Toward an Appeal for Revision on ACI Stress Block Parameters for High Strength Concrete

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Abstract

The continuous increase in the concrete strength has called for adapting and amending the rectangular stress block parameter values recommended by provisions. This study reviews several different mathematical stress-strain models, and adopts a numerical technique through which a complete stress-strain curve characteristic for any strength is determined. The study highlights the effect of stress-strain model on equivalent stress block parameters.

The paper illustrates the variation of the new parameter values against concrete strength together with those of provisions and of other studies. For the purpose of safe, economic, and vast use; the author suggests two formulas to calculate modified stress block parameters.

الخلاصة

ان دراسة عدة نماذج رياضية أقترحت على فترات مختلفة لتمثيل مخطط الاجهاد-الانفعال للخرسانة في الانضغاط، أمكن من اختيار النموذج الاصلح لتغطية مدى واسع من الخرسانة عالية الانضغاط. ان اختيار النموذج الرياضي الامثل ومن ثم اقتراح تكنيك رياضي يوصل الى تغطية خصائص كل نوع ويعطي تصور كامل لمنحنى الاجهاد-الانفعال وذلك بالاستعانة بالبرنامج الرياضي **Computer Software Maple 9.5®** لانجاز صيغ دقيقة **Closed form** للمشتقات والتكاملات التي يتطلبها العمل. هذه المعطيات كانت كافية لحساب معاملات جديدة للبلوكة (القطعة) المستطيلة المكافئة لاجهادات الضغط. تم اعطاء استنتاجات كافية لتغير المعاملات مع المقاومات المختلفة، معتدلة، عالية، وفائقة الانضغاط.

تمت مقارنة تلك المعاملات مع أخرى توصي بها المواصفات العالمية الشائعة، ومع أخرى اقترحت -حديثا- لاخرين. اقترحت الدراسة صيغا معدلة لتلك التي تقرّها المواصفات. يعتقد الباحث ان الاوان قد حان للاخذ بها مع الازدياد المتصاعد في مقاومة الانضغاط للخرسانة.

1. Introduction

In the 50 years since the American Concrete Institute (ACI) 318-56 Code introduced the ultimate strength design method; this method has become the primary method for reinforced concrete design in the United States. The concept of the ultimate or limit state design has also been incorporated into building codes around the world.

The modern analytical approach to predict the (ultimate) concrete beam strength originated by F. Stüssi in 1932 [1], called for the proliferation of mathematical representations of concrete stress block which exert the concrete section from the extreme fiber to the neutral axis. Historically, a number of simplified, fictitious equivalent stress distributions have been proposed in various periods. **Fig.(1)** [2] demonstrates a historical graphic review that takes place on compression stress block. The actual geometrical shape of compression stress distribution may be complex as well as variable. However, its complete and precise knowledge is not required if the magnitude of the compressive force C and its location are known. Now any other convenient geometrical shape may replace the actual distribution. If the new geometrical shape maintains magnitude and location of the compressive force, the final answer is not affected.

In the 1942 Whitney proposed the use of a rectangular compressive stress distribution to replace the actual distribution [3]. The ACI code and other international codes allow the use of a rectangular block to replace the more exact stress distribution because of its simplicity in application especially for irregular sections, **Fig.(2)**. The equivalent rectangular stress block, assumes a uniform stress of γ f'_c over a depth ($a = \beta_1 * c$), determined so that ($a/2 = \beta * c$).

Due to the different characteristics of high strength concrete HSC some design procedures which are customary used in normal strength concrete NSC structures have to be changed. The stress-strain curve for HSC is different than that for NSC. This has an effect on the equivalent rectangular stress block parameters. Modifications are necessary for the efficient use of very HSC.

The actual stress distribution in a very HSC section is almost triangular as shown in **Fig.(2)**. If the equivalent stress block depth factor, β_1, is set equal to 0.65, the coefficient, γ needs to be equal to 0.75 in order to maintain an equivalent force level between the triangular and the rectangular stress blocks. To maintain equivalent force level between the triangle and the rectangle, the γ coefficient should be 0.75 rather than the conventional 0.85 [4].

