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# *Iterative Decoding of Serial Concatenated Convolutionally Encoded Signals over Frequency Selective Fading Channels*

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## Abstract

***Serial concatenated convolutional coding with iterative decoding is examined for data transmission employing BFSK (binary frequency shift keying) modulation and non-coherent detection receiver.***

***The core of the iterative decoding structure is soft-input soft-output (SISO) module. In this paper, the performance of serial concatenated convolutional codes (SCCCs) is evaluated by simulation over frequency selective Rayleigh and Rician fading channels.***

***Simulation results show that, for both frequency selective Rayleigh and Rician fading channels, better BER (bit error rate) performance could be obtained when the outer encoder is a non-recursive (NR) encoder and the inner encoder is a recursive (R) encoder.***

الخلاصــــــة

*تم فحص المشفرات الالتفافية التلاصقية المتسلسلة مع فاتح التشفير التكراري للمعلومات المرسلة باستخدام المنغم نوع* ***BFSK*** *والتحسس غير الملتحم* ***(non-coherent)*** *في المستلم* ***(receiver)****.*

*ان القلب البنائي لفاتح التشفير التكراري هو فاتح المشفر ذو الدخل الناعم و الخرج الناعم* ***SISO decoder****. في هذا البحث، تم تقييم كفاءة اداء المشفرات الالتفافية التلاصقية المتسلسلة خلال قنوات الخفوت الانتقائية* ***frequency selective fading channels*** *من نوع* ***Rayleigh*** *و* ***Rician****.*

*بينت نتائج المحاكاة* ***simulation results****، ان اقل معدل خطأ* ***BER*** *للاداء يمكن الحصول عليه عندما يكون التشفير الخارجي هو من النوع غير المرجع* ***non-recursive (NR)*** *و التشفير الداخلي هو من النوع المرجع* ***recursive (R)*** *.*

## 1. Introduction

Since their presentation in 1993 [1], turbo codes were shown to have astonishing performance close to the theoretical Shannon capacity limit in AWGN (additive white Gaussian noise) channel with relatively simple iterative decoding technique [2]. As a powerful coding technique, turbo codes are a prime candidate for wireless applications and mobile radio communications [3].

On AWGN channels, serial concatenated codes performance has been discussed and evaluated by using analytical average upper bounds [4,5]. In the past few years, a new concept, called "turbo equalization" has emerged as a way of efficiently fighting against strong channel intersymbol interference (ISI) caused by limited bandwidth, multipath propagation and motion [6,7].

SCCCs admit a suboptimum decoding process based on iterations of the maximum a posteriori (MAP) algorithm or soft-output Viterbi algorithm (SOVA) applied to each constituent code [2,8]. It has been shown in [8], that for long channel impulse responses and/or high modulation orders, the SOVA algorithm becomes prohibitively complex, therefore, SISO algorithm that implements MAP algorithm offers very good performance with low complexity.

The purpose of this paper is to achieve low BER using iterative decoders based on SISO algorithm over frequency selective Rayleigh and Rician fading channels. It is also shown that, better BER performance can be obtained when the outer encoder is a non-recursive (NR) encoder and the inner encoder is a recursive (R) encoder, as compared when both encoders are recursive encoders.

## 2. SCCC and Iterative Decoding

The block diagram of SCCC encoder is shown in **Fig.(1a)** [4].


**Figure (1a) Block diagram of SCCC**

The information ($u^o$) bits are first encoded by an outer convolutional encoder. The output code words ($c^o$) are then permuted by an interleaver of length N before encoding by an inner encoder. $u^i$ and $c^i$ are the information and code words of inner convolutional encoder.

