

## ***Vibration of Non-rotating Blades Experimental and Numerical Investigation***

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### **Abstract**

***In this work, the vibration characteristics of cantilever blades are studied; natural frequencies are evaluated for different aspect ratios. Other parameters like thickness taper and width are considered.***

***The experimental results are compared with numerical results using finite element method (ANSYS 5.4 package) these results give good agreement and show the effect of the aspect ratio on the natural frequency.***

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### **الخلاصة**

**في هذا البحث تمت دراسة خصائص الاهتزازات لريش كابولية، تم ايجاد الترددات الطبيعية لنسب مظهرية (نسبة الطول إلى العرض) مختلفة وأخذت عوامل أخرى في الاعتبار كالسمك المتناقص وكذلك العرض المتزايد بالتدرج.**

**فورنت النتائج العملية مع نتائج عددية مستنتجة بطريقة العناصر المحددة باستخدام برنامج ANSYS 5.4 . وقد بينت النتائج توافقا ملحوظا وهي توضح ما للنسبة المظهرية من تأثير على التردد الطبيعي.**

## 1. Introduction

A large class of engineering structural components, such as, compressor and turbine balding, helicopter rotor blades installed on spacecraft are idealized as cantilever beams mounted on rigid supports for the purpose of vibration analysis. Accurate prediction of natural frequencies and mode shapes of these components along with the stress and displacement distribution along the length of the blade is of much importance.

Many investigators have studied the vibration characteristics under non-rotating conditions. Others studied the stress and deformation of blades under rotating conditions. The solution for a straight uniform beam was well known early since 1763. According to Singh <sup>[1]</sup>, many researchers gave solutions of lateral vibrations of beams with variable cross-section. Thomson <sup>[2]</sup> gave the solution of vibration of non-uniform beams by Matrix Method.

Martin <sup>[3]</sup> derived the correction factors for the effect of taper on flexural frequencies by expressing the frequency as a double power series on two taper parameters of breadth and thickness. Housner and Keightly <sup>[4]</sup> gave the solution of the natural frequencies and mode shapes of variable cross-section cantilever by means of a digital computer.

Carnegie and Dawson <sup>[5]</sup> found theoretical and experimental natural frequencies and mode shapes up to the fifth mode of vibration. The theoretical procedure consists essentially of transforming the differential equations of motion in a set of simultaneous first order equations and solving step-by-step finite difference procedure. Walker <sup>[6]</sup> studied the vibration of combined helicoidally fan blade, in his study a conforming finite shell element suitable for the analysis of curved twisted fan blades was developed and applied to a number of blade models. The thin shell element was used in this study to predict the natural frequencies and mode shapes of a number of fabricated fan blade structures and the results were compared with the experimental results. Gill and Ucmaklioglu <sup>[7]</sup> used finite element method to determine the natural frequencies of an oval cross-section blade and hollow airfoil section turbine blade, employing the eight-node isoparametric shell element with a reduced number of integration points. In this study an oval cross-sectional cylindrical shell represented hollow blade was manufactured and the natural models of vibration when mounted as a cantilever were determined. Lee <sup>[8]</sup> determined the frequencies and mode shapes of turbo machinery blade having both camber and twisted. The body forces are considered to be centrifugal forces generated within the shell due to rotation of the blade. Another conclusion was that the Ritz method is capable to deal with camber and twisted blade. Leisa, et. al. <sup>[9]</sup> used the shallow shell theory and Ritz method to determine the frequencies and mode shapes of turbo machinery blade having both camber and pre-twisted, rotating with non-zero angles of attack.

The body forces are considered to be centrifugal forces generated within the shell due to the rotation of blade. Another conclusion was that the Ritz method is capable to deal with camber and twisted blade. Rao and Gupta <sup>[10]</sup> used the classical bending theory of thin shells to determine the free vibration characteristics of a rotating pre-twisted small aspect ratio blade. Differential geometry of the blade in curvilinear coordinates was analyzed and strain-displacement relations were established. The strain and kinetic energies of the rotating

and vibrating blade were determined; and Lagrangian function was set up. Following the Ritz procedure, the natural frequencies and mode shapes of the blade were determined. H. H. Yoo, et. al. <sup>[11]</sup> studied the vibration analysis of rotating pre-twisted blade with a concentrated mass.

The blade has an arbitrary orientation with respect to the rigid hub to which it is fixed. The equations of motion are derived based on a modeling method that employs hybrid deformation variables. The resulting equations for the vibration analysis is transformed into the dimensionless parameters on the modal characteristics of the rotating blades are investigated through numerical analysis. J. Chung and H. H. Yoo <sup>[12]</sup> studied the dynamic analysis of rotating cantilever beam by using finite element method. This study based on a dynamic modeling method using stretch deformation instead of the conventional axial deformation, the behaviors of the natural frequencies are investigated for the rotating speed. In addition the time responses and distributions of the deformations and stresses are computed when the rotating speed prescribed. The effects of the rotating speed profile on the vibrations of the beam are also investigated.