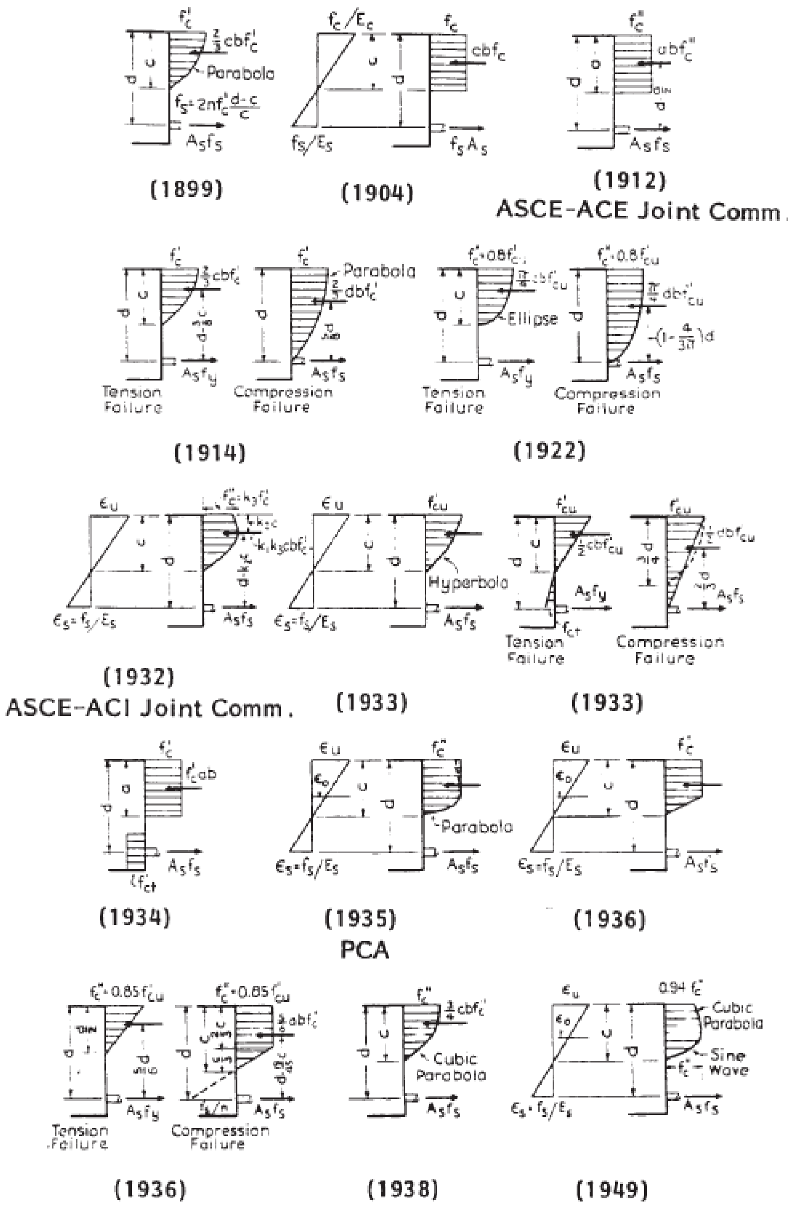

Figure (1) Development of ultimate strength theories of flexure [2]

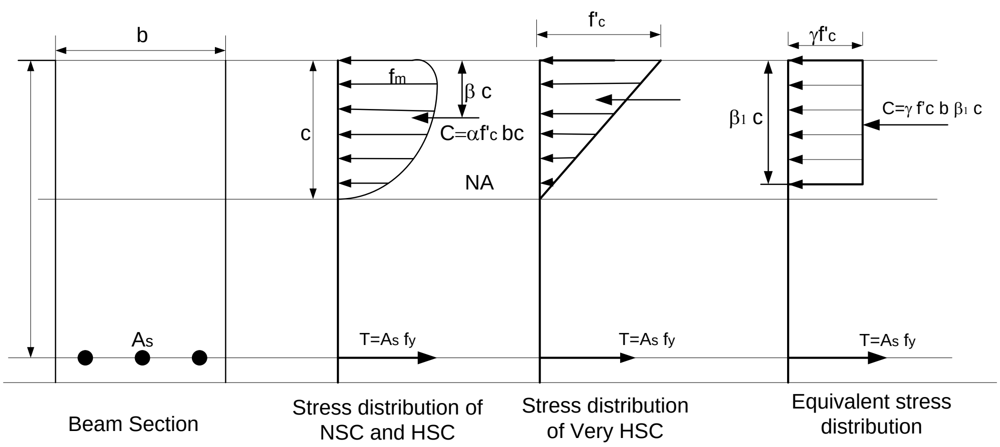

Figure (2) Stress distribution diagrams for various concrete strengths

This paper presents a series of curves for equivalent stress block by reviewing five different stress-strain models. The analysis will include computation of concrete compressive strain at failure, the depth of the neutral axis and the equivalent stress block parameters β_1 and γ. In order to formulate equivalent stress block parameters, different stress-strain models are investigated and the significance of the calculations on high strength concrete HSC is focused. The widespread use of HSC worldwide with rare revisions occurring at ACI code stress block parameters through the period (1977-2005) motivated the interest of this paper in studying these parameters from both economical and safety views. In this work comparisons are made between the parameters produced by the modified stress-strain curves and those given by international codes, and other theoretical studies, to present an idea about degrees of safety obtained by using those curves.

2. Compressive Stress-Strain Relations

The experimental investigation of the uniaxial stress-strain behavior of concrete in compression together with the analytical representation of the behavior by proper mathematical forms were the matter of interest and the subject of extensive study during the last century.

Generally, the stress-strain relation is composed of a strain-hardening portion extending to the maximum stress (f_m), which is of the same shape for various testing conditions, followed by a strain-softening portion extending to the ultimate compressive strain (ε_{cu}), which is much affected by the testing machine (constant rate of strain or constant rate of stress) and the aggregate characteristics. The essential features defining the stress-strain relation are: (1) maximum compressive strength f'_c, (2) initial modulus (E_{ci}), (3) the strain (ε_m) corresponding to the maximum stress, and (4) the descending branch which shows steeper unloading for higher strength concrete, and it is much affected by the testing machine.

Since 1899, many mathematical functions have been proposed to represent the ascending part of the stress-strain relation in compression; linear, parabola, polynomial, etc. The importance of introducing the descending part was recognized in the early fifties of the last century. Since then many equations were proposed to represent the complete stress-strain relation. These are found in several forms (parabolic-constant [2], parabolic-linear [5], exponential [6], fractional [7]). Other investigators proposed similar relations with certain revisions, modifications and refinements [8, 9].