**Figure (1b)** illustrates the block diagram of an iterative decoder for SCCC [4].

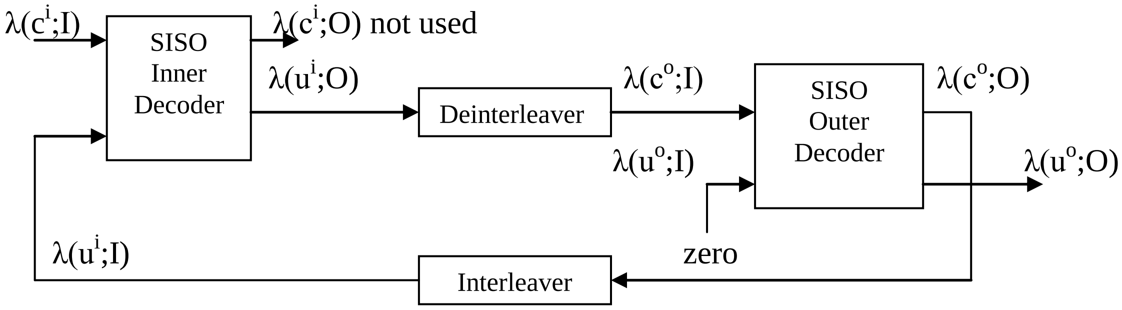


**Figure (1b) Block diagram of an iterative decoder for SCCC**

The block SISO operates on log-likelihood ratio (LLR) of information and coded bits [4]. Assuming the information and code symbols are defined over a finite time index set [1,…., K]. At time k, k=1,…., K, the output extrinsic LLR's are computed as [4]:

$$\lambda_k(c;O) = \max_{e:c(e)=c}^{*} \{\alpha_{k-1}[s^S(e) + \lambda_k[u(e);I] + \beta_k[s^E(e)]\} + h_c \quad \text{……………… (1)}$$

$$\lambda_k(u;O) = \max_{e:u(e)=u}^{*} \{\alpha_{k-1}[s^S(e) + \lambda_k[c(e);I] + \beta_k[s^E(e)]\} + h_u \quad \text{……………... (2)}$$

The symbols $\lambda_k(.;I)$ and $\lambda_k(.;O)$ are the LLR's at the input and output ports of SISO, $h_c$ and $h_u$ are normalization constants. The quantities α(.) and β(.) are obtained through the forward and backward recursions, respectively, as [4]:

$$\alpha_k(s)= \max_{e:s^E(e)=s}^{*} \{\alpha_{k-1}[s^S(e) + \lambda_k[u(e);I] + \lambda_k[c(e);I]\} \quad \text{…………………….. (3)}$$

$$\beta_k(s)= \max_{e:s^S(e)=s}^{*} \{\beta_{k+1}[s^E(e) + \lambda_{k+1}[u(e);I] + \lambda_{k+1}[c(e);I] \quad \text{………………….. (4)}$$

With initial values:

$$\alpha_0(s)= \begin{cases} 0 & s = S_0 \\ -\infty & \text{otherwise} \end{cases} \quad \text{……………………………………………………… (5)}$$

$$\beta_K(s) = \begin{cases} 0 & s = S_K \\ -\infty & \text{otherwise} \end{cases} \quad \text{…………………………………………………… (6)}$$

where the symbol e denotes the trellis edge, u(e), and c(e) are the information and code symbols associated to the edge e. $s^S(e)$ and $s^E(e)$ are the starting and ending states of the edge e.

The "SISO inner" is fed with the demodulator soft outputs. The second input $\lambda_k(u^i;I)$ of the SISO inner is set to zero during the first iteration, since no a priori information is available on the input symbol $u^i$ of the inner encoder [4].

The LLR's $\lambda_k(c^i;I)$ computes the extrinsic LLR's of the information symbols of the inner encoder $\lambda_k(u^i;O)$ conditioned on the inner code constraints. The extrinsic LLR's are passed through the deinterleaver whose outputs are $\lambda_k(c^o;I)$. These LLR's are then set to the "SISO outer", which in turn, processes the LLR's $\lambda_k(c^o;I)$ and computes the LLR's of both code and information symbols based on the code constraints. The input $\lambda_k(u^o;I)$ of the SISO outer is always set to zero, which implies assuming equally likely transmitted source information symbols. The output LLR's of information symbols will be used in the final iteration to recover the information bits. The LLR's $\lambda_k(c^o;O)$, after interleaving are fed back to the lower entry of "SISO inner" to start the second iteration [4].

## 3. System Model

The block diagram of the system model for simulation is shown in **Fig.(2)**.

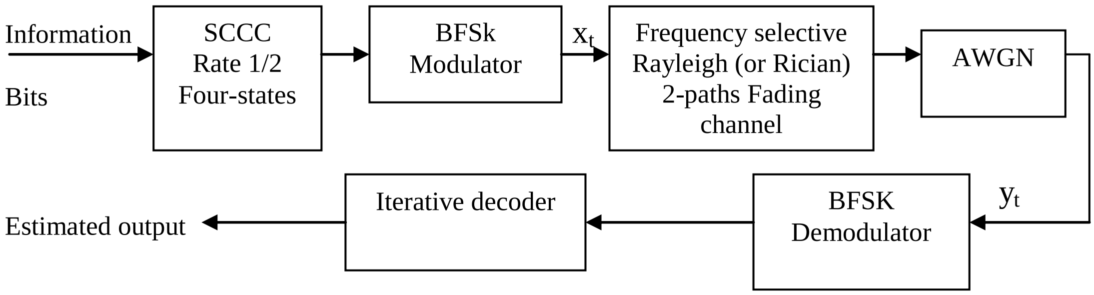


**Figure (2) Block diagram of system model for simulation**

The output bits for the inner encoder are BFSK modulated and transmitted through a frequency selective Rayleigh (or Rician) fading channel. The channel is modeled using Jake's model [9]. Let $x_t$ be the transmitted signal at time t. The received signal at time t is given by:

$$\mathbf{y_t = \boldsymbol{a}_t\, x_t + n_t} \quad \text{.....} \quad \mathbf{(7)}$$

where:

$n_t$: is AWGN and,

$\boldsymbol{a}_t$: is a random variable represents the channel fading variation.

The probability density function (pdf) of the fading amplitude ***a*** is given by [9]:

$$\mathbf{P(\boldsymbol{a}) = 2\boldsymbol{a}\,(1+K)\,e^{-\boldsymbol{a}^2\,(1+K)-K}\; I_0\,(2\,\boldsymbol{a}\,\sqrt{K\,(1+K)}\;)}\;,\; \boldsymbol{a} \ge 0 \quad \text{.....} \quad \mathbf{(8)}$$

where:

K: is a Rician factor denoting the ratio of the deterministic signal power component to the power of the diffuse fading signal component.

$I_o$: is the zero-order modified Bessel function of first kind.

Small values of **K** indicates severely faded channel. For **K**= 0, the Rician pdf becomes a Reyleigh pdf, and for large values of **K**, there is no fading at all, resulting in AWGN channel.

The received signal $y_t$ is fed to BFSK demodulator (non-coherent). The demodulated sequence is passed to an iterative decoder for computing soft-output values.

## 4. Simulation Results

The BER performance of the system model (**Fig.(2)**), on 2-paths frequency selective Rayleigh and Rician fading channels is estimated by simulation. The fade rate, $f_d$ $T_b$ =0.001 is normalized by symbol rate, where, $f_d$ is the Doppler shift and $T_b$ is bit duration. The Rician factor, **K**=7. A random interleaver of lengths 3000 and 1000 is used with the number of iterations is six. SCCC is of rate 1/4, with four-states. **Table (1)** lists the design parameters of the constituent codes (CC's).

**Table (1) Design parameters of SCCCs [4]**

| Code | Outer code | | | | | Inner code | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Rate | G | M | $d_f$ | $w_m$ | Rate | G | M | $d_f$ | $w_m$ |
| SCCC1 | 1/2 NR | (7 5) | 2 | 5 | 1 | 1/2 R | (1 5/7) | 2 | 5 | 2 |
| SCCC2 | 1/2 R | (1 5/7) | 2 | 5 | 2 | 1/2 R | (1 5/7) | 2 | 5 | 2 |

Where, G is the generator polynomial (octal form) of each code, M is the number of memories, $d_f$ is the free distance (or minimum Hamming weight of error events for convolutional CC's), $w_m$ is minimum weight of an input sequence generated by a constituent code [4].

**Figure (3)** and **(4)** show the BER performance versus SNR of SCCC1 and SCCC2 in frequency selective Rayleigh and Rician fading channels respectively with random interleaver of lengths N=3000 and 1000. The performance shows a significant interleaver gain, i.e., lower values of BER for a higher value of N. At BER= $10^{-4}$, there is 0.5 dB and 0.7 dB interleaver gain obtained of SCCC1 over frequency selective Rayleigh and Rician fading channels respectively. So for the values of N where the SCCC performance is dominated by its free distance $d_f^{SCCC}$, increasing the interleaver length yields again in performance for both fading channels.

