Image coding using Wavelet Theory

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Abstract

In this paper image coding for (Computer Generated Hologram) CGH technique, is investigated ideal analysis is undertaking to build and to constructed hologram by using wavelet theory and the images taking in to construction binary cell oriented holograms. This model is used to test performance for the image of CGH. The success of any transform coding technique depends on how well the biases functions represent the signal features.

The discrete wavelet transform (DWT) performs analysis of a signal; this enables an efficient representation of smooth and detailed signal regions. Furthermore, computationally efficient algorithms exit for computing the DWT. For this reasons, recent image standards such as TIF us the wavelet transform. It is well known that orthogonally and symmetry is desirable transform properties in image CGH application.

This paper discus analyzes the impact of these wavelet characteristics on image performance; the analysis allows us to the first reasons for the small performance gap between the scalar wavelet and another method.

All the results indicate the wavelet properties that more important to image properties, Moreover, the CGH quality results depict similar performance for the best wavelet technique.

الخــلاصـــــة

في هذا البحث نستخدم طريقة تشفير الهولوكرامات باستخدام تقنية تحويل المويجة و كيفية بناء هذا الهولوكرام لصورة معينة. أن نجاح أي تقنية للتشفير تعتمد على هكذا طريقة وتعتمد أيضا على طريقة التمثيل الجيد للميزات البارزة للصورة.

أن طريقة DWT تحلل الإشارة التي يمكن تمثيلها بنعومة وان المناطق البارزة في الصورة يمكن وصفها بهذه الطريقة علاوة على ذلك يمكن تمثيلها على شكل خوارزمية ، ولهذا السبب أخذنا معيار الصورة من نوع TIF في تحويلات wavelet.

في هذا البحث تم تحليل الصورة بطريقة (wavelet)، يسمح لنا هذا التوضيح بتحليل الصورة للميزات المذكورة للاداء الجيد في هذه الطريقة.

كل النتائج تشير إلى خواص طريقة wavelet لما لها من خواص في السرية والأمنية العالية كما ولها أهمية لوصف خواص الصورة، علاوة على ذلك، أن نتائج CGH تكون بأفضل أداء عندما تكون ممثلا بطريقة wavelet مما عليه في بقية الطرق .

1. Introduction

For image of CGH method of optical imagery is not really new. Nearly five decades ago British research scientist Dennis Gabor (1948) first conceived of, as he called it, "a new two-step method of optical imagery" [1,2]. However, it is only in the past a few years that his method has become widely know and used.

An image of original object can be obtained from the recorded interference pattern, Denis Gabor called the recorded interference pattern a hologram as shown in **Fig.(1)**, meaning total recording.
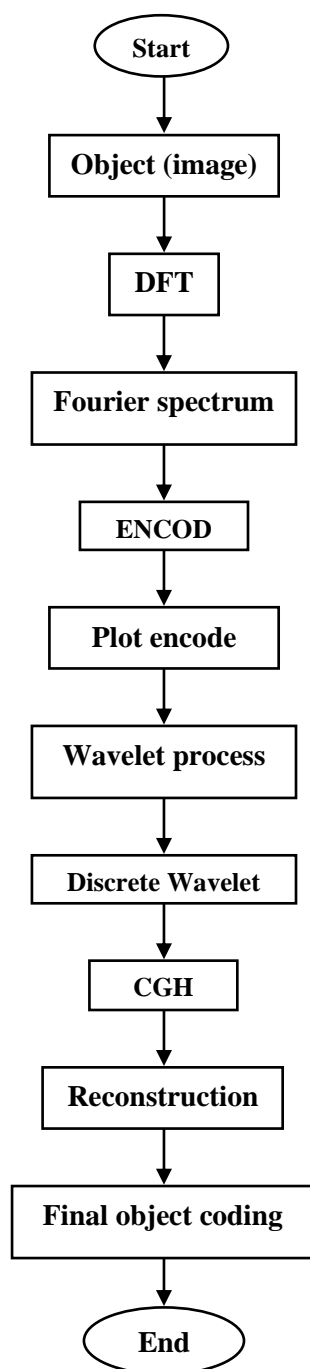
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Figure (1) Flowchart for the object to image sequence of events

A subarea of holography is the so-called digital or synthetic holography, which simulates the recording process of optical holography, on the basis of diffraction theory the wave fronts scattered by objects specified or by means of mathematical models.

