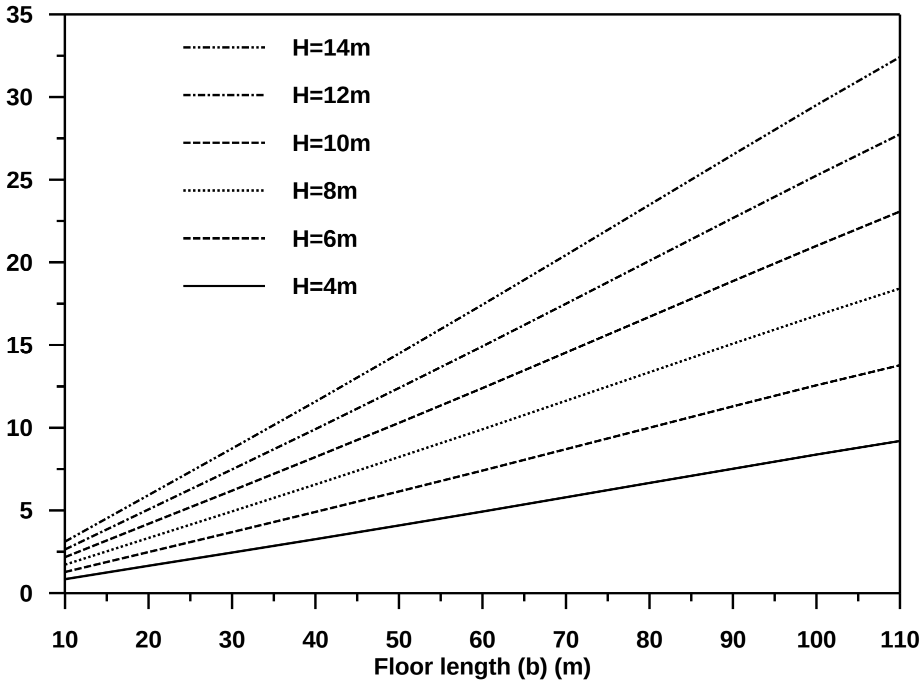
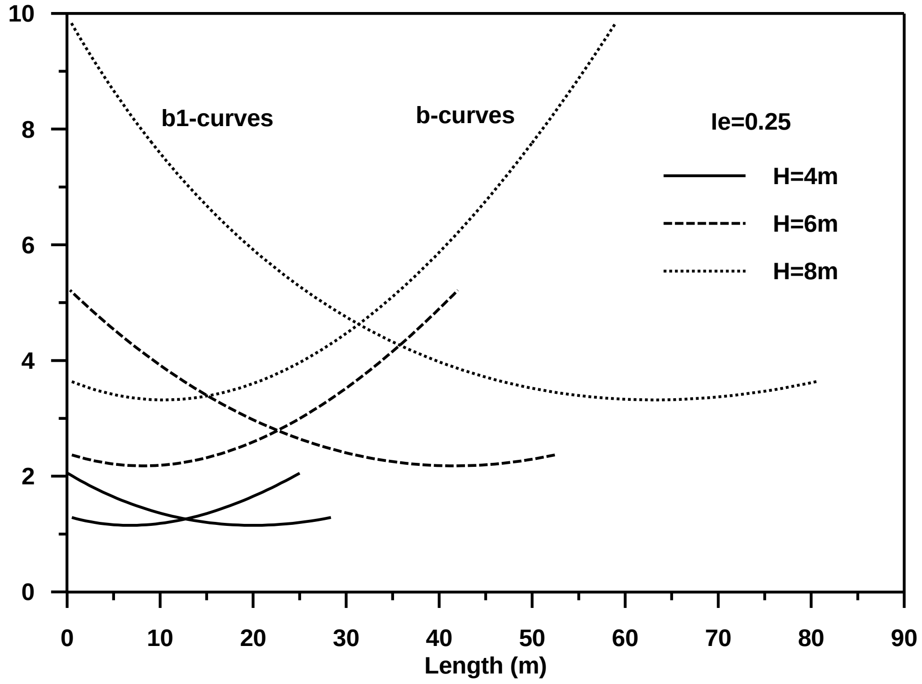
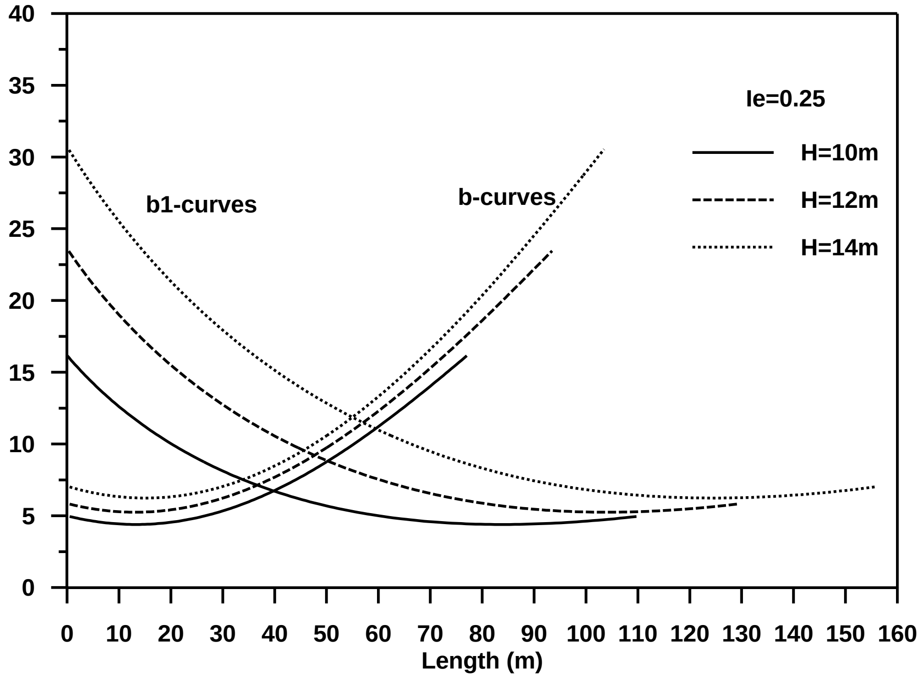