## 2. Experimental Instruments

The instruments used for natural frequency measurement are shown in **Fig.(1)**:

1. Electromagnetic shaker type B & K 4810.
2. Sine generator type B&K1023.
3. Accelerometer type B&K 4344.
4. Conditioning amplifier type B&K 2626.
5. Oscilloscope.

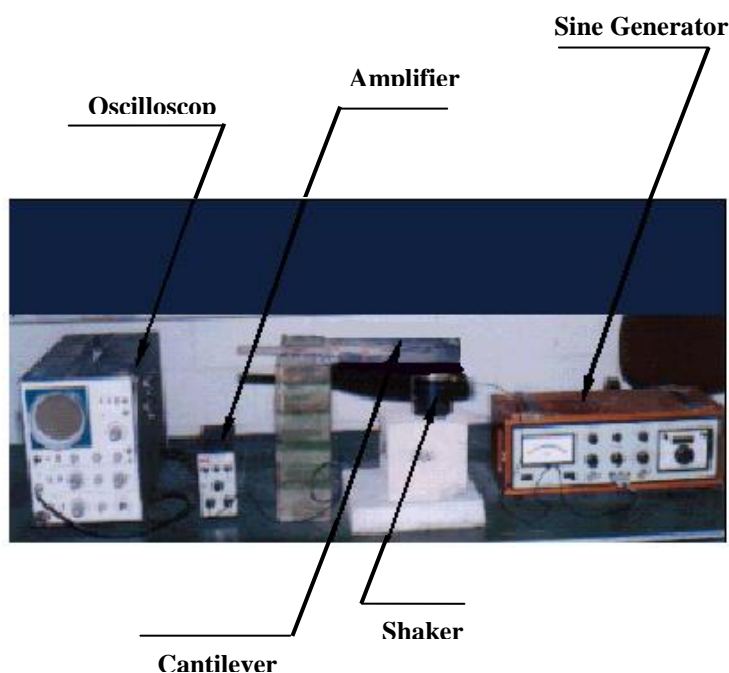


Figure (1) Instruments used for natural frequency measurements

### 3. Validity of the used Devices

Different lengths of rectangular blades were used to calibrate the device that used to measure the natural frequencies in this work. The dimensions and properties of these blades are ( $E=200$  GP,  $\rho=7850$  Kg/m<sup>3</sup>,  $\nu=0.3$ ,  $b=0.025$  m,  $t=0.002$  m).

The natural frequency, which measured from the calibration process agreed with the results, obtained by ANSYS PACKAGE (5.4) as shown in **Table (1)**.

**Table (1) Comparison of the natural frequency for the experimental and analytical methods**

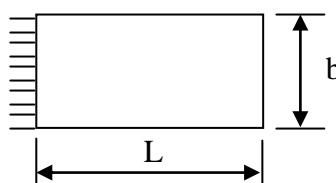
Length (m)	Natural frequency (HZ)		Percentage error %
	Exp.	ANSYS (5.4)	
0.3175	15.80	16.25	2.79
0.1588	61.70	65.24	5.43
0.1058	137.50	147.53	6.79
0.0794	241.70	262.70	7.99
0.0635	375.50	411.78	8.80
0.0529	529.00	594.47	11.01
0.0454	662.70	808.23	18.00
0.0397	852.50	1057.80	19.40
0.0353	1031.50	1339.60	22.99
0.03175	1193.40	1657.50	27.35

### 4. Experimental Work and Laboratory Measurements

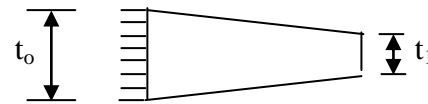
The experimental investigation carried out by a series of tests performed to measure the natural frequency using different types of specimens. The frequency response for the fan blades are investigated by increasing the driving frequency of the vibrator by means of the sine generator holding the shaker in stationary. The natural frequency is distinguished by observing the sharp increment in amplitude of the picked output which is amplified and displayed on the oscilloscope, and by the intensity of the tone emitted. The shaker and pickup accelerometer were placed at different positions in order to avoid the possibility of having the accelerator and/or the shaker at a nodal line. Mode shapes (nodal line) are observed by placing fine sand on the surface.

Various aspect ratios specimens are used with various thicknesses and tapered thickness and width. The first three mod shapes are observed.