It is well known from theory that a signal can be expressed as the sum of possibly infinite series of sines and cosines. This sum is also referred to as an expansion. The big disadvantage of an expansion however is that it has only frequency resolution and no time resolution. This means that although we might be able to determine all the frequencies present in a signal, we do not know when they are present. To overcome this problem in the past decades, several solutions have been developed which are more or less able to represent a signal in the time and frequency at the same time. The idea behind these time-frequency joint representations is to cut the signal of interest into several parts and then analyze the parts separately. It's clear that analyzing a signal this way will give more information about the different frequency components occur. If I want to know exactly all the frequency components presented at a certain moment in time, it must cut out only this very short time using a pulse and transform it to the frequency domain [11].

The problem here is that cutting the signal corresponds to a convolution between the signal and the cutting pulse. Since convolution in the time domain is identical to multiplication in the frequency domain and since the transform of a pulse contains all possible frequencies. The frequency components of the signal will be spread out all over the frequency axis. In fact this situation is the opposite of the standard transform since we now have time resolution but no frequency resolution whatsoever [12].

The principle of the phenomena just described is due to heisenberg's uncertainty principle, which, in signal processing terms, states that it is impossible to know the exact frequency and the exact time of occurrence of this frequency in a signal. In other words, a signal can simply not be represented as a point in the time-frequency space.

The wavelet transform or wavelet analysis is probably the most recent solution to overcome the shortcomings of the transform. In wavelet analysis the use of a fully scalable modulated window (pulse) solves the signal-cutting problem. The window is shifted along the signal and for every position the spectrum is calculated. Then this process is repeated many times with a slightly shorter (or longer) window for every new cycle. In the end the result will be a collection of time-frequency representations of the signal, all with different resolution. Because of this collection of representation we can speak of a multiresolution analysis. In the case of wavelet we normally do not speak about time-frequency representations but about time-scale representations, scale being in a way the opposite of frequency [13].

2. Binary Cell-Oriented Holograms

The binary hologram has a transmittance which is, at any point on its surface. Either clear or opaque. This simplified photographic processing since the necessity to obtain precise gray scale values is eliminated. Photographic materials may be used in the saturation regions of their characteristic curves. Film grain noise is minimized [5,6,7].

A resolution cell containing a single aperture, with width W_{ik} and height H_{jk} (**Fig.(2)**) has transmittance.
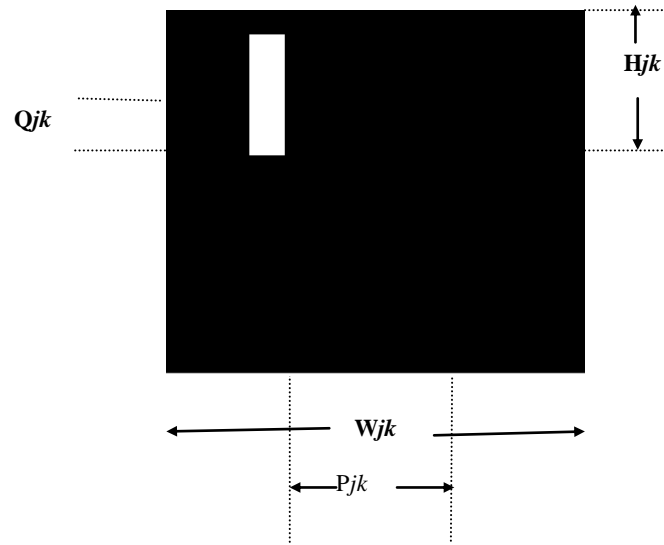
Figure (2) Aperture in resolution cell of binary hologram

The associated space-variant DFT coefficient is as follows:

$$\tilde{C}_{jk} = \mathbf{rect}[(\mu - P_{jk}\,\delta\mu)] \,/\, (W_{jk}\,\delta v)\,\mathbf{rect}[v - Q_{jk}\,\delta v)/(H_{jk}\,\delta v)] \ \text{.........} \ (1)$$

