Effect of Beam Size on Shear Strength of Reinforced Concrete Normal Beams

Asst. Lect. Hadi Nasir Ghadhban
Civil Engineering Department, College of Engineering
Al-Mustansiriya University, Baghdad, Iraq

Abstract

This paper studies the shear capacity of normal strength concrete (NSC) reinforced beams with and without web reinforcement, as indicated by available research and explain the effect of beam size on shear strength of reinforced concrete beams. Two equations are proposed for design which is simple to use, safe and have low coefficient of variation (COV) when used for NSC.

The results of analysis for NSC indicate that the proposed equations are conservative and give lower COV and smaller range of results when compared with the available methods of design which are based on the ACI.318M-02 [1], Canadian [2], B.S.8110-97 [3], New Zealand [4] codes and one equation proposed by Zsutty [5].

By applying regression analysis was applied to both types of beams, with and without web reinforcement, two design shear equations are proposed, one for each type of beam.

The computer programs which are used for this purpose are (QBASIC, Graph Win, and SPSS).

الخـلاصـــــــة

هذا البحث يدرس مقاومة القص للعتبات الخرسانية الاعتيادية المسلحة مع او بدون حديد القص المؤشرة في البحوث المتوفرة وبيان تأثير حجم العتبة على مقاومة القص للعتبات الخرسانية الاعتيادية المسلحة. المعادلات المقترحة للتصميم هي بسيطة الاستخدام وأمينة للعتبات الخرسانية الاعتيادية.

ان نتائج التحليل للعتبات الخرسانية الاعتيادية تبين إن المعادلات المقترحة هي منطقية وتعطي معامل تغاير قليل ومدى قليل عند المقارنة مع معادلات التصميمية المتوفرة والمعتمدة على كل من الكود (الأمريكي والبريطاني والكندي والنيوزلندي) ومعادلة الباحث زوتي.

باستخدام التحليل الإحصائي تم اقتراح معادلات لتخمين مقاومة القص لهذه العتبات بنوعيها الحاوية وغير الحاوية على تسليح قص.

برامج الحاسوب التي استخدمت لهذا الغرض هي **(QBASIC, Graph Win, and SPSS).**

1. Introduction

In the design of concrete structures beams are important part of structures, which resist the loading and distribute these loads to the columns or supports.

There are different types of failure. In this study the behavior of reinforced concrete beams under shear failure is studied.

689 Beams with two point concentrated loads taken from references [6,7,8] are used in this study.

The large number of parameters affecting shear strength of beams has led to a large number of shear design methods. These parameters include the proportions and shape of beams, loading and support conditions, amount and arrangement of tensile, compressive and web reinforcement, as well as the concrete and steel properties.

2. Factors Affecting Beam Shear Strength

Several factors influence the shear strength of reinforced concrete beams. The most important ones are the following:

- Concrete Strength (fc`).
- Longitudinal Reinforcement Ratio (ρ_w).
- Shear Reinforcement (ρ_v).
- Shear Span to Depth Ratio (a/d).
- Beam Size.

Following is a historical review on the effect of shear span to depth ratio a/d and beam size on shear strength of beams:

2.1 Shear Span to Depth Ratio (a/d)

In simple beam with a single point load or with two symmetrical point loads, the term (a) is the distance from the load point to the nearest support but for numerous other loading conditions the term (a) has no direct physical meaning. The difficulty was later overcome by a slight modification of the general concepts of the diagonal tension. The length to depth ratio in reality relates the effect of horizontal flexural tension with diagonal tension. This led to the development of theories based on the ratio (M/Vd), involving bending moment (M), shear force (V) and the effective depth (d). For any other loading conditions (M/Vd) still has physical significance at any cross section of a beam.

The development of the (M/Vd) concept may be considered as breakthrough toward an empirical solution of shear and diagonal tension as a design problem.

A number of researchers such as reference [9] have presented relationship similar to **Fig.(1)** showing that (M/Vd) is an important variable in defining the shear strength of a beam.

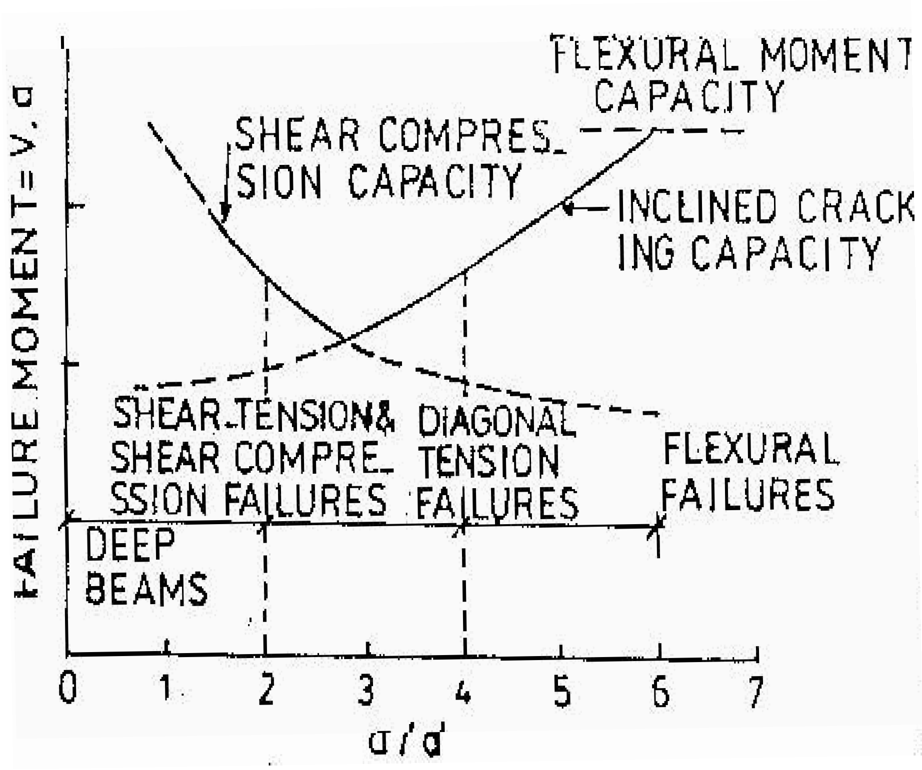


Figure (1) Variation in shear capacity with a/d for rectangular beams [9]

The codes of practice and some researchers incorporate the effect of a/d differently. Reference [10] uses an enhancement factor for shear strength predictions of beams of a/d <2- namely the multiplier (2d/a). However, references [5,6,11,12] expect ACI.318M-02 [1] give an additional role for (a/d) by using it in original equation ,as well as in the enhancement factor which may take different forms while reference [11] uses (3.5-2.5 (a/d)). Haddadin et. al. [13] tested T-beams and they observed that the diagonal tension cracks in beams with a/d greater than 2.5 tended to become more steeply inclined to the axis of the beam as the amount of web reinforcement increased.