Several equations were proposed for the simulation of unconfined compression behavior, some of which are reviewed below.

2-1 Hognestad Relation [5]

Hognestad 1951 [5] proposed his well-known parabolic relation with a straight line representation of the descending part.

$$\sigma = f_m \left[2\frac{\varepsilon}{\varepsilon_m} - \left(\frac{\varepsilon}{\varepsilon_m}\right)^2 \right] \qquad 0 \le \varepsilon \le \varepsilon_m \quad \text{......} \quad (1)$$

$$\sigma = f_m \left[1 - 0.15\left(\frac{\varepsilon - \varepsilon_m}{\varepsilon_{cu} - \varepsilon_m}\right) \right] \qquad \varepsilon_m \le \varepsilon \le \varepsilon_{cu} \quad \text{......} \quad (2)$$

$$\frac{E_{ci}}{f_m / \varepsilon_m} = 2 \quad \text{......} \quad (3)$$

$$E_{ci} = 12418 + 460 f'_c \quad \text{......} \quad (4)$$

The above relations were widely used by investigators due to their simplicity. The stress-strain curves representing Hognestad model is shown in **Fig.(3)**.

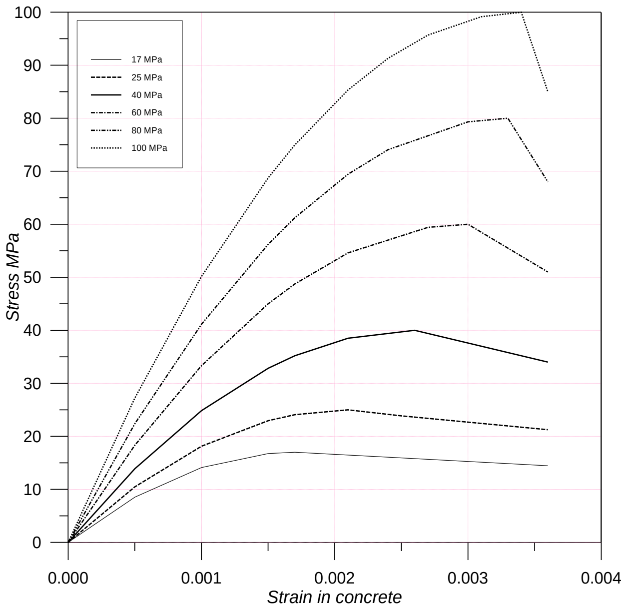

Figure (3) Stress-strain relationship of concrete based on Hognestad model (1951)

2-2 Desayi and Krishnan Relation [7]

Desayi and Krishnan 1964 [7] proposed a new general relation, utilizing Eq.3. Their model showed slightly better fit to the experimental data of Smith and Young than their exponential function. The model is simple in form such that closed-form integration can be evaluated to calculate the stress-block parameters.

$$\sigma = f_m \left[\frac{2\,\varepsilon/\varepsilon_m}{1+\left(\varepsilon/\varepsilon_m\right)^2} \right] \quad \text{......} (5)$$

The stress-strain curves representing Desayi and Krishnan model is shown in Fig.4. It becomes obvious from **Figs.(3)** and **(4)** that Hognestad and Desayi relations were proposed to simulate the stress-strain curves of NSC. They did not reflect the behavior of HSC (sharper peak and steeper downward section beyond the peak stress). This may attributed to the common low levels of strength at that period.

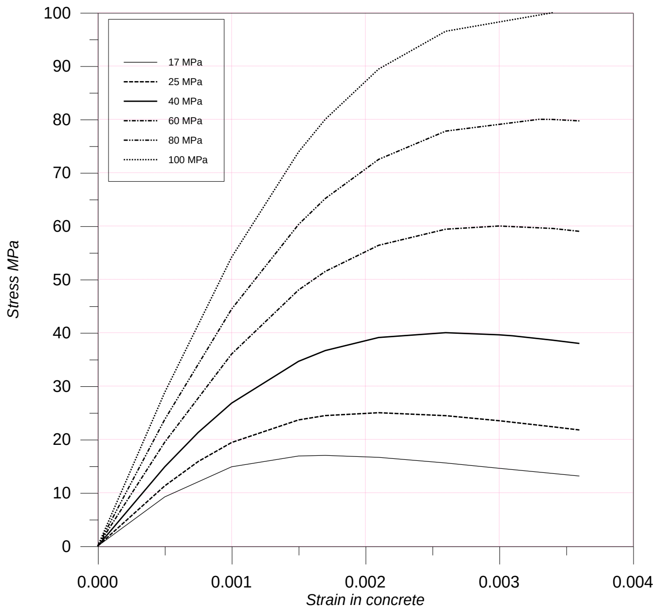

Figure (4) Stress-strain relation of concrete based on Desayi and Krishnan model (1964)

2-3 Popovics Relation [8]

Popovics 1973 [8] suggested the use of variables ($E_{ci}/\ f_m/\ \varepsilon_m$) to include the effect of concrete composition in his formula.

$$\sigma = \frac{\varepsilon}{\varepsilon_m} \cdot \frac{n \cdot f_m}{(n-1)+\left(\frac{\varepsilon}{\varepsilon_m}\right)^2} \quad \text{......} (6)$$

where:

$$\frac{E_{ci}}{f_m/\varepsilon_m} = \frac{n}{n-1} \quad \text{......} (7)$$

He proposed a relation to evaluate (n) in terms of *f'c* depending on the values of ($E_{ci}\ /\ f_m/\ \varepsilon_m$):

$$n = k.f'_c + 1 \qquad (8)$$

where: $k = 0.058\,(MPa)^{-1}$

The proposed stress-strain curves are shown in **Fig.(5)**.