(The integration is carried out over entire aperture, a necessary constriction because the aperture may "overflow" into an adjacent cell).

$$C_{jk}^{mn} = W_{jk}\,H_{jk}\,\mathbf{sinc}(W_{jk}\,m/M)*\mathbf{sinc}(H_{jk}\,n/N)\exp[2\pi i\,P_{jk}\,(m/M)] \ \text{......} \ (2)$$
$$*\exp[2\pi i\,Q_{jk}\,(n/N)]$$

Limiting ourselves to observing the reconstructed image only in the region about the center of the first diffraction order, $m \cong M$, $n \cong 0$, then:

$$C_{jk} = (1/\pi)\sin(W_{jk})\,H_{jk}*\exp(2\pi i\,P_{jk}) \ \text{..............................} \ (3)$$

The detour phase is determined by the lateral shift P_{jk} of the aperture. Changing either the height or width of the aperture can control the modulus [8]. Choosing to vary the height H_{jk} and fix the width at:

$$W_{jk} = 1/2 \ \text{...} \ (4)$$

to maximize C_{jk} and therefore the hologram diffraction efficiency, gives

$$C_{jk} = (1/\pi) H_{jk} \exp(2\pi i \, P_{jk}) \dots\dots\dots\dots\dots\dots (5)$$

The cell aperture height and lateral displacement would then be related to the approximately normalized object DFT modulus phase by:

$$H_{jk} = \tilde{A}_{jk} \dots\dots\dots\dots\dots\dots\dots\dots (6)$$

$$P_{jk} = \tilde{\phi}_{jk} / 2\pi \dots\dots\dots\dots\dots\dots\dots\dots (7)$$

If the aperture width W_{jk} were varied and the aperture height H_{jk} made a constant then

$$C_{jk} = (1/\pi)\sin(\pi W_{jk}) * \exp(2\pi i P_{jk}) \dots\dots\dots\dots\dots (8)$$

Equations (6, 7) would be changed to:

$$W_{jk} = \arcsin(\tilde{A}_{jk} / \pi) \dots\dots\dots\dots\dots\dots\dots (9)$$

$$P_{jk} = \tilde{\phi}_{jk} / 2\pi \dots\dots\dots\dots\dots\dots\dots\dots (10)$$

3. Wavelet Properties

The most important properties of wavelets are the admissibility and the regularity conditions and these are the properties which gave wavelets their name As shown in **Fig.(3)**, Sinusoids do not have limited duration-they extend from minus to plus infinity, and where sinusoids are smooth and predictable, wavelets tend to be irregular and asymmetric [11]**.** The wavelet analysis described in the introduction is known as the continuous wavelet transform. It is written a:

$$\gamma(s,\tau) = \int_{-\infty}^{\infty} f(t)\,\psi_{s,t}^{*}(t)\,dt \dots\dots\dots\dots\dots\dots (11)$$

where:

*: denotes complex conjugation. This equation shows how Function f(t)is decomposed into a set of basis functions Ψs,τ called the wavelets. The variables s and τ, scale and translation, are the new dimensions after the wavelet transform. This inverse wavelet transform is:

$$f(t) = \int\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} \gamma(s,\tau)\,\psi_{s,\tau(t)}\,d_\tau d_s \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(12)}$$

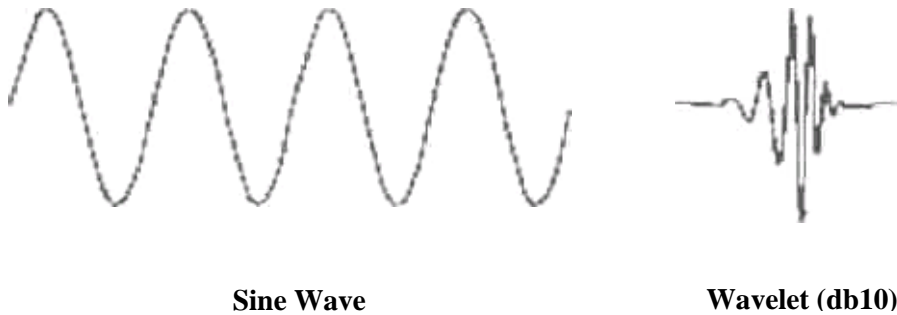**Sine Wave**   **Wavelet (db10)**

Figure (3) Basic function

The wavelets are generated from a signal basis wavelet $\psi(t)$ the so-called mother wavelet, by scaling and translation [12]:

$$c_h = \int \frac{\left|\psi(\omega)^2\right|}{|\omega|}\,d\omega \quad \langle \; +\infty \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(13)}$$

where:

s: is the scale factor τ the translation factor and the factor

$s^{-1/2}$: is for energy normalization across the different scales. It can be shown that square intertribal function $\psi(t)$. Satisfying the admissibility condition

$$\psi_{s,\tau}(t) = \frac{1}{\sqrt{s}}\left(\frac{t-\tau}{s}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(14)}$$

where:

Ch: The admissibility constant.

$\psi(\omega)$: The fourier transform of f(t).

Can be used first to analyze and then reconstruct a signal without loss of information. In the Eq.(14), $\psi(\omega)$ stands for the transform of f(t). The admissibility condition implies that the transform of f(t) vanishes at the zero frequency, i.e.

$$\left|\psi(\omega)^2\right|\;\Big|_{\omega=0} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(15)}$$

This means that wavelets must have a band-pass like spectrum. A zero at the zero frequency also means that the average value of the wavelet in the time domain must be zero,

$$\int \psi(t)\,dt = 0 \ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (16)$$

and therefore it must be oscillatory. In other words, $\psi(t)$ must be a wave. As can be seen from Eq.(11), the wavelet transform of a one-dimensional function is two-dimensional; the wavelet transform of a two-dimensional function is four-dimensional [13].

4. Discrete Wavelet Transform

We know what the wavelet transform is, we would like to make it practical. However, the wavelet transform as described so far still has three properties that make it difficult to use directly in the form of Eq.(11). The first is the redundancy of the CWT. In Eq.(11), the wavelet transform is calculated by continuously shifting a continuously scalable function over a signal and calculating the correlation between the two. Even without the redundancy of the CWT. Discrete wavelets are not continuously scalable and translatable but can only be scaled and translated in discrete steps. This is achieved by modifying the wavelet representation Eq.(9) to create:

$$\psi_{j,k}(t) = \frac{1}{\sqrt{s_0^j}} \psi\left(\frac{t - k\tau_0 s_0^j}{s_0^j}\right) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (17)$$

Although it is called a discrete wavelet, it is normally a (piecewise) continuous function. In Eq.(17), j and k are integers and so>1 is a fixed dilation step. The translation factor τ_0 depends on the dilation step. The effect of discretizing the wavelet is that the time-scale space is now sampled at discrete intervals. We usually choose so=2. This is a very natural choice for computers, the human ear and music for instance. For the translation factor we usually choose $\tau_0 = 1$. It is proven that the necessary and sufficient condition for stable reconstruction is that the energy of the wavelet coefficients must lie between two positive bounds, i.e.

$$A\|f\|^2 \leq \sum_{j,k} \left|\langle f, \psi_{j,k}\rangle\right|^2 \leq B\|f\|^2 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (18)$$

where:

$\|f\|^2$: is the energy of f(t), a>0 and

a, b: are independent of f(t).
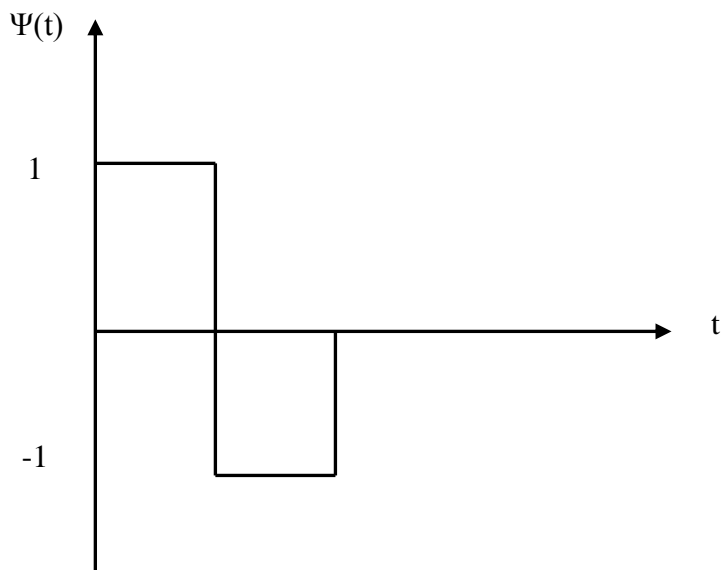
When Eq.(22) is satisfied, the family of basis function $\Psi_{j,k}(t)$ with j,k,f is referred to as a frame with frame bounds A and B. When A=B the frame is tight and the discrete wavelets behave exactly like an orthonormal basis. When A<B, exact reconstruction is still possible at the expense of a dual frame. In a dual frame discrete wavelet transform, the decomposition wavelet is different from the reconstruction wavelet. The discrete wavelets can

be made orthogonal to their own dilations and translations by special choices of the mother wavelet [12,13], which mean:

$$\int \psi_{j,k}(t)\psi^*_{m,n}(t)dt = \left\{ \begin{matrix} 1 & \text{if j=m and k=n} \\ 0 & \text{otherwise} \end{matrix} \right. \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (19)$$

The Haar wavelet transform, is one of the earliest examples of what is now called a compact support, dyadic orthonormal wavelet transform. As far back as 1910, Haar described the following function as providing an orthonormal basis. The analyzing wavelet of a continuous variable is a step function shown in F**ig.(4)** [12], and is given by:

$$\psi(t) = \left\{ \begin{matrix} 1 & o \le t \le 1/2 \\ -1 & 1/2 \le t \le 1 \\ 0 & \text{otherwise} \end{matrix} \right. \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (20)$$

Figure (4) Haar wavelet

An arbitrary signal can be reconstructed by summing the orthogonal wavelet basis functions, weighted by the wavelet transform coefficients:

$$\mathbf{f(t)} = \sum_{j,k} \gamma(j,k)\psi_{j,k}(t) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (21)$$

This transform for discrete wavelets. An efficient way to implement this scheme using filters was developed in 1988 by Mallat. The Mallat algorithm is in fact a classical scheme known in the signal processing community as a two-channel sub and coder [13].

5. Results and Discussion

Image Coding for the Computer generated hologram The First image was taken of an image object (tire image). The image background was removed and white space added around the object to obtain higher transmission.

Then the image was propagated as shown in F**ig.(5)**. This figure represents the hologram coding of the image. The theoretical reconstruction of the image was created by MATLAB software (version 7).
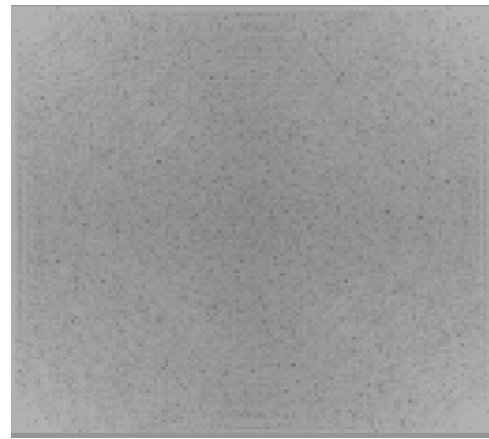

