Kinematics Modeling and Development of a Nonholonomic Wheeled Mobile Robot

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Abstract

The main contribution of this paper is to present and discusses a new approach for development of a kinematics model and control strategy for a nonholonomic Wheeled Mobile Robot WMR.

Dynamic model is involved, the linearization of the model is also presented, and stability analysis is discussed. Extensive simulation results for the proposed controller are presented.

الخـلاصــــــة

المساهمة الرئيسية لهذا البحث تكمن في عرض ومناقشة تقارب جديد لتطوير الموديل الكينماتيكي وإستراتيجية القيادة للـ nonholonomic روبوت جوال بعجلات.

تضمن البحث الموديل الديناماتيكي والموديل الخطي وتم مناقشته تحليل الاستقرارية. تضمن البحث ايضاً نتائج التمثيل الشامل للمسيطر المقترح.

1. Introduction

Robotics is one of the most exciting emerging fields in modern science. Robots and other automated device have the potential to improve our economy, health, standard of living and our knowledge about the world we live in [1]. During the last decade, there has been a considerable increase in the research devoted to autonomous mobile robots. The most fundamental ability for a mobile robot is the ability to move around its environment safely. The robot must know where it is (localization), where it wants to go (mapping), and how to get there (path planning and obstacle avoidance). Each of these component technologies has challenged the robotics community for decades. Evolution Robotics provides breakthrough technologies that enable low-cost and reliable navigation in realistic environments, such as homes and workplaces [2]. The term nonholonomic planning was introduced by Laumond [3] to describe the problem of motion planning for wheeled mobile robots.

There are many types of moving systems for WMR and every type have its own kinematics equations. One of the easiest examples is the like simple car; car cannot drive sideways because the back wheels would have to slide instead of roll. This is why parallel parking is challenging. If all four wheels could be turned simultaneously toward the curb, it would be trivial to park a car. The complicated maneuvers for parking a simple car arise because of rolling constraints [4]. The second type is classic three-wheeled vehicle, which uses a single controlled angle and speed wheel to move to a desired position and orientation [5]. The third type is differential configuration wheeled vehicle. The differential configuration use independent velocities in both wheels left and right to move in the 2D plane to a specific point and specific orientation [5]. Another type is a four-wheel drive mobile robot in which each wheel can turn freely around its horizontal axis, and its direction can be freely controlled as well [6].

2. Wheeled Mobile Robot Modeling

From the idea that the differential drive WMR cannot move in the direction along the axis, this is a singularity, and differential drive vehicles are very sensitive to slight changes in velocity in each of the wheels. Differential WMR is also very sensitive to small variation in the ground. From all a new nonholonomic WMR which we named Roc was modeling and implemented as illustrated in **Figs.(1)** and **(2)**. In this model extra wheels are added in addition more than the differential wheels (groups 2). Groups 3 are used for support and make the error slightly scarce, for the notation, this group working as a coaxial to minimize its effect in the rotation. Groups 1 are the most important for steering the robot when it comes out from the trajectory; also it helps the robot in the rotation.

