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The Performance of Orthogonal Wavelet Division Multiplexing (OWDM) in Flat Rayleigh Fading Channel

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Abstract

The aim of this paper is to investigate the effect of flat Rayleigh fading channel on the performance of the OWDM (Orthogonal Wavelet Division Multiplexing) system. This scheme is shown to be overall quite similar to Orthogonal Frequency Division Multiplexing (OFDM), but with some interesting additional features and improved characteristics.

The system performance is investigated for various Doppler frequencies for a channel. OWDM system is compared with OFDM. A detailed analysis of the system's implementation complexity is also reported.

الخلاصــــــة

الهدف من هذا البحث هو دراسة تاثير قناة الاضمحلال المسطح الريلي (Rayleigh) على نظام تضمين قسم المويجات المتعامدة (OWDM) . يشبه هذا النظام نظام تضمين تقسيم الترددات المتعامدة (OFDM) ولكن مع أضافة بعض السمات و الخصائص المحسنة.

تناول البحث إداء المنظومة بقيم مختلفة من ترددات (Doppler) للقناة. كما تم مقارنة نظام (OWDM) مع نظام (OFDM). كما تم التطرق الى التحليلات المفصلة لتعقيد تشكيل النظام.

1. Introduction

Although the principle of multicarrier modulation is not recent, its actual use in commercial system had been delayed until the technology required to implement it became available at reasonable cost [1]. Similarly, the ideal of using more advanced transform than Fourier's as the core of multicarrier system has been introduced more than a decade ago [2]. In wireless communication systems, we are entitled to wonder about the possible improvement that wavelet-based modulation could exhibit compared to OFDM systems.

Recently, a new class of multicarrier system based on wavelet transform has been proposed [3] and for simplicity, called Orthogonal Wavelet Division Multiplexing (OWDM) in this paper (also called Wavelet Packet Modulation (WPM)). The major advantage of OWDM is its flexibility. This feature makes it eminently suitable for future generation of communication systems [4].

Wavelet theory has been foreseen by several authors as a good platform on which to build multicarrier waveform bases [5-7]. Wavelet packet bases therefore appear to be more logical choice for building orthogonal waveform sets usable in communication. In the review on the use of orthogonal trnasmultiplexers in communications [2]. Akansu et. al. emphasize the relation between filter banks and multiplexer theory and predict that OWDM has a role to play in future communication systems.

2. Flat Rayleigh Fading Generation

The transmitted Signal will be transferred through the channel to the receiver. For type of fading channel which is chosen for simulation, the signal will be affected by the flat fading with addition to AWGN, in this case all the frequency components in the signal will be effect by a constant attenuation and linear phase distortion of the channel, which has been chosen to have a Rayleigh's distribution. To generate Rayleigh fading channel gain is by using mathematical functions, which is called Jake fading generator [8]. a_c and a_s are given as the following:

$$a_c = \frac{2}{M_0}\left(\sum_{n=1}^{M_0} \cos\beta_n \cos\omega_n t + \sqrt{2}\cos\zeta\cos\omega_m t\right) \quad \text{.....} (1)$$

$$a_s = \frac{2}{M_0+1}\left(\sum_{n=1}^{M_0} \sin\beta_n \cos\omega_n t + \sqrt{2}\sin\zeta\cos\omega_m t\right) \quad \text{.....} (2)$$

$$A_1 = \frac{\left[(a_c)^2 + (a_s)^2\right]^{\frac{1}{2}}}{\sqrt{2}} \quad \text{.....} (3)$$

$$\beta_n = \frac{\pi \cdot n}{M_0}, \zeta = \pi/4, \text{ and } \omega_n = \omega_m \cos\left(\frac{2\cdot\pi\cdot n}{4M_o+2}\right) \quad \text{.....} (4)$$

where:

M_0: is the number of low frequency oscillators with frequencies equal to ω_n.

a_s, a_c: are approximately Gaussian random processes with zero means and unit variances.

A_1: is Rayleigh distributed.

In the simulations, M_0 is 8 (in practice the ranges of M_o are between 8 and 20) [8].

3. Orthogonal Wavelet Division Multiplexing (OWDM) System

OWDM shares all the benefit of Multicarrier (MC) technique and exhibits further benefit such as higher efficiency due to elimination of guard interval (GI). It is considered as one of wavelet transforms which are well localized both in time and frequency domain, while sinusoid waveforms, are only localized in frequency but not in time domain.

Thus, time domain diversity of sinusoid waveform within one symbol period is difficult to achieve. Therefore, a guard interval (GI) is needed to eliminate residual Inter Symbol Interference (ISI). This addition of GI signal, then introduces overhead that decrease bandwidth efficiency [9].

Figure (1) illustrates a generic block diagram of OWDM transceiver, while the details are shown in **Figs.(2)** and **(3)** respectively. OWDM system employs two filter banks i.e. Inverse Wavelet Packet Transform (IWPT) placed at the transmitter side, and Wavelet Packet Transform (WPT) placed at the receiver side.

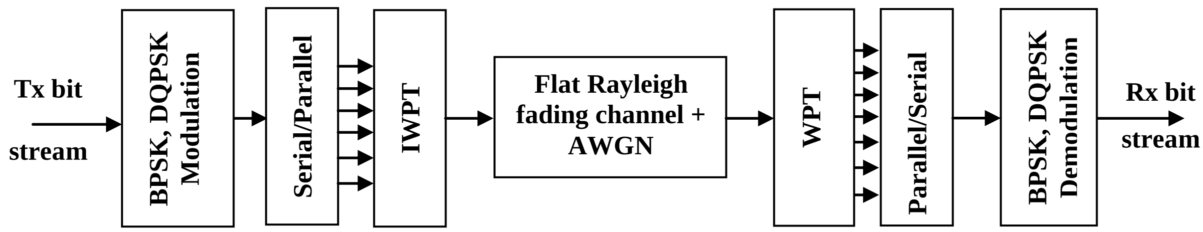


Figure (1) Block diagram of WPDM transceiver

As described in **Fig.(1)**, modulated data in BPSK or DQPSK is entered to the Inverse Wavelet Packet Transform (IWPT) to generate the multicarrier signal, and then transmitted over the transmission medium (note that BPSK is used with real WPDM and DQPSK is used with complex WPDM). The transmitted signal is corrupted by additive white Gaussian noise (AWGN) and flat Raleigh fading channel. The signal is received by WPT, demodulated and converted back to serial-sequence for recovering the transmitted data. The subband or subcarrier overlapping in the structure of IWPT-WPT (then called transmultiplexer [10]) is adjusted by controlling the wavelet filter in each subband.