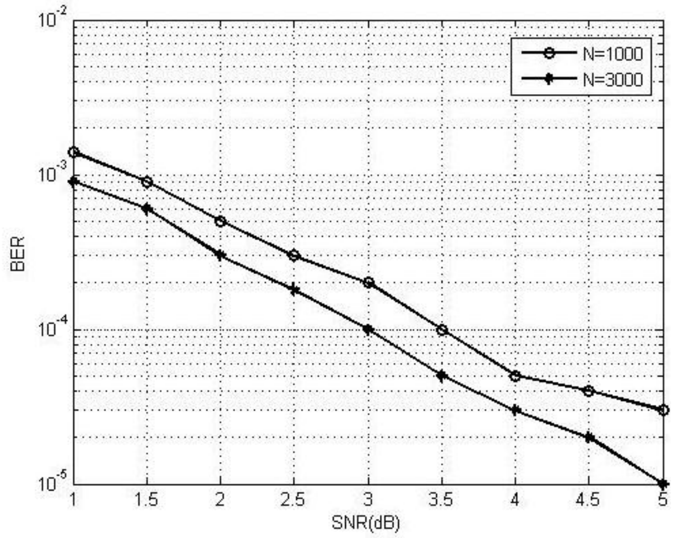


**Figure (3) BER performance of SCCC1 for random interleaver lengths, N=1000 and N=3000 in frequency selective Rayleigh fading channel**

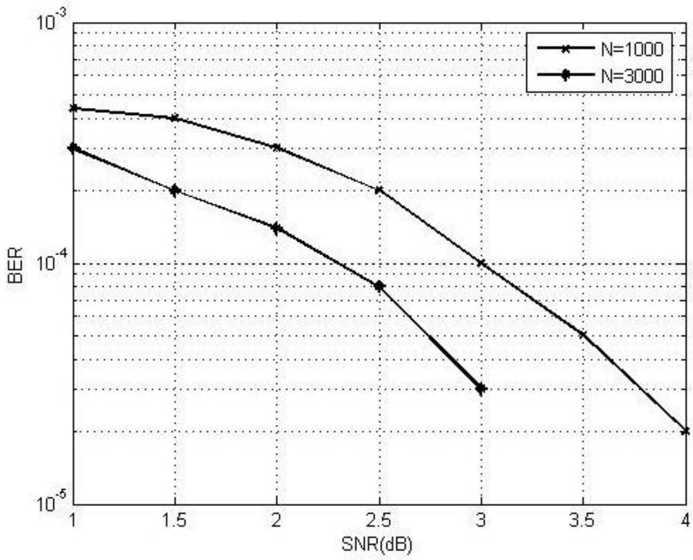


**Figure (4) BER performance of SCCC1 for random interleaver lengths, N=1000 and N=3000 in frequency selective Rician fading channel**

**Figure (5)** and **(6)** show BER performance versus SNR of SCCC1 and SCCC2 with random interleaver of length N=3000, in frequency selective Rayleigh and Rician fading channels respectively. For a recursive inner convolutional encoder, the minimum weight of input sequences generating error event is 2 and thus always yields an interleaver gain. Moreover, as to an outer code, one should have minimum number of input sequences generating free distance error, and since nonrecursive encoders have error events with weight 1, it is convenient to choose as outer code a nonrecursive convolutional encoder. Therefore, SCCC1 has better BER performance than SCCC2 for both fading channels.