Figure (12) Cost of Floor for Various Head Difference



(a) For (H=4 , 6 , and 8 m)

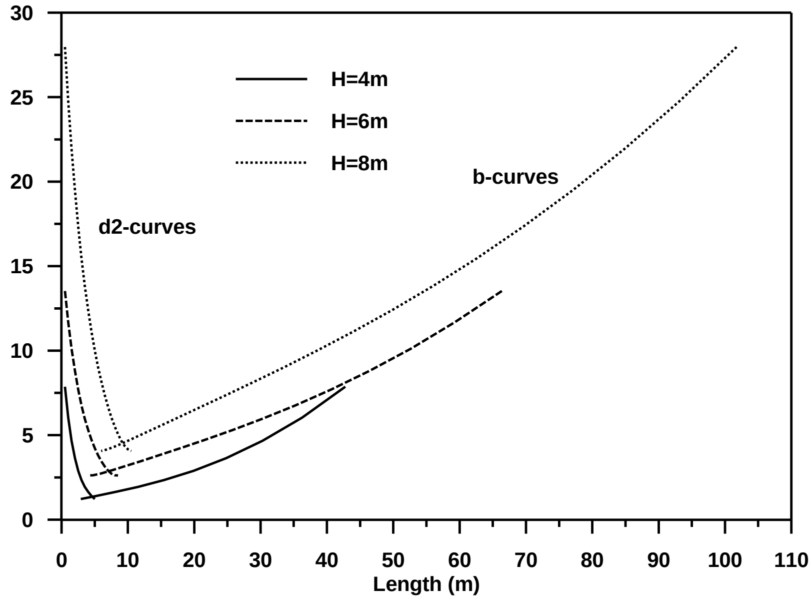


(b) For (H=10 , 12 , and 14 m)