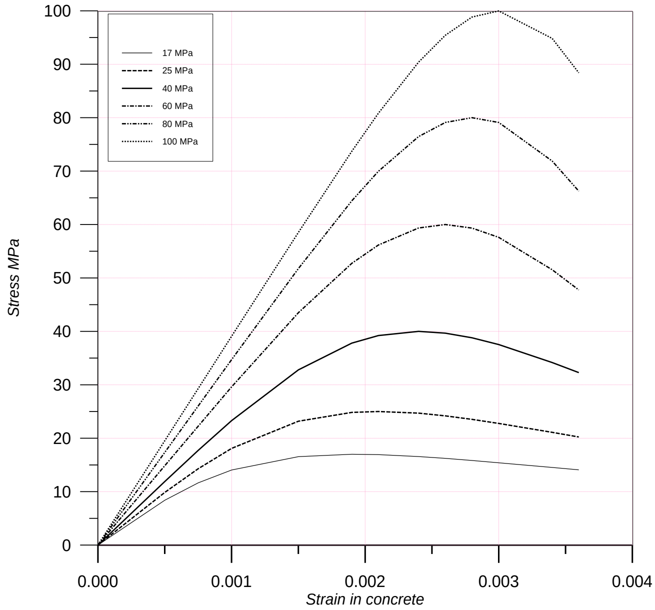

Figure (5) Stress-strain relationship of concrete based on Popovics model (1973)

2-4 Carreira and Chu Relation [9]

Carreira and Chu 1985 [9] proposed Eq.6 again to represent the complete unconfined stress-strain relation. A large number of experimental data was used to fit the equation and to derive a nonlinear relation for evaluating *n* in terms of *f'c*. But the results of their equation differ much from those of Eq.8.

$$\sigma = \frac{\varepsilon}{\varepsilon_m} \cdot \frac{n \cdot f_m}{(n-1) + \left(\frac{\varepsilon}{\varepsilon_m}\right)^n} \qquad (9)$$

$$n = \left(\frac{f'_c}{k}\right)^3 + 1.55 \qquad (10)$$

where: k= 32.4 MPa

$$\varepsilon_m = \left(0.71 f'_c + 168\right) * 10^{-5} \qquad (11)$$

This model depicts better the stress-strain behavior of wide range of strengths. It shows relatively flat top for lower strength, and relatively straight line ascending to the peak for HSC, **Fig.(6)**.

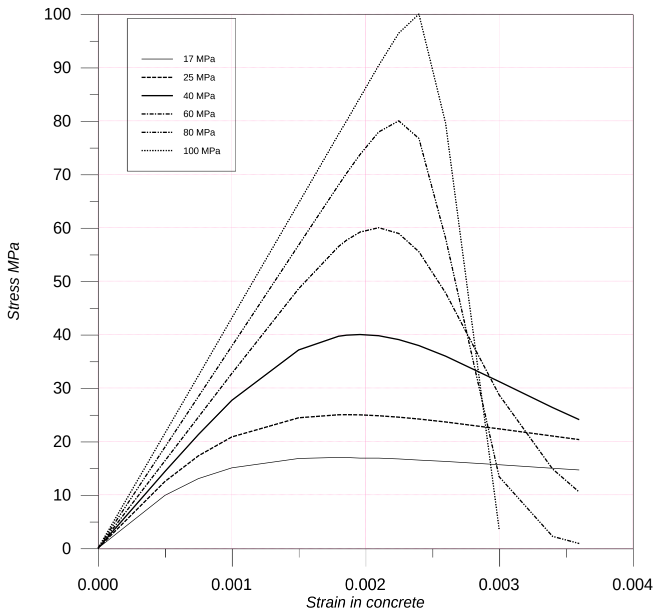

Figure (6) Stress-strain relation of concrete based on Carreira and Chu (1985)

2-5 Collins et. al.

According to Kumar [10], Collins et. al. (1993) proposed a new equation which may be considered as a generalization to Eq.6 of Popovics. It consists of two parameters, one to control the ascending part and the other to control the descending part, which may refine the accuracy of the relation.

$$\sigma = \frac{\varepsilon}{\varepsilon_m} \cdot \frac{n \cdot f_m}{(n-1) + \left(\frac{\varepsilon}{\varepsilon_m}\right)^{nk}} \quad \text{.......} \quad (12)$$

where:

$$\frac{E_{ci}}{f_m / \varepsilon_m} = \frac{n}{n-1} \quad \text{.......} \quad (13)$$

$$n = 0.8 + \frac{f_m}{17} \quad \text{.......} \quad (14)$$

$k = 0.67 + \frac{f_m}{62}$ *for the descending part*

k=1 *for the ascending part*

$$E_{ci} = 3320\sqrt{f_c'} + 6900 \quad \text{...} \quad (15)$$

This model accounts for the fact that the stress-strain curves drop at higher rate after the peak stress for HSC compared to NSC, **Fig.(7)**.