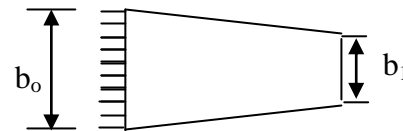
The aspect ratios  $\frac{L}{b} = 1, 1.5 \text{ and } 2.5$



The tapered thickness ratios are  $\frac{t_o}{t_1} = 2 \text{ and } 4$



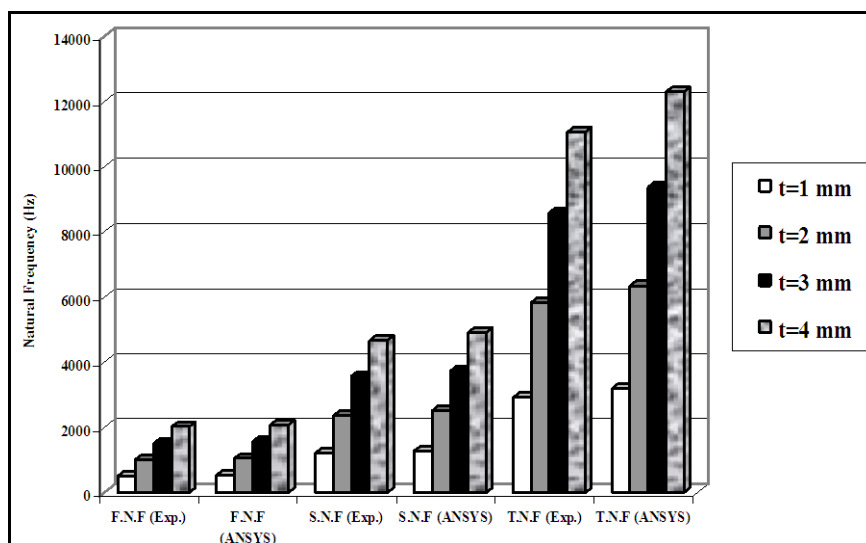
The tapered width ratios are  $\frac{b_o}{b_1} = 2 \text{ and } 4$



By coupling the measured data for first three mode shapes with estimates from the ANSYS 5.4 model, it was possible to find several natural frequencies.

**Figure (2)** shows the variation of the first three natural frequencies with four thicknesses and aspect ratio ( $l/b = 1$ ) and **Fig.(3)** Takes the aspect ratio ( $l/b = 1.5$ ) while, **Fig.(4)** Takes ( $l/b = 2.5$ ). **Figures (5)** and **(6)** give the natural frequencies for the last two aspect ratios with two tapered thickness ratios ( $t_o/t_1 = 2 \text{ and } 4$ ). **Figures (7)** and **(8)** repeat the test for tapered width with ratios ( $b_o/b_1 = 2 \text{ and } 4$ ).

**Figures (9)** and **(10)** show the suitable mesh size for cantilever plate with various dimensions with ANSYS 5.4 package. **Figures (11 to 15)** show the first three mode shapes for each case using the same package.



**Figure (2)** The variation of the first three natural frequencies with thickness ( $l/b=1$ )

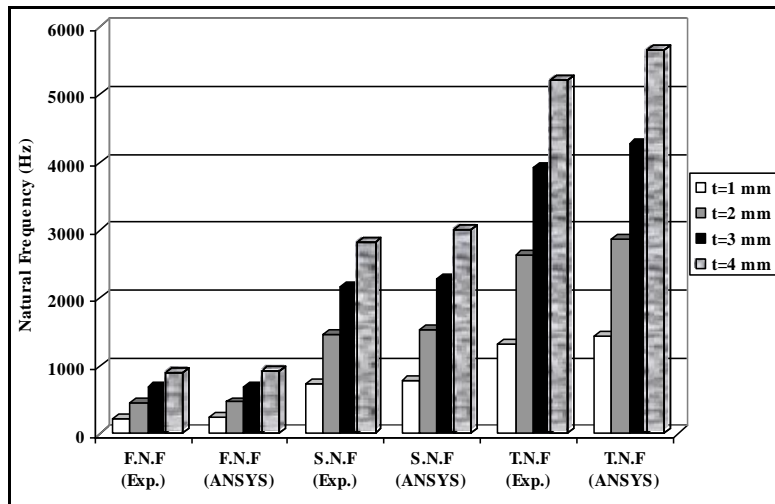


Figure (3) The variation of the first three natural frequencies with thickness ( $l/b=1.5$ )

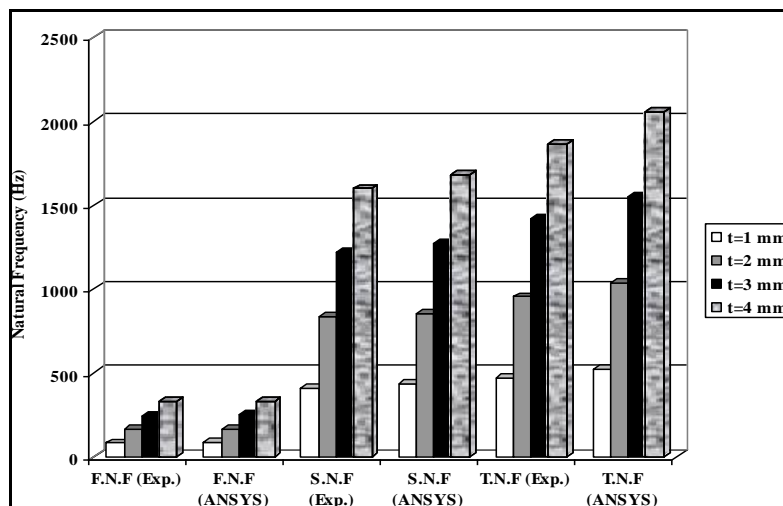


Figure (4) The variation of the first three natural frequencies with thickness ( $l/b=2.5$ )