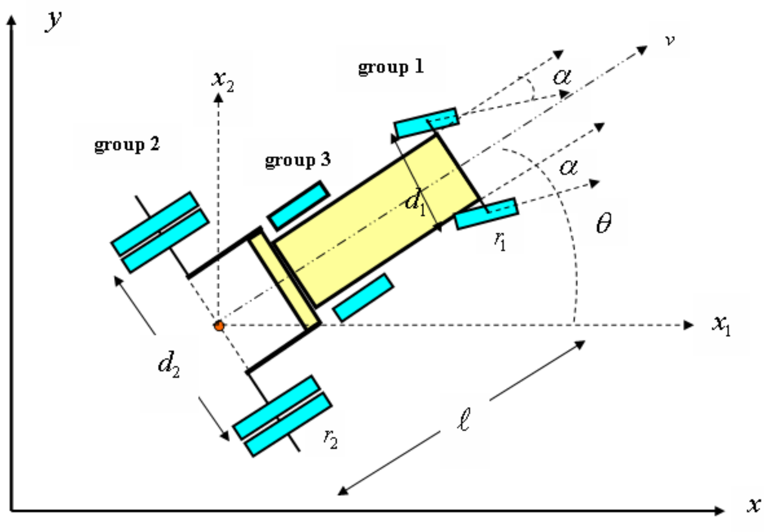


Figure (1) Free body diagram for Roc WMR

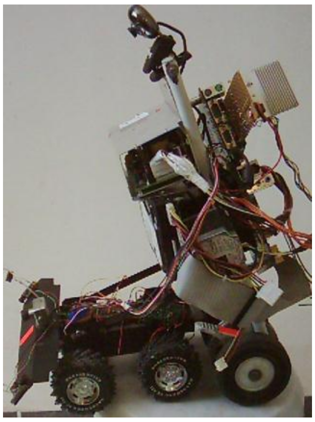


Figure (2) Roc WMR implemented

2-1 Kinematic Model

Assume that (P) represents a point in the space having (n) generalized coordinates, and q a vector of (m) actuation variables (for n>m), and assume $\dot{p}$ and $\dot{q}$ are the respective derivatives of such vectors, then the jacobian expression in the form (Zhao and Bennet [5]):

$$\dot{\mathbf{p}} = \mathbf{f}(\mathbf{p}) + \sum_{i=1}^{m} \mathbf{g}(\mathbf{p})_i \dot{\mathbf{q}}_i \quad \text{.......... (1)}$$

$$\dot{\mathbf{p}} = \begin{bmatrix} \cos\theta \\ \sin\theta \\ 0 \end{bmatrix} \mathbf{v} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \varpi \quad \text{.......... (2)}$$

where,

v: is the linear velocity of the vehicle, and

ϖ : is its angular velocity.

These equations can also be changed to the form of equation (1) as:

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix}=\begin{bmatrix}\cos\theta & 0\\ \sin\theta & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}v\\ \varpi\end{bmatrix} \quad \text{.......... (3)}$$

The new joint coordinates for the wheels according to the WMR that illustrated in **Fig.(1)** is:

$$\begin{bmatrix}v\\ \omega\end{bmatrix}=\begin{bmatrix}\frac{r_2}{2} & \frac{r_2}{2}\\ \frac{-r_2}{d_2}+\frac{r_1^2[\mathrm{sgn}(\alpha)\sin(\alpha)]}{d_1\ell} & \frac{r_2}{d_2}+\frac{r_1^2[\mathrm{sgn}(\alpha)\sin(\alpha)]}{d_1\ell}\end{bmatrix}\begin{bmatrix}u_1\\ u_2\end{bmatrix} \quad \text{.......... (4)}$$

where,

α : is front steering angle of constant value.

The steering inputs angular velocities:

$$u_1=\varpi_L \quad \text{.......... (5)}$$

$$u_2=\varpi_R \quad \text{.......... (6)}$$

and, the two outputs:

$$x_1=x+l\cos\theta \quad \text{.......... (7)}$$

$$x_2=y+l\sin\theta \quad \text{.......... (8)}$$

The sign function is used to indicate the application of the front steering angle α , to the right (-) or to the left (+).

Now, to get the jacobian matrix substituting equations (5), (6) and (4) into equation (3):

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix}=\begin{bmatrix}\frac{r_2\cos\theta}{2} & \frac{r_2\cos\theta}{2}\\ \frac{r_2\sin\theta}{2} & \frac{r_2\sin\theta}{2}\\ \frac{-r_2}{d_2}+\frac{r_1^2[\mathrm{sgn}(\alpha)\sin(\alpha)]}{d_1\ell} & \frac{r_2}{d_2}+\frac{r_1^2[\mathrm{sgn}(\alpha)\sin(\alpha)]}{d_1\ell}\end{bmatrix}\begin{bmatrix}\varpi_L\\ \varpi_R\end{bmatrix} \quad \text{.......... (9)}$$

