Journal of Engineering and Development, Vol. 12, No. 2, June (2008) ISSN 1813-7822

Direct and Inverse Position Kinematics of 3-UPS Parallel Manipulators

Asst. Lect. Mazin Ismail Jabar
Technical Instructors Training Institute
Foundation of Technical Education, Baghdad, Iraq

Abstract

In this paper, the equations for the inverse and direct kinematics problems of a 3-UPS spatial parallel mechanism (universal-prismatic-spherical) of six degrees of freedom with two actuated joints and four passive joints in each of three parallel branches have been studied.

The number of solutions of the inverse kinematics problem is shown to be not more than 64, and the solution of the direct kinematics problem has been shown to reducible to a 8th order polynomial. This implies that for a given set of actuated angles, this 3-UPS parallel mechanism can be assembled in at most 8 different configurations. Further numerical computations were performed to check the algebra and numerical examples were solved to demonstrate the procedure.

الخـلاصـــــــة

في هذا البحث تمت دراسة معادلات الحركة المباشرة و المعكوسة المجردة للآلية الفضـــــائية 3-UPS المتوازية ذات درجات الحرية الستة والمتصفة بفعالية مفصلين و خمول المفاصل الأربعة المتبقية لكل ساق من السيقان الثلاث.

لقد تم التوصل إلى إن عدد الحلول هي ٦٤ للحالة الحركية المعاكسة بينما أتضح بان حل معادلة الحركة المباشرة هي معادلة متعددة الحدود من الدرجة الثامنة، أي يمكن الحصول على ثمانية هيئات مختلفة لمجموعة زوايا مفـعّلة. تم إجراء مختلف الحسابات الرياضية الأخرى للتأكد من دقة المعادلات التي استنتجت، كذلك تم حل العديد من الأمثلة الرقمية لإثبات صحة ودقة الحلول.

1. Introduction

For the past few years, parallel robotic systems have received considerable research attention in the robotic field. This is because of their high stiffness where the load is usually carried in compression-traction mode only. Also it offers a high load capability since the payload is carried by several links in parallel and hence quick dynamic response and good position accuracy due to non-cumulative join error. Among the many aspects of parallel robotic systems, forward position analysis has been studied extensively but remains a challenging problem for researchers. Forward position analysis is to determine the position and orientation of the moving platform provided that a set of actuated joint variables is specified. Forward position analysis of a parallel mechanism is necessary under the following circumstances: (1) the parallel system is controlled by a Cartesian scheme; (2) the parallel system is used as a position-orientation sensor or force-torque sensor; and (3) the parallel system is utilized as the master manipulator in a teleoperation system [1].

A six degrees of freedom parallel manipulator was introduced by Stewart in 1965 and since then has been commonly known as the "Stewart-Gough platform" Merlet [2]. An atlas of parallel robots was composed by Merlet and can be found in [3]. Several researchers have analyzed the direct kinematics of Stewart-Gough platform. Kim and Tsai [4] studied the kinematics of 3-RPS parallel manipulator using the method of prescribed positions. Song and Kwon [5] solved the kinematics of parallel mechanism using the tetrahedron configuration. Hopkins and Williams [6] designed a PSU parallel mechanism. The limitation of these methods is due to their dependent on the estimation of the initial configuration. Hence the solutions of the forward kinematics are generally calculated and preferred by numerical methods. For example, Tahmasebi [7] studied a Novel Tip Tile Piston parallel manipulator, Ben-Horin et. al. [8] analyzed a planarly actuated parallel robot. All these methods were without initial estimation.

2. Description of the Mechanism

In this paper the equations for the direct and inverse position kinematics for 3-UPS mechanism. **Figure (1)** have been developed and solutions have been obtained It is shown that, at most, sixty-four solutions exist for the inverse position kinematics problem. While the direct position kinematics solution has been shown to be reducible to an eighth order polynomial equation.

The mechanism under discussion consists of two platforms connected to each other by three serial chains as shown in **Fig.(2)**. One of the platforms is fixed to the ground while the other one is free. Each of the three serial chains has a total of six degrees of freedom. All the three serial chains are connected to the top platform by passive spherical joints at P_1, P_2, and P_3, respectively. Each chain is connected to the bottom platform by two active perpendicular revolutes, forming the universal joint at O_1, O_2, and O_3 respectively, also each chain has a passive prismatic joint L_i connecting the passive ball joint at P_i to the active universal joint at

O_i as shown in **Fig.(3)**. The angle of rotation about the Xg axis which is needed to align the Zg axis with Z_{oi} is (270-αi). This fixes the position and orientation of $X_{oi}Y_{oi}Z_{oi}$ relative to the base frame XgYgZg. The overall mobility of this mechanism can be evaluated by applying the kutzbach criterion [9]:

$$\mathbf{D.O.F} = \lambda(\mathbf{n}-\mathbf{j}-1)+\sum_{i=1}^{j=1}\mathbf{f}_i = 6(8-9-1)+3(2+1+3) = -12+18 = 6$$

Therefore the mechanism is **6** D.O.F.