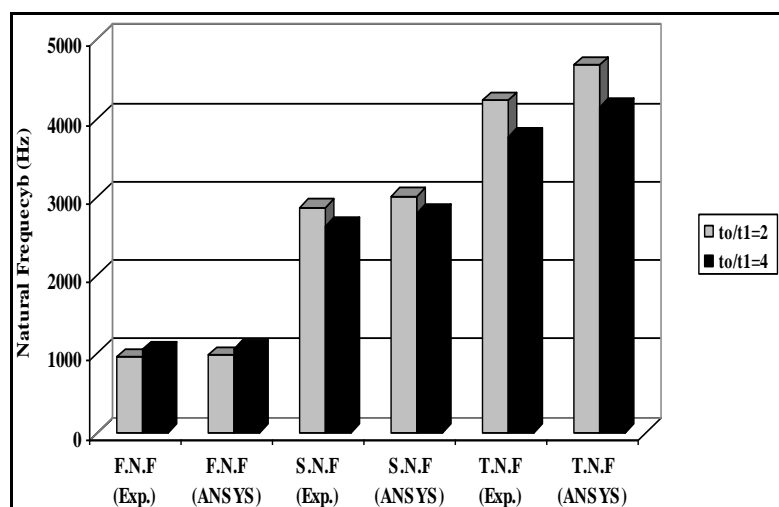


Figure (5) The variation of the first three natural frequencies with ( $t_o/t_1$ ), ( $l/b=1.5$ )

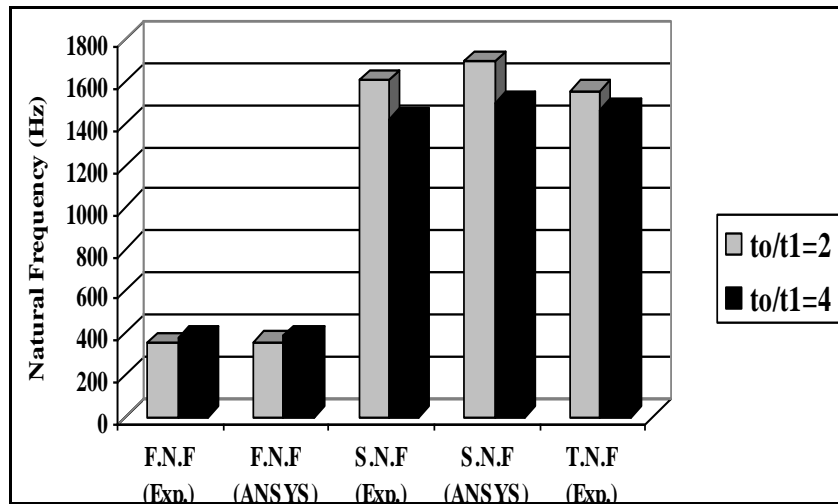


Figure (6) The variation of the first three natural frequencies with (to/t1), (l/b=2.5)

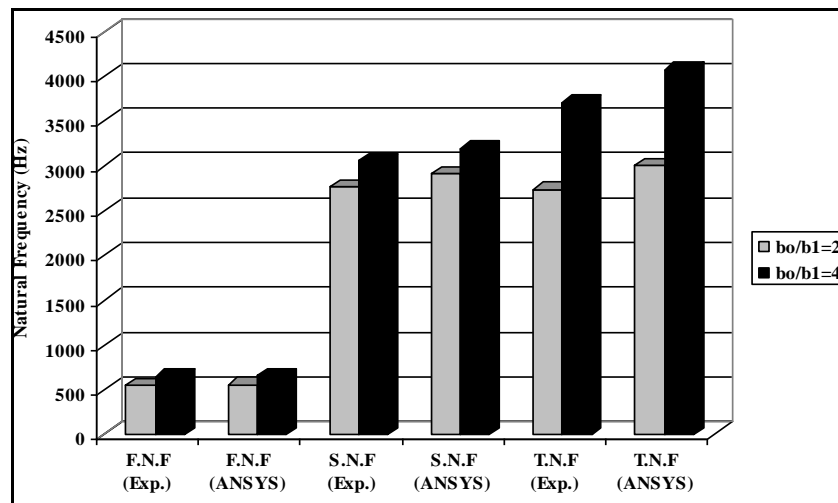


Figure (7) The variation of the first three natural frequencies with (bo/b1), (l/b=1.5)

