

The Effects of Changes in Manning's Roughness Coefficients and Eddy Viscosity on a Constrained Flume

Dr. Thair Sharif Khayyun

Building and Construction Engineering Department

University of Technology, Baghdad, Iraq

Abstract

Any numerical model is designed to address a specific class of problems. The results of these numerical models are often sensitive to the values of the Manning's roughness and eddy viscosity.

This study analyzes the effects of changes in Manning's roughness coefficients and the eddy viscosities on a constrained flume. Understanding these effects is useful in models calibration. A computer program called RMA2 has been used for this purpose. The effects of five values of Manning's (n) and five values of eddy viscosity (E) are investigated, for determining the water depth and flow velocities. The results indicate the fact that as the roughness increases, the upstream water depth increases and the eddy viscosities have a much larger effect when there are large longitudinal velocity gradients.

For realistic values of eddy viscosity, differences in depth at the upstream end of the channel are small. The effect of Manning's roughness and eddy viscosity in models calibration are presented by an equation. This equation involves the dimensionless parameters (Peclet No. and Froude No.) and for different values of Manning's (n). In this study and for subcritical flow, the Peclet No. was less than 28.0 and a good agreement was found between the result of the present work and that of the published studies.

الخلاصة

تصمم النماذج العددية لإيجاد حلول لمشاكل هندسية محددة. بعضها يقوم بحساب بيانات هيدروديناميكية مثل مستوى سطح الماء و سرعة الجريان. والبعض الآخر يقوم بحساب ميكانيكية الموجات مثل ارتفاع الموجة واتجاهها و تقوم أخريات بتتبع انتقال الملوثات أو تراكيز الرسوبيات العالقة.

في هذه الدراسة تم تحليل تأثير التغيرات في معامل خشونة ماننك و اللزوجة الدوامية على مسيل متقلص المقطع. إن فهم طبيعة تلك التأثيرات هو ضروري في معايرة النماذج، ولهذا الغرض فقد تم استخدام النموذج عددي يدعى (RMA2). تم اختيار خمسة قيم لمعامل خشونة ماننك وخمسة قيم للزوجة الدوامية لحساب عمق الماء وسرعة الجريان و استكشاف تأثيرهما. أوضحت النتائج حقيقة زيادة عمق الماء في المقدم عندما تزداد الخشونة.

إن اللزوجة الدوامية لها تأثير كبير عندما توجد تغيرات كبيرة للسرعة في الاتجاه الطولي. ولقيم حقيقية للزوجة الدوامية فإن الاختلافات في أعماق الماء في مقدم القناة تكون قليلة. في هذه الدراسة تم تحويل تأثير خشونة ماننك و اللزوجة الدوامية على معايرة النماذج إلى معادلة رياضية. هذه المعادلة تتضمن معاملات غير بعدية (رقم بيكلت ورقم فرويد) ولمختلف القيم لخشونة ماننك. لقد بينت الدراسة بأن قيمة رقم بيكلت هي أقل من (28.0) وحالة الجريان تحت الحرج، وأن هناك تطابق جيد بين نتائج هذه الدراسة والدراسات الأخرى.

1. Introduction

Roughness and eddy viscosity affect the water surface profile of a steady state river simulation. It can affect the speed of the flow or the velocity distribution in the river passage.

Wetting and drying may be affected. Mannings (n) is used to describe the resistance to flow due to channel roughness caused by sand or gravel bed-forms, bank vegetation and obstructions, bend effects and circulation-eddy losses ^[1]. The Manning (n) is user-defined for each channel or river reach between cross-sections. The Manning n is users-specified either constant or as a function of (stage or discharge).

In the absence of necessary data (observed stages and discharges), n can be estimated, however, best results are obtained when n is adjusted to reproduce historical observations of stage and discharge. The adjustment process is referred to as calibration. This may be either a trial-error process or an automatic iterative procedure available within any model.

Basic references for selecting the Manning n may be found in ^[2,3]. Also, some other reports should be considered in selecting n value, such as, Arcement and Schneider ^[4] for wooded flood plains and Jarrett ^[5] for relatively steep slope streams with gravel beds, ($0.002 \leq S_o \leq 0.040$).