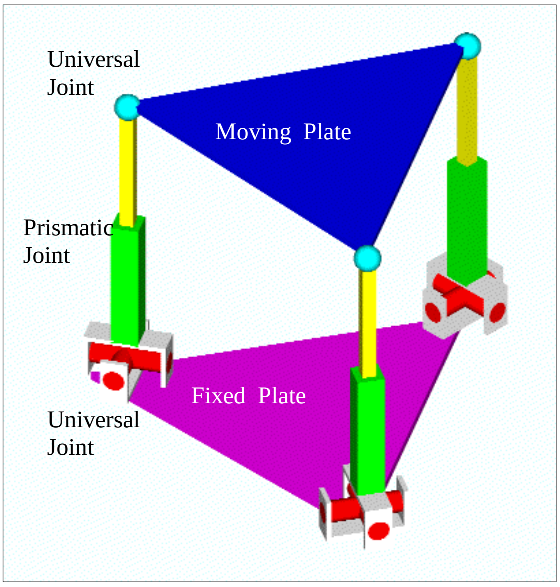

Figure (1) Prototype of 3-UPS

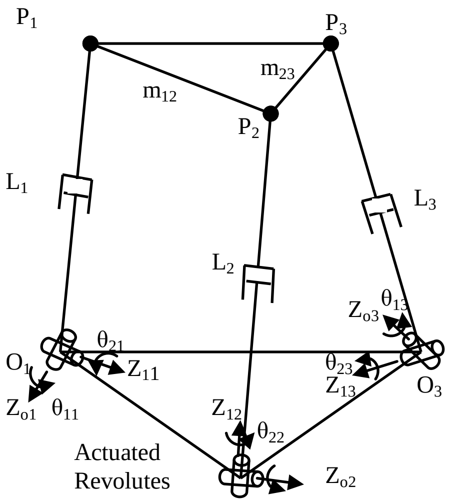

Figure (2) Schematic of 3-UPS parallel mechanism

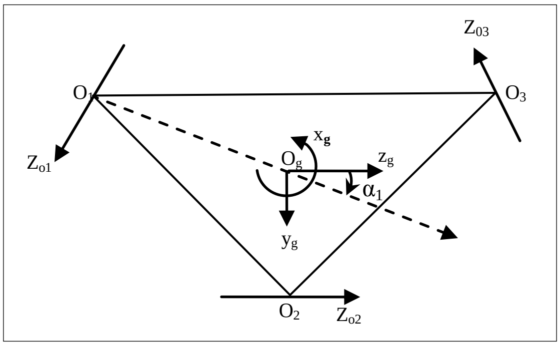

Figure (3) Location of the base coordinate system

3. Inverse Position kinematics

The inverse position kinematics problem for this structure can be stated as: given the position and orientation of the moving platform $P_1P_2P_3$ with respect to the base platform $O_1O_2O_3$, find the intermediate actuator angles.

Since the position and orientation of a moving body in space can be uniquely determined by specifying the position of three noncollinear points embedded in the moving body, we can restate the inverse position kinematics problem as follows: given The position of the points P_1,P_2 and P_3 in the base coordinate frame $X_gY_gZ_g$, and $\theta_{11},\theta_{21},L_1,\theta_{12},\theta_{22},L_2,\theta_{13},\theta_{23}$ *and* L_3, find the intermediate actuator angles.

Using standard, serial chain techniques, the vector from O_g to P_i can be written in the form [10]:

$$\mathbf{p_{pi}} = [{}^{g}\mathbf{R_{oi}}]\mathbf{q_i} + \boldsymbol{\rho}_{\mathbf{oi}} \qquad \textbf{for } \mathbf{i = 1,2,3} \text{ .... } \mathbf{(1)}$$

where:

[${}^{g}R_{oi}$]: is the coordinate transformation matrix of $X_{oi}Y_{oi}Z_{oi}$ frame with respect to $X_gY_gZ_g$ frame and is given by:

$$ {}^{g}\mathbf{R_{oi}} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & -\mathbf{sin}\alpha_i & \mathbf{cos}\alpha_i \\ \mathbf{0} & -\mathbf{cos}\alpha_i & -\mathbf{sin}\alpha_i \end{bmatrix} $$

ρ_{oi} : is the vector from O_g to O_i and is given by:

$$\boldsymbol{\rho}_{\mathbf{oi}} = [\mathbf{0} \quad (\boldsymbol{\rho}_{\mathbf{oi}})_{\mathbf{y}} \quad (\boldsymbol{\rho}_{\mathbf{oi}})_{\mathbf{z}}]^{T}$$

Detailed expressions for all of the three limbs are given below. The notation and conventions used here are those of Husain and Waldron [10]. They are consistent with Hartenberg and Denavit [11] notation, but the inhomogeneous matrix from which has separate matrices for the notation and the position of the origin is preferred [12]. The first subscript refers to the position of a revolute in the chain, and the second subscript refers to the chain itself.

$$\mathbf{q_i = U_{1i} J U_{2i} J S_i} \quad \text{.......... (2)}$$

where:

$$\mathbf{U_{ij}} = \begin{bmatrix} \cos\theta_{ij} & -\sin\theta_{ij} & 0 \\ \sin\theta_{ij} & \cos\theta_{ij} & 0 \\ o & 0 & 1 \end{bmatrix} \qquad \mathbf{J} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \qquad \mathbf{S_i} = \begin{bmatrix} 0 \\ 0 \\ L_i \end{bmatrix}$$

Combining Eq.(1),with Eq.(2), we get:

$$[{}^g\mathbf{R_{oi}}]^{-1}(\mathbf{p_{pi}} - \rho_{0i}) = \mathbf{U_{1i} J U_{2i} J S_i} \quad \text{.......... (3)}$$

The above matrix equation can be decomposed to 3 independent scalar equations in 3 unknowns which can be solved using stander techniques. The numbers of solutions obtained are as follows:

If i =1 equation (3) gives: 2 solutions for L_1, 2 solutions for θ_{12}, and 1 solution for θ_{11},

If i =2 equation (3) gives: 2 solutions for L_2, 2 solutions for θ_{22}, and 1 solution for θ_{12},

If i =3 equation (3) gives: 2 solutions for L_3, 2 solutions for θ_{32}, and 1 solution for θ_{13}.