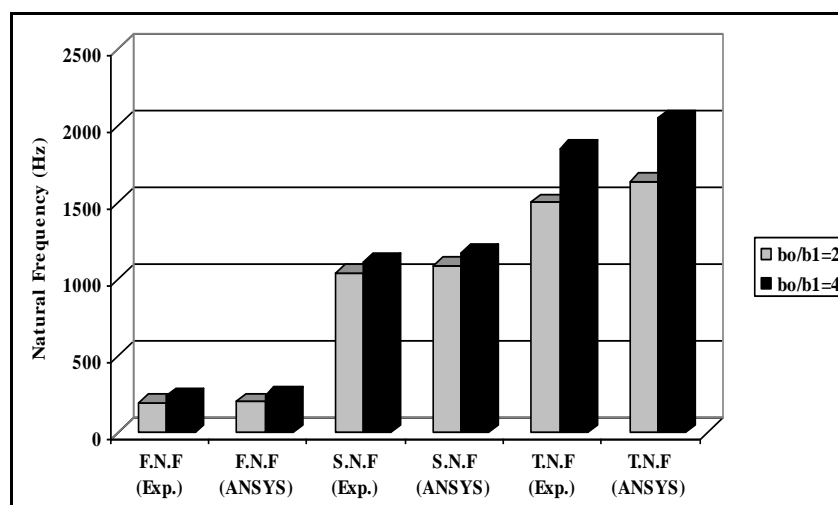


Figure (8) The variation of the first three natural frequencies with (bo/b1), (l/b=2.5)

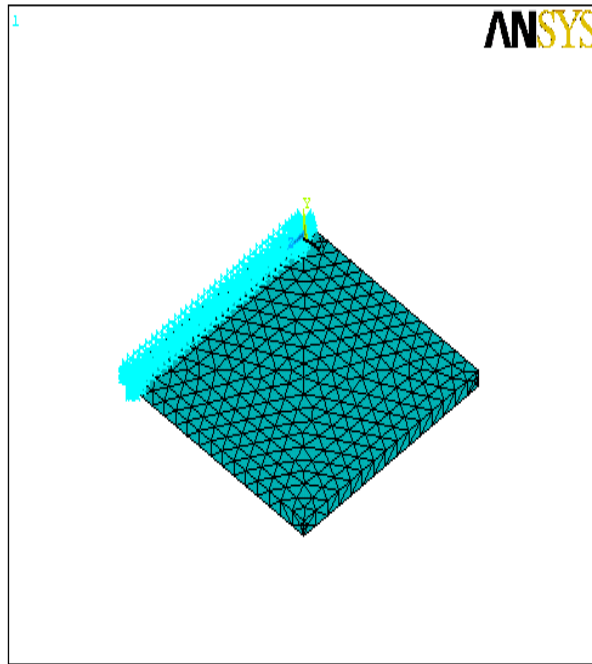


Figure (9) Suitable mesh size for cantilever plate  
( $L/b=1$ ,  $t=2$  mm,  $b=40$  mm)

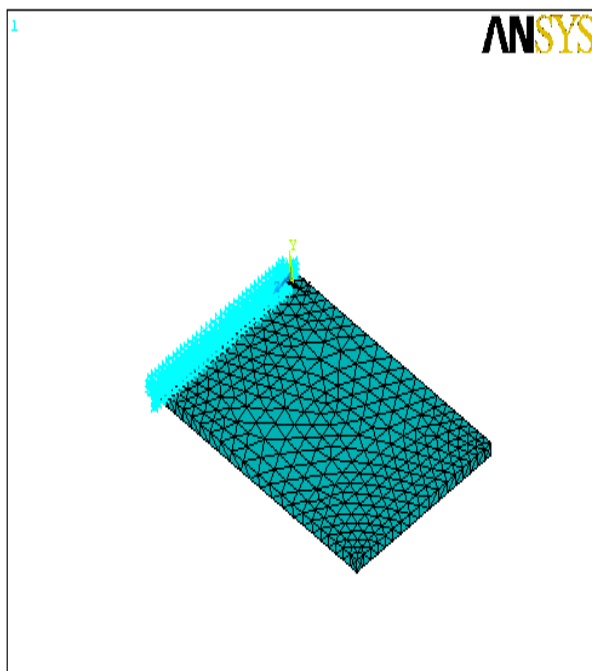


Figure (10) Suitable mesh size for cantilever plate  
( $L/b=1.5$ ,  $t=2$  mm,  $b=40$  mm)