Taking the time derivative of equations (7) and (8):

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}\dot{x}-l\sin\theta\dfrac{d\theta}{dt}\\ \dot{y}+l\cos\theta\dfrac{d\theta}{dt}\end{bmatrix}=\begin{bmatrix}1 & 0 & -l\sin\theta\\ 0 & 1 & l\cos\theta\end{bmatrix}\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} \quad \text{.......} \quad (10)$$

Substituting equation (9) in to equation (10)

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}1 & 0 & -\ell\sin\theta\\ 0 & 1 & \ell\sin\theta\end{bmatrix}\begin{bmatrix}\dfrac{r_2\cos\theta}{2} & \dfrac{r_2\cos\theta}{2}\\ \dfrac{r_2\sin\theta}{2} & \dfrac{r_2\sin\theta}{2}\\ \dfrac{-r_2}{d_2}+\dfrac{r_1^2[\operatorname{sgn}(\alpha)\sin(\alpha)]}{d_1\ell} & \dfrac{r_2}{d_2}+\dfrac{r_1^2[\operatorname{sgn}(\alpha)\sin(\alpha)]}{d_1\ell}\end{bmatrix}\begin{bmatrix}\varpi_L\\ \varpi_R\end{bmatrix} \quad \text{......} \quad (11)$$

The forward velocity can be obtained by simplifying equation (11):

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=\begin{bmatrix}\dfrac{r_2\cos\theta}{2}-\ell\sin\theta\left[\dfrac{-r_2}{d_2}+\dfrac{r_1^2[\operatorname{sgn}(\alpha)\sin(\alpha)]}{d_1\ell}\right] & \dfrac{r_2\cos\theta}{2}-\ell\sin\theta\left[\dfrac{r_2}{d_2}+\dfrac{r_1^2[\operatorname{sgn}(\alpha)\sin(\alpha)]}{d_1\ell}\right]\\ \dfrac{r_2\sin\theta}{2}+\ell\cos\theta\left[-\dfrac{r_2}{d_2}+\dfrac{r_1^2[\operatorname{sgn}(\alpha)\sin(\alpha)]}{d_1\ell}\right] & \dfrac{r_2\sin\theta}{2}+\ell\cos\theta\left[\dfrac{r_2}{d_2}+\dfrac{r_1^2[\operatorname{sgn}(\alpha)\sin(\alpha)]}{d_1\ell}\right]\end{bmatrix}\begin{bmatrix}\varpi_L\\ \varpi_R\end{bmatrix} \quad \text{..} \quad (12)$$

2-2 Dynamic Model

This subsection is demonstrated the control task to solve the stabilization problem for a simplified dynamic model of the robot with saturation on the control torques. More precisely, consider the following dynamic extension of the WMR:

$$\left.\begin{aligned}\dot{x}&=v\cos\theta\\ \dot{y}&=v\sin\theta\\ \dot{\theta}&=\omega\\ \dot{\omega}&=u_{T1}\\ \dot{v}&=u_{T2}\end{aligned}\right\} \quad \text{.......} \quad (13)$$

where,

u_{T1} and u_{T2}: are generalized torque-inputs subject to the constraints:

$$|u_{T1}|\le u_{T1,max} \quad \text{.......} \quad (14)$$

$$|u_{T2}|\le u_{T2,max} \quad \text{.......} \quad (15)$$

$$\theta=f(\alpha,\varpi_L,\varpi_R) \quad \text{.......} \quad (16)$$

with; $u_{T1,\max} > 0$ and $u_{T2,\max} > 0$ arbitrary positive constants.