Smith and Vantisots [14] used the same mathematical model of reference [1] and derived Eq.(1a,1b).

$$V_{cr} = [0.2(f_c')+23.5\rho_w V_u.d/Mu]\ b_w.d \quad \text{.. (1a)}$$

$$V_{cr} = [0.14(f_c')+31.4\rho_w V_u d/Mu]\ b_w d \quad \text{.. (1b)}$$

Eq.(1a) was obtained using test data of beams with and without web reinforcement while Eq.(1b) was obtained using only the test data of beams without web reinforcement .

Elzanaty et. al. [15] studied the behavior of HSC beams with fc`=65.5MPa and longitudinal steel ratio of 1.2%-2.5%. They found that the shear strength of test beams decreased with rising a/d. They observed a high shear strength of the beams with a/d=2.

Increasing a/d from 4 to 6 leads to 9% and 35% decrease in shear strength in beams with ρ_w values 2.5% and 1.2%, respectively.

Kani [16] studied shear strength various values of shear span to depth ratio a/d. Beams with very low a/d ratios had a very high strength in shear.

2.2 Beam Size

Kani [17] tested four series of beams with depth of (152,305,610 and 1220 mm). It was found that considerable influence of absolute depth became apparent to such an extent that the safety factor for the largest beams was approximately 40% lower than the smaller beams. The relative strength (ru) was chosen rather than shear strength as an indicator of failure. Eq.(2) is semiempirical which includes the three major parameters affecting beams shear strength: ρ_w, a/d and the absolute beam-depth.

$$ru=[0.215/(100\rho_w*\sqrt{d/25.4})]^{0.5}*a/d=Mc/Mf \quad \ldots (2)$$

with the reduction factor (ru) known, the ultimate bending (Mc) can be calculated.

Tests by Taylor [18] have shown much less size effect if the size of the coarse aggregate is changed in the same proportion as the beam size. Taylor also showed that in large beams with normal d/bw ratio (d/bw<4), the loss of strength is not as serious as that reported by Kani. For beams with d/bw>4, Taylor has proposed that the design value of vc (the shear stress carried by the concrete) should be reduced by 40%.

Ahmed et. al. [12] proposed Eq.(3) which includes the depth factor (η) to account for the observed drop in shear capacity with increasing beam depth.

The derivation of the depth factor (η) was obtained from results of beams of the same a/d ratio and different depths, which were tested by Kani [20] for concrete strength up to 26MPa.

$$V_u=1.8(f_c'*\rho_w*d/a)^{0.333}*\eta*b_w.d \quad \ldots (3)$$

where:

$$\eta=1-0.00265[(d-135.9)^{0.85}/(a/d)^{0.63}] \quad \text{for } 3 \le a/d \le 6 \quad \ldots (4)$$

$$\eta=1-0.03985[(d-135.9)^{0.80}/(a/d)^{2.84}] \quad \text{for } a/d \le 3 \quad \ldots (5)$$

Bazant [6] found that the shear strength of concrete beams is equally sensitive to fracture energy as to the tensile strength of the concrete, there fore he proposed Eq.(6).

$$V_c=[10(\rho_w)^{0.333}+(d/25da)](0.083\sqrt{f_c'}+20.69\sqrt{\rho_w/(a/d)^5})\, b_w.d \quad \ldots (6)$$

where:

da: is the maximum size of aggregate.

Bazant and Kim [18] studied the size effect as well as the effect of the maximum aggregate size on the shear strength of normal reinforced concrete beams by means of a non-linear fracture mechanics model. Structure size represented by the depth as well as the maximum aggregate size was normalized to intrinsic length parameters of the concrete. This length parameter is proportional to the fracture energy of the concrete.

3. Theoretical Analysis

A total of 689 reinforced concrete beams (402 and 287) without and with web reinforcement respectively were selected from the literature and studied. These beams are used to investigate the influence of some parameters such as(a/d , b_w, d) on shear strength of reinforced concrete beams and to evaluate the efficiency of proposed design method and the existing code equations for beams with and without web reinforcement. Here, the proposed design equations are based on applying the statistical regression analysis to the data obtained from the literature, from which the shear strength of reinforced concrete beams without web reinforcement.

Many design equations were proposed or used in codes [1,2,3,4,5]. Only six simple ones will be considered as shown in the following

Summary of previous methods for calculating the shear strength of beams with and without web reinforcement.

1. ACI.318M-02 [1]

$$V_u = 0.75[\sqrt{f_c'} + 120 * p_w . V_u . d / Mu] b_w d + 0.75 * p_v f_{yv} b_w . d \quad \text{..................... (7)}$$

2. Canadian Code 1984 [2]

$$V_u = [\, 0.6 * 0.2 * \lambda * \sqrt{f_c'}\,]\, b_w . d + 0.85 * p_v f_{yv} b_w . d \quad \text{.................................. (8)}$$

3. British Standard (B.S. 8110) [3]

$$V_u = [0.79 * (100\, p_w)^{(1/3)} * [400/d]^{(1/4)} * [\, f_c'/20]^{(1/3)}] b_w . d + 0.95 * p_v f_{yv} b_w . d \quad \text{..... (9)}$$

where, it is assumed that $f_c' = 0.8 f_{cu}$.

4. New Zealand Code [4]

$$V_u = 0.85[(0.07 + 10\rho_w)\sqrt{f_c'}\, bw.\, d] + 0.85 * p_v fy_v b_w . d \quad \text{.......................... (10)}$$

5. Zsutty Method [5]

$$V_u = 0.75[2.2 * (f_c' .\, \rho_w . d/a)^{1/3}\, b_w . d] + 0.75 * p_v f_{yv} b_w . d \quad \text{........................ (11)}$$

where, a material reduction factor of 0.75 is used.