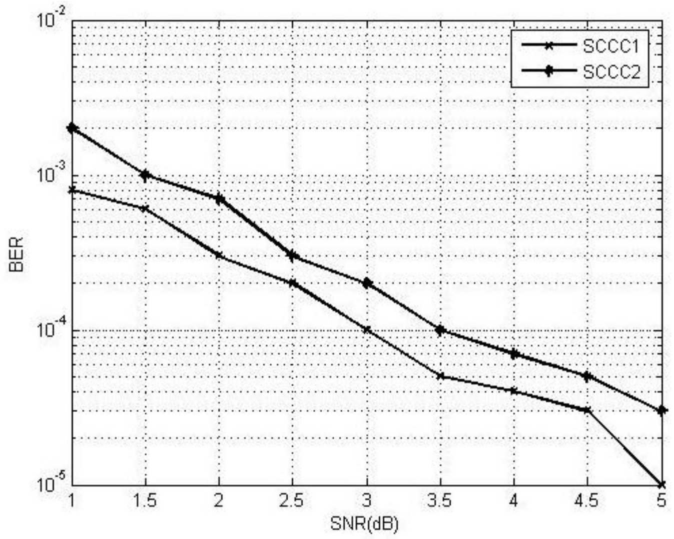


**Figure (5) BER Performance of SCCC1 and SCCC2 in frequency selective Rayleigh fading channel, N=3000**

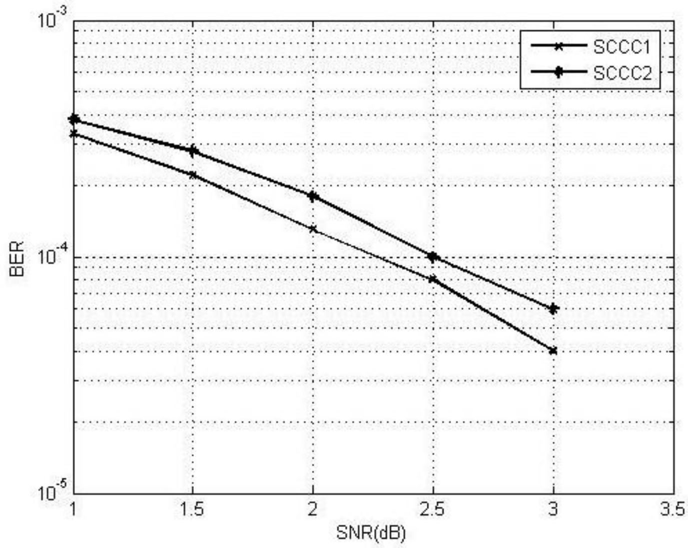


**Figure (6) BER Performance of SCCC1 and SCCC2 in frequency selective Rician fading channel, N=3000**

**Figure (7)** and **(8)** show BER performance versus SNR with random interleaver of length N=3000, and number of iterations is four, in frequency selective Rayleigh, and Rician fading channels as well as AWGN channel for each SCCC1 and SCCC2. SCCC1 performs a slightly better than SCCC2 for all channels especially at high SNR values, also it can be concluded that at iteration six, the BER performance is zero for both SCCC1 and SCCC2 in AWGN channel.

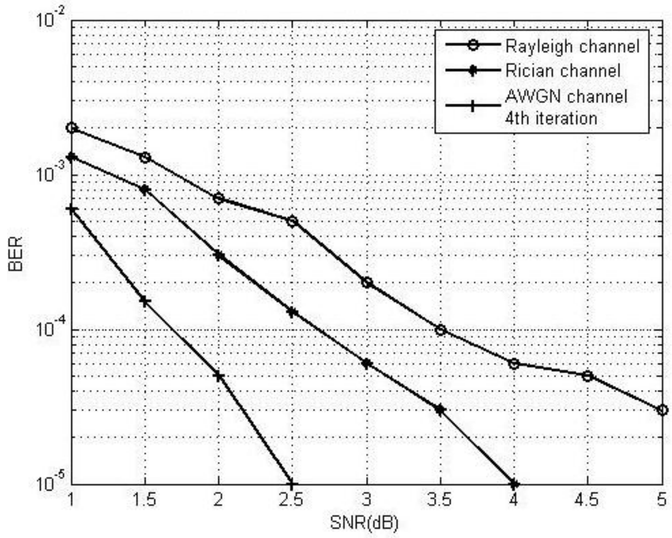


**Figure (7) BER performance of SCCC1 in frequency selective fading channels and AWGN channel, N=3000**

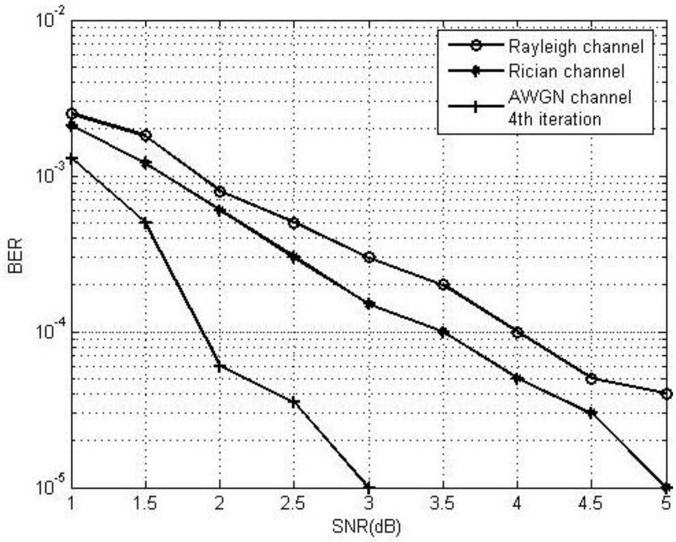


**Figure (8) BER performance of SCCC2 in frequency selective fading channels and AWGN channel, N=3000**

## 5. Conclusion

In this paper, iterative decoding using SISO decoding algorithm of convolutionally encoded signals with FSK modem under multipath (2-paths) Rayleigh and Rician channels are estimated by simulation.

Simulation results show that the performance of SCCC depends on the selection of the outer and inner convolutional codes as well as the interleaver size. It has been shown that, the most favorable SCCC is when the outer encoder is a non-recursive (NR) encoder and the inner encoder is a recursive (R) encoder.

## 6. References

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