Figures (2) and **(3)** show the detail structure of a transmultiplexer with the number of inputs and outputs of (2^j), where j is the number of stages. Wavelet filters, g_i are high pass filters (HPF), and g_o are low pass filters (LPF) at the transmitter, while h_i and h_o are HPFs and

LPFs respectively at receiver. All filters should be designed so that the occurred aliasing are exactly cancelled out. This condition leads to the construction of perfectly reconstruction (PR) filter bank, which is called quadrature mirror filter (QMF). Considering the carrier allocation which is divided into 2^j subcarriers or subbands, the symbol a_1 is the symbol with the highest frequency, while symbol a_{2^j} is the symbol with the lowest frequency among all of the transmultiplexer subbands. Upsampling is performed at the transmitter before entering the filter of its stage, and downsampling is held at the receiver after each data passed a filter of its stage.

In the QMF case, the filters are specified in terms of a single lowpass filter, $h_o(i)$, and the others are time reversal. These are [11]:

$$h_i(i) = (-1)^i h_o(L-1-i) \quad \text{.......... (5)}$$

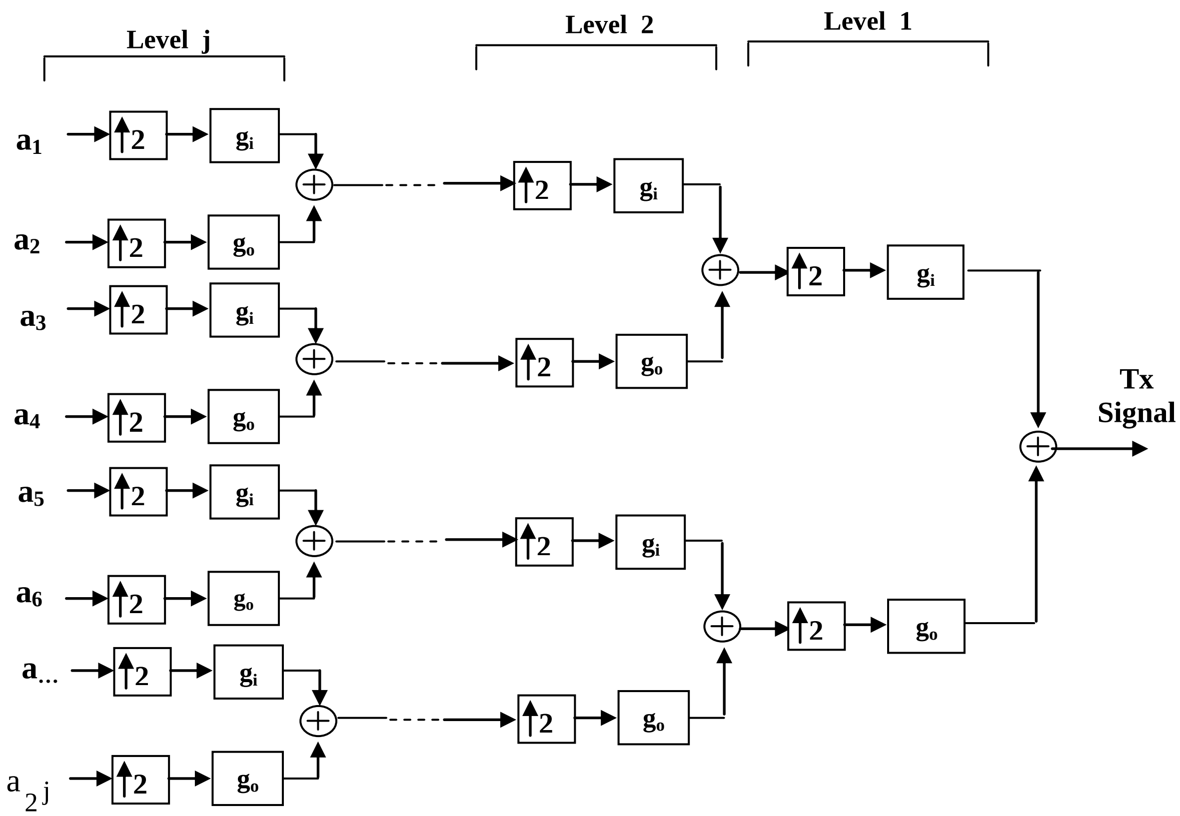
$$g_o(i) = h_o(L-1-i) \quad \text{.......... (6)}$$

$$g_i(i) = -(-1)^i h_o(i) \quad \text{.......... (7)}$$

where:

L: is the length of the filter and $0 \le i \le L-1$

In this paper, h_i, g_i, h_o, and g_o use Haar wavelet. Haar wavelet is selected due to its simplicity in design and calculation.