The range of the values of the considered parameters are compressive strength fc` was in the range (6.0 to 101.8MPa), shear span to depth ratio for these beams ranged from (2.0 to 8.6), longitudinal reinforcement ratio ρw ranged from (0.46 to 7.00%), beam width, bw was ranges from (127 to 612mm), and effective depth d, ranged from (132 to 1200mm). All beams were tested under two equal top point loads.

3.1 Analysis of Data

Theory of regression analysis is used to derive the proposed equations. The objective of regression to evaluate the coefficient of an equation relating the criterion variables to one or more other variables, which are called predictor variables. The predictor variables are variables whose variation is believed to cause variation in the criterion variables.

Regression analysis is broadly used to analyze the relationship among variables. It is one of the most widely used statistical tools because it provides a simple method of establishing a functional relationship among variables.

After the regression equation is calibrated, it is very important to examine the rationality of the regression coefficient.

In addition to checking for rationality, the goodness of fit, statistical range (R), mean deviation (MD), standard deviation (σ_{n-1}), and coefficient of variation (COV) should be computed to assess the accuracy of predictions.

3.2 Derivation of the Proposed Equation

Results of beams failing in shear are selected from the literature; beams without stirrups were used to obtain the basic format of the proposed equation. This equation is compared with existing shear design relationships.

3.2.1 Selection of the Basic Format

The basic format of the empirical equation is based on dimensional analysis, while the based format of the semi empirical equations is derived analytically. Regression analysis was used to evaluate the regression constant for best agreement with test data. The test results included 402 and 287 without and with web reinforcement respectively, for beams having a/d $\geq$2.

To test these equations the relative shear strength values RSSV ($Vc_{test}/\phi Vc_{calculated}$) were found for 402 beams using each of the equations, then the values of the mean (μ), standard deviation (σ_{n-1}), and the coefficient of variation (COV) were calculated for these equations as shown in Table (4-1). It is obvious that the first equation given in **Table (1)** has the lowest values σ_{n-1} and COV, this means that the power format of this equation gives the best representation for the shear strength prediction.

Table (1) Selection of the basic format

No.	Equation	Vc test/ϕVc estimate		
		μ	σ_{n-1}	COV%
1	$Vc=65\ (fc`)^{0.37}(\rho w)^{0.44}(d/a)^{0.79}\ (b_w d)^{0.77}$	2.13	0.48	22.8
2	$Vc=5.2\ (fc`)^{0.49}(\rho w)^{0.54}(d/a)^{0.87}\ (b_w d)$	1.53	0.40	26.33
3	$Vc=32\ (fc\ \rho w\, d/a)^{0.44}\ (b_w d)^{0.77}$	1.4	0.33	23.6
4	$Vc= 4.74\ (fc)^{0.49}\ (\rho w .d/a)^{0.6}\ (b_w d)$	1.55	0.4	25.89

The regression analysis was used to evaluate the empirical constants that weight the beam properties in this format, in order to get best agreement with test results and thus the general equation [Eq.(12)] becomes Eq.(13).

$$Vc=a\times(f_c`)^b\times(\rho w)^c(a/d)^e\times(b_w\times d)^w \quad \ldots\ldots (12)$$

$$\phi Vc=\phi\, 65(f_c)^{0.37}\times(\rho w)^{0.44}(d/a)^{0.79}\times(b_w\times d)^{0.77} \quad \ldots\ldots (13)$$

This equation was tested and the values of μ, σ_{n-1}, and COV, were found equal (2.13,0.48,22.8) respectively. Eq.(13) proposed to predict the shear strength capacity.

For the beams with web reinforcement the same method is used to predict the shear strength as shown in the following:

$$V_u=\phi\, 65\ [(f_c`)^{0.37}\times(\rho_w)^{0.44}\times(d/a)^{0.79}\times (b_w.d)^{0.77}+\rho_v f_{yv}\ (b_w.d)^{0.64}] \quad \ldots\ldots (14)$$

By using strength reduction factor ϕ equal to (0.48), the two equations become:

$$Vu=31.2(f_c`)^{0.37}\times(\rho w)^{0.44}(d/a)^{0.79}\times(b_w\times d)^{0.77} \quad \ldots\ldots (13)$$

$$V_u=31.2[(f_c`)^{0.37}\times(\rho_w)^{0.44}\times(d/a)^{0.79}\times(b_w.d)^{0.77}+\rho_v f_{yv}\ (b_w.d)^{0.64}] \quad \ldots\ldots (14)$$

4. Evaluation of Theoretical Results Shear Design Equations

To evaluate the existing design methods and proposed equations, RSSV is used for all methods to determine (μ, σ_{n-1}, COV, Max, Min and No.<1). The results are listed in **Tables (2) and (3)**. These tables clearly show that the proposed equations (Eqs 13,14) give the smallest values of COV and number of beams having RSSV < 1 and therefore the proposed equations can be considered as safe and give the best estimation of shear strength.

Table (2) Comparison between ($v_{c\ test}/\phi v_{c\ calculated}$) for 402 beams without web reinforcement

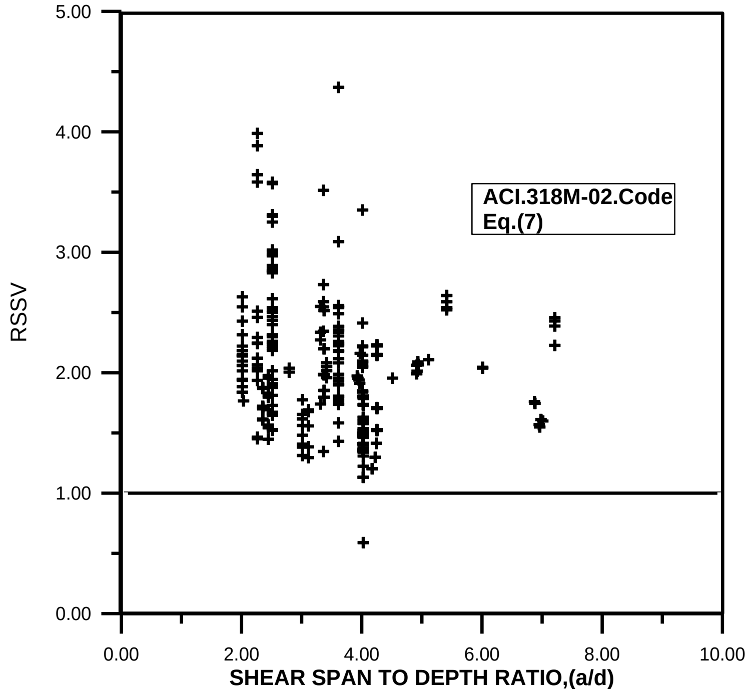
No.	Equation	μ	σ_{n-1}	COV	Min	Max	No.<1
1	ACI Code [1] Eq.(7)	2.21	0.91	41.0	0.60	11.02	7
2	CAN Code [2] Eq.(8)	2.35	1.15	48.81	0.55	14.52	8
3	B.S. 8110 [3] Code Eq.(9)	1.69	0.92	54.44	0.64	11.25	9
4	New Zealand [4] Code Eq. (10)	1.69	0.79	46.83	0.45	10.25	20
5	Zsutty [5] Method Eq. (11)	1.53	0.53	34.60	0.68	6.67	13
6	Proposed Eq.(13)	2.13	0.48	22.80	0.99	5.59	1