By taking a quick look at the family of stress-strain curves of the five reviewed models **Figs.(3-7)**, which represent different periods of concrete strength development, one may easily decides the limited applicability of some models and the large scale of others.

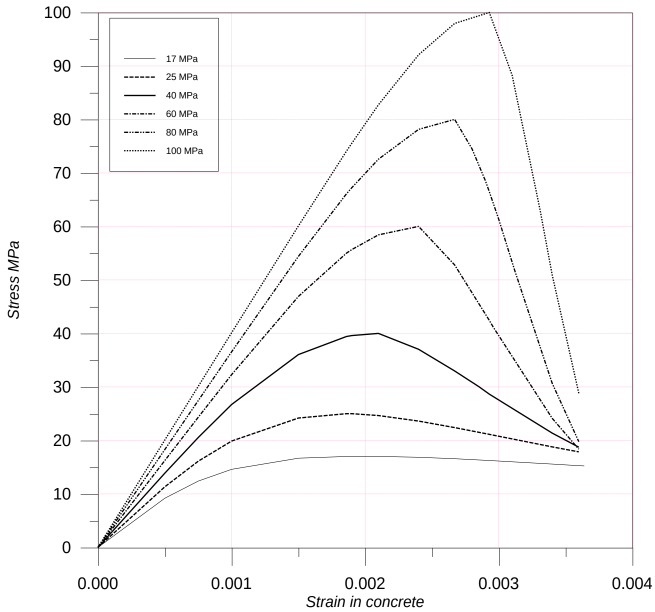

Figure (7) Stress-strain relationship of concrete based on Collins et. al. model (1993)

3. Equivalent Rectangular Stress Block Parameters

The major code provisions on the parameters are summarized in **Table (1)**. It is obvious that these parameters were adopted for NSC and the inconsistency noticed in **Table (1)** is mainly due to inclusion of HSC.

Table (1) Code provisions of equivalent rectangular stress block

Code	γ	β_1
ACI 318M-02 [11]	0.85	$0.85-0.00714(f'_c-30)$ $\beta_1 \geq 0.65$
CAN3-M,1994 [12]	$0.85-0.00015\ f'_c$ $\gamma \geq 0.67$	$0.97-0.0025\ f'_c$ $\beta_1 \geq 0.67$
BS8110-1985 [13]	0.67	0.90
EC2-1992 [14,15]	0.85	0.80

The equivalent rectangular stress block parameters mentioned in **Fig.(8)**, have been determined analytically. The strategy requires adoption of rational stress-strain relation to cover wide range of concrete strengths, and to well-defined stress-strain behavior of concrete in compression. For the purpose of comparison the five reviewed stress-strain models have been used. The computer software MAPLE 9.5® [16] has been utilized to derive the necessary closed-form integration formulas. Maximum crushing strain ε_{cu} at concrete extreme fiber of 0.003 has been used as ACI Code assumes [11]. Contemporary practical strength ranges (17-100 MPa) have been used to calculate the corresponding rectangular stress block parameters β_1 and γ. **Table (2)** shows the results of this analytical process.

Table (2) Equivalent rectangular stress block parameters according to models reviewed in literature (ε_{cu}=0.003)

f_m		17 MPa	25 MPa	40 MPa	60 MPa	80 MPa	100 MPa
Hognestad model [5]	β_1	0.84	0.81	0.78	0.75	0.74	0.74
	γ	0.94	0.93	0.90	0.89	0.86	0.84
Desayie &Krishnan model[7]	β_1	0.86	0.82	0.78	0.76	0.75	0.75
	γ	0.93	0.95	0.94	0.91	0.88	0.87
Popovics model [8]	β_1	0.84	0.80	0.76	0.72	0.70	0.68
	γ	0.94	0.94	0.91	0.90	0.86	0.83
Carriera & Chu model [9]	β_1	0.86	0.84	0.80	0.80	0.80	0.80
	γ	0.96	0.94	0.90	0.78	0.70	0.65
Collins et al. model [10]	β_1	0.84	0.84	0.80	0.76	0.72	0.69
	γ	0.97	0.91	0.87	0.85	0.85	0.84

4. Present Study Strategy and Mathematical Model

As it is known as a concept, also this paper intends to replace the actual stress distribution in a reinforced concrete flexural member with a simpler rectangular shape, which maintains magnitude and location of the resultant compressive force. **Figure (8)** represents the basic concept of stress block replacement, with most notations used in this study.

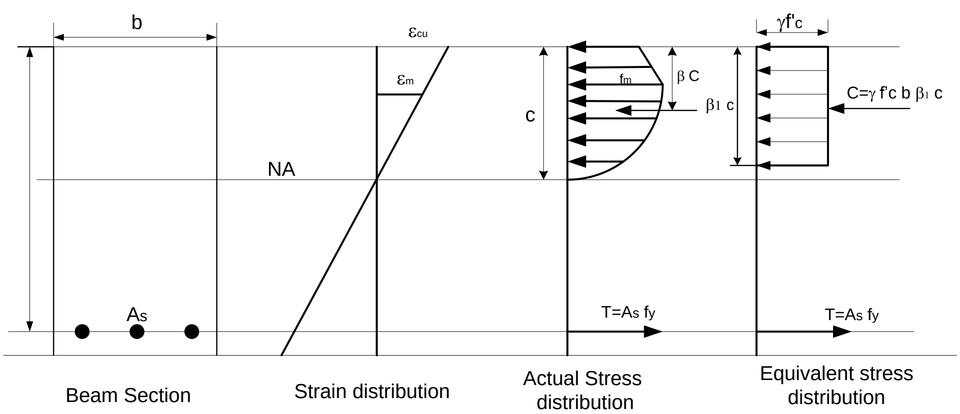

Figure (8) Concept of equivalent rectangular stress block

The following mathematical model is adopted in this paper (Carreira and Chu 1985) [9]. The stress σ and strain ε are used in non-dimensional form, $\mathbf{y} = \sigma / \mathbf{f_m}$ and $\mathbf{x} = \varepsilon / \varepsilon_m$:

$$\mathbf{y} = \frac{\mathbf{n\,.x}}{(\mathbf{n}-1)+(\mathbf{x})^{\mathbf{n}}} \quad \text{.. (16)}$$

where:

n and ε_m: are as suggested by Carreira and Chu and given in Eq.10 and Eq.11.

The curvatures above and below the inflection point on the descending part of the stress-strain curve are of opposite sign, the curvature must vanish at the inflection point. This condition has been used in the present study utilizing the computer software MAPLE 9.5® [16] to generate the following equation in order to calculate the strain ratio $\varepsilon_{cu}/\varepsilon_m$:

$$\mathbf{x} = e^{\frac{\ln(\mathbf{n}+1)}{\mathbf{n}}} \quad \text{.. (17)}$$

Or in terms of explicit strain values:

$$\varepsilon_{cu} = e^{\left(\frac{\ln(\mathbf{n}+1)+\mathbf{n}\ln\varepsilon_m}{\mathbf{n}}\right)} \quad \text{.. (18)}$$

The inflection point may be located from Eq.17, hence ε_{cu} for any value of strength is known. By using this mathematical technique a complete depiction of stress-strain curves for any compressive value is achieved.

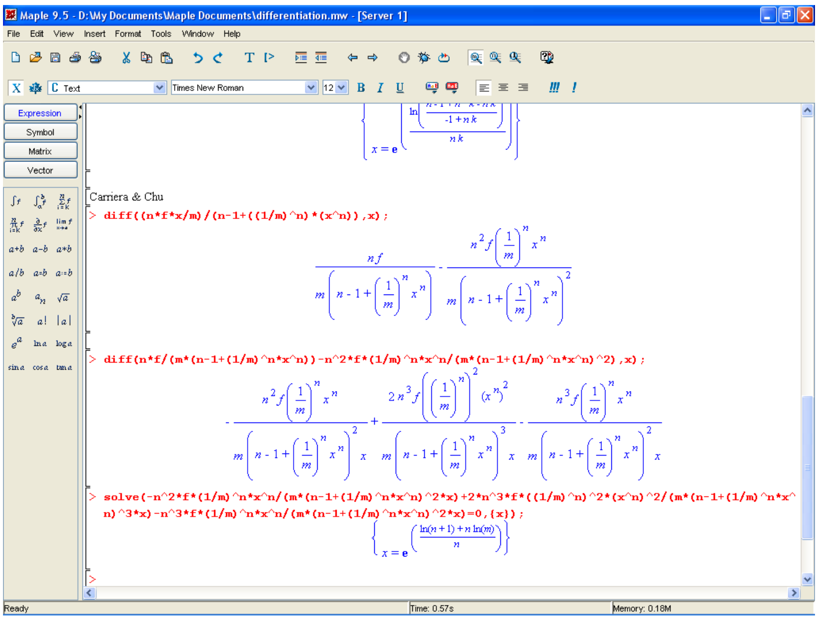

Figure (9) Computer software MAPLE 9.5® interface

5. Factors Affecting Stress Block Parameters β1 and γ

This study has confirmed that the parameters **γ** and β_1 are affected mainly by two factors; the maximum compressive stress, and the ratio of ultimate to peak strain (strain ratio=$\varepsilon_{cu}/\varepsilon_m$). While the beam section dimensions do not govern values of stress block parameters, when the section is either rectangular or triangular. The strain ratio ($\varepsilon_{cu}/\varepsilon_m$) has been included the contribution of size of the descending part of the stress-strain curve. Calculations show that parameter **γ** becomes constant for the same strain ratio regardless of changing of maximum compressive stress. Experimental evidence suggests that ε_m increases, whereas ε_{cu} decreases with increasing concrete strength [1]. As proposed in this study, determination of ε_{cu} by location of inflection point of the stress-strain curve gives quite reasonable and comparable results. By using Eq.17 or Eq.18 value of $\varepsilon_{cu}/\varepsilon_m$=1.79 applies to 17 MPa concrete (ε_{cu}=0.00323 and ε_m=0.0018) and this ratio reduces to 1.05 for 140 MPa concrete (ε_{cu}=0.0028 and ε_m=0.00267), ultimate strain becomes practically too close to peak strain. This simulation ensures that the shape of actual stress block is approximately triangular for very high strength concrete. **Figure (10)** gives strain ratio variation which has been calculated by using the inflection point technique. Meanwhile this study suggests a practical empirical formula to determine the strain ratio for any concrete strength.

$$\varepsilon_{cu}/\varepsilon_m = -0.389\ln f_m + 2.934 \quad \text{.......... (19)}$$