Turbulence issues can create problems during a simulation. Since viscosity is involved in the equations of motion, it affects the velocity distribution, which will affect the depth and which will determine the effects of roughness.

The eddy viscosity terms in the governing equations of motion, actually represent the molecular viscosity and the effects of turbulence from the Reynold's stress terms. The eddy viscosity, E, includes both effects. But under the flows of interest the Reynold's stress terms are several orders of magnitude larger than the effects of the molecular viscosity. Therefore, the molecular viscosity is effectively ignored ^[6]. Although it is difficult to establish a value for E, analogy with physical conditions suggests that eddy viscosities depend on the momentum of the fluid, gradients of the velocity and the scale of the flow phenomenon (i.e. length of the element). Therefore as the element size increases, E should increase or as the velocity increases, E should increase.

Table (1) below, provides some representative ranges of eddy viscosity ^[7].

Table (1) Ranges of eddy viscosity

Type of problem	E, lb-sec/ft ²	E, Pascal-sec
Homogenous horizontal flow around an island	10-100	480-4800
Homogenous horizontal flow at a Confluence	25-100	1200-4800
Steady-state flow for thermal to a Slow moving river	20-1000	950-48000
Tidal flow in a marshy estuary	50-200	2400-9580
Slow flow through a shallow pond	0.2-1.0	10-50

Note: 1 lb-sec/ft² = 48 Pascal-sec

The eddy viscosity can be assigned by allowing the models to automatically adjust E after each iteration, based upon a provided Peclet No., which defines the relationship between the average elemental velocity magnitude, elemental length, fluid density and E. The Peclet N. is recommended between 15 and 40 ^[8].

Joseph ^[9], showed that, the turbulent shear stress affects by a concept of mixing length, which is defined as the distance one must move transversely to the direction of the flow.

2. Description of Numerical Model

RMA2 (Resource Management Associates) is a two-dimensional depth averaged finite element hydrodynamic numerical model. It computes water surface elevations and horizontal velocity components for subcritical, free-surface two-dimensional flow fields.

RMA2 represents a finite element solution of the Reynolds form of the Navier-Stokes equations for turbulent flows. Friction is calculated with the Manning or Chezy equation and eddy viscosity coefficients are used to define turbulence characteristics. Both steady and unsteady (dynamic) problems can be analyzed by this package. RMA2 operates under the hydrostatic assumption, meaning accelerations in the vertical direction are negligible.

The generalized computer program RMA2 solves the depth-integrated equations of fluid mass and momentum conservation in two horizontal directions ^[10]. The forms of the solved equations are:

$$\begin{aligned} h \frac{\partial u}{\partial t} + hu \frac{\partial u}{\partial x} + hv \frac{\partial u}{\partial y} - \frac{h}{\rho} (E_{xx} \frac{\partial^2 u}{\partial x^2} + E_{xy} \frac{\partial^2 u}{\partial y^2}) + gh (\frac{\partial a}{\partial x} + \frac{\partial h}{\partial x}) \\ + \frac{gum^2}{(1.486h^{\frac{1}{2}})^2} \times \xi(u^2 + v^2) - \zeta Va^2 \cos \psi - 2hvw \sin \phi = 0 \end{aligned} \quad \dots\dots\dots (1)$$

$$\begin{aligned} h \frac{\partial v}{\partial t} + hu \frac{\partial v}{\partial x} + hv \frac{\partial v}{\partial y} - \frac{h}{\rho} (E_{xx} \frac{\partial^2 v}{\partial x^2} + E_{xy} \frac{\partial^2 v}{\partial y^2}) + gh (\frac{\partial a}{\partial y} + \frac{\partial h}{\partial y}) \\ + \frac{gum^2}{(1.486h^{\frac{1}{2}})^2} \times \xi(u^2 + v^2) - \zeta Va^2 \sin \psi + 2hvw \sin \phi = 0 \end{aligned} \quad \dots\dots\dots (2)$$

$$\frac{\partial h}{\partial t} + h (\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}) + u \frac{\partial h}{\partial x} + v \frac{\partial h}{\partial y} = 0 \quad \dots\dots\dots (3)$$

where:

h: water depth.

u,v: velocities in the Cartesian directions.

x,y,t: cartesian coordinates and time.