Table (3) Comparison between ($v_{n\ test}/\phi v_{u\ calculated}$) for 287 beams with web reinforcement

No.	Equation	μ	σ_{n-1}	COV	Min	Max	No.<1
1	ACI Code [1] Eq.(7)	1.99	0.54	27.11	0.59	4.37	1
2	CAN Code [2] Eq.(8)	1.93	0.54	28.15	0.54	4.65	1
3	B.S. 8110 [3] Code Eq.(9)	1.56	0.41	26.57	0.53	3.24	7
4	New Zealand [4] Code Eq. (10)	1.61	0.46	28.77	0.48	3.68	10
5	Zsutty [5] Method Eq. (11)	1.61	0.42	26	0.53	3.34	4
6	Proposed Eq.(13)	2.25	0.49	21.77	0.92	4.15	1

5. Comparison of Design Methods

5.1 Graphs for Beams with Web Reinforcement



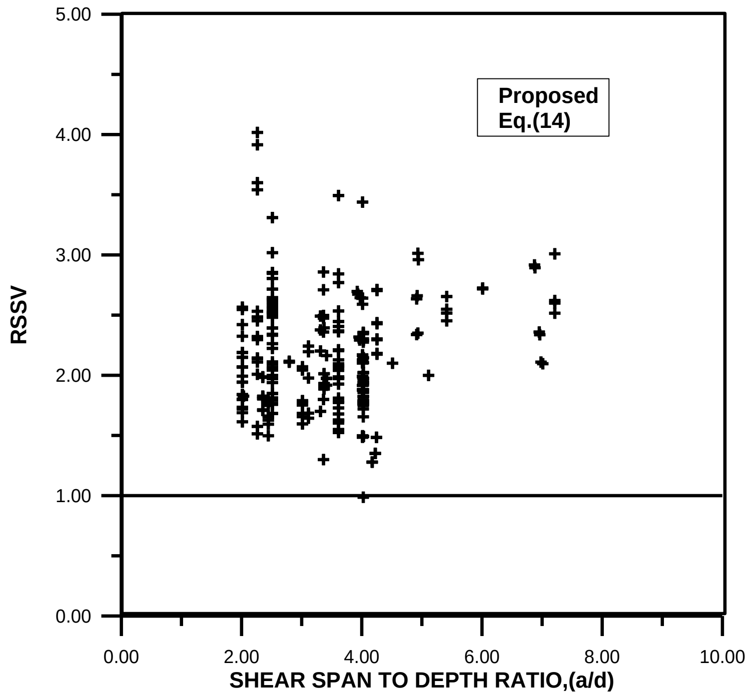
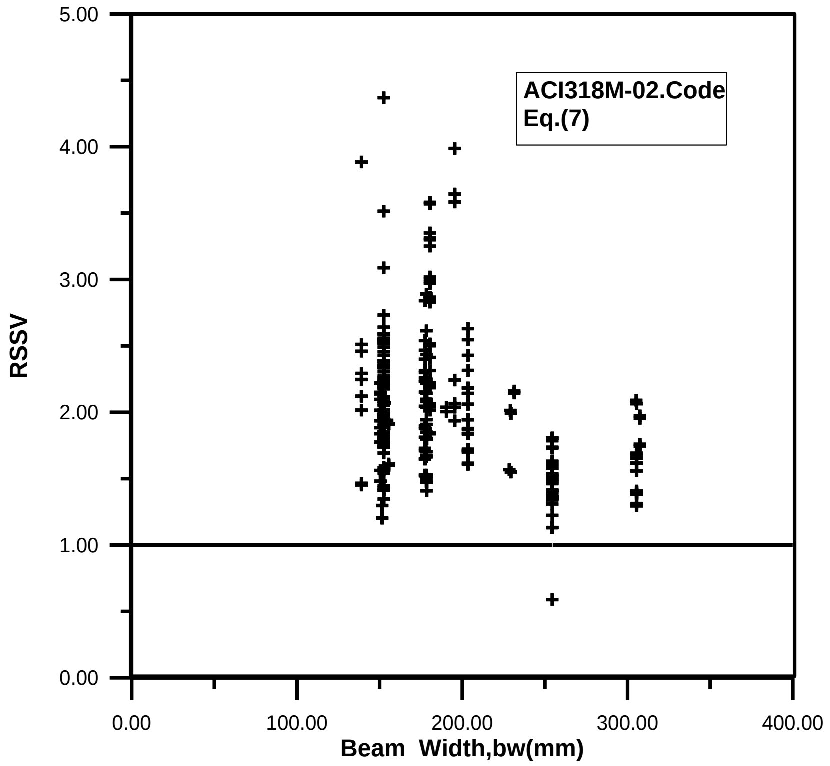


