# Design of Complex Dynamical Systems

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### Abstract

The design of complex dynamical systems involving multi loop configurations demands appropriate choice of parameters which must match together to provide an acceptable system.

This is investigated by the development of a simulation which provide a conceptual implementation of such systems and verifies the told behavior before its construction. The potential of the approach is explored by applying it to a problem in engineering practice. The approach permits experimentation with different combination of system elements, parameters and constraints.

The efficiency of the approach suggests its suitability as a general way to start the practical design of these systems.

Index terms design of complex control systems, simulation complex dynamical systems, nonlinear control, and digital control systems.

الخلاصــــة إن تصميم الأنظمة الديناميكية المعقدة التي تنطوي على الدارة متعددة الأشكال يتطلب اختيار المعابير المناسبة التي يجب أن تكون متطابقة معا لتوفير نظام مقبول. و هذا يتحقق بايجاد المحاكاة التي توفر إطارا مفاهيميا لتنفيذ هذه النظم، والتحقق من سلوكها قبل بنائها. إن إمكانات هذا النهج تتبين في استكشاف تطبيقه على أي مشكلة في ممارسة الهندسة كما انه يتيح تجربة بمزيج مختلف من عناصر النظام، والمعابير والقيود. كفاءة النهج تكمن في ملائمته كطريقه عامه للبدء بتصميم عملي لهذه الأنظمة .

### 1. Introduction

A control system with a nonlinear controller forms a complex dynamical system. A preliminary design of such a system is usually found to be unsatisfactory in meeting all the required specifications.

Consideration alteration of the parameters of the original design is usually needed. This is so as a change of one parameter may frequently be used to meet characteristic but such a change may another performance characteristic in an undesirable way or beyond a permissible limit <sup>[1,2,3]</sup>. These considerations suggest a need for formulation of the basic relationship between the significant parameters to enable the appropriate choice of the parameters that match together and produce an acceptable system. This is sought in this work by simulation of the systems to provide a conceptual implementation and to verify its total behavior, before its construction. A simulation scheme is developed for complex dynamical systems with the following specific) features.

- (i) The scheme provides the necessary information and relieves the designer from involvement in a rigorous theoretical, mathematical or complete computational treatment needed for these stems.
- (ii) To a control engineer, having a good background in computer related topics of programming and mathematical skills, the scheme brings programming and' mathematical economy of thought and considerations without the task: of learning a simulation language
- (iii) It allows modification of the model to reflect new problems and situations of interest for a variety of control strategies and. engineering constraints.
- (iv) It is developed from the time domain view point and it is inherently suitable for the digitally controlled version of the system.

### 2. Model Structure

The complete description of the scheme and its concepts is outlined in this section to present the approach in a unified manner; the model is based and developed for systems which incorporate optimal control strategies as these are found to be suitable to describe the various aspects of the scheme. Starting with the general diagram of multi loop control systems shown in **Fig.** (1) action based on a more realistic appraisal of the system performance and objective than is possible with the measurements and control of a single variable as with conventional elements. In general, the dynamics of the plant can always be formulated as a set of first or differential equations of the form:

#### with yj(0) = yj, as the initial conditions and j=1,2,

Generally there are 1 two sorts of constrains in complex control systems. They can be represented by inequalities that concern both the state and the control variables. These may be written as, j = 1, 2, ..., p

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where: Myj and Muj are constants these together with the functions Nyj and Nuj must be determined for every practical case. The control strategy provides a set of control variables ui(t), or synthesizing functions, so that

#### where:

i=1,2,.....p



Figure (1) General structure of a complex dynamical system

The discrete model and the sequence of computation to pivotal values along a single increment of time is now developed. An appropriate recursion relation for equation (1) has the form:

yj(tr)=Fj(yl (tr1),.....yq(tri),ui(tr), .....up(tr), At.)

where:

yj(o)yj, j=1,2,...,q

At is a small time step and r is the scanning index. The nonlinear transfer characteristics are incorporated in suitable subroutines <sup>[1,4]</sup> Hence, the constants given in equation (2) may be described by the symbolic discrete equation;

Ny (l (tr), 
$$\dots$$
 yq(tr) = Myj

where:

j=1,2,...,q

The developed model shows that the variables of similar functions, such as command variables x, control variables or state variables associated with the multiple lines feeding or connecting the blocks of the system are seen to correspond to the same the inputs are all available to a particular block, a sequence of is then made to provide the output variables of the block. The linear plant is treated through the state transient equation representing the

differential equation of the plant, generally the discrete model, provides the current state value as a function of the previous state values and the current input values. This is implemented by any desired integration routine available. The nonlinear trans associated with the practical constraints the control strategy on the system control and state variables provide a modification of their current values.