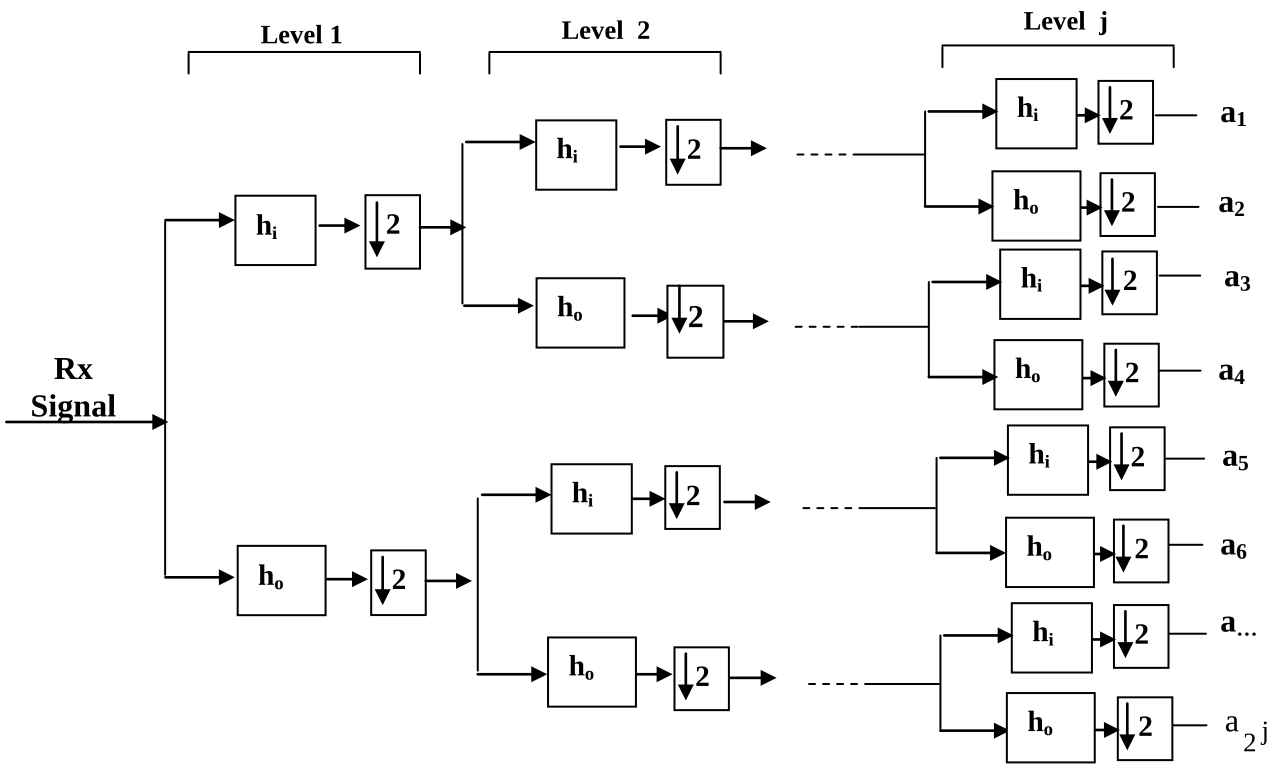


Figure (2) Inverse wavelet packet transform at the transmitter side

Figure (3) Wavelet packet transform (WPT) at the receiver side

4. Complex Wavelet Packet

Recently, Adhikary [11] proposed approaches for designing complex wavelet packets. These assume direct parallel implementation of the wavelet system. Also, there is strong constraints in the design of wavelet bases and sometimes yield large filters, this mean high complexity and time consuming.

However, any complex wavelet bases, to be used for Multicarrier Modulation (MCM), should exhibit the desired orthogonality. Further, the real and imaginary parts of the complex base should be spectrally similar and also orthogonal to each other. The idea of complex wavelet based MCM is exactly similar to the quadrature carriers centered at the same carrier frequency, but they are in phase quadrature [11].

The complex wavelet system requires two sets of bases functions and then filters. Consider **Figs.(2)** and **(3)**, the complex bases are denoted by $h_k(i)$ and $h'_k(i)$ corresponding to the analysis filters, and by $g_k(i)$ and $g'_k(i)$ corresponding to the synthesis filters (note that in the complex Wavelet packet the signal is split into real and imaginary part, the real part is entered alone and the imaginary part is entered alone to IWPT and then combined together to trans through channel, therefore the same **Figs.(2)** and **(3)** are used for both real and imaginary part, where the symbol dash refer to imaginary part). The complex filters occupy the same frequency band and thus should be orthogonal.

Consider Eq. (5), the wavelet filter $h_i(i)$ is orthogonal to the scaling filter $h_o(i)$ that is, $h_i(i) \perp h_o(i)$, where $\perp$ denotes orthogonality.

However, the second requirement is the orthogonality of the complex bases that occupy the same band. To ensure this, $h'_i(i)$ and $h'_o(i)$ are given by [11]:

$$\left.\begin{aligned} h'_o(i) &= (-1)^i h_o(L-1-i) \\ h'_i(i) &= (-1)^i h_i(L-1-i) \end{aligned}\right\} \qquad (9)$$

Eq.(9) imply that $h'_o(i) \perp h'_i(i)$ and $h'_i(i) \perp h_i(i)$ respectively. Therefore, the design of wavelet-based multicarrier system that satisfy these equations ensure orthogonal subchannels and orthogonal complex bases. Similarly, the synthesis filters are [11]:

$$\left.\begin{aligned} g'_o(i) &= (-1)^i g_o(L-1-i) \\ g'_i(i) &= (-1)^i g_i(L-1-i) \end{aligned}\right\} \qquad (10)$$

5. Peak to Average Power Ratio (PAPR)

One practical problem with multicarrier signal, which is often cited as a major drawback of a multicarrier system is its large envelope fluctuations, usually measured by the parameter called PAPR. PAPR is defined as the ratio of the peak instantaneous power to the average power i.e., mathematically expressed as [9]:

$$\mathrm{PAPR} = \frac{\max_{0<t<T_s} |x(t)|^2}{\mathrm{mean}_{0<t<T_s} |x(t)|^2} \qquad (11)$$

where:

$x(t)$: is a multicarrier symbol with interval $0< t <T_s$.

Also, the discrete PAPR of a multicarrier signal is defined as [12]:

$$\mathrm{PAPR}(x) = \frac{\max |x_k|^2}{\mathrm{mean} |x_k|^2} \qquad (12)$$

where:

x_k: is the discrete of the multicarrier symbol.

The discrete PAPR is a good approximation of an exact PAPR, if the sampling rate is high enough. One more important measurement, which can be helpful in analyzing the performance of a multicarrier modulation system, is Cumulative Distribution Function (CDF) of PAPR i.e.

$$P_{PAP}=Pr\ (PAPR > PAPR_t) \qquad (13)$$

where:

($PAPR_t$): is a threshold value of PAPR and P_{PAP} is CDF of PAPR.