Figure (2) a/d versus the relative shear strength prediction



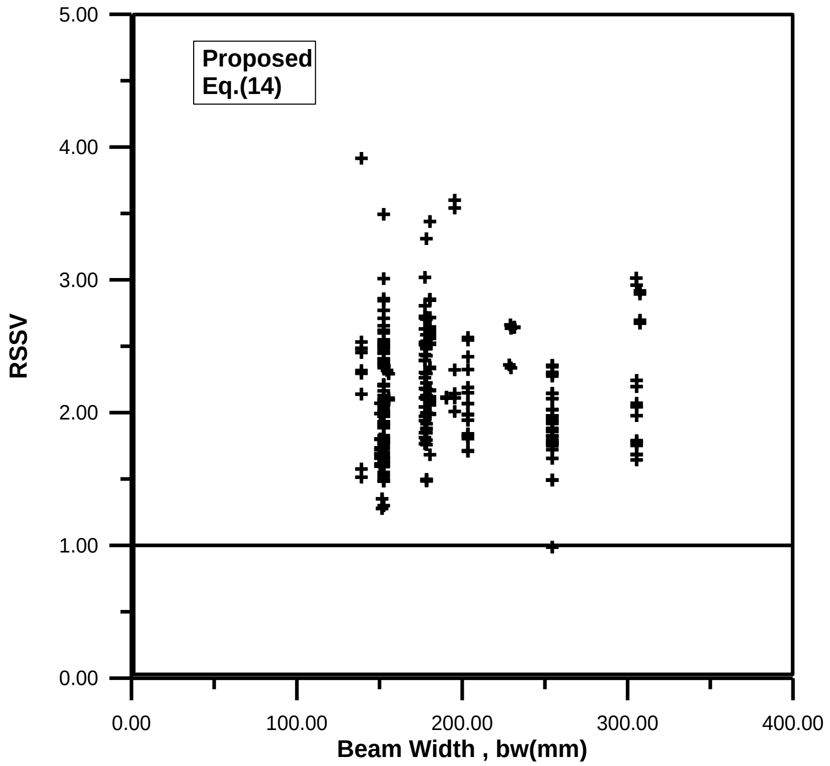


Figure (3) b_w, Versus the relative shear strength predictions

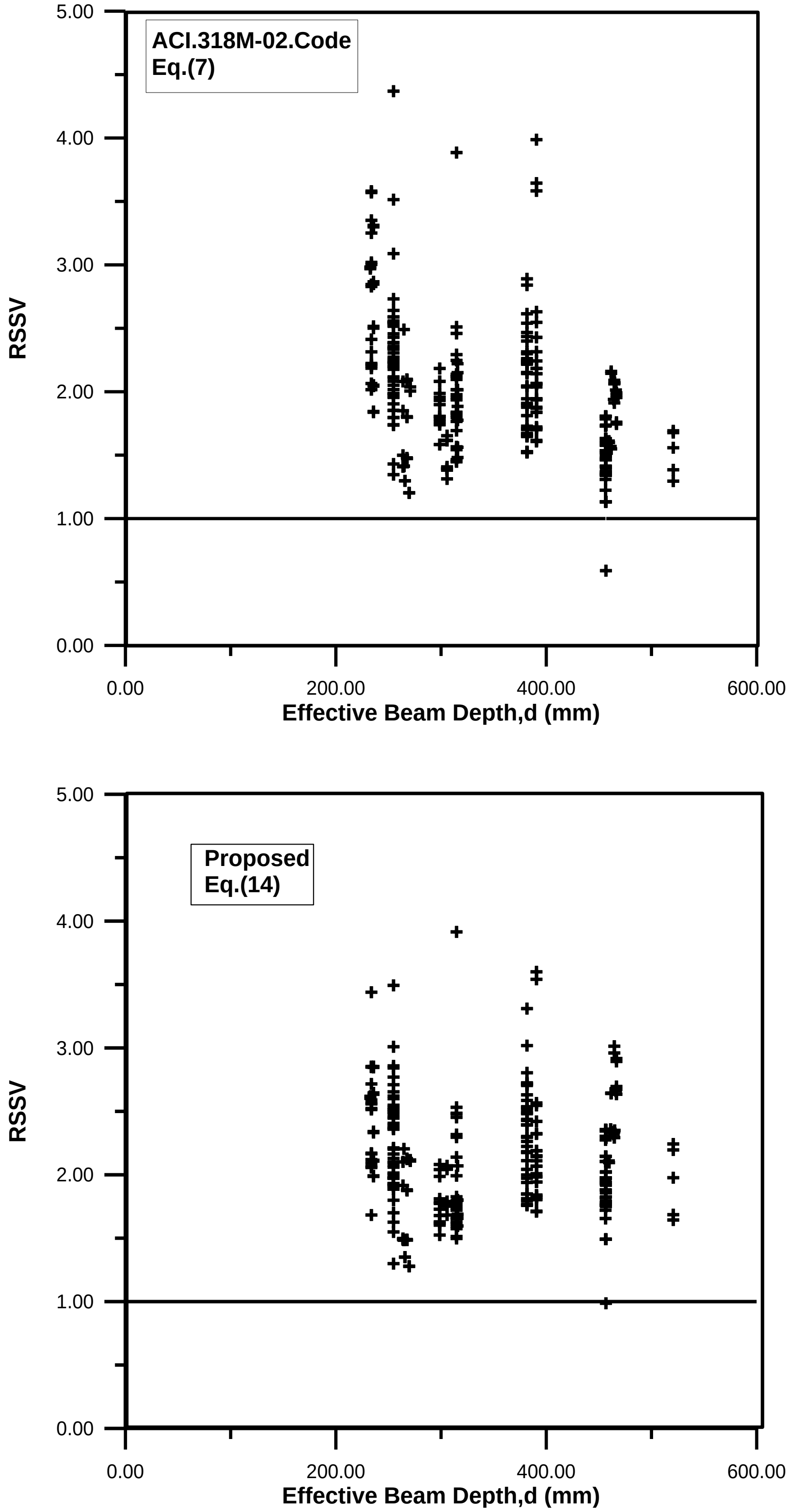
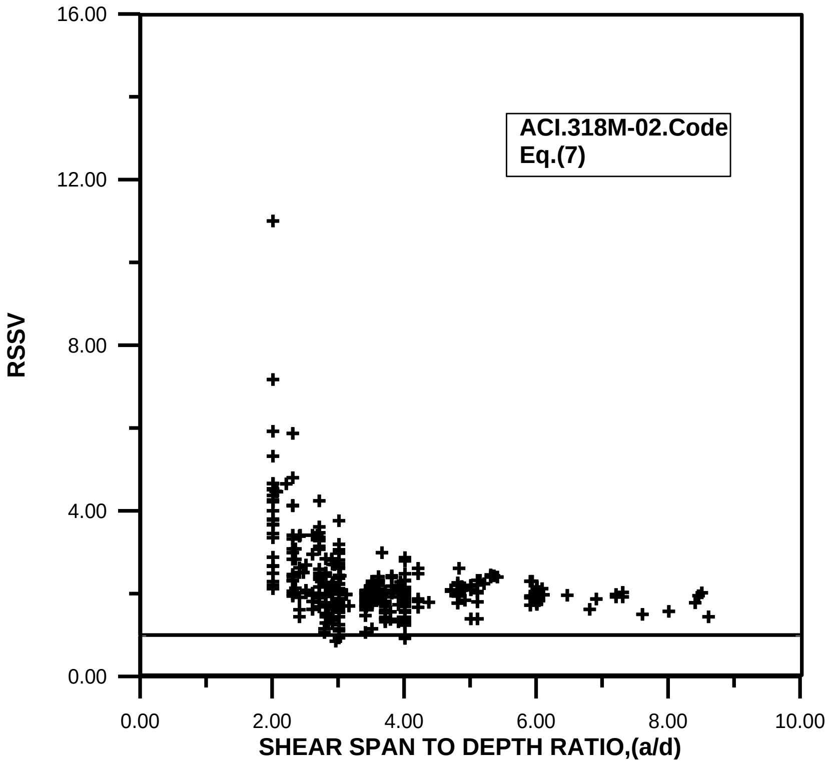