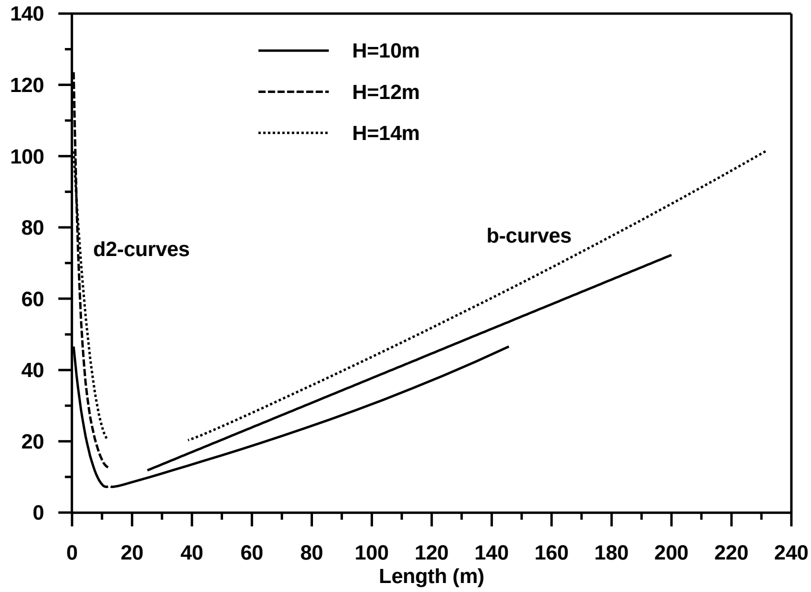
Figure (13) Total Cost for U/S Blanket Length(b1) and Floor Length(b),with Safe Exit Gradient

Figure (14) shows the total cost (Z) for both downstream cut-off and the floor, for the considered head difference. The total cost decreases with the increase of the downstream cut-off depth whereas it increases with the increase of floor length. The figure could be used to find the necessary downstream cut-off depth (d_2) for any floor length (b).

For each head difference value (H), an optimum design (minimum total cost) could be found from the feasible solutions.



(a) For (H= 4 , 6 , and 8 m)



(b) For (H= 10 , 12 , and 14 m)

Figure (14) Total Cost for Both D/S Cut-Off (d2) and the Floor (b), with Safe Exit Gradient

Table (1) summarizes the final results of running the optimization model. The table indicates that the filter trench gives a more economical cost than the upstream blanket, upstream cut-off and downstream cut-off.

Table (1) Optimum Design of Control Devices (Lagrange-Multiplier Method)

Diff . Head (H) (m)	Floor Length (b) (m)	U/S Blanket (b_1) (m)	U/S Cut-Off (d_1) (m)	D/S Cut-Off (d_2) (m)	Total Cost (10^6 in I.D./m)	
					With Filter Trench	Without Filter Trench
4	3.0	8	3	5	1.0504	1.2503
6	8.5	10	3.5	6.5	1.7894	2.1973
8	15	13	4.5	7	2.7437	3.4261
10	20	15	6	8.5	3.3936	5.2810
12	32	18	9	9.5	4.2226	7.1978
14	40	22	10	11	5.6693	11.4581

7. Conclusions

The following conclusions have been abstracted:

1. The uplift pressure decreases with the increase in U/S length of the blanket; the same holds true with the exit gradient. However, the uplift pressure and exit gradient increase when using a D/S blanket.
2. The D/S cut-off is more effective than the U/S cut-off in terms of the reduction of the exit gradient.
3. The pressure head U/S and D/S of the filter trench decreases as the width or depth of filter increases; this remains true with the exit gradient. Also, the exit gradient decreases as the filter trench is moved from U/S to D/S.
4. The uplift pressure and exit gradient would decrease with increasing the (width / depth) ratio for the same sectional area of filter.
5. For a hydraulic structure with U/S blanket of various lengths, a blanket of (zero) length gives the maximum floor cost for all applied head differences.
6. For a hydraulic structure with U/S cut-off of various depths, the total cost decreases with increasing the depth of the U/S cut-off and decreasing the length of the floor. The minimum total cost is attained when the maximum U/S cut-off depth is used. This remains true regardless of using a D/S cut-off.
7. Using the U/S cut-off is more economical than using the D/S cut-off.
8. For a hydraulic structure with different control devices, the minimum total cost could be achieved when a filter trench is used.

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