Hence, the number of solutions to the inverse kinematics problem is not greater than 64, although all of these solutions may not be physically realizable.

4. Direct Position Kinematics

The direct position kinematics problem is much more involved. It can be stated as: Given the actuator angles for all of the actively controlled joints, find the position and orientation of the moving platform $P_1P_2P_3$ with respect to the base platform $O_1O_2O_3$.

From the previous discussion of inverse position kinematics it is obvious that the direct position kinematics problem can be re-stated as:

GIVEN: The actuator angles $\theta_{11}, \theta_{21}, \theta_{12}, \theta_{22}, \theta_{13}$ *and* θ_{23}, FIND: The prismatic joint lengths L_i.

Expansion of Eq.(1) with the help of Eq.(2) gives the vector from O_g to P_1 as:

$$\mathbf{p_{p1}} = \begin{bmatrix} \mathbf{P_{11}.L_1} \\ \mathbf{P_{12}.L_1 + R_{12}} \\ \mathbf{P_{13}.L_1 + R_{13}} \end{bmatrix} \quad \text{.......... (4)}$$

where:

$$\mathbf{P_{11} = \cos\theta_{11}\sin\theta_{21}}$$

$$\mathbf{P_{12} = -\cos\alpha_1\cos\theta_{21} - \sin\alpha_1\sin\theta_{11}\sin\theta_{21}}$$

$$\mathbf{R_{12} = (\rho_{o1})_y}$$

$$\mathbf{P_{13} = \sin\alpha_1\cos\theta_{21} - \sin\alpha_1\sin\theta_{11}\sin\theta_{21}}$$

$$\mathbf{R_{13} = (\rho_{o1})_z}$$

Similarly, the vector from O_g to P_2 is:

$$\mathbf{p_{p2}} = \begin{bmatrix} \mathbf{P_{21}.L_2} \\ \mathbf{P_{22}.L_2 + R_{22}} \\ \mathbf{P_{23}.L_2 + R_{23}} \end{bmatrix} \quad \text{.. (5)}$$

where:

$$\mathbf{P_{21} = \cos\theta_{12}\sin\theta_{22}}$$

$$\mathbf{P_{22} = -\cos\alpha_2\cos\theta_{22} - \sin\alpha_2\sin\theta_{12}\sin\theta_{22}}$$

$$\mathbf{R_{22} = (\rho_{o2})_y}$$

$$\mathbf{P_{23} = \sin\alpha_2\cos\theta_{22} - \cos\alpha_2\sin\theta_{12}\sin\theta_{22}}$$

$$\mathbf{R_{23} = (\rho_{o2})_z}$$

and the vector from O_g to P_3 is:

$$\mathbf{P_{p3}} = \begin{bmatrix} \mathbf{P_{31}.L_3} \\ \mathbf{P_{32}.L_3 + R_{32}} \\ \mathbf{P_{33}.L_3 + R_{33}} \end{bmatrix} \quad \text{.. (6)}$$

where:

$$\mathbf{P_{31} = \cos\theta_{13}\sin\theta_{23}}$$

$$\mathbf{P_{32} = -\cos\alpha_3\cos\theta_{23} - \sin\alpha_3\sin\theta_{13}\sin\theta_{23}}$$

$$\mathbf{R_{32} = (\rho_{o3})_y}$$

$$\mathbf{P_{33} = \sin\alpha_3\cos\theta_{23} - \cos\alpha_3\sin\theta_{13}\sin\theta_{23}}$$

$$\mathbf{R_{33} = (\rho_{o3})_z}$$

All of these P's and R's can be considered to be known for the direct kinematics problem.

For the formulation of the direct kinematics equations, we use geometric constraint that triangle $P_1P_2P_3$ embedded in the moving platform is invariant [10], i.e.,

$$\mathbf{\left|p_{p1} - p_{p2}\right|^2 = \left|P_1P_2\right|^2 = m_{12}^2} \quad \text{.. (7)}$$

$$\left|\mathbf{p}_{p2}-\mathbf{p}_{p3}\right|^2=\left|\mathbf{P}_2\mathbf{P}_3\right|^2=\mathbf{m}_{23}^2 \quad \text{.. (8)}$$

$$\left|\mathbf{p}_{p1}-\mathbf{p}_{p3}\right|^2=\left|\mathbf{P}_1\mathbf{P}_3\right|^2=\mathbf{m}_{13}^2 \quad \text{... (9)}$$

Using Eqs.(4), (5), and (6) we can simplify Eqs.(7), (8) and (9) giving Eqs.(10), (11), and (12), respectively, which are stated below:

$$\mathbf{A_1L_1^2+A_2L_2^2+A_3L_1L_2+A_4L_1+A_5L_2+A_6=0} \quad \text{................................ (10)}$$

$$\mathbf{B_1L_2^2+B_2L_3^2+B_3L_2L_3+B_4L_2+B_5L_3+B_6=0} \quad \text{................................ (11)}$$

$$\mathbf{C_1L_1^2+C_2L_3^2+C_3L_1L_3+C_4L_1+C_5L_3+C_6=0} \quad \text{............................... (12)}$$

where:

$$\mathbf{A_1=P_{11}^2+P_{12}^2+P_{13}^2}$$
$$\mathbf{A_2=P_{21}^2+P_{22}^2+P_{23}^2}$$
$$\mathbf{A_3=-2P_{11}P_{21}-2P_{12}P_{22}-2P_{13}P_{23}}$$
$$\mathbf{A_4=2P_{12}(R_{12}-R_{22})+2P_{13}(R_{13}-R_{23})}$$
$$\mathbf{A_5=2P_{22}(R_{22}-R_{12})+2P_{23}(R_{23}-R_{13})}$$
$$\mathbf{A_6=(R_{12}-R_{22})^2+(R_{13}-R_{23})^2-m_{12}^2}$$

and,

$$\mathbf{B_1=P_{21}^2+P_{22}^2+P_{23}^2}$$
$$\mathbf{B_2=P_{31}^2+P_{32}^2+P_{33}^2}$$
$$\mathbf{B_3=-2P_{21}P_{31}-2P_{22}P_{32}-2P_{23}P_{33}}$$
$$\mathbf{B_4=2P_{22}(R_{22}-R_{32})+2P_{23}(R_{23}-R_{33})}$$
$$\mathbf{B_5=2P_{32}(R_{32}-R_{22})+2P_{33}(R_{33}-R_{23})}$$
$$\mathbf{B_6=(R_{22}-R_{32})^2+(R_{23}-R_{33})^2+m_{23}^2}$$

and,

$$\mathbf{C_1=P_{11}^2+P_{12}^2+P_{13}^2}$$
$$\mathbf{C_2=P_{31}^2+P_{32}^2+P_{33}^2}$$
$$\mathbf{C_3=-2P_{11}P_{31}-2P_{12}P_{32}-2P_{13}P_{33}}$$
$$\mathbf{C_4=2P_{12}(R_{12}-R_{32})+2P_{13}(R_{13}-R_{33})}$$
$$\mathbf{C_5=2P_{32}(R_{32}-R_{12})+2P_{33}(R_{33}-R_{13})}$$
$$\mathbf{C_6=(R_{12}-R_{32})^2+(R_{13}-R_{33})^2-m_{13}^2}$$

Eqs.(10),(11) and (12) cab be rewritten as:

$$T_1L_2^2 + M_1L_2 + N_1 = 0 \quad \text{.......... (13)}$$

$$T_2L_2^2 + M_2L_2 + N_2 = 0 \quad \text{.......... (14)}$$

$$V_1L_1^2 + V_2L_1 + V_3 = 0 \quad \text{.......... (15)}$$

where:

$T_1 = A_2$ $\quad M_1 = A_3L_1 + A_5$ $\quad N_1 = A_1L_1^2 + A_4L_1 + A_6$

$T_2 = B_1$ $\quad M_2 = B_3L_3 + B_4$ $\quad N_2 = B_2L_3^2 + B_5L_3 + B_6$

$V_1 = C_1$ $\quad V_2 = C_3L_3 + C_4$ $\quad V_3 = C_2L_3^2 + C_5L_3 + C_6$

We can eliminate L_2 from Eqs.(13) and (14) using Bezout's method [9,10] and the resulting equation will contain only L_1 and L_3 ; as follows:

- **Step 1- Elimination of L_2:**

Multiplying eq.(13) by T_2 and eq.(14) by T_1 and subtracting we obtain:

$$(T_2M_1 - T_1M_2)L_2 + (T_2N_1 - T_1N_2) = 0 \quad \text{.......... (16)}$$

Multiplying eq.(13) by N_2 and eq.(14) by N_1 and subtracting then dividing by L_2 we obtain:

$$(N_2T_1 - N_1T_2)L_2 + (N_2M_1 - N_1M_2) = 0 \quad \text{.......... (17)}$$

Equations (13) and (14) represent two linear equation in one unknown:

$$\begin{vmatrix} T_2M_1 - T_1M_2 & T_2N_1 - T_1N_2 \\ T_1N_2 - T_2N_1 & N_2M_1 - N_1M_2 \end{vmatrix} = 0 \quad \text{.......... (18)}$$

Expanding equation (18) and substituting the expressions for L_1, M_1, N_1, L_2, M_2, and N_2 results in the following equation.

$$O_1L_1^4 + O_2L_1^3 + O_3L_1^2 + O_4L_1 + O_5 = 0 \quad \text{.......... (19)}$$

where:

$O_1 = G_1$

$O_2 = G_2L_3 + G_3$

$O_3 = G_4L_3^2 + G_5L_3 + G_6$

$$O_4 = G_7L_3^3 + G_8L_3^2 + G_9L_3 + G_{10}$$
$$O_5 = G_{11}L_3^4 + G_{12}L_3^3 + G_{13}L_3^2 + G_{14}L_3 + G_{15}$$

where:

$$G_1 = A_1^2B_1^2$$
$$G_2 = -A_1A_3B_1B_3$$
$$G_3 = -A_1A_3B_1B_4 + 2A_1A_4B_1^2$$
$$G_4 = A_3^2B_1B_2 + A_1A_2B_3^2 - 2A_1A_2B_1B_2$$
$$G_5 = A_3^2B_1B_5 - A_1A_5B_1B_3 - A_3A_4B_1B_3 + 2A_1A_2B_3B_4 - 2A_1A_2B_1B_5$$
$$G_6 = A_3^2B_1B_6 - A_1A_5B_1B_4 - A_3A_4B_1B_4 + A_1A_2B_4^2 - 2A_1A_2B_1B_6 + 2A_1A_6B_1^2 + A_4^2B_1^2$$
$$G_7 = -A_2A_3B_2B_3$$
$$G_8 = 2A_3A_5B_1B_2 - A_2A_3B_2B_4 - A_2A_3B_3B_5 + A_2A_4B_3^2 - 2A_2A_4B_1B_2$$
$$G_9 = 2A_3A_5B_1B_5 - A_2A_3B_4B_5 - A_2A_3B_3B_6 - A_4A_5B_1B_3 - A_3A_6B_1B_3 + 2A_2A_4B_3B_4 - 2A_2A_4B_1B_5$$
$$G_{10} = 2A_3A_5B_1B_6 - A_2A_3B_4B_6 - A_4A_5B_1B_4 - A_3A_6B_1B_4 + A_2A_4B_4^2 - 2A_2A_4B_1B_6 + 2A_4A_6B_1^2$$
$$G_{11} = A_2^2B_2^2$$
$$G_{12} = -A_2A_5B_2B_3 + 2A_2^2B_2B_5$$
$$G_{13} = A_5^2B_1B_2 - A_2A_5B_2B_4 - A_2A_5B_3B_5 + A_2A_6B_3^2 - 2A_2A_6B_1B_2 + 2A_2^2B_2B_6 + A_2^2B_5^2$$
$$G_{14} = A_5^2B_1B_5 - A_2A_5B_4B_5 - A_2A_5B_3B_6 - A_5A_6B_1B_3 + 2A_2A_6B_3B_4 - 2A_2A_6B_1B_5 + 2A_2^2B_5B_6$$
$$G_{15} = A_5^2B_1B_6 - A_2A_5B_4B_6 - A_5A_6B_1B_4 + A_2A_6B_4^2 - 2A_2A_6B_1B_6 + A_6^2B_1^2 + A_2^2B_6^2$$

Step 2- Elimination of L_1:

Multiplying equation (19) by V_1 and eq.(15) by $O_1L_1^2$, and subtracting, we obtain:

$$(O_2V_1 - O_1V_2)L_1^3 + (O_3V_1 - O_1V_3)L_1^2 + O_4V_1L_1 + O_5V_1 = 0 \quad \text{.......................... (20)}$$

Multiplying equation (19) by $V_1L_1 + V_2$, and equation (15) by $O_1L_1^3 + O_2L_1^2$, and subtracting, we get:

$$(O_3V_1 - O_1V_3)L_1^3 + (O_4V_1 + O_3V_2 - O_2V_3)L_1^2 + (O_5V_1 + O_4V_2)L_1 + O_5V_2 = 0 \quad \text{.... (21)}$$

Multiplying equation (15) by L_1, we obtain:

$$V_1L_1^3 + V_2L_1^2 + V_3L_1 = 0 \quad \text{.. (22)}$$

We can think of equations (20), (21), (22), and (15) as four linear equations in three unknown's L_1^3, L_1^2, *and* L_1. Vanishing of their eliminate yields [7]:

$$\begin{vmatrix} \mathbf{O_2V_1 - O_1V_2} & \mathbf{O_3V_1 - O_1V_3} & \mathbf{O_4V_1} & \mathbf{O_5V_1} \\ \mathbf{O_3V_1 - O_1V_3} & \mathbf{O_4V_1 + O_3V_2 - O_2V_3} & \mathbf{O_5V_1 + O_4V_2} & \mathbf{O_5V_2} \\ \mathbf{V_1} & \mathbf{V_2} & \mathbf{V_3} & \mathbf{0} \\ \mathbf{0} & \mathbf{V_1} & \mathbf{V_2} & \mathbf{V_3} \end{vmatrix} = \mathbf{0} \quad \text{....................} \quad \mathbf{(23)}$$

If we substitute the expressions for V_1, V_2, V_3, O_1, O_2, O_3, O_4, and O_5 into eq.(23), and expand, we obtain:

$$\mathbf{E_1L_3^8 + E_2L_3^7 + E_3L_3^6 + E_4L_3^5 + E_5L_3^4 + E_6L_3^3 + E_7L_3^2 + E_8L_3 + E_9 = 0} \quad \text{...............} \quad \mathbf{(24)}$$

Detailed expressions for the Oi's and Ei's are not given here due to space limitation and for further information please contact the author.

After solving for L_3, L_1 and L_2 can be computed from Eqs.(14) and (15), respectively. Each of these equations is quadratic, and yields two solutions for L_1 and L_2 into Eq.(13) reveals that only one of the four possible combinations of solutions satisfies the equation. Hence we have a total of eight valid solutions for L_1, L_2 and L_3.

The positions of points P_1, P_2 and P_3 relative to the fixed frame are now given by Eqs. (4), (5) and (6), respectively. This is sufficient for formulating the transformation from the moving to the fixed reference frame.

5. Numerical Examples

5-1 Validation Example

Using the example of Cruz, P., et. al. [13] after modification to be 3-UPS mechanism, so:

5-1-1 Inverse Position Kinematics Problem

The dimensions of the mechanism were chosen to be:

$\alpha_1 = 30^0$, $\alpha_2 = 270^0$, $\alpha_3 = 150^0$

$m_{12} = 1.5$, $m_{23} = 1.5$, $m_{13} = 1.5$

Let the coordinates of the members of the mechanism in the base coordinate frame be O1= (0,-0.5,-0.866), O_2= (0,1.0,0), O_3= (0,-0.5,0.866)

P_1= (1.9365,0,-0.866), P_2= (1.9365,0.75,0.433), P_3= (1.9365,-0.75,0.433)

These coordinate of P_1, P_2, and P_3 do satisfy the constraint that $m_{12}=m_{23}=m_{13}=1.5$. Solving Eq.(3) when i=1 we get the following set of solutions for θ_{11}, θ_{21} and L_1 stated below:

L_1= (+2.0,-2.0)

θ_{21}= (+102.50392^0, - 102.50392^0) when L_1= +2.0

θ_{21}= (+77.49608^0, - 77.49608^0) when L_1= -2.0
θ_{11}= -7.356155^0 when L_1= +2.0 and θ_{21}= +102.50392^0
or,
when L_1= -2.0 and θ_{21}= - 77.49608^0.
θ_{11}= +172.64575^0 when L_1= +2.0 and θ_{21}= -102.50392^0
or,
when L_1= -2.0 and θ_{21}= + 77.49608^0.