Figure (4) d, Versus the relative shear strength predictions

5.2 Graphs for Beams without Web Reinforcement



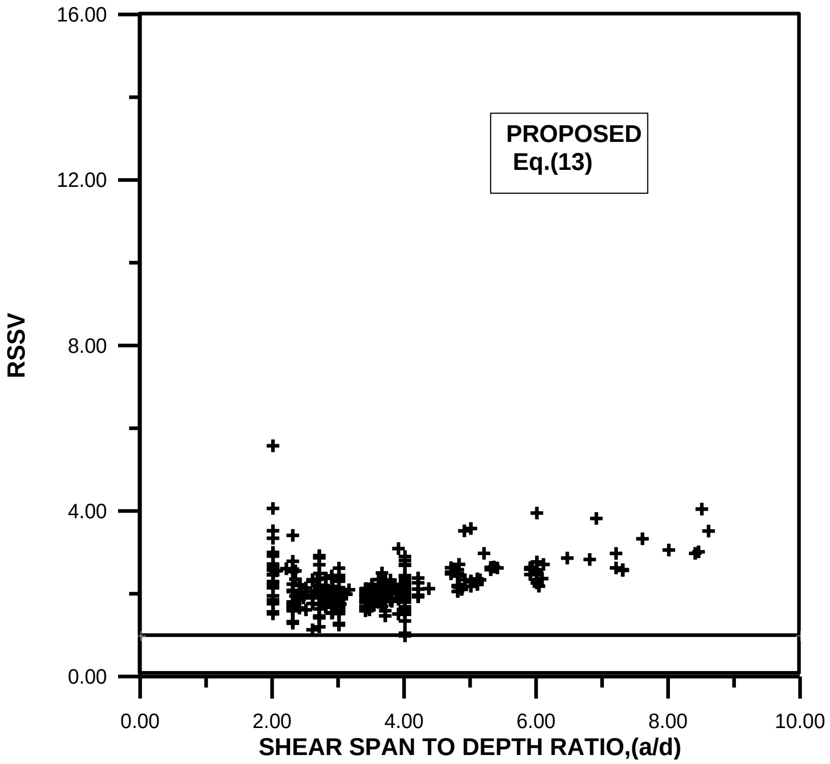
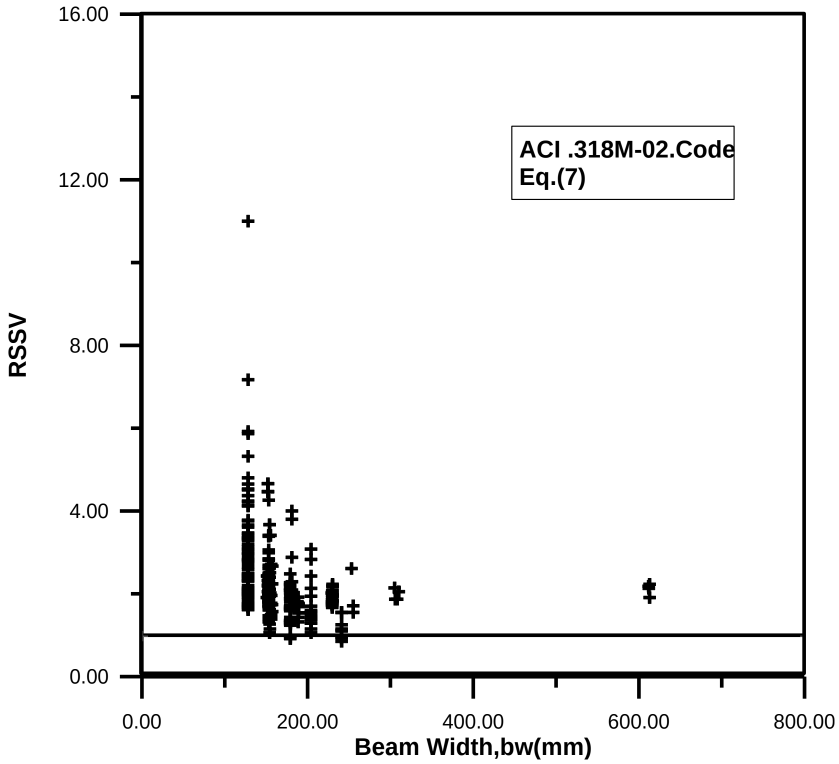