ρ : density of fluid,

E: eddy viscosity coefficient (for *xx*= normal direction on *x* axis surface, for *yy*= normal direction on *y* axis surface, for *xy* and *yx*= shear direction on each surface).

g: acceleration due to gravity.

a: elevation of bottom.

n: Manning's roughness *n*-value, 1.486= conversion from SI (metric) to customary (English).

ζ : empirical wind shear coefficient.

V_a : wind speed.

ψ : angle between x-direction and wind direction.

w : rate of earth's angular rotation and

ϕ : local latitude.

Equation 1, 2, and 3 solved by the finite element method using the Galerkin method of weighted residuals. Two-dimensional quadrilaterals elements (4 corners and 4 midside nodes) are used in this study. The shape (or basis) functions are quadratic for velocity and linear for depth. Integration in space is performed by Gaussian integration. Derivatives in time are replaced by a nonlinear finite difference approximation. Variables are assumed to vary over each time interval in the form:

$$f(t) = f(t_0) + at + bt^c \dots\dots\dots to \leq t < t_0 + \Delta t \dots\dots\dots (4)$$

Which is differentiated with respect to time, and cast in finite difference form, Letter a, b and c is constants. It has been found by experiment that the best value for c is 1.5^[7]. The solution is fully implicit and the set of simultaneous nonlinear equations are solved by Newton-Raphson scheme.

3. Constrained Flume

To show the effect of roughness coefficients and eddy viscosities, a flume (L=800 m) by (W=100 m) has been considered. The flow rate is set at (100 m³/sec). The downstream water surface elevation is (1.0 m). The channel is constricted to (20 m) wide through the middle and has gradual contractions and expansions above and below the constricted section.

4. Importing Bed Topography and Mesh

The program in this study operates by first reading in a bed topography file which contains digitized position, elevation and information for node numbers for each element. Data points for a finite elements mesh can be generated directly from topographic data, such as a list of survey points by using meshing techniques. A8-noded quadrilateral elements were used in this study. These elements are often more stable for numerical analysis^[11]. The use of it also reduces the number of elements. The use of quadrilaterals also reduces the number of elements. In this program two options were provided for converting from triangular to quadrilateral elements. The total numbers of nodes and elements were (283) and (80) respectively, **Fig (1)**.

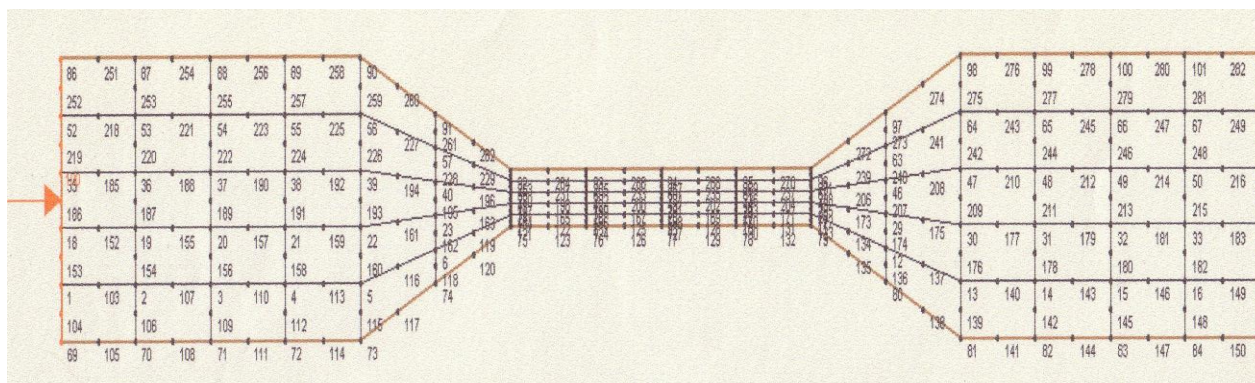


Figure (1) Flume mesh with nodes

5. Assigning the Boundary Conditions

When the finite element mesh are generated, boundary conditions can now be assigned by selecting the specified flow rate as the boundary condition type and enter in a constant flow rate of ($100 \text{ m}^3/\text{sec}$) at the inflow cross section. The same procedure is used for assigning the water surface boundary condition at the outflow cross section of the flume. The water surface elevation as the boundary condition type was selected and enters in a constant water surface elevation of (1m).