This is a total of 4 real solutions for θ_{11}, θ_{21} and L_1. Similarly solving Eq.(3) when i=2 we get the following set of solutions for θ_{12}, $\theta_{22}$and L_2 stated below:

L_2= (+2.0,-2.0)
θ_{22}= (+102.50392^0, - 102.50392^0) when L_2= +2.0
θ_{22}= (+77.49608^0, - 77.49608^0) when L_2= -2.0
θ_{12}= -7.356155^0 when L_2= +2.0 and θ_{22}= +102.50392^0
or,
when L_2= -2.0 and θ_{22}= - 77.49608^0:
θ_{12}= +172.64575^0 when L_2= +2.0 and θ_{22}= -102.50392^0
or,
when L_2= -2.0 and θ_{22}= + 77.49608^0:

This is a total of 4 real solutions for θ_{12}, $\theta_{22}$and L_2.

Solving Eq.(3) when i=3 we get the following set of solutions for θ_{13}, θ_{23} and L_3 stated below:
L_3= (+2.0,-2.0)
θ_{23}= (+102.50392^0, - 102.50392^0) when L_3= +2.0
θ_{23}= (+77.49608^0, - 77.49608^0) when L_3= -2.0
θ_{13}= -7.356155^0 when L_3= +2.0 and θ_{23}= +102.50392^0
or,
when L_3= -2.0 and θ_{23}= - 77.49608^0:
θ_{13}= +172.64575^0 when L_3= +2.0 and θ_{23}= -102.50392^0
or,
when L_3= -2.0 and θ_{23}= + 77.49608^0:

This is a total of 4 real solutions for θ_{13}, θ_{23} and L_3. Hence, for the inverse kinematics problem of this particular example-we have a total of 64 real solutions from the maximum possible 64 solutions.

5-1-2 Direct Position Kinematics Problem

As an example of solution of a direct kinematics problem, we shall take the same dimensions as were taken in the inverse kinematics problem ,and also for the angles of the actuated joints we shall take one of the solutions obtained from the inverse kinematics problem so as to verify our result.

Hence we select: $\theta_{11} = -7.355^0$, $\theta_{21} = +102.503^0$

$$\theta_{12} = -7.355^0 \ , \ \theta_{22} = +102.503^0$$

$$\theta_{13} = -7.355^0 \ , \ \theta_{23} = +102.503^0$$

The polynomial equation obtained for L_3 is:

$$-0.0098 L_3^8 + 0.1563 L_3^7 - 1.1341 L_3^6 + 4.8551 L_3^5 - 13.3947 L_3^4$$
$$+ 24.3754 L_3^3 - 28.5839 L_3^2 + 19.7692 L_3 - 6.1811 = 0$$

The solutions of this equation are presented in **Table (1)**. The lengths in this table in meter. The final mechanism configuration obtained from this simulation is presented in **Fig.(4)**.

Hence for this particular problem, 2 equal real solutions are obtained. All the solutions were checked by back-substitution into Eqs.(13), (14), and (15). They are all valid. This shows that there are no spurious solutions and the minimum order of the polynomial Eq.(24) is eight. Also we can clearly verify that two of the real solutions obtained for direct kinematics problem do match with the solution obtained to the inverse kinematics problem, which was expected.

Table (1) The numerical results of ex. 1

No.	L_3	L_1	L_2
1	2.3778+1.1794i	-----	-----
2	2.3778-1.1794i	-----	-----
3	2.0000+1.2580i	-----	-----
4	2.0000-1.2580i	-----	-----
5	1.6224+1.1790i	-----	-----
6	1.6224-1.1790i	-----	-----
7	2.0044	2.0004	2.0005
8	1.9954	1.9956	1.9958

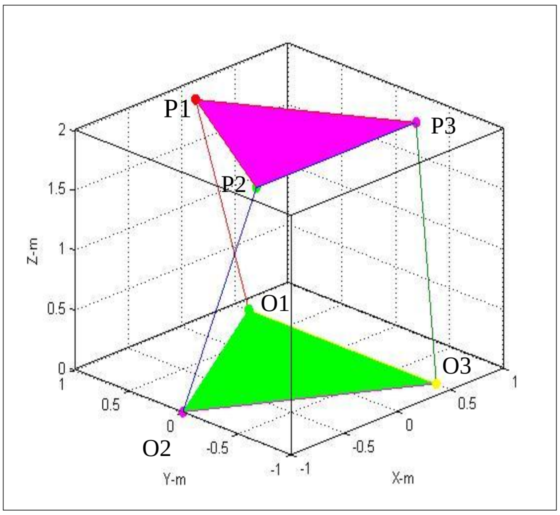

Figure (4) The resulting 3-UPS mechanism (ex_1) using MATLAB

5-2 Validation Example

5-2-1 Inverse Position Kinematics Problem

The dimensions of the mechanism were chosen to be:

$\alpha_1 = 30^0, \ \alpha_2 = 270^0, \ \alpha_3 = 150^0$

$m_{12} = 1.5$, $m_{23} = 1.5$, $m_{13} = 1.5$

Let the coordinates of the members of the mechanism in the base coordinate frame be O_1=(0,-0.5,-0.866), O_2=(0,1.0,0), O_3=(0,-0.5,0.866)
P_1=(2.586,0,-0.6919), P_2=(1.9365,0.75,0.433), P_3=(1.9365,-0.75,0.433)

These coordinate of P_1, P_2, and P_3 do satisfy the constraint that $m_{12}=m_{23}=m_{13}=1.5$. Solving Eq.(3) when i=1 we get the following set of solutions for θ_{11}, θ_{21} and L_1 stated below:

L_1= (+2.639,- 2.639)
θ_{21}= (+97.531^0, - 97.531^0) when L_1= +2.639
θ_{21}= (+82.468^0, - 82.468^0) when L_1= -2.639
θ_{11}= -8.8095^0 when L_1= +2.639 and θ_{21}= +97.531^0
or,
when L_1= -2.639 and θ_{21}= - 82.468^0 .
θ_{11}= +171.19^0 when L_1= +2.639 and θ_{21}= -97.531^0
or,
when L_1= -2.639 and θ_{21}= + 82.468^0 .

This is a total of 4 real solutions for θ_{11}, θ_{21} and L_1.

Similarly solving Eq.(3) when i=2 we get the following set of solutions for θ_{12}, θ_{22} and L_2 stated below:

L_2= (+2.0,-2.0)
θ_{22}= (+102.50392^0, - 102.50392^0) when L_2= +2.0

θ_{22}= (+77.49608^0, - 77.49608^0) when L_2= -2.0

θ_{12}= -7.356155^0 when L_2= +2.0 and θ_{22}= +102.50392^0

or,

when L_2= -2.0 and θ_{22}= - 77.49608^0:

θ_{12}= +172.64575^0 when L_2= +2.0 and θ_{22}= -102.50392^0

or,

when L_2= -2.0 and θ_{22}= + 77.49608^0 :

This is a total of 4 real solutions for θ_{12}, θ_{22} and L_2.

Solving Eq.(3) when i=3 we get the following set of solutions for θ_{13}, θ_{23} and L_3 stated below:

L_3= (+2.0,-2.0)

θ_{23}= (+102.50392^0, - 102.50392^0) when L_3= +2.0

θ_{23}= (+77.49608^0, - 77.49608^0) when L_3= -2.0

θ_{13}= -7.356155^0 when L_3= +2.0 and θ_{23}= +102.50392^0

or,

when L_3= -2.0 and θ_{23}= - 77.49608^0:

θ_{13}= +172.64575^0 when L_3= +2.0 and θ_{23}= -102.50392^0

or,

when L_3= -2.0 and θ_{23}= + 77.49608^0:

This is a total of 4 real solutions for θ_{13}, θ_{23} and L_3.

Hence, for the inverse kinematics problem of this particular example, we have a total of 64 real solutions from the maximum possible 64 solutions.

5-2-2 Direct Position Kinematics Problem

As an example of solution of a direct kinematics problem, we shall take the same dimensions as were taken in the inverse kinematics problem, and also for the angles of the actuated joints we shall take one of the solutions obtained from the inverse kinematics problem so as to verify our result.

Hence we select:

$$\theta_{11} = -8.8095^{\,0} \;,\; \theta_{21} = +97.531^{\,0}$$

$$\theta_{12} = -7.355^{\,0} \;,\; \theta_{22} = +102.503^{\,0}$$

$$\theta_{13} = -7.355^{\,0} \;,\; \theta_{23} = +102.503^{\,0}$$

The polynomial equation obtained for L_3 is:

$$-0.0049L_3^{\;8} + 0.0988L_3^{\;7} - 0.859L_3^{\;6} + 4.2026L_3^{\;5} - 12.6068L_3^{\;4}$$
$$+ 23.55L_3^{\;3} - 26.4917L_3^{\;2} + 16.2723L_3 - 4.163 = 0$$

The solutions of this equation are presented in **Table (2)**. The lengths in this table are in meter. The final mechanism configuration obtained from this simulation is presented in **Fig.(5)**.

Table (2) The numerical results of ex. 2

No.	L_3	L_1	L_2
1	4.5068+1.3043i	-----	-----
2	4.5068-1.3043i	-----	-----
3	2.5414+1.718i	-----	---
4	2.5414 -1.718i	-----	-----
5	2.0099	2.6510	2.0090
6	1.9239	-----	-----
7	1 .0257+1.1790i	------	-----
8	1.0257-1.1790i	------	-----

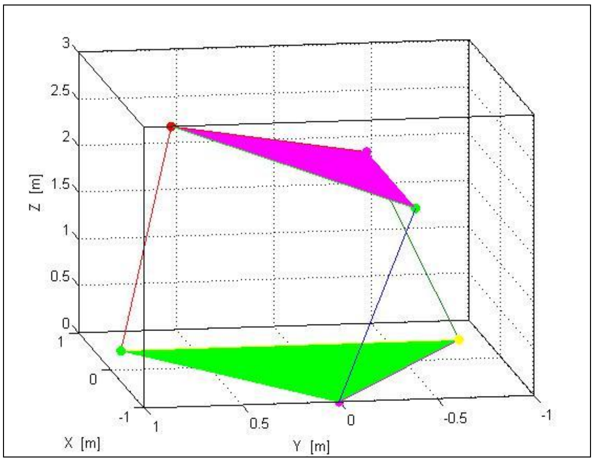

Figure (5) The resulting 3-UPS mechanism (ex_2) using MATLAB

Hence for this particular problem, we get 1 equal real solution. All the solutions were checked by back-substitution into Eqs.(13), (14), and (15). They are all valid. This shows that there are no spurious solutions and the minimum order of the polynomial Eq.(24) is eight. Also we can clearly verify from the spatial configuration and movement that this parallel mechanism has 6 D.O.F. as was expected.