Figure (5) a/d Versus the relative shear strength predictions



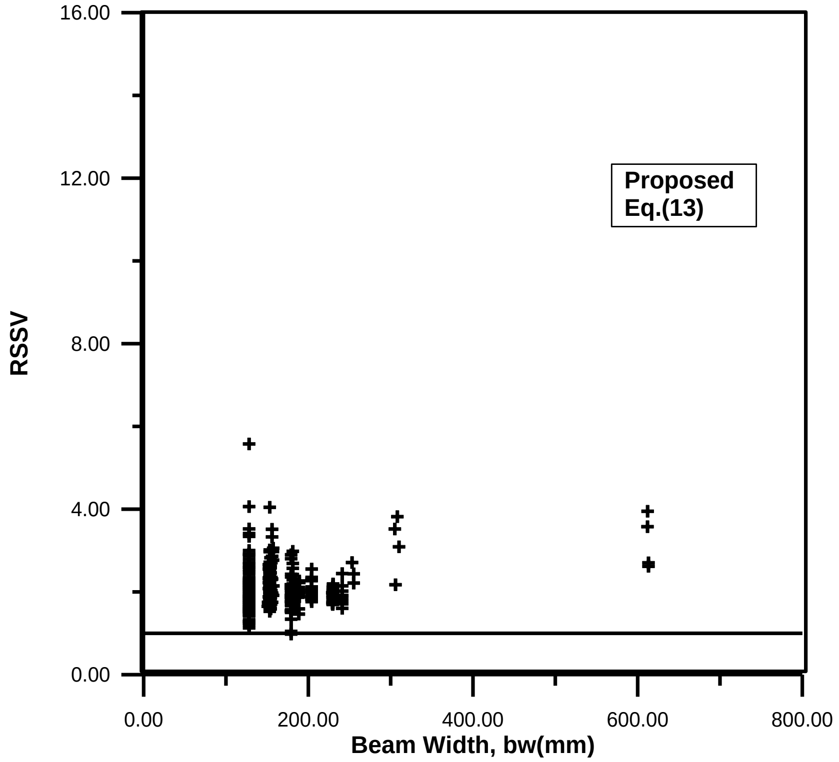
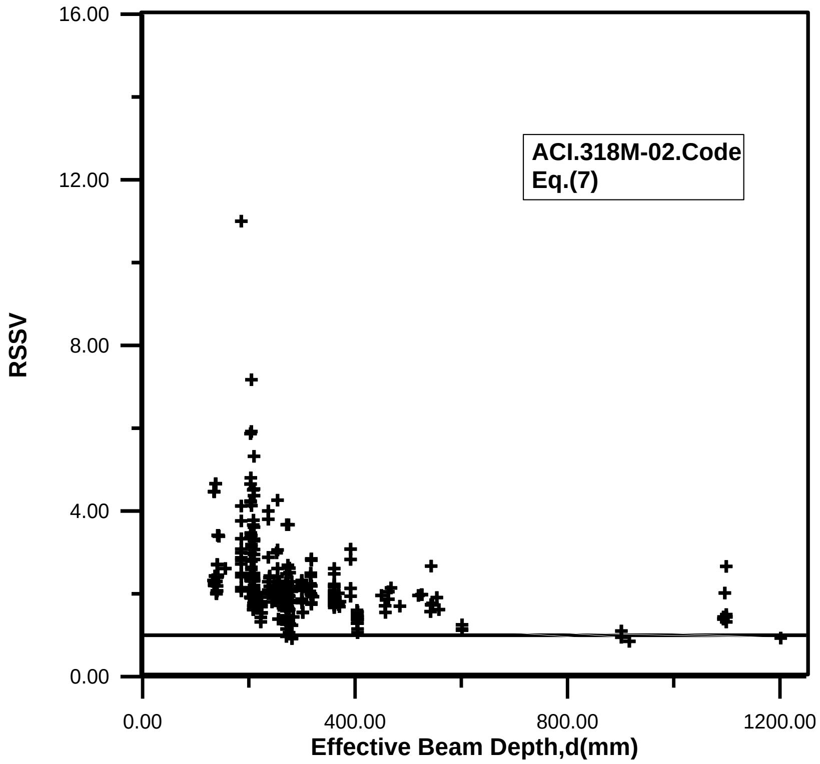


Figure (6) b_w, Versus the relative shear strength predictions



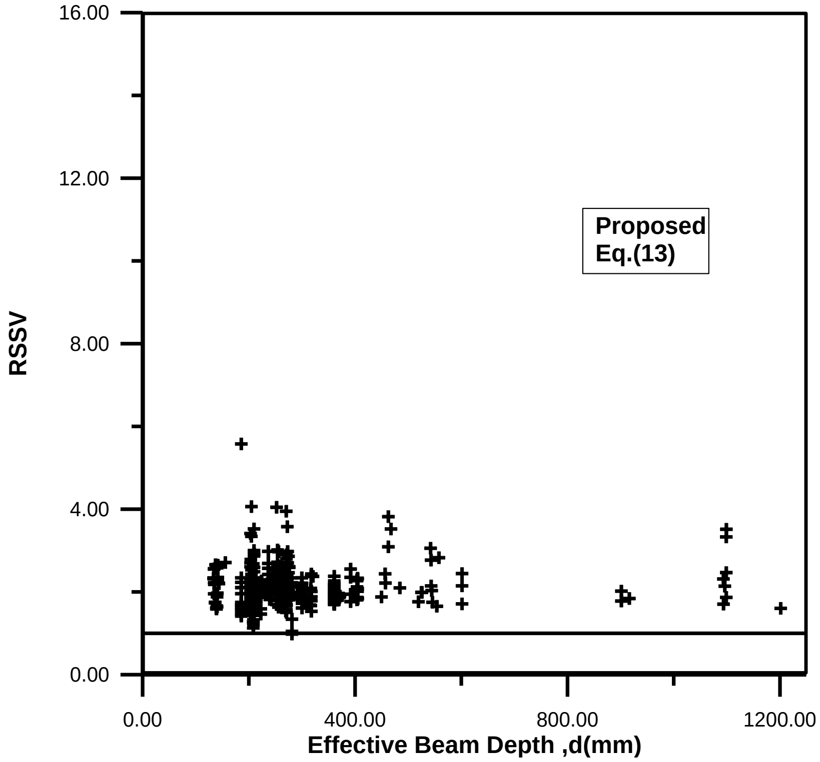


Figure (7) d, Versus the relative shear strength predictions

6. Conclusions

The following conclusions can be drown from this investigation and **Figs.(2,3,4,5,6,7)**:

1. In general the proposed equations (Eq.(13)for beams without web reinforcement and Eq.(14) for beams with web reinforcement) led to the lowest COV values compared to 5 existing design methods, based on RSSV (Relative Shear Strength Value) values. For beams without stirrups the COV was (34.60-54.44)% compared with proposed Eq.(13) that gives COV (22.8)% and for beams with stirrups the COV was (26.00-28.77)% compared with proposed Eq.(14) that gives COV (21.77)%.

2. The tested beams have showed that:

The RSSV of beams increased significantly by increasing concrete compressive strength (fc`).The RSSV of beams increased when a/d ratio decreased. Increasing the longitudinal reinforcement ratio ρ_w caused increased in RSSV. Beams size effect on shear strength of concrete beams, this shown in Eq.(13) shows that ($b_w d$) power to 0.77. Eq.(13) gives safe values of RSSV greater than 1.0 except one value being less than 1.0. For beams with stirrups the beam size is raised to the same power as beams without stirrups to determine the shear strength provided by concrete, Eq.(13). Eq.(14) shows that the (bw.d) power is (0.64) in the part that determines the shear strength provided by web reinforcement V_s., This compares with 5 methods that the beam size power value is equal to 1.0 in determining shear strength provided by concrete, **Tables (2) and (3)** show some unsafe results from existing methods. In general with the increase in beam size the RSSV of beams decreases [6,17,19]. All existing design methods were safe (RSSV≥1.0) except that a few result are unsafe points (RSSV<1.0). The proposed method has 1 result with RSSV<1.0.