6. Creating Profile Plots

In RMA2 model, a profile plots can be created to visualize the results of a model run. It is necessary to create an observation coverage with an observation are to define the profile to plot. An arc was created down the center of the flume.

7. Varying Manning's Roughness

In order to compare the results, it must be change the material properties in the model by selecting different values of Manning's roughness coefficients. Wide ranges of Manning's n values were used to represent an envelope of possible water surface elevations within the range of uncertainty associated with estimated n values.

The Manning's roughnesses considered in this study were 0.010, 0.030, 0.045, 0.060, and 0.100 respectively.

For each run, the effects of various Manning's roughness on water depths were analyzed while keeping eddy viscosity constant, **Table (2)** and **Fig.(2)**.

Table (2) Water depth with varied mannings roughness coefficients and turbulent eddy viscosity

Distance m	Mannings Roughness Coefficients ($E_{xx}=5$)					Turbulent Eddy Viscosity , $E_{xx}(n=0.030)$				
	0.01	0.03	0.045	0.06	0.1	5	50	500	1000	2000
0	0.847	1.95	2.246	2.499	3.076	1.95	1.991	2.532	3.079	3.963
50	1.845	1.948	2.243	2.495	3.07	1.948	1.989	2.531	3.079	3.964
100	1.843	1.946	2.24	2.491	3.064	1.946	1.987	2.53	3.078	3.963
150	1.841	1.944	2.237	2.487	3.059	1.944	1.985	2.531	3.08	3.966
200	1.839	1.941	2.233	2.483	3.054	1.941	1.984	2.535	3.088	3.978
250	1.829	1.931	2.223	2.473	3.042	1.931	1.979	2.571	3.152	4.082
276.25	1.796	1.899	2.194	2.445	3.014	1.899	1.936	2.468	3.007	3.87
300	1.765	1.871	2.167	2.42	2.988	1.871	1.898	2.378	2.881	3.687
350	1.524	1.644	1.963	2.229	2.773	1.644	1.661	1.934	2.286	2.876
400	1.538	1.623	1.876	2.108	2.594	1.623	1.612	1.863	2.164	2.652
450	1.292	1.383	1.64	1.858	2.332	1.383	1.425	1.933	2.365	3.009
500	0.884	0.99	1.226	1.409	1.789	0.99	1.062	1.5	1.834	2.294
523.75	0.974	1.039	1.203	1.346	1.67	1.039	1.067	1.276	1.47	1.772
550	1.069	1.088	1.174	1.274	1.537	1.088	1.068	1.024	1.066	1.19
600	1.043	1.069	1.46	1.228	1.448	1.069	1.071	1.034	0.989	0.958
650	1.046	1.059	1.117	1.185	1.378	1.059	1.057	1.057	1.048	1.014
700	1.025	1.039	1.083	1.135	1.291	1.039	1.04	1.042	1.044	1.039
750	1.016	1.021	1.044	1.075	1.79	1.021	1.021	1.023	1.026	1.029
800	0.999	1.00	1.001	1.002	1.009	1.00	1.00	1.003	1.006	1.012