6. Simulation Results

The proposed system of the OWDM is shown in **Fig.(1)**. The simulation results of this system are obtained by using MATLAB version 7. These results are compared with OFDM over flat Rayleigh fading channel environment. The effects of several parameters of wireless channels on the proposed system will be investigated. **Table (1)** shows the parameters are used in OWDM system (note that BPSK is used with real OWDM and DQPSK is used with complex OWDM).

Table (1) Simulation parameters

Modulation Type	BPSK, DQPSK
Number of subcarriers	32,64,128,256
Channel model	AWGN + Flat fading
Bit rate	2 Mbps
Carrier frequency (f_c)	2 GHz
Doppler frequency (f_d)	(20, 75, 150) Hz
Number of transmission bit	50000

6-1 Real OWDM Results

Figure (4) shows the performance of OWDM system and a single carrier system in a flat Rayleigh fading channel, where a BPSK modulation scheme is used in these two systems. The number of subcarriers is 128 for OWDM system. The bit error rate (BER) performance of OWDM system is better than the performance of single carrier system.

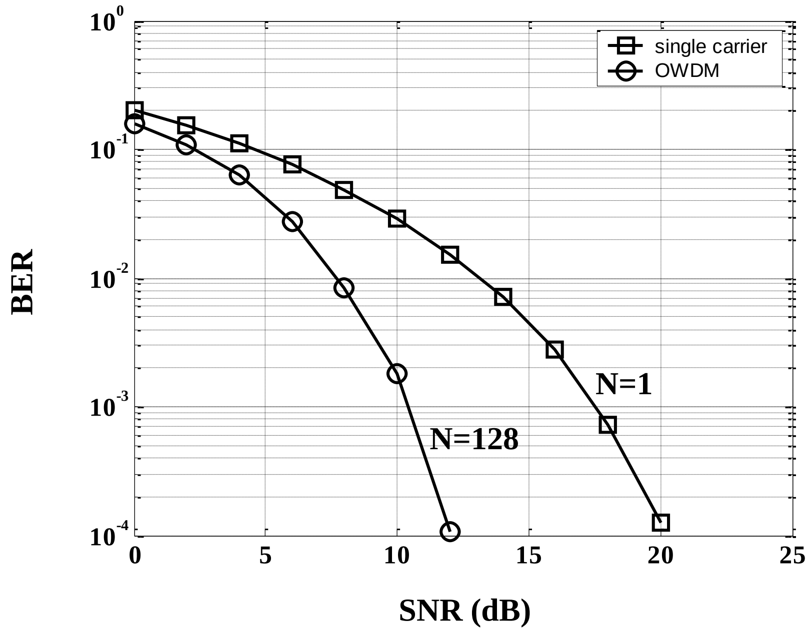


Figure (4) Comparison of BER performance between OWDM system and single carrier system (in a flat Rayleigh fading channel with fd=20)

Figure (5) shows the (BER) performance of OWDM system in flat fading channel with BPSK modulation for different values of Doppler frequency (f_d=20, 75, 150 Hz) and N=64. From this figure, it can be noticed that increase Doppler frequency f_d will effect on the performance of OWDM system. At BER $=10^{-3}$ the SNR of OWDM system over flat fading channel with f_d=75 is about 11 dB, while SNR for f_d =150 is about 14 dB. This means that increase Doppler frequency affect badly on BER.

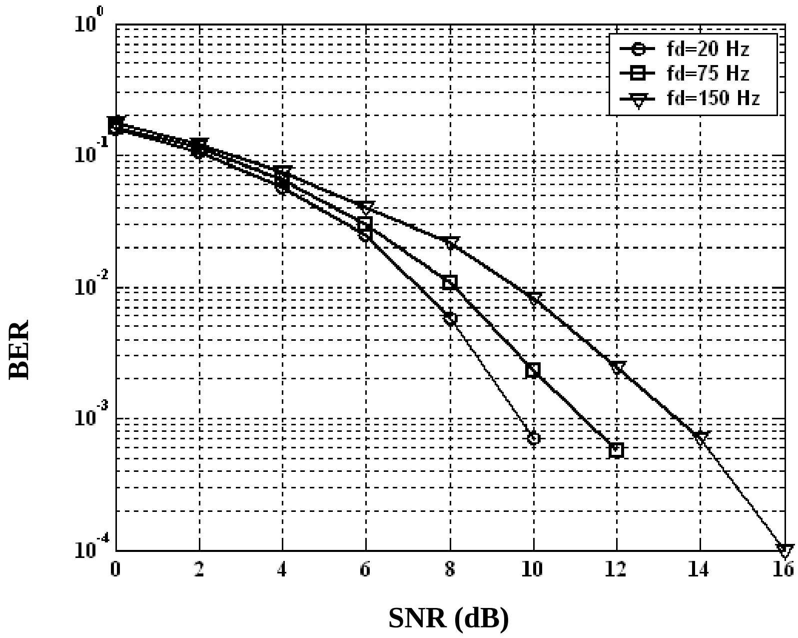


Figure (5) Performance of OWDM system over flat Rayleigh fading channel, N=64 and different values of Doppler frequency.

Figure (6) shows the (BER) performance of OWDM system for various number of subcarriers with BPSK modulation and Doppler frequency, f_d =75 Hz. At BER=10^{-4} SNR=12 dB for N=256 and SNR=18 dB for N=32, therefore increase N will increase the gain in SNR.