The power format Eq.(13) and Eq.(14) were found to give the best estimation for the shear strength (Vu) of beams with and without web reinforcement, when compared with other commonly used (analytically or dimensionally derived) format for V_u .

7. References

1. ACI Committee 318, ***"Building Code Requirement for Structural Concrete and Commentary (ACI 318M-02 /ACI 318 RM-02)"***, American Concrete Institute, Detroit, 2002, 443 pp.

2. Canadian Standards Association, ***"Design of Concrete Structures for Buildings (CAN.3- A23.3-M84)"***, National Standard of Canada, Rexdale, Canada, 1984.

3. British Standards Institution, ***"Code of Practice for Design and Construction (BS 8110: Part 1: 1997)"***, British Standards Institution, London, 1997.

4. Standards Association of New Zealand, ***"Code of Practice for Commentary on: The Design of Concrete Structures (NZS 3101:1982, Part 1 and 2)"***, Standard Council, New Zealand, 1982.

5. Zsutty, T. C., ***"Beam Shear Strength Prediction by Analysis of Existing Data"***, ACI Journal, Vol. 65, No. 11, November 1968, pp. 943-951.

6. Bazant, Z. P., and Sun, H. H., ***"Size Effect in Diagonal Shear Failure: Influence of Aggregate Size and Stirrups"***, ACI Materials Journal, Vol. 84, No. 4, July-August, 1987, pp. 259 - 272.

7. AL-Musawi, J. M. S., ***"Shear Capacity of High Strength Concrete Beams with Web Reinforcement"***, M.Sc. Thesis, University of Technology, Baghdad, Iraq, 1989, 121 pp. (in Arabic).

8. Bunni, Z. K., ***"The Effect of the Shear Span to Depth Ratio on Shear Strength of the High-Strength Concrete Beams"***, M.Sc. Thesis, University of Technology, Baghdad, 1994, 120 pp.

9. Kani, G. N. J., ***"The Riddle of Shear Failure and its Solution"***, ACI Journal, Vol. 61, April 1964, pp. 441-467.

10. Kotsovos, M. D., ***"Mechanisms of Shear Failure"***, Magazine of Concrete Research, Vol. 35, No. l23, June l983, pp. 99-106.

11. ACI Committee 318, ***"Building Code Requirement for Reinforced Concrete and Commentary (ACI 318M-99/ACI318 RM-99)"***, American Concrete Institute, Detroit, 1999.

12. Ahmed, S. H., Khaldo, A. R., and Poveda, A., ***"Shear Capacity of High-Strength Concrete Beams"***, ACI Journal, Vol. 83, No. 2, March-April, 1986, pp. 297-305.

13. Haddadin, M. J., Hong, S. T., and Mattock, A. H., ***"Stirrup Effectiveness in Concrete Beams with Axial Force"***, Proceeding, ASCE, Vol. 97, No. ST9, September 1971, pp. 2277-2297.

14. Smith, T. N., and Vantsotis, A. S., ***"Deep Beam Test Results Compared with Present Building Code Models"***, ACI Journal, Vol. 79, July-August 1982, pp. 280-287.

15. Elzanaty, A. H., Nilson, A. H., and Slate, F. O., ***"Shear Capacity of Reinforced Concrete Beams Using High Strength Concrete"***, Report No.85-1, Department of Structural Engineering, School of Civil Engineering, Cornel University, Feb. 1985, 216 pp.

16. Kani, G. N. J., ***"Basic Facts Concerning Shear Failure"***, ACI Journal, Vol. 63, June 1966, pp. 675-692.

17. Kani, G. N. J., ***"How Safe Are Our Large Reinforced Concrete Beams?"***, ACI Journal, Vol. 64, No. 3, March 1967, pp. 121-148.

18. Bazant, Z. P., and Kim, J. K., ***"Size Effect in Shear Failure of Longitudinally Reinforced Beams"***, ACI Journal, Vol. 81, Sept.-Oct. 1984, pp. 456-468.

19. Taylor, H. P. J., ***"Shear Strength of Large Beams"***, Journal of the Structural Division, ASCE, Vol. 98, ST. 11, November 1972, pp. 2473-2490.

20. Kani, G. N. J., ***"A Rational Theory for the Function of Web Reinforcement"***, ACI Journal, Vol. 66, No. 3, March 1969, pp. 185-196.

Notations

a:	Shear span, distance between concentrated load and face of support, mm
a/d:	Shear span to depth ratio
As:	Area of tension reinforcement, mm^2
Av:	Area of vertical reinforcement within a distance s, mm^2
b_w:	Width of the beam, mm
d:	Effective depth of the beam, mm
d_{agg}:	Maximum size of aggregate, mm
Es:	Modulus of elasticity of steel, MPa
f_{cu}:	Compressive strength of concrete based on B.S.8110 specifications, MPa
f_c':	Compressive strength of concrete based on ASTM specifications, MPa
f_y:	Yield strength of steel reinforcement, MPa
f_{yv}:	Yield strength of vertical shear reinforcement, MPa
l:	Clear span of the beam, mm
Mu:	Ultimate moment at the section, kN.m
S:	Spacing of vertical and inclined Shear reinforcement, mm
V:	Shear strength, kN
V_{test}:	Test shear strength, kN
Vc:	Shear strength carried by concrete, kN
Vs:	Shear strength carried by dowel action, kN
Vu:	Ultimate shear strength of reinforced concrete beams, kN
ρ_w:	Reinforcement ratio of main steel equal to ($As/b_w d$)
ρ_v:	Reinforcement ratio of vertical stirrups steel equal to ($A_v/b_w S$)
ϕ_c:	Resistance factor for concrete
ϕ_s:	Resistance factor for Steel
RSSV:	Relative shear strength value
R:	Statistical range
μ:	Mean value of (V_{test}/V_u)
σ_{n-1}:	Standard deviation
ru:	Reduction factor
η:	Depth factor