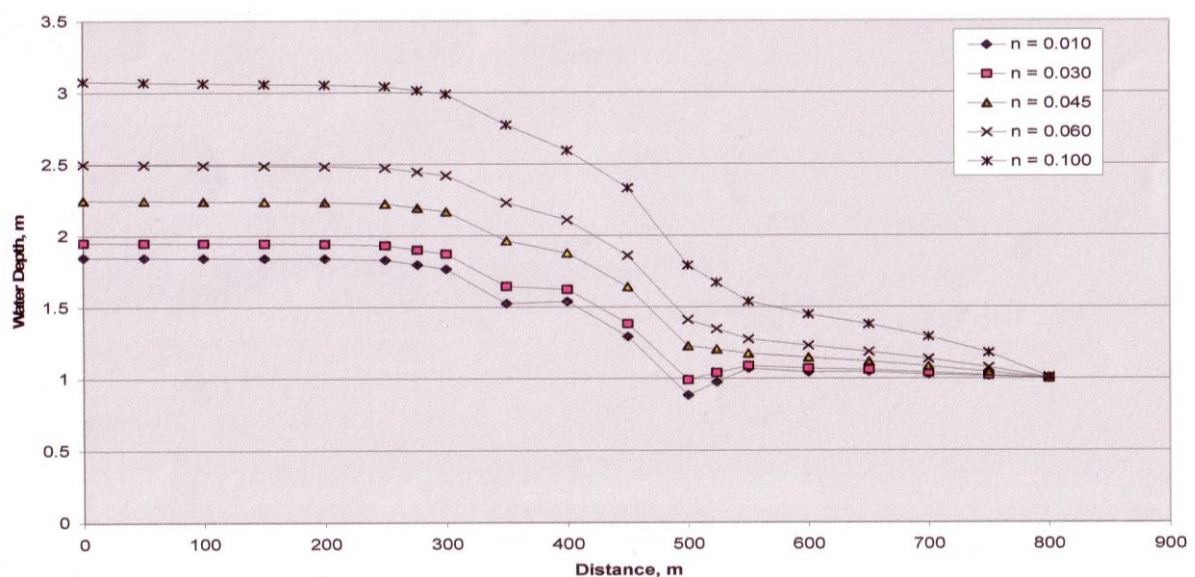


Figure (2) Constricted flume water depths with various Manning's roughness coefficients

Figure (2) demonstrates the fact that as the roughness increases, the upstream water surface elevation increases. Also, the recession curve of the water depth in the constricted section of the flume was unstable for the minimum values of Manning's roughness.

As one can see in **Table (2)**, the flow depths at section 400.0 m (contraction section) vary between 1.623-2.652 m. The critical depth corresponding to a given rate of flow in a rectangular channel with a width of 20.0 m, is 1.36 m. Hence, the flow is subcritical (streaming flow).

8. Changes in Eddy Viscosity

Eddy Viscosity is another parameter that can be modified to alter the model's solution and sensitivity analysis. Before creating solutions for different values of eddy viscosities, Constrained flume velocities with various Manning (n) were created through an arc through the center of the flume, **Table (3)** and **Fig.(3)** and for $E_{xx}=50.0$.

Table (3) Velocity with varied mannings roughness coefficients

Distance m	Mannings Roughness Coefficients, n				
	0.01	0.03	0.045	0.06	0.1
0.0	0.643	0.513	0.443	0.393	0.333
50	0.641	0.516	0.445	0.395	0.334
100	0.64	0.521	0.45	0.398	0.335
150	0.632	0.541	0.466	0.412	0.343
200	0.639	0.618	0.538	0.479	0.408
250	0.828	0.951	0.836	0.757	0.668
267	1.639	1.276	1.108	0.987	0.824
300	1.815	1.921	1.643	1.461	1.224
350	3.965	3.541	2.924	2.513	1.946
400	0.406	2.74	2.441	2.208	1.918
450	6.227	3.746	3.065	2.649	2.072
500	7.429	4.436	3.499	2.912	2.094
542.04	2.786	1.843	1.784	1.667	1.384
550	1.49	1.405	1.562	1.52	1.284
600	1.32	1.138	0.95	0.895	0.801
650	0.767	0.869	0.886	0.84	0.733
700	1.279	0.993	0.925	0.888	0.795
750	0.685	0.962	0.961	0.938	0.885
800	1.029	1.004	1.00	0.999	0.991