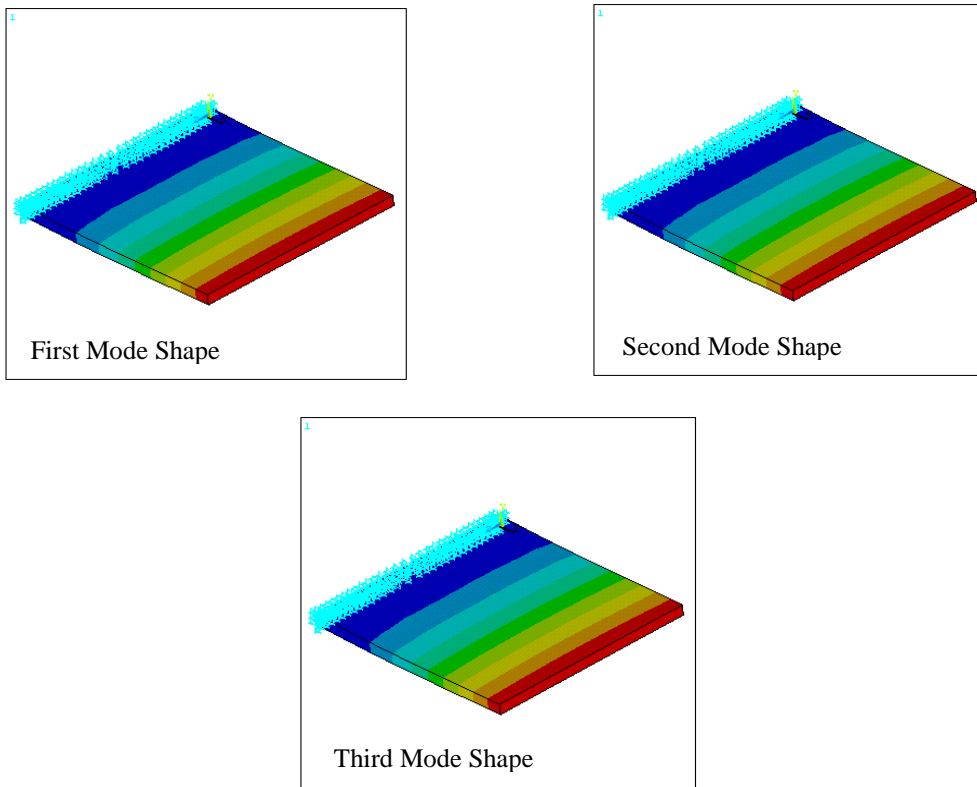


Figure (11) The first three modes shapes for cantilever plate, ( $L/b=1$ ,  $t= 2$  mm,  $b= 40$  mm)

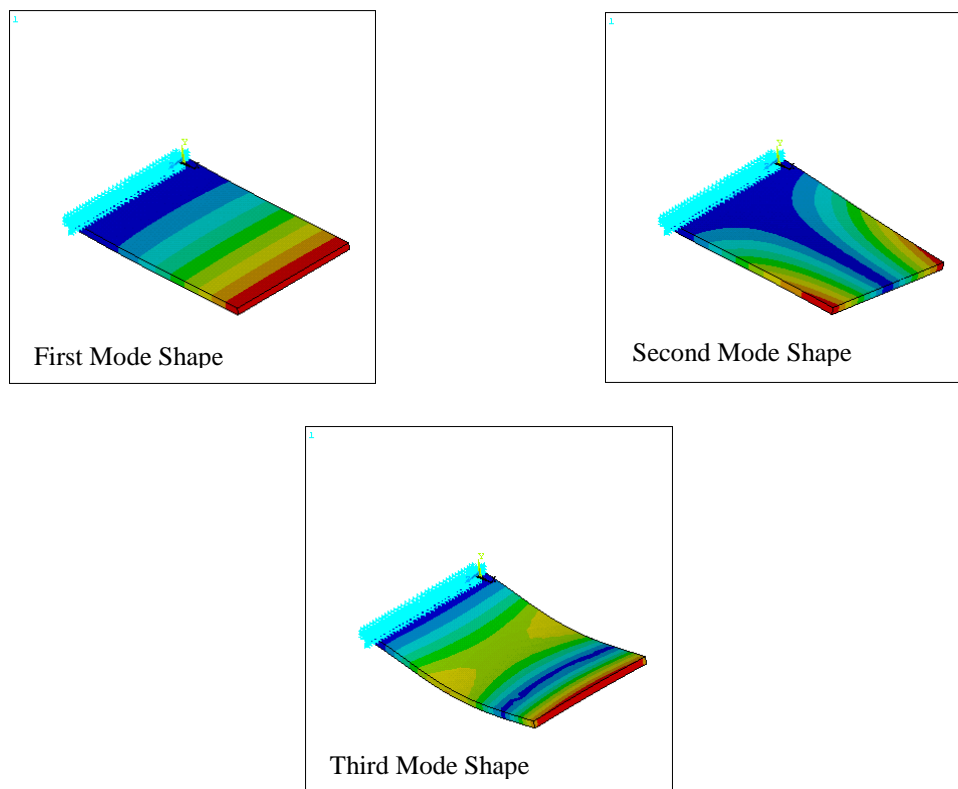


Figure (12) The first three modes shapes for cantilever plate, ( $L/b=1.5$ ,  $t= 2$  mm,  $b= 40$  mm)