with; $\alpha_1(t,\theta,x,y)$ and $\alpha_2(t,\theta,x,y)$. Introduce two new variables ϖ and $\bar{v}$ as:

$$\varpi = \omega - \alpha_1(t,\theta,x,y) \quad \text{.......... (17)}$$

$$\bar{v} = v - \alpha_2(t,\theta,x,y) \quad \text{.......... (18)}$$

Consider the positive definite proper Lyapunov function candidate for system (13)

$$V_2(t,X) = \mu_4 \log(1 + V_1(t,\theta,x,y)) + \frac{1}{2}\varpi^2 + \frac{1}{2}\bar{v}^2 \quad \text{.......... (19)}$$

where,

$X = (x^T, \omega, v)^T = (x, y, \theta, \omega, v)^T$ *and*

$\mu_4 > 0$ *is a design parameter.*

Therefore, we choose the time-varying control laws as:

$$u_{T1} = -h_{\mu_5}(\varpi) + \dot{\alpha}_1 - \frac{\mu_4(\theta + \mu_1 k_1 \cos t)}{1 + V_1(t,x)} \quad \text{.......... (20)}$$

$$u_{T2} = -h_{\mu_6}(\bar{v}) + \dot{\alpha}_2 - \frac{\mu_4(x\cos\theta + y\sin\theta)(1 + 2\mu_1(\theta + \mu_1 k_1 \cos t)k_1' \cos t)}{1 + V_1(t,x)} \quad \text{. (21)}$$

where,

$\mu_5 > 0$, $\mu_6 > 0$ *are design parameters and*

h_{μ_5}, h_{μ_6} *corresponding set of saturation functions and*

$k_1' = \frac{dk_1}{ds}(x^2 + y^2)$, *and*

$$\dot{\alpha}_1 = \frac{\partial\alpha_1}{\partial t} + \frac{\partial\alpha_1}{\partial t}\omega + \left(\frac{\partial\alpha_1}{\partial x}\cos\theta + \frac{\partial\alpha_1}{\partial y}\sin\theta\right)v \quad \text{.......... (22)}$$

$$\dot{\alpha}_2 = \frac{\partial\alpha_2}{\partial t} + \frac{\partial\alpha_2}{\partial t}\omega + \left(\frac{\partial\alpha_2}{\partial x}\cos\theta + \frac{\partial\alpha_2}{\partial y}\sin\theta\right)v \quad \text{.......... (23)}$$

and,

$$V_1(t,x) = \frac{1}{2}(\theta + \mu_1 k_1 (x^2 + y^2)\cos t)^2 + \frac{1}{2}x^2 + \frac{1}{2}y^2 \quad \text{.......... (24)}$$

3. Linearization

The functions of kinematics equations of model (9) are:

$$x = f(\varpi_L, \varpi_R, \theta) \quad \text{......} \quad (25)$$

$$y = f(\varpi_L, \varpi_R, \theta) \quad \text{......} \quad (26)$$

$$\theta = f(\varpi_L, \varpi_R, \alpha) \quad \text{......} \quad (27)$$

After taken the Tailer series of equations (25),(26) and (27) and using the linearization procedures [7] for the kinematics equations of model (9), a set of three transfer functions are obtained:

$$F^{X}_{\varpi+\alpha} = \frac{x(s)}{\varpi_{+\alpha}(s)} = \frac{As - B}{s^2} \quad \text{......} \quad (28)$$

$$F^{y}_{\varpi+\alpha} = \frac{y(s)}{\varpi_{+\alpha}(s)} = \frac{Cs + D}{s^2} \quad \text{......} \quad (29)$$

$$F^{\theta}_{\varpi+\alpha}(s) = \frac{\theta(s)}{\varpi_{+\alpha}(s)} = \frac{a_{23} + a_{24} + a_{25}}{s} \quad \text{......} \quad (30)$$

where,

$\varpi + \alpha$ & $\varpi_{+\alpha}$: is the angular velocities of ϖ_L and ϖ_R including α effects, and,

$$A = a_{03} + a_{04} \quad \text{......} \quad (31)$$

$$B = a_{02}a_{23} + a_{02}a_{24} + a_{02}a_{25} \quad \text{......} \quad (32)$$

$$C = a_{13} + a_{14} \quad \text{......} \quad (33)$$

$$D = a_{12}a_{23} + a_{12}a_{24} + a_{12}a_{25} \quad \text{......} \quad (34)$$