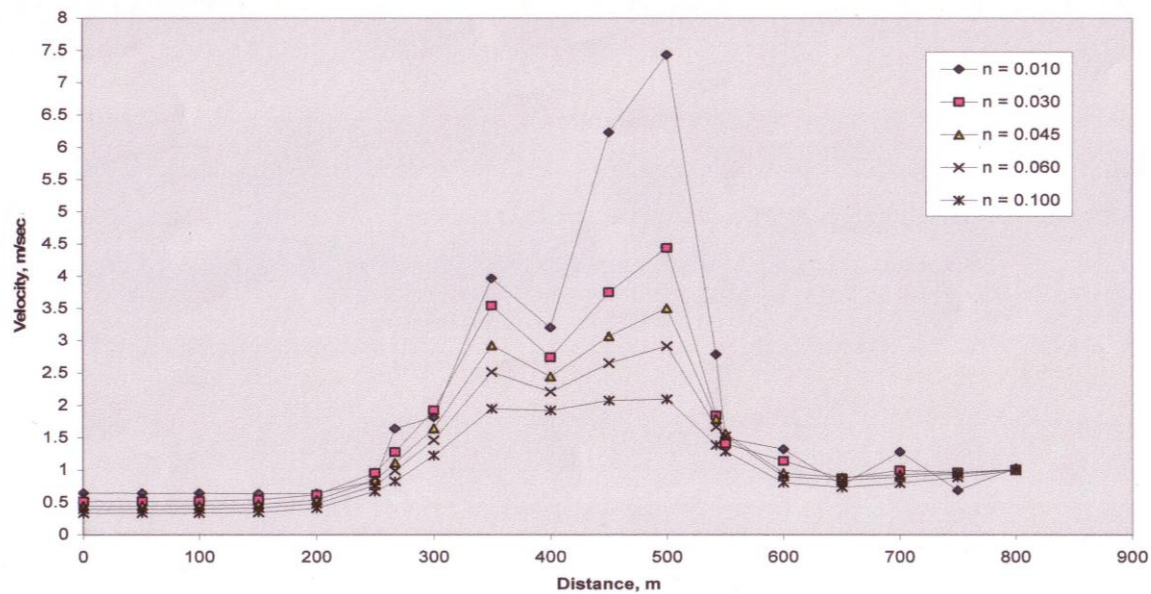


Figure (3) Constricted flume velocities with various Manning's roughness coefficients

As one can see in the graph, smaller Manning's roughness allows larger longitudinal velocity gradients to appear in the solution.

It is most important that eddy viscosity (E) in the model be properly selected in order to avoid computational difficulties and to achieve an acceptable level of numerical accuracy for computed flow depths and velocities. Eddy viscosity values ranging from 0.20-48000 Pascal-sec were used for this purpose. The computations indicated that the appropriate values for eddy viscosities to achieve the stability were varied between 5.0-2000.0 Pascal-sec.

Varies eddy viscosities values were selected while keeping Manning's roughness constant. For running the model, viscosities of 5, 50, 500, 1000 and 2000 were used, **Table (2)**. The selection of these values depend on the fact that the eddy viscosity is a function of turbulence of flow and its value is equal to the absolute viscosity multiply by many thousands and on other test of models. As a guide line for selecting reasonable values for E , several models choose to expedite this process. A utility program called MAKE-EV-DF will calculate the average elemental size of each material type in the mesh and create a table of recommended E values^[7].

As shown in **Fig.(4)**, eddy viscosities have a much larger effect when there are large longitudinal velocity gradients, **Fig.(3)**, and for realistic values of eddy viscosity, differences in depth at the upstream end of the channel are small.