The controller output variables are calculated in terms of the current values of the input variables and the previous values of the state. Variables provided by the feedback loops. Thus calculation for the current values of output variables of the controller block may be carried first. This is then followed by these variable values in accordance with the constrain of the control strategy. Then the current state variables are calculated and finally the state variable in accordance with the practical constrain are notified thus, a complete act of system variable: values at the current time to be then developed. Repeating this cycle, each time r is increased by one, solution over the desired time range is provided.

The sequence of calculations is indicated in the flowchart of the **Fig. (2)**. This provides the idea of implementation of the model structure and indicates how the variable values are provided through the multiple lines connecting the blocks of the system.

The approach is promoted further by simulating a DDC version of the mathematical model, conforms ideally with the DDC version of systems operation because the parameters arc taken at the respective timing when incorporating the digital elements in the feedback loop of these systems. Here signals that are feedback from the output are sampled and quantified in magnitude by sampler and the encoder respectively. The encoder of the AID converter may be depicted by a staircase (nonlinear) characteristic of the number of setups is associated with the fixed word length of the conversion. The digital element processes the incoming digital signal according to the implemented control algorithm. Finally, the D/A converter then produce the Analog signal and hold the converted value.



Figure (2) Flow Chart for the simulation program

### 3. Application to Engineering Problem

The simulation scheme is applied to a normalized model of a rocket and a reaction a motor for compensation the rocket angular deviation and keeping it on course. This comes to anon to release the rotor speed from being river rated variable is also kept at level when the system n at shady state.

The analytical expression of the optimum switching function is represented by a nonlinear expression <sup>[5,6,7]</sup>. This provides the feedback variable in terms of the angular position, rotational speed, gain and the time constant of the plant. The mathematical expressions are not given as they can be found in the above mentioned references. The designer requires considering the following aspects; firstly; the errors reflected on the response of the system from the ideal case by the use of (1) the logarithmic amplifier to accommodate the controller logarithm term (2) an approximate controller which frees the system from implementing the logarithmic amplifier which always has imperfections. The digital controller which introduces time and magnitude quantization. Secondly; the change in the time response duration. Due to a change in the gain magnification of the control variable feeding the (1) motor (2) a changes in the rated speed, when the motor torque is fixed, which changes the power, size, mass and efficiency of the chosen motor.

### 4. The Simulation Accuracy and Results

In this section, a numerical expression for the determination of the model accuracy is presented first. This is followed by the simulation results that are carried out. An iterative two value problem approached described. This utilizes the establish a reference solution 40 which the simulation solution is to be compared. The solution of the problem consists of two segments. The transition instant value (Ts) marks the termination of the first segment and the commencement of the second segment solution. The process started by setting an approximate value for the transition instant. This determines the equation for the second segment as indicated above. At an output position to the required input position the speed value is checked. If the value is different from zeroed. Its steady state value, the initial value for the transition instant is then adjusted such as to make the final speed value move towards zero. This iteration is repeated until the speed equals or is as close to zero as desired. The start and end of the second segment, by this way to establish with the first segment the whole reference solution. Using the above iterative approach a reference solution was established that provided a diminished speed of an order of a simulation program is constructed for the system on the bases of the model structure developed earlier. The percentage error is computed by calculating the r.m.s. value of the differences between the response of the simulated system and that of the reference numerical solution described above.

The differences are computed over a fixed set of points, twenty points in the present case that covers the transient range of the response.

The error is then referred to the maximum value of the response. The deviation

percentage for the position and speed of the model is shown in **Fig. (3)**. They model and reference solution are not significantly different and that higher accuracy of the model is achieved at fast rate when he pivotal integration steps are increased from 200 poi Its to 700 points on a solution range of 0.4 sec, of course, the advantage of higher accuracy is to increase the margin of convergence and stability of the model solution, the degree of the agreements of the simulation with the reference solution, verifies the validity of the simulation model for the problem.



Figure (3) Numerical experimentation for the model validation

The optimum responses of the simulated system are shown in **Fig.** (4). The graphs marked "a" display the response of the system for a step command of ten degrees. They indicate that the motor speed rise to certain extent, but remains below the rated speed then falls to zero. The graphs marked "b" display the response of the system for a step command of thirty degrees. They indicate that the motor speed rises to its value It remains at this speed for a certain time, then falls to zero.

The speed is normalized with respect to the rated speed. However, the position variable is normalized -with respect to the value of the applied input.

The information needed for the designer to obtain various combinations of the system parameters to modify the preliminary design and choose parameters values are given below. When the impart of the logarithmic amplifier on the respect of the continuous system is needed, the errors of this unit is included to modify the logarithmic function operation by the corresponding error percentage errors reaching art order of 9.8% is recorded. This indicates a preference towards the implementation of a controller which avoids the use of the logarithmic amplifier for a digital controller.



Figure (4) States of the optimal system when rated speed is a. not exceeded, b. exceede

The effect of replacing the controller by another which approximate the function of the first is investigated by:

This is replacing controller avoids the use of logarithmic amplifiers. The percentage deviation is calculated as the error of the system with the original controller and that of the system using the approximate controller. Here the control action is obtained from the synthesizing function approximation. Approximation is exercised through replacement of In1+Y/K by its series expansion. The results are given below for successively increasing number of terms o-f the expansion.