| Object (image) | Amplitude |
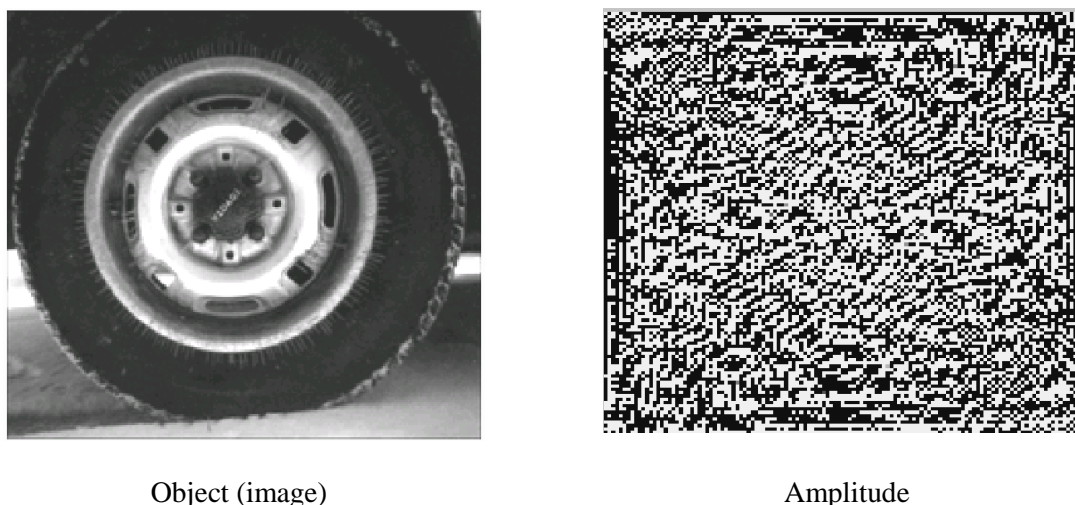
**Figure (5) Complex definition of the object**

The twin images interference with the reconstruction was show in F**ig.(6)** (Amplitude and Phase) which represented the intensity (Histogram) for the image dimension (128*128) pixel ,but the image dimension (256*256) pixel the intensity was very clearness representing the "Lena" coding and histograms as shown in F**ig.(7).** The two images types (TIF). The class unit 8 array (No. bytes 262144) for the first image (tire image), the second image (Lena image) class unit double array (No. bytes 524288).
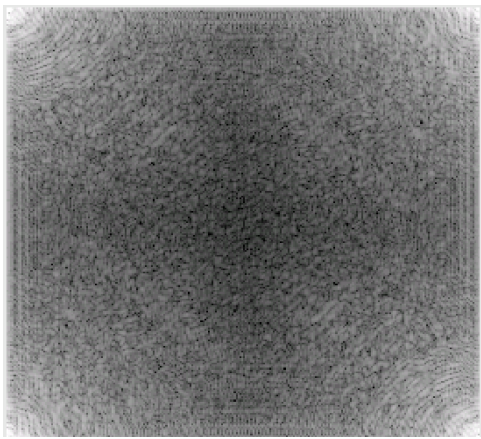
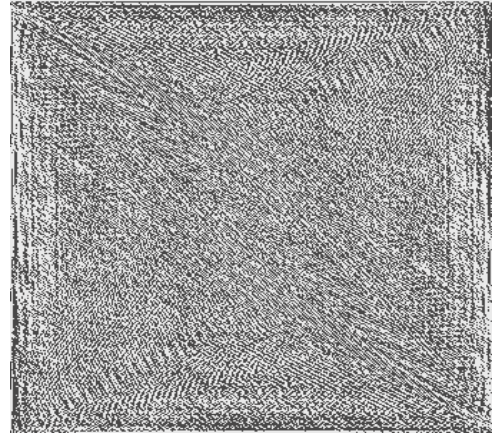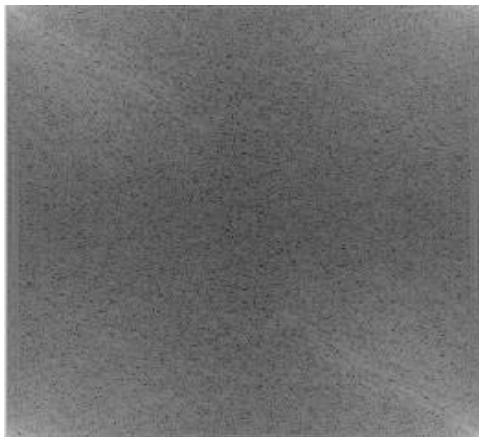Amplitude   Phase
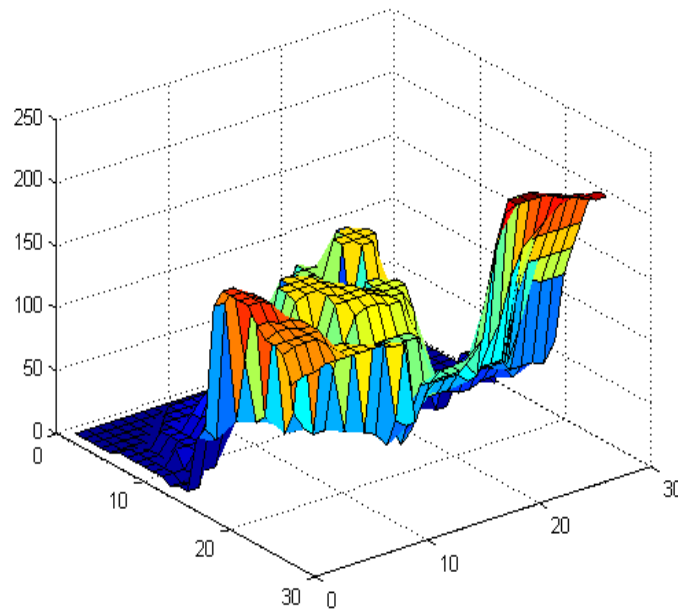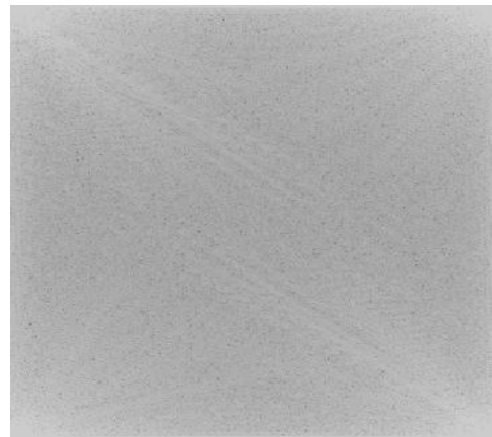
Figure (6) Interference with reconstruction of images