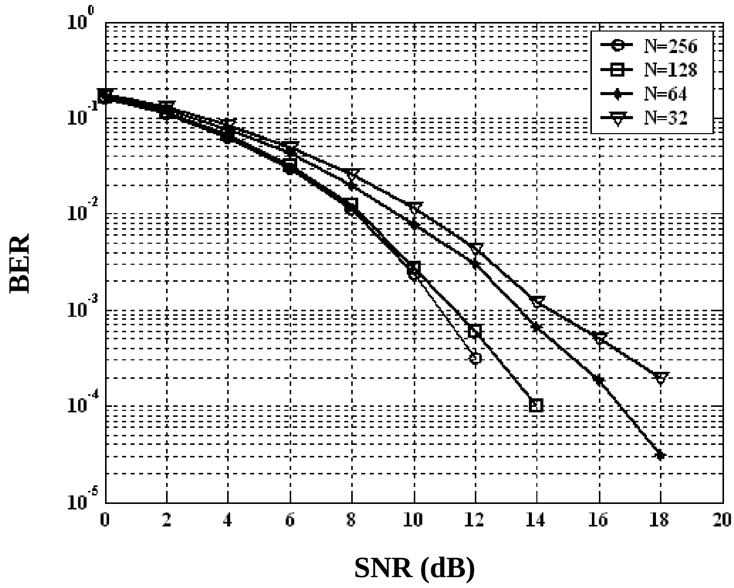


Figure (6) BER Performance of OWDM system for different numbers of subcarriers and fd=75 Hz

A comparison between the (BER) performance of real OWDM and OFDM system is shown in **Figs.(7)** and **(8)** respectively using Doppler frequency, f_d=75 Hz and subcarrier number, N=32 and 128 respectively. From these figures it can be observed that the performance of OFDM is better than of the performance of real OWDM. At BER=10^{-4} and N=128, there is about 2dB difference between OWDM system and OFDM system.

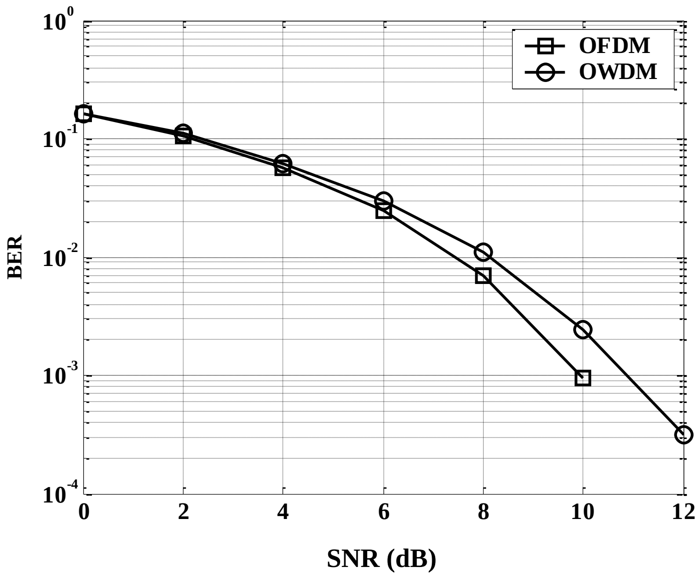


Figure (7) Comparison of BER performance between OWDM and OFDM systems using fd=75 Hz and N=32 in flat Rayleigh fading channel

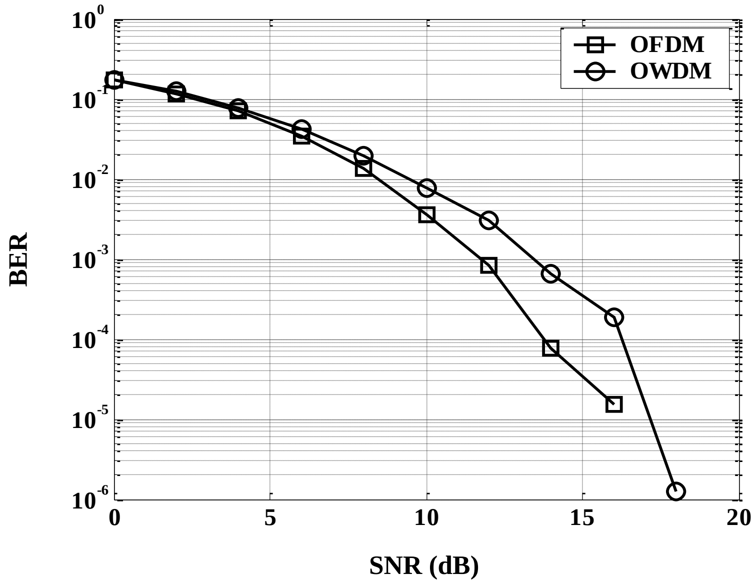


Figure (8) Comparison of BER performance between OWDM and OFDM systems using fd=75 Hz and N=128 in flat Rayleigh fading channel

6-2 Complex OWDM Results

Figure (9) shows the (BER) performance of complex OWDM system in flat fading channel using DQPSK modulation, N=128, and different values of Doppler frequency (f_d=20, 75, 150 Hz). Increasing Doppler frequency will affect badly on the performance of OWDM system.

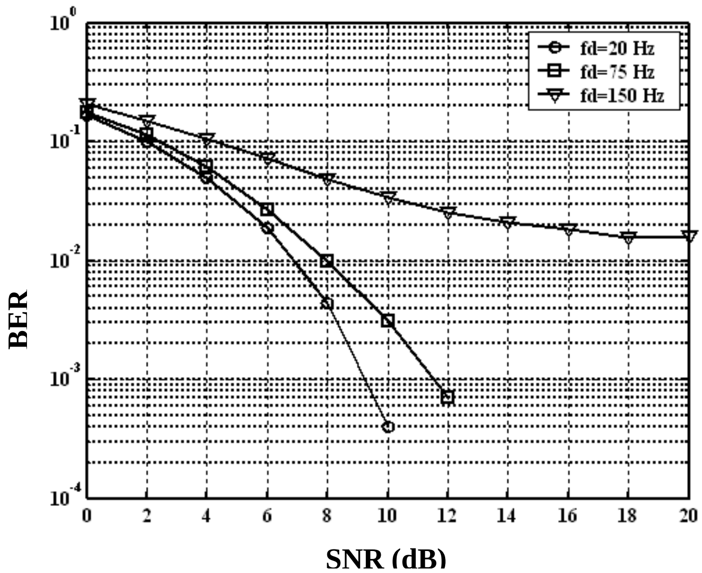


Figure (9) BER performance of complex OWDM system for various value of Doppler frequency and N=128

A comparison between the (BER) performance of complex OWDM and OFDM system for DQPSK modulation is illustrated in **Figs.(10)** to **(12)** respectively. A Doppler frequency of f_d=75 Hz is used with subcarriers number, N=32, 64 and 128 respectively. From these figures can be noticed that the performance of complex OWDM is the same with that of OFDM system.