#### No. of terms 1 2 3 4 % deviation 5.2 1.7 0.5 0.03

Although some of the resulting systems are acceptable, the complexity involved in implementing the approximate controllers are still present.

Other methods may also be used to approximate the controller action. However, for any method the process shows that the developed simulation approach lends itself to the experience of making a design decision, testing it, observing the results, and trying again, until a desired performance of the system is achieved.

This encourage the use of this technique in future work to involve the sensitivity analysis which plays an important role in the design and implementation of suboptimal system which approach the sub-optimal system as an initial value problem.

The simulation of the digital controller version of the first system is also carried out. The time response (measured from the application of the command input to the last different combinations of gain magnification and motor rated i speed is obtained by experimenting with this simulation. Typical curves are shown in **Fig. (5)**.

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The integers on the x-axis indicate the 25 where as they indicate the rated speed of the motor in radius /second. The informs these curves assist the designer on the Ohio desired rate speed c: the motor and the gain magnification of the voltage being fed to the motor to provide the desired time response. The percentage error of the response for different combination of word size, and sampling frequencies is obtained. A word size of 8 bits and a sampling frequency, of 1 KHz are considered acceptable as they produced an error of 1:45%.

### 5. Practical Implementation

Experimentation with the simulation scheme lead to the design of the first system with a total time response of 0.4 second to 30 degree step input. The choice is set for a d. c motor with rated speed of 5 r/s and for a gain factor of 25 for a plant with a time constant of 0.25 seconds. The system is constructed. Voltages proportional to the output angular speed and position via a tacho generator and a potentiometer respectively are obtained. These are fed to an electronic unit built to provide a feedback voltage proportional to the value of the synthesizing function. A logic circuit is also used to provide the third state at the output of the relay. The measure (errors of this system are found to have a maximum error of 6.8%. These errors are mainly due to the loss of the linearity of the logarithmic amplifier, used for implementing the synthesizing function and are particularly severe when high input steps are applied connected at the output stage of the selection unit. The switching unit is constructed to compute the switching function, at each clock cycle.

The natural logarithms of the speed values are converted to binary and stored in an EPROM; type 2716, at increments o (10/29Y) r/s and a total range of speed. The EPROM is addressed the output of speed latch.

The control signal circuit is constructed to produce the action of a three level controller. **Fig. (6)** shows the block diagram of the system.



(A) Control signal unit. (B) Pre-Amplifier. (C) Servo amplifier. (D) D.C. servo motor.(E) Selection unit. (F) Switching curve unit. (H) Absolute value unit. (G) Tachogenerator

#### Figure (6) Sampled data laboratory version of the system

The percentage error of the response for the laboratory prototype of the hybrid system, using word size of 8-bits is too tabulated below:

FS	0.1	1.0	10
% Error (practical	8.1	4.62	3.2
% Error (simulation)	3.51	1.50	1.32

As can be seen the simulation program for die hybrid version has already predicted an approximate order of errors.

The constructed systems, described above, provide an example which shows that for some control system applications, better system performance may be achieved by a hybrid system design over a continuous system. This is achieved here by the digital logarithmic implementation. Therefore, the accuracy of the simulation results, in any can lead to the proper choice of the practical version to be implemented, as well as provide the necessary information concerning the various system parameters needed for the design.

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## 6. Conclusion

A scheme involving simulation is developed to permit the synthesis or design of complex control system. This is achieved as an inverse analysis problem in which design is sought to cause desired performance of the system.

An overall response percentage deviation of an order of 0.2% is typical of the scheme accuracy with increments of 0.02 sec.

The provision of the system performance under the variation of the main parameter that influence the system response and dynamics is demonstrated. These are obtained over a range which enables the design to meet all specifications with permissible element parameters and within an acceptable cost.

# 7. References

- 1. Chen, C., "Linear System Theory and Design", Holt, Reinhart and Winston, 1984.
- Stewart, R. K., and Julka, V., "A General Purpose Simulator for Continuous and Sampled Data Control", Simulation Conference, Boston, Massachusetts, Elsevier pp. 1101-1106.
- **3.** Tomavic, R., *"Introduction to Nonlinear Automatic Control Systems"*, John Wiley and Sons Ltd., London, 1996.
- 4. Athens, M., and Flab, P. L., "Optimal Control", Mc-Grow Hill, New York, 1966.
- Doll, H. G., and Stout, T. M., "Design and Analog Computer Analysis of an Optimal Third Order Nonlinear Servomechanism", Transaction of the ASME, Vol. 97, No. 2, 1977.
- 6. Sage, A. P., and With, C. C., "Optimum System Control", Prentice Hall, 1977.
- **7.** D. H., Jones, "Choosing the Right Servo Amplifier Control Engineering", Jan 1973, pp. 40-43.