5-3 Discussion

When studying and comparing the solution procedure it can be observed that it is not only simple but also hasn't got a complex programming as well as it doesn't need initial trial and error values, this program gives accurate results compared with Cruz, P. [13] and Song, S. [5] who studied the 3-UPS and used the iteration method, which leads to non accurate results because of the initial guess besides the long period where the desired solutions were obtained (converged) in at least 4 iterations.

6. Conclusion

The number of solutions of the inverse kinematics problem is shown to be not more than 64, and the solution of the direct kinematics problem has been shown to reducible to a 8th order polynomial. This implies that for a given set of actuated angles, this 3-UPS parallel mechanism can be assembled in at most 8 different configurations.

The algebra used in this work was verified using MATLAB-7^{TM} program. Further numerical computations were performed to check the algebra and numerical examples were solved to demonstrate the procedure. It has been shown that the mechanism has a translational and rotational motion and this verify that the mechanism is a 6-D.O.F.

In spite of, the solution method is independent on the iteration technique, the solution results has been achieved simply where the initial configuration is not important, emphazing all the equations of motion are trigonometric functions, and as a result there has been sensitivity for input values.

It can be found that there are a positive value for each limb (Li) corresponding to an equal negative value and they both constitute a pair of mutual mirror image configuration with respect to base. This is unavoidable in mathematics since the sign of the UPS limb cannot be constrained to be positive in the problem formulation. The mirror image solutions merely make sense in mathematics and they thereby should be regarded as extraneous solutions [14].

7. References

1. Zhang, C., and Song, S., ***"Forward Position Analysis of Nearly General Stewart Platforms"***, Journal of Mech. Design, Vol. 116, March 1994.

2. Merlet, J., ***"Parallel Robots"***, Kluwer Academic (Book on CD-ROM), 2000.

3. Merlet, J., website<http://www.inria.fr/prisme/personal/merlet/merlet_eng.html.

4. Kim, H., and Tsai, L., ***"Kinematic Synthesis of Spatial 3-RPS Parallel Manipulators"***, ASME 2002 Design Eng. Technical Conferences and Computers and Information in Engineering Conference, Montreal, Canada, Sep.29-Oct.2, 2002.

5. Song, S., and Kwon, D., ***"New Methodology for the Forward Kinematics of 6-DOF Parallel Manipulators Using Tetrahedron Configurations"***, Mech. Eng. Dept., Korea Advanced Institute of Science and Technology, 2001.

6. Hopkins, B., and Williams, R., ***"Kinematics, Design and Control of the 6-PSU Platform"***, Industrial Robot: International Journal, Vol. 29, No. 5, 2002.

7. Tahmasebi, F., ***"Direct and Inverse Kinematics of a Novel Tip-Tilt-Piston Parallel Manipulator"***, Report: NASA/TM-2004-212763, October 2004.

8. Ben-Horin, R., Shoham, M., and Djerassi, S., ***"Kinematics, Dynamics and Construction of a Planarly Actuated Parallel Robot"***, Journal Robotics and Computer-integrated Manufacturing 14, 1998.

9. Callegari, M., Palpacelli, M., and Scarponi, M., ***"Kinematics of the 3-CPU Parallel Manipulator Assembled for Motions of Pure Translation"***, International Conf. on Robotics and Automation IEEE, Spain, April 2005.

10. Husain, M., and Waldron, K. J., ***"Position Kinematics of a Three-Limbed Mixed Mechanism"***, Journal of Mechanical Design, Vol. 116, Sep., 1994.

11. Hartenberg, R., and Denavit, J., ***"Kinematics Synthesis of Linkages"***, McGraw-Hill, 1964.

12. Waldron, K. J., Wang, S. L., and Bolin, S. J., ***"A Study of the Jacobian Matrix of Serial Manipulators"***, ASME Journal of Mechanisms, Transmit Ion and Automation in Design, Vol. 107, 1985, pp. 230-238.

13. Cruz, P., Ferreira, R., and Sequeira, J., ***"Kinematic Modeling of Stewart-Gough Platforms"***, Robotics and Automation-ICINCO 2005.

14. Huang, T., Chetwynd, D. G., and Hu, S., ***"Forward Position Analysis of the 3-DOF Module of the Tri Variant: A 5-DOF Reconfigurable Hybrid Robot"***, Journal of Mechanical Design, Vol. 128, Jan., 2006.

Notations

θ_{1i}, θ_{2i}:	the actively controlled joints are the two perpendicular revolutes at Oi.
Li:	the passively controlled limb length.
m_{12}, m_{23}, m_{13}:	the length of the sides of the moving triangle P_1P_2, P_2P_3 and P_1P_3.
$X_gY_gZ_g$:	the fixed coordinate frame has its origin Og at the centroid of the base triangle $O_1O_2O_3$., Xg axis points outward from the plane paper.
Z_{o1}, Z_{o2}, Z_{o3}:	the unit vectors along the passive revolute O_1, O_2 and O_3 respectively.
α_i:	the angle made by the normal of the Zoi axis with the Zg axis (positive in the clockwise sense).
λ:	degree of freedom of the task space.
n:	total number of links.
j:	number of joints.
f_i:	degrees of relative motion permitted by joint i.
q_i:	the position vector of P_i relative to O_i.