and,

$$a_{02} = \left(\frac{\varpi_L r_2}{2} \sin\theta + \frac{\varpi_R r_2}{2} \sin\theta \right) \quad \text{......} \quad (35)$$

$$a_{03} = \frac{r_2}{2} \cos\theta \quad \text{......} \quad (36)$$

$$\mathbf{a_{04}} = \frac{\mathbf{r_2}}{\mathbf{2}}\cos\theta \quad \text{(37)}$$

$$\mathbf{a_{12}} = \left(\frac{\varpi_L \mathbf{r_2}}{\mathbf{2}}\cos\theta + \frac{\varpi_R \mathbf{r_2}}{\mathbf{2}}\cos\theta\right) \quad \text{(38)}$$

$$\mathbf{a_{13}} = \frac{\mathbf{r_2}}{\mathbf{2}}\sin\theta \quad \text{(39)}$$

$$\mathbf{a_{14}} = \frac{\mathbf{r_2}}{\mathbf{2}}\sin\theta \quad \text{(40)}$$

$$\mathbf{a_{23}} = \left(\frac{-\mathbf{r_2}}{\mathbf{d_2}} + \frac{\mathbf{r_1^2}}{\mathbf{d_1}\ell}[\mathrm{sgn}(\alpha)\sin(\alpha)]\right) \quad \text{(41)}$$

$$\mathbf{a_{24}} = \left(\frac{\mathbf{r_2}}{\mathbf{d_2}} + \frac{\mathbf{r_1^2}}{\mathbf{d_1}\ell}[\mathrm{sgn}(\alpha)\sin(\alpha)]\right) \quad \text{(42)}$$

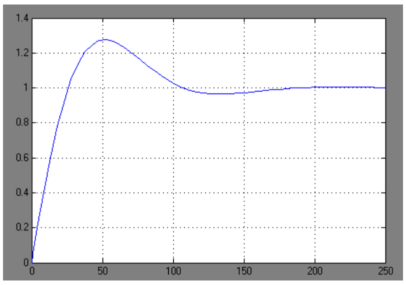
$$\mathbf{a_{25}} = \left(\frac{\varpi_L \mathbf{r_1^2}}{\mathbf{d_1}\ell}[\mathrm{sgn}(\alpha)\cos(\alpha)] + \frac{\varpi_R \mathbf{r_1^2}}{\mathbf{d_1}\ell}[\mathrm{sgn}(\alpha)\cos(\alpha)]\right) \quad \text{(43)}$$

4. Stability Analysis

The system stability is studied in to two phases: first for the angular velocities ϖ only ($\alpha = 0$), and the second for the coupled of ϖ in addition with the corrected steering angle α as illustrated in **Fig.(3)**. The nominal values that are used for stability analysis:

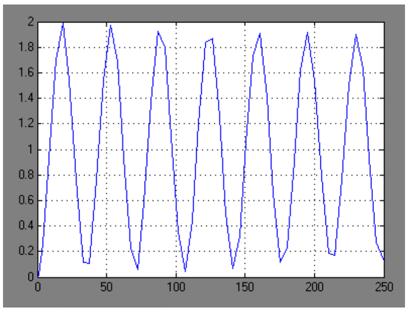
- Robot parameters: $r_1 = 0.04m$, $r_2 = 0.05m$, $d_1 = 0.13m$, $d_2 = 0.17m$ and $\ell = 0.22m$.
- Angular velocities: $\varpi_L = \mathrm{var}iable \;\; rad/\sec$, $\varpi_R = \mathrm{var}iable \;\; rad/\sec$
- Moving conditions: $\theta = 0.174533 rad$.