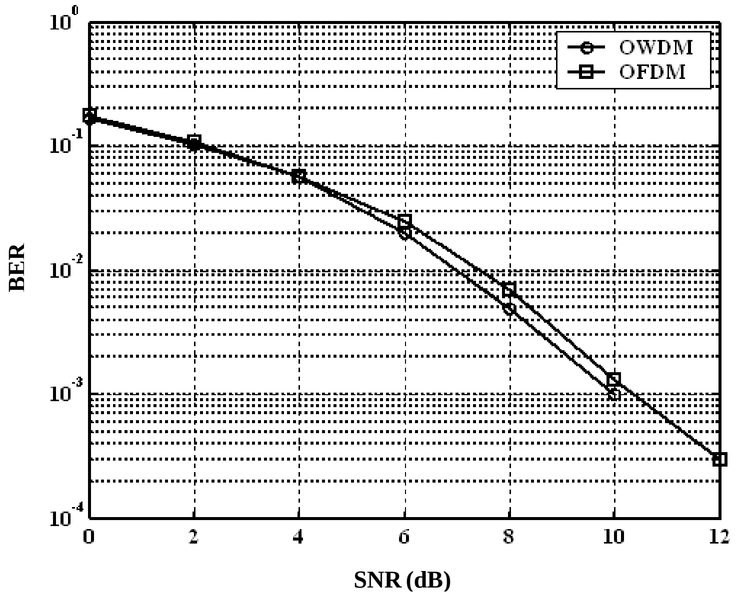


Figure (10) Comparison of BER performance between complex OWDM and OFDM systems using fd=75 Hz and N=32

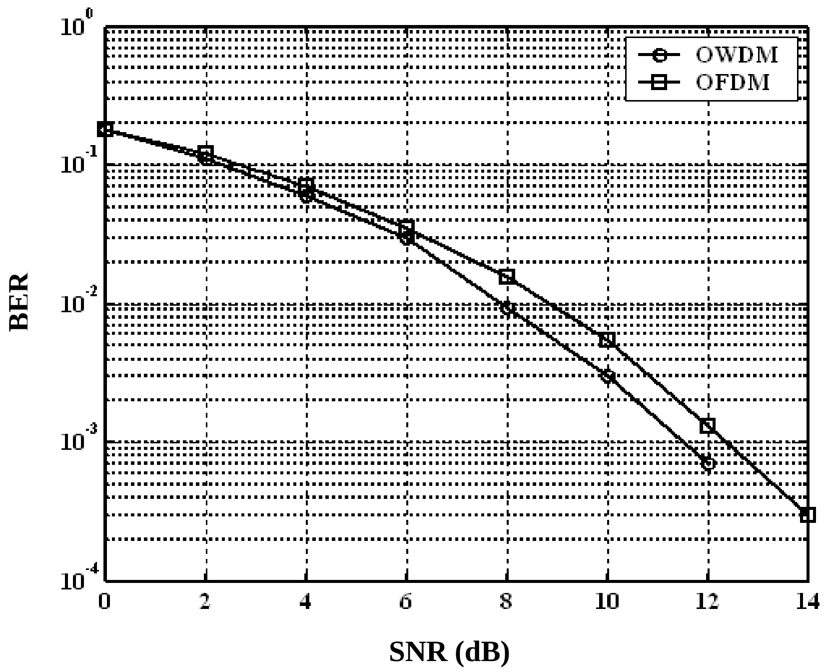


Figure (11) Comparison of BER performance between complex OWDM and OFDM systems using fd=75 Hz and N=64

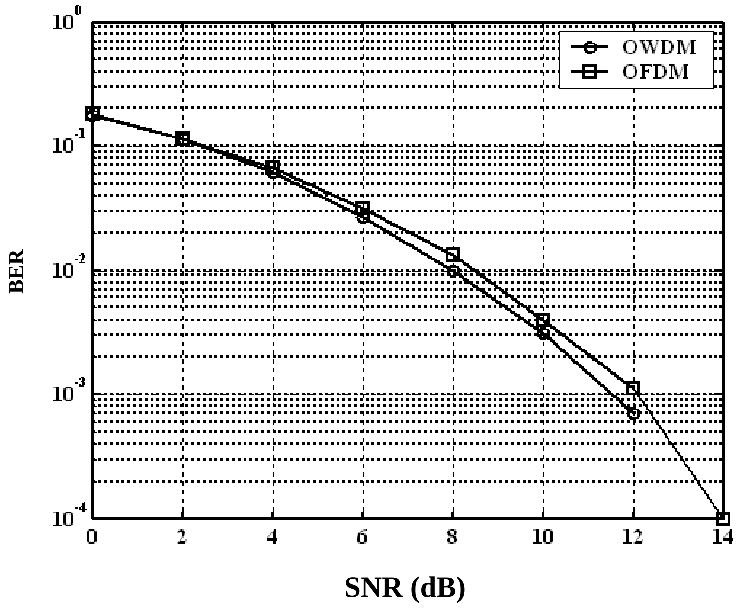


Figure (12)
Comparison of BER performance between complex OWDM and OFDM systems using fd=75 Hz and N=128

6-3 Implementation Complexity Estimates

For Wavelet Packet Transform (WPT) with filters of length L, the number of operations required by each filter is actually equivalent to the complexity required by a L/2-tap long filter banks to the zero values samples inserted by the up sampler. Hence, the number of operations per input sample required by one elementary block is [4]:

$$\rho_{\substack{\text{direct}\\ \text{WPT}}} = \begin{cases} 4\left\lceil \frac{L}{2} \right\rceil - 2 & \text{adds} \\ 4\left\lceil \frac{L}{2} \right\rceil & \text{mults} \end{cases} \quad \text{.......... (14)}$$

where:

$\lceil . \rceil$: *denotes the smallest integer higher than its argument.*

Overall, a transform of size M=2^j is composed of $\nu(j)$ elementary blocks per stage j. Denoting R(j) as the input rate of the block of stage j, the number of operations for the stage j:

$$C_{WPT}(j) = \nu(j)\, R(j)\, \rho_{\substack{\text{direct}\\ \text{WPT}}} \quad \text{.......... (15)}$$

$$\text{with} \begin{cases} \nu(j) = 2^{j-1} \\ R(j) = 2^{J-j} \end{cases}, \quad \text{.......... (16)}$$

where:

J: is the number of stage and the transform input signal

R(J): is assumed to be equal unity.