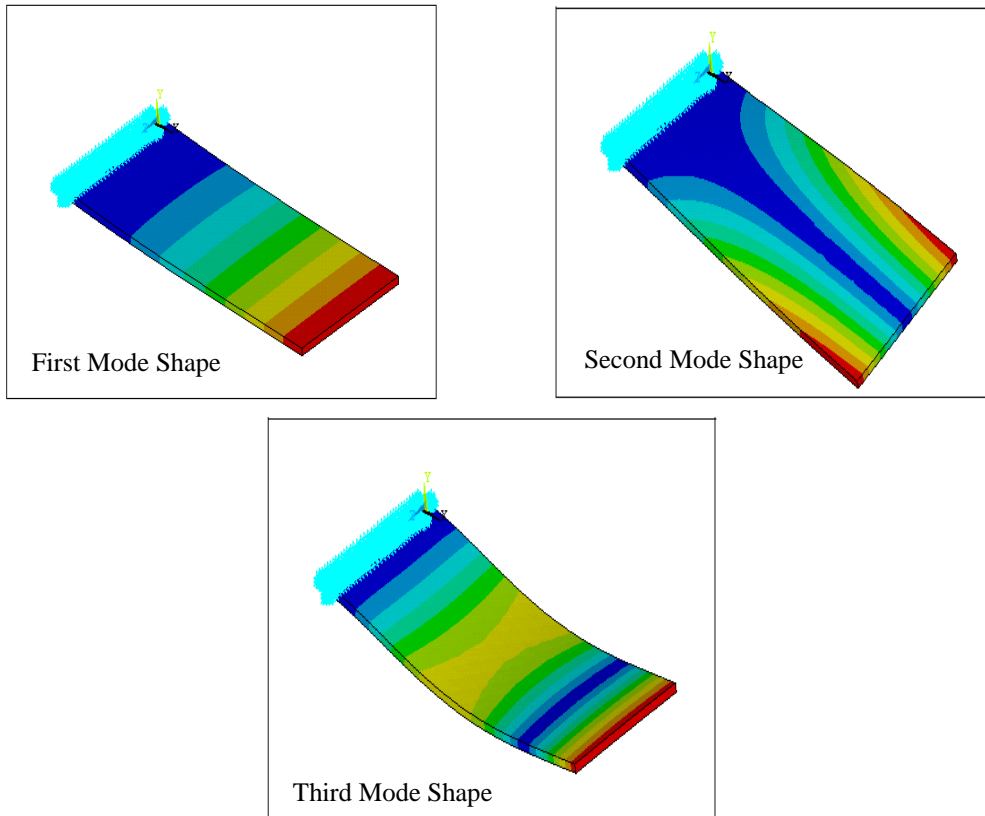


Figure (13) The first three modes shapes for cantilever plate, ( $L/b=2.5$ ,  $t= 2$  mm,  $b= 40$  mm)