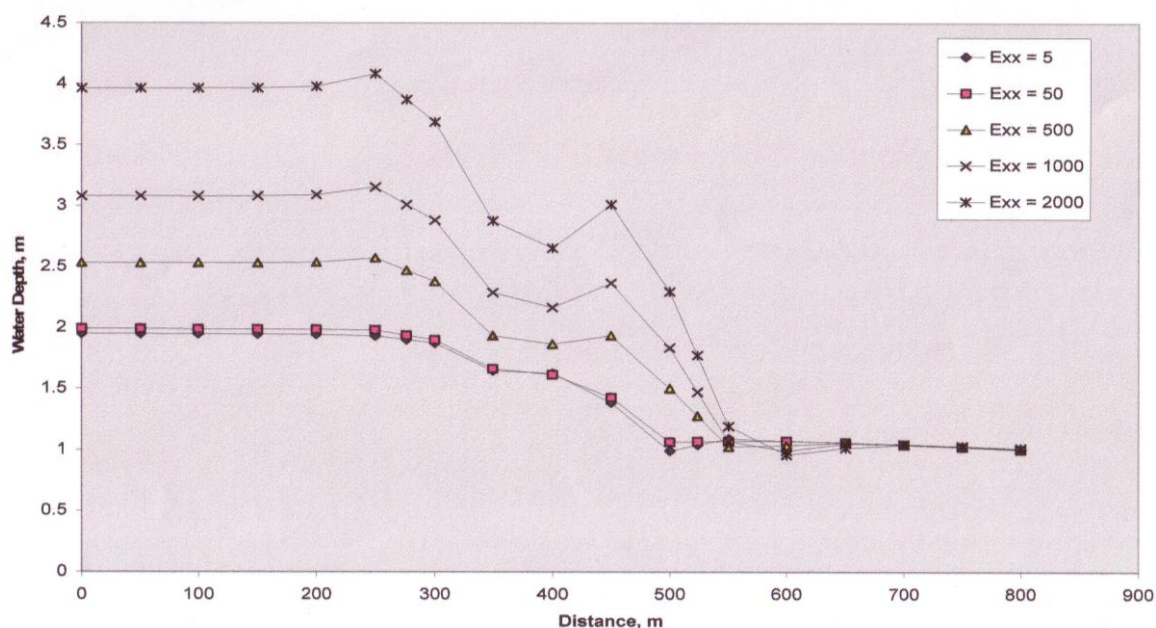


Figure (4) Constricted flume water depths with various Eddy viscosities

9. Dimensionless Parameters

Water depth and velocities are sensitive to changes in the Manning's roughness and eddy viscosity values. The results of the models are numerically unstable problem for which the programs will diverge rather than converge to a solution. One changes the values of Manning (n) and eddy viscosity until a stable solution is achieved. So, to illustrate the sensitivity between the model output (water depth and velocity) and the parameters (Manning(n) and eddy viscosity (E)), a dimensionless parameters which include the Peclet number ($\rho v d/E$) and Froude number ($v/\sqrt{gd}$) are used.

It was found that a multiple regression model for the dimensionless parameter ($\rho v d/E$) versus the dimensionless parameter ($v/\sqrt{gd}$) and Manning's roughness coefficient, fitted all the data well with a value of correlation coefficient being (0.87). The regression equation was:

$$\left(\frac{\rho v d}{E}\right) = (n)^{-0.72} * \left(\frac{v}{\sqrt{gd}}\right)^{0.32} \dots\dots\dots (5)$$

This equation is converted to a dimensionless parameters graph **Fig.(5)**. Its speed and easy of use, make it appropriate for use as a preliminary estimated of the eddy viscosity and for the models calibration in the constrained flume. The value of Peclet number in this study is less than (28.0) for subcritical flow (water depth is used instead of element length). So, a close agreement between the original Peclet N . and Peclet N . by this study strongly suggests that the equation is valid.