Object (image)   Amplitude
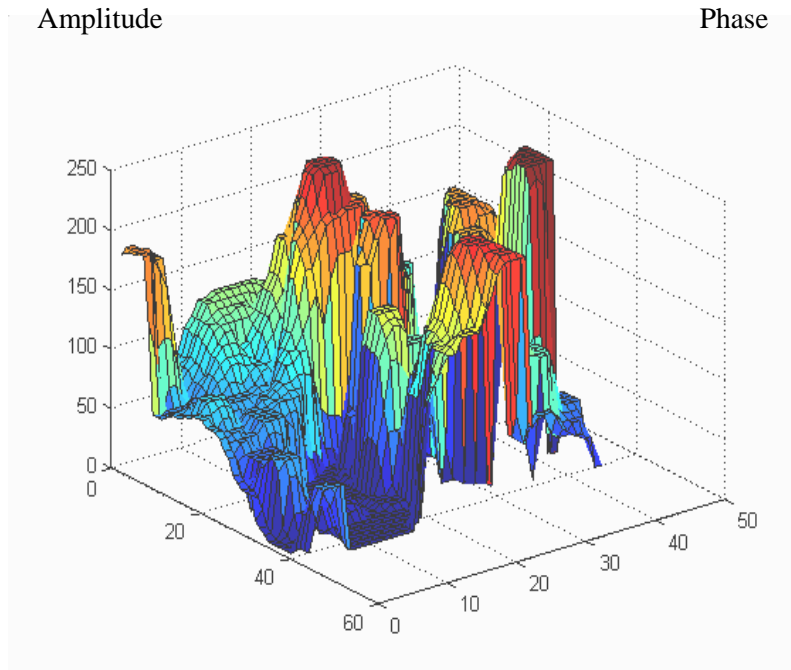

Amplitude   Phase

Figure (7) Object image coding and histograms

6. Conclusion

The goal of this paper is to develop theory and methods that can be applied to coding frequency components of time series may take general periodic shapes that include sinusoids as special cases (Images).

We present conclusions based on our evaluations of the wavelet properties, the analysis of the DWT computation methods. This paper implement discrete wavelet transform for generate interfacing amplitude and phases were discussed.

7. References

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