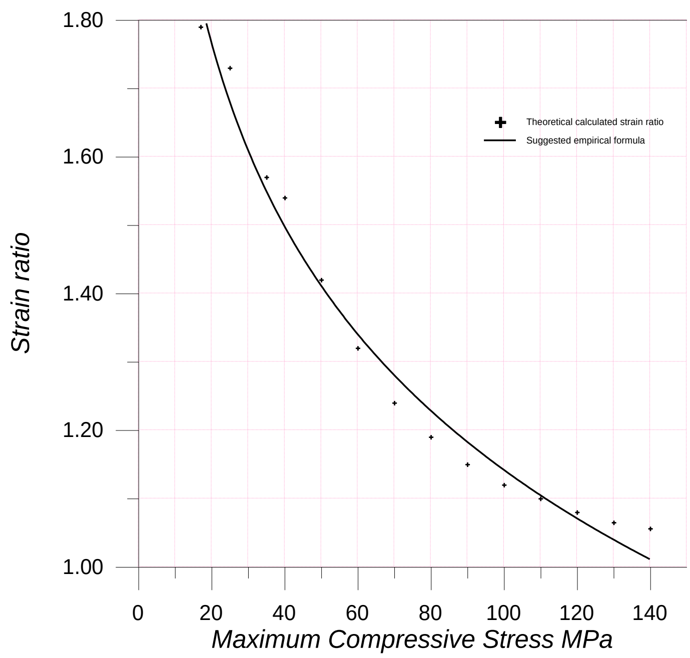

Figure (10) Variation of strain ratio used in analysis

6. Analysis of Results

The analytically computed parameters γ, β_1 and the product γ β_1 for beams of rectangular section are plotted in **Figs.(11)** to **(13)**. The adopted modification and the parameters provision, together with analytical parameters of others are shown in these figures for purpose of comparison. Present study values of parameter γ show uniform descent from upper limit of 0.95 applying for f'_c=17 MPa, to lower limit value of 0.775 for f'_c=140 MPa. Values of γ are greater than the existed ACI recommended values in normal strength range, and smaller in high strength range **Fig.(11)**. Present study suggests the following equation to determine the value of γ for any value of stress:

$$\gamma = 0.95 - 0.0021\left(f'_c - 17\right) \geq 0.775 \quad \text{.. (20)}$$

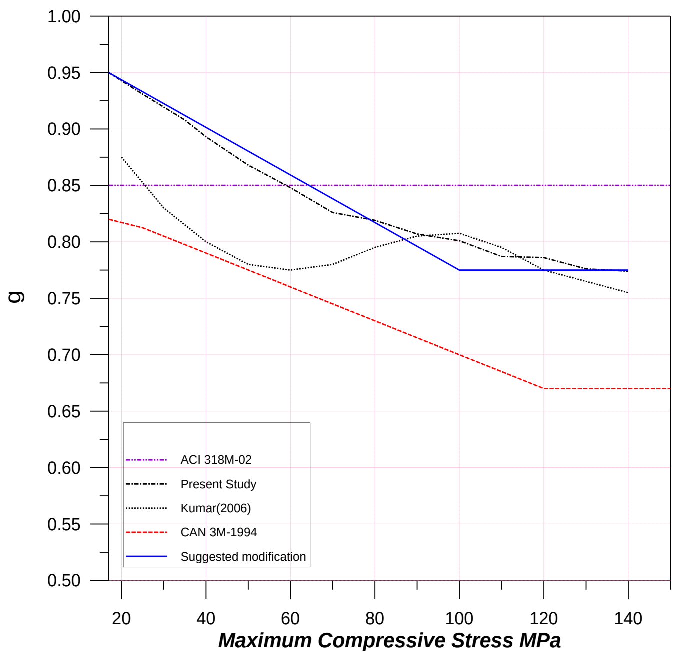

Figure (11) Variation of the parameter γ

Parameter β_1 having the same tendency as Canadian provision but of lower values for all strength range, **Fig.(12)**. The decrease in γ and $\beta_1$for HSC is related to the fact that such concretes are more brittle, and show more sharply curved stress-strain plot with a smaller near-horizontal portion, **Fig.(6)**. When the concrete strength is very high, the compressive stress block is approximately triangular. Theoretical value of rectangular equivalent stress block parameters for triangular stress distribution are (γ =3/4 and β_1=2/3) [4]. These results almost are achieved for concrete strength of 140 MPa. This reality may validate the proposed analysis technique, and makes the adoption of Carriera stress-strain model acceptable.

The effect of increase in concrete strength is to increase the altitude of the peak and to decrease the horizontal distance of stress-strain curve. The net effect of this behavior is to reduce the area under the curve. So the present study results of γ and β_1 together with Canadian provision seem to be rational.