The overall number of operations for WPT is then:

$$C_{WPT}(J) = \sum_{j=1}^{J} 2^{J-1} \rho_{\substack{direct \\ WPT}} \quad \text{(17)}$$

$$C_{WPT}(J) = \begin{cases} (2^J - 1)(4\left\lceil \frac{L}{2} \right\rceil - 2) & \text{adds} \\ (2^J - 1)(4\left\lceil \frac{L}{2} \right\rceil) & \text{mults} \end{cases} \quad \text{(18)}$$

This computational complexity can be further compared to that required by the DFT used in OFDM system. We assume here the implementation of the DFT considered in [13], leading to complexity of:

$$C_{DFT}(J) = \begin{cases} 2^J(2^J - 1) & \text{adds} \\ J2^J & \text{mults} \end{cases} \quad \text{(19)}$$

Figure (13) displays the complexity of the WPT relative to the DFT in terms of additions and multiplication. The number of additions lower for WPT for transform sizes higher than 8 with L=4, and for a transform size greater than 32 in the case where a longer filter with L=14 is used. The number of multiplications on the other hand is generally higher for the WPT. With the shorter filter (L=4), the number of multiplications is equal for both. In the case of an implementation with a generic processor, the cost of both operations is identical. Hence, **Fig.(14)** shows the total number of operations required by the WPT, relative to the DFT for different lengths L of the wavelet filter. The WPT complexity is higher for small size transform, but decreases as the size of a transform size increases. Hence, for a transform size over 64, the number of operations for the WPT is lower than for the DFT, even in the case of L=14.

Overall, the WPT implementation complexity is on the same order of the one required by the DFT, and might even be lower for medium size transforms and wavelet filters of moderate length. The repartition between the addition/multiplication operations leads to the conclusion that the complexity is in favor of the WPT in the case of an implementation in a general purpose signal processor.

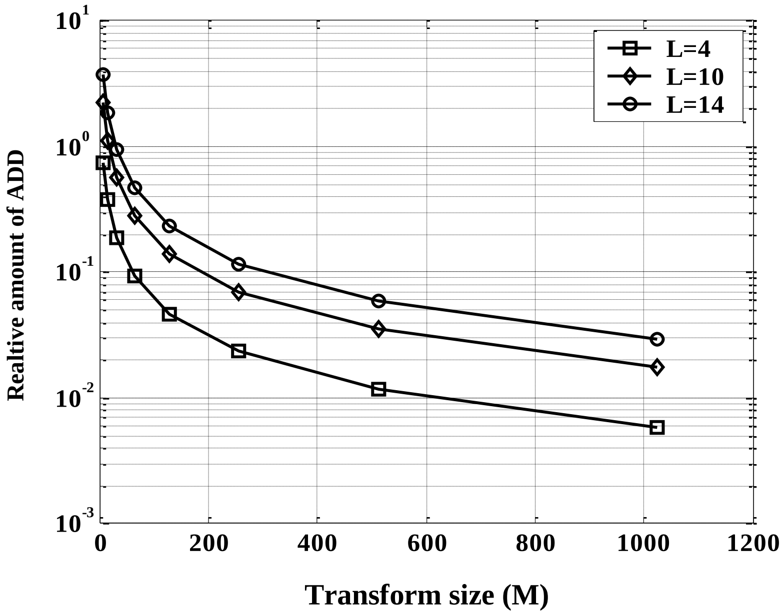


Figure (13) Number of additions required by the WPT to the DFT as a function of the transform size M, for different wavelet filter length L

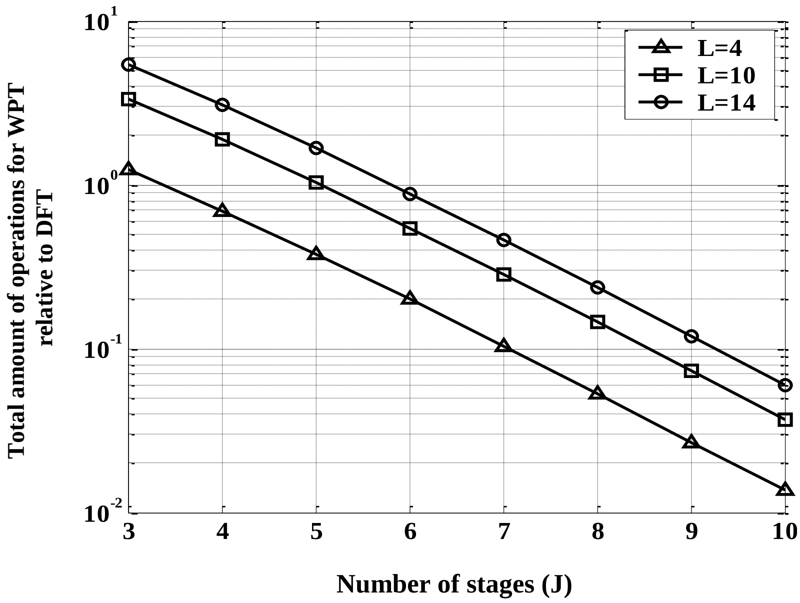


Figure (14) WPT complexity relative to that of DFT as a function of the transform size M, for different wavelet filter length L

6-4 PAPR Results

Figures (15) and **(16)** show the discrete PAPR for f_d=75 Hz and various values of N for OWDM and OFDM respectively. These figures show that increasing in PAPR is due to increasing in subcarrier numbers.

Figure (17) shows a discrete PAPR performance of OWDM system compared with OFDM system for N=256. It is easy to see that PAPR of OWDM is similar to that of OFDM system, where difference is small and possible to be neglected.