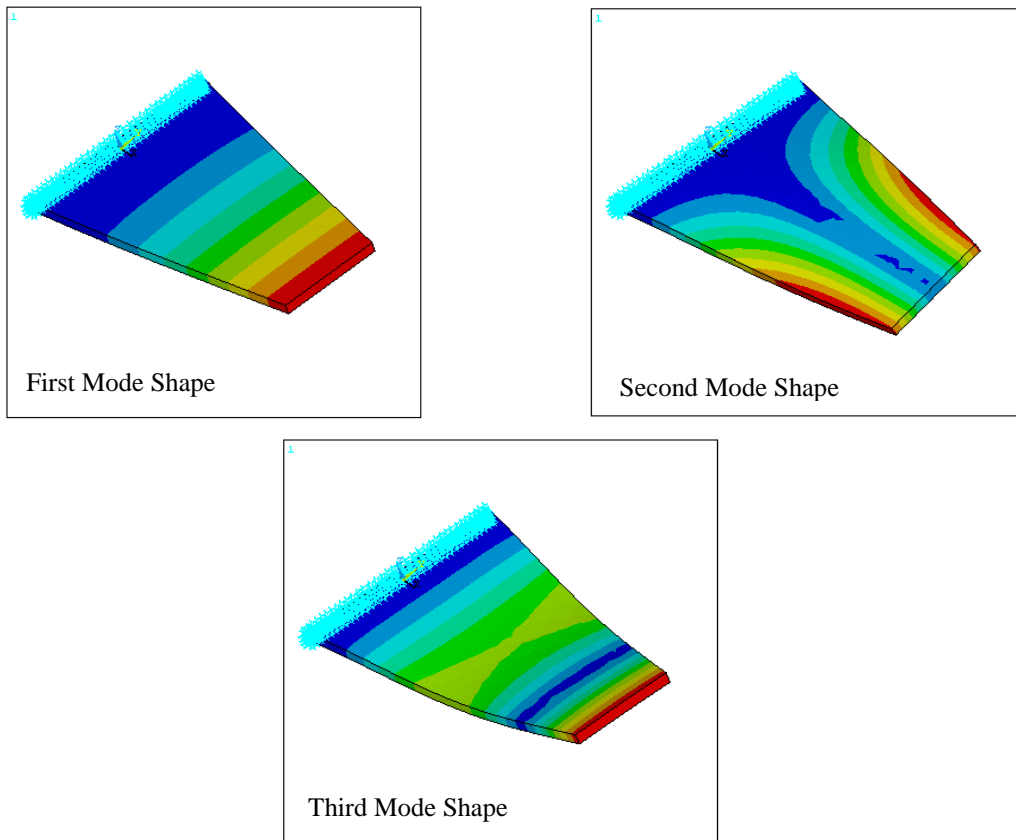


Figure (14) The first three modes shapes for cantilever plate, ( $L/b=1.5$ ,  $b_o/ b_1=2$ ,  $t= 2$  mm,  $b_o = 40$  mm)

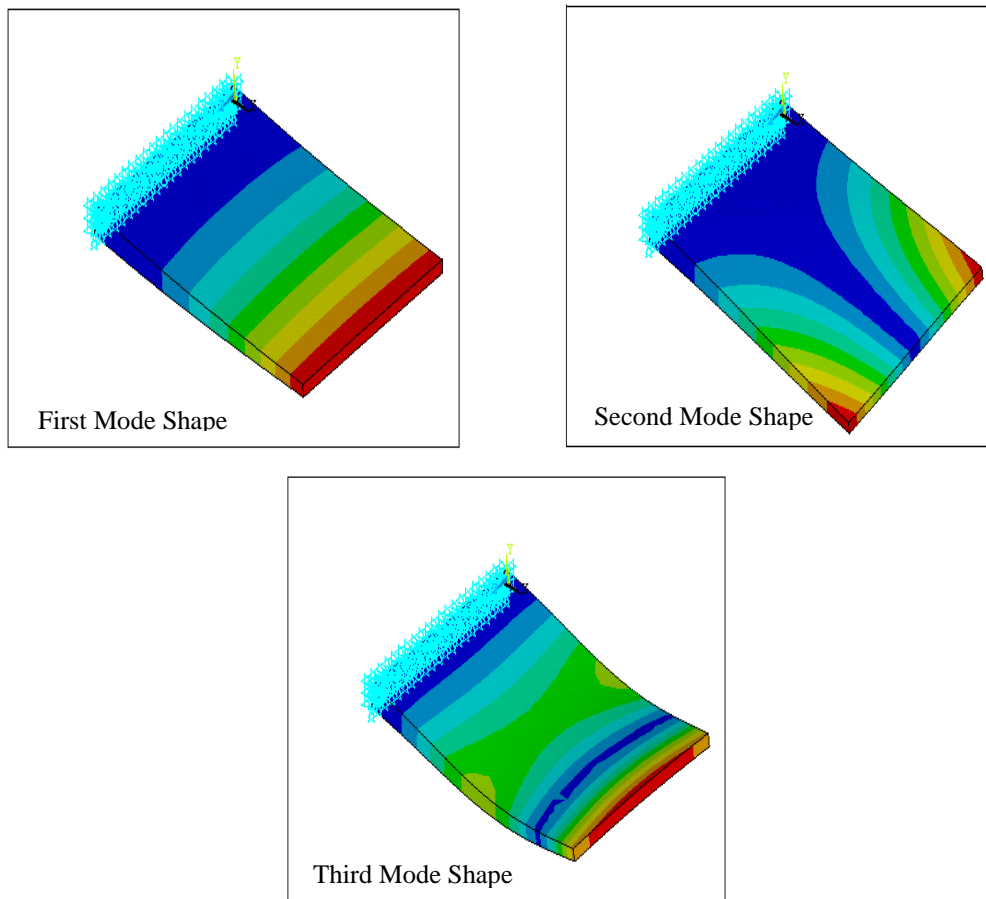


Figure (15) The first three modes shapes for cantilever plate, ( $L/b=1.5$ ,  $t_0=2$  mm,  $t_0 / t_1= 2$  ,  $b= 40$  mm)

## 5. Discussion and Conclusion

From **Figs.(2, 3, and 4)** we conclude that the natural frequency (for the three mode shapes) increases with the increment of the thickness for the same aspect ratio while, it decreases with the increment of aspect ratio for the same thickness. **Figures (5) and (6)** give us an idea, which is, the natural frequency increases with the increment of the tapered thickness ratio ( $t_o/t_1$ ) for any aspect ratio, however, the increment of the aspect ratio decreases the natural frequency for the same thickness ratio. **Figures (7) and (8)** declare that the natural frequency increases with the increment of the tapered width ratio  $b_o/b_1$  for the same aspect ratio and decreases with the decrease of aspect ratio for the same width ratio.

This paper documents the initial results of ongoing efforts to understand the behavior of non-rotating blades as an initial test to study the centrifugal effects on the rotating blades in future work.

## 6. References

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