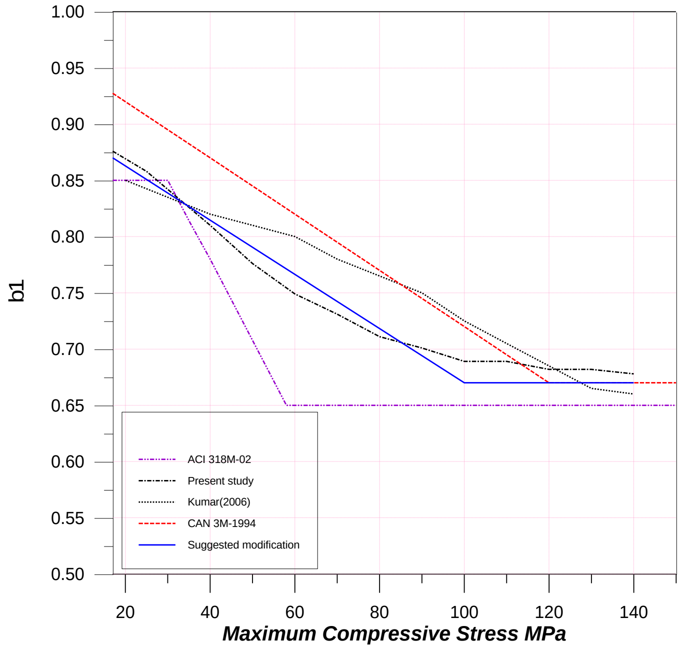

Figure (12) Variation of the parameter β_1

The low values of parameter β_1 to maintain equivalent compressive force location, lead to high values of parameter γ to match equality in the area. Therefore, the exaggerations in reducing the value of $\beta 1$ even to low strength level; definitely urge the same source to elevate values of parameter γ.

Canadian provision as well as present study suggests an increase in the value of parameter β_1 over the current ACI recommended value. Selection of appropriate value of β_1 is important in design of flexural members, especially for the heavily reinforced section. The under-estimated β_1 leads to lower value of theoretical ultimate moment capacity. Present study suggests the following equation to determine the value of β_1 for any value of stress:

$$\beta_1 = 0.87 - 0.0024\left(f'_c - 17\right) \geq 0.67 \quad \text{.. (21)}$$

The results of product parameter $\gamma\ \beta_1$ in **Fig.(13)** show much less discrepancy than the results in **Figs.(11)** and **(12)**. Effect of large γ and small β_1 and the opposite does not appear in their product. Present study results agree with Canadian recommended values for NCS and HSC. The ACI recommended values are smaller for the previous range, whereas ACI recommended values are higher for very HSC.

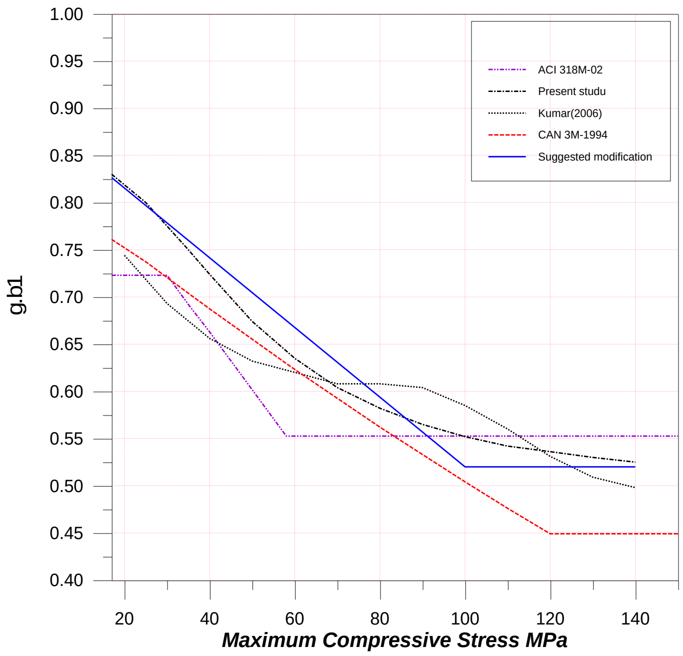

Figure (13) Variation of the parameters product $\gamma\ \beta_1$

7. Conclusions

1. This study suggests a mathematical technique to include adequately the amount of the descending part of the stress-strain curve.
2. The proposed technique has resulted in deriving a logarithmic equation to calculate the strain ratio. The ratio decreases from 1.83 to 1.0 as concrete strength increases from 17 MPa to 140 MPa.
3. The proposed logarithmic variation of strain ratio agrees with results available for comparison. Thus, an empirical formula to estimate strain ratio has been found useful, and has not been mentioned through the published literature.
4. The results show comparable behavior with the Canadian provisions and another recent study than with ACI recommended parameters. This corroborates the strategy of the study. The ACI recommendation on parameters in the HSC and very HSC needs to be amended.
5. The extensive available calculations through this study together with provisions and published works have allowed suggesting two formulas. These formulas may be studied as expected modifications on ACI recommended parameters.

8. References

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16. Maple 9.5®, A Division of Waterloo Maple Inc., 2004, Maple Soft and Maple are Trademark of Waterloo Maple Inc.

Notations

A_s	Bottom steel reinforcement area.
C	Compressive force
E_{ci}	Initial elastic modulus of compressive concrete
f'_c	Maximum compressive strength of concrete (15*30) cm cylinders
f_m	Maximum stress of concrete in flexural compression
f_y	Yield stress of reinforcing steel
T	Tensile force
X	Dimensionless strain
Y	Dimensionless stress
α, β	Actual stress block parameters
γ, β_1	Equivalent rectangular stress block parameters
ε	Strain in concrete
ε_{cu}	Ultimate strain in concrete
ε_m	Strain corresponding to maximum compressive stress
$\varepsilon_{cu}/\varepsilon_m$	Strain ratio
σ	Stress in concrete