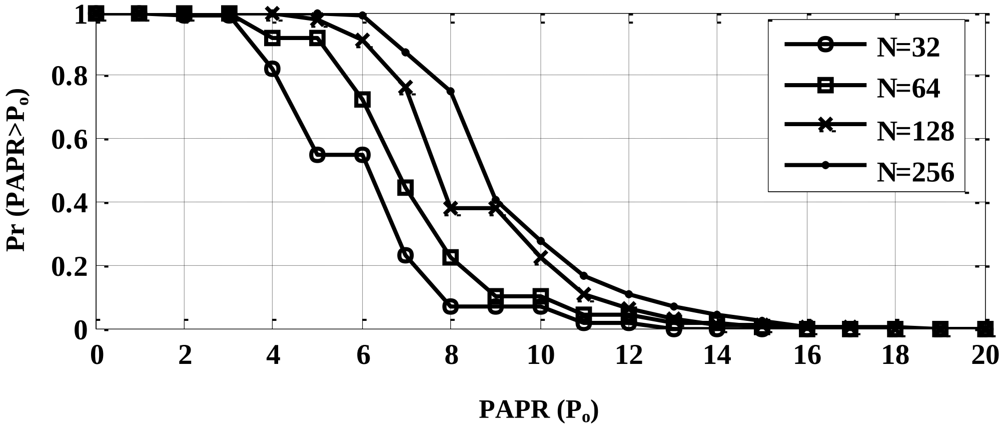


Figure (15) CDF of PAPR for OWDM system for different values of N

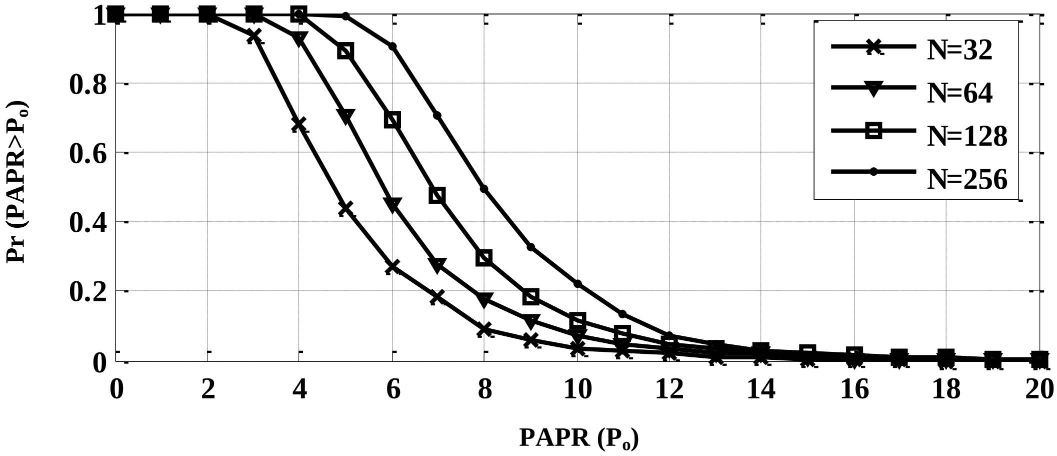
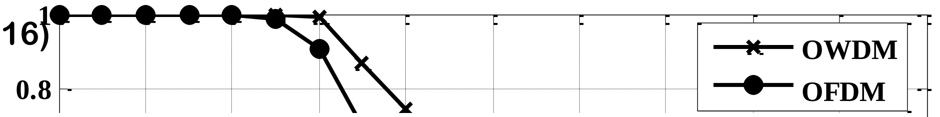


Figure (16)



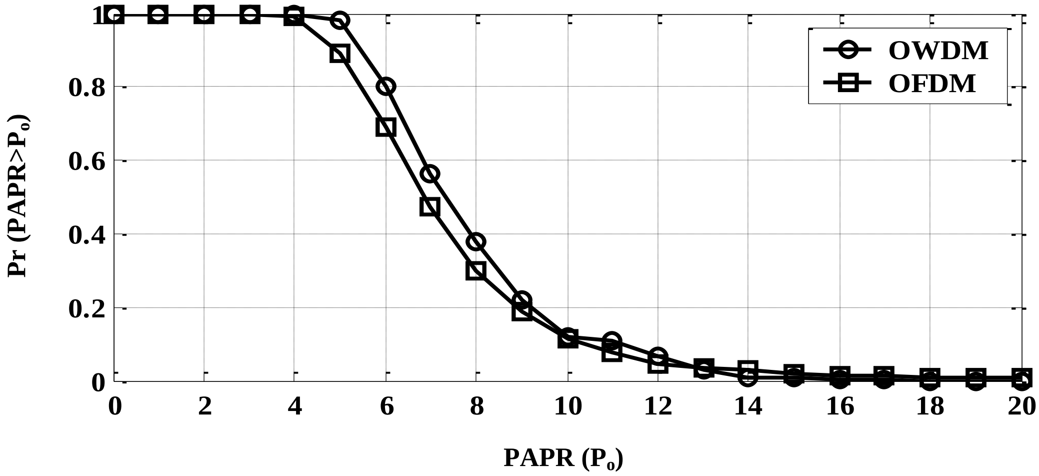


Figure (17) Comparison of CDF of PAPR between OFDM and OWDM systems for N=256

7. Conclusions

The important points were during simulation and discussion of the results are given below:

1. The behavior of the OWDM system is studied. The study shows that the OWDM system is the efficient way to deal with flat Rayleigh fading channel in comparison with single carrier system.
2. OWDM system is capable of supporting high data rate transmission in comparison with single carrier system.
3. The simulation shows that when the number of subcarriers is increased, the BER performance is better and PAPR is increased.
4. The PAPR of OWDM is similar to that of OFDM system and is recommended to solve the problem of PAPR in OWDM by using Direct Sequence–Code Division Multiplexing (DS-CDM) system.
5. The WPT implementation complexity is the same to the one required by the DFT, and might even be lower for medium size transforms and wavelet filters of moderate length.
6. The performance results of OWDM lead us to conclude that this new modulation scheme is viable alternative to OFDM to be considered for today's communication systems. OFDM remains nevertheless a strong competitor thanks to its capability to cope with multipath effects efficiently.

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