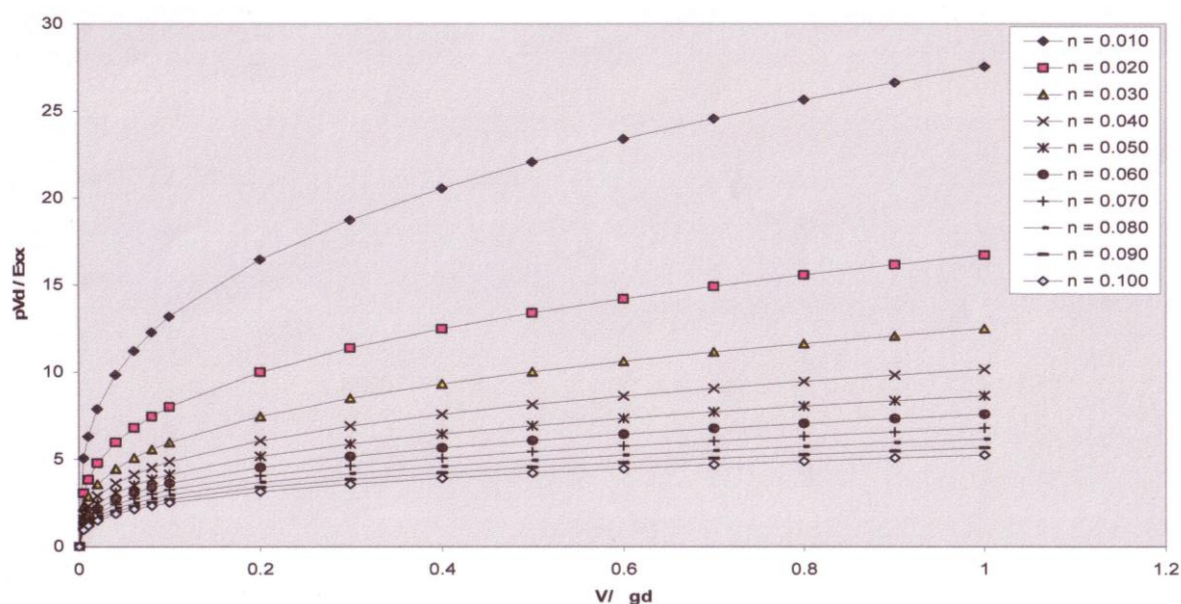


Figure (5) Peclet No. versus Froude No. for different values of Manning's roughness

10. Conclusions

The effects of changes in Manning's roughness and eddy viscosity in the finite element hydrodynamic models calibrations may permits to draw the following conclusions:

1. As the roughness increases, the upstream water surface elevation increases. Also, the recession curve of the water depth in the constricted section of the flume was unstable for the minimum values of Manning's roughness.
2. Smaller Manning's roughness allows larger longitudinal velocity gradients to appear in the solution.
3. Eddy viscosities have a much larger effect when there are large longitudinal velocity gradients and for realistic values of eddy viscosity, differences in depth at the upstream end of the channel are small.
4. A dimensionless parameters graph was created, which depends on a multiple regression model for the dimensionless parameters $(\rho v d / E)$, $(v / \sqrt{g d})$ and Manning's roughness (n). Its speed and easy of use, make it appropriate for use as a general solution of the finite element hydrodynamic numerical model and for the models calibration.
5. For the subcritical flow, it was found that the Peclet number is less than 28.0 when the water depth is used instead of element length.

11. References

1. Fread, D. L., ***“Calibration Technique for 1-D Unsteady Flow Models”***, Journal of Hydraulic Division, ASCE, Vol. 104, No. Hyd, 1998, pp. 1027-1044.
2. Chow, V. T., ***“Open Channel Hydraulic”***, McGraw-Hill, New York, 1959.
3. Barnes, H. H., ***“Roughness Characteristics of Natural Channels”***, Geological Survey Water Supply Paper, U.S., Washington, D.C., 1967, 223pp.
4. Arcement, G. J., Schneider, V. R., ***“Guide for Selecting Manning’s Roughness Coefficients for Natural Channels and Flood Plains”***, U.S. Geological Survey for Federal Highway Administration, PB84, 1984, 61pp.
5. Jarrett, R. D., ***“Determination of Roughness Coefficients for Streams in Colorado”***, U.S. Geological Survey Water Resources Investigation Report, 1985, pp.54.
6. Thomas, T. G., ***“Large Eddy Simulation of a Symmetric Trapezoidal Channel at a Reynolds Number of 430000”***, Journal of Hydraulic Research, Vol. 33, No.6, 1995.
7. Norton, W. R., and King, ***“Operating Instructions for the Computer Program for Depth-Averaged Flow Calculation”***, Resource Management Associates, Lafayette, CA, 1997.
8. Olsen, R. B., ***“A Three-Dimensional Model for Calculation of Hydraulic Parameters”***, Conference on Hydraulics, Trondheim, Norway, 1994.
9. Joseph, B. F., ***“Fluid Mechanics with Engineering Application”***, McGraw-Hill, New York, 1997.
10. Barbara, P. D., ***“Users Guide To RMA2”***, U.S Army, Engineer Research and Development Center, Water Ways Experiment Station, 2005.
11. Smith, I. M., ***“Programming the Finite Element Method”***, John Wiley, England, 1998.