$F^{X}_{\varpi+\alpha}$



a.1

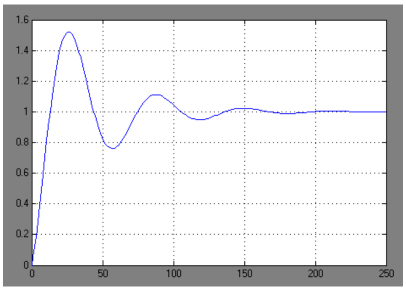
$F^{y}_{\varpi+\alpha}$



a.2

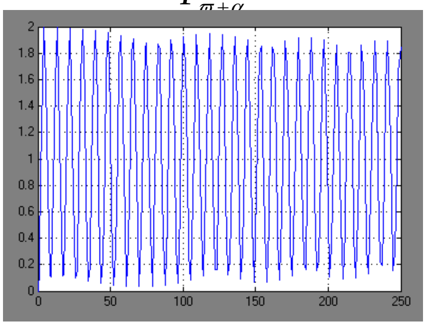
(a)

$F^{X}_{\varpi+\alpha}$



b.1

$F^{y}_{\varpi+\alpha}$



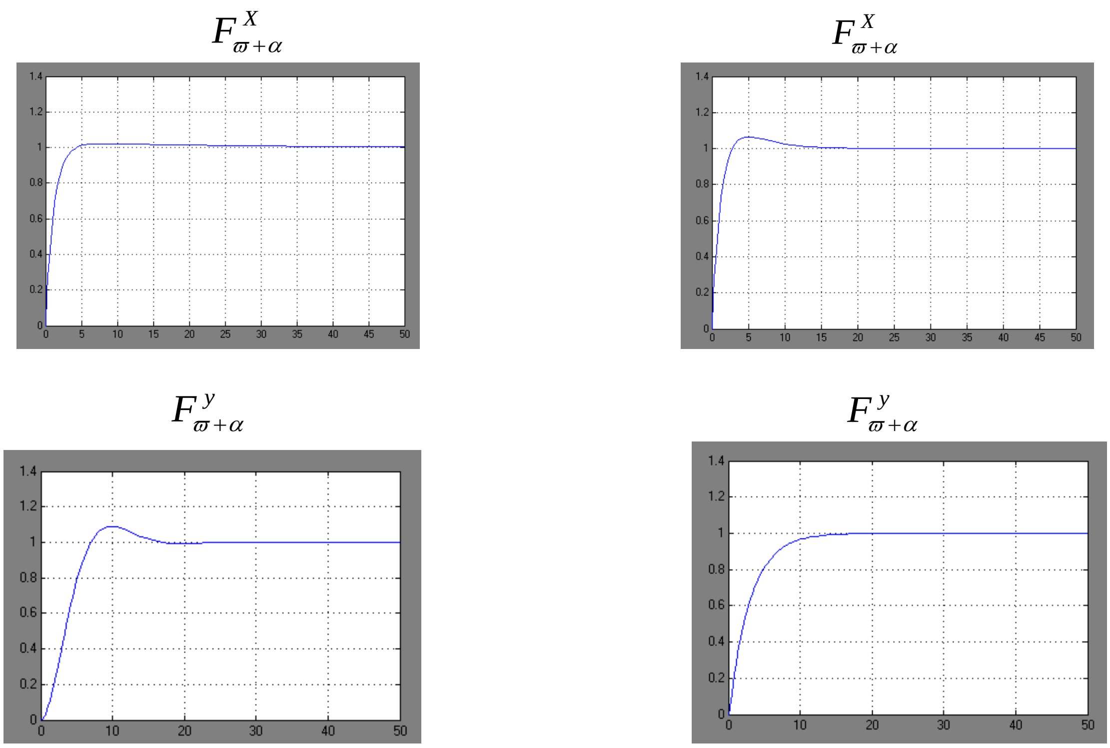
b.2

(b)

Figure (3) Stability responses a) when $\alpha = 0$ b) when $\alpha = 1.22 rad$

From **Figs.(3.a.1)** and **(3.b.1)** it can seen that the system in the X-directional channel is under damping, and its very slow convergence to the unity step. Critical damping appears in Y-directional channel; see **Figs.(3.a.2)** and **(3.b.2)**.

The WMR model as a result of stability needs controllers to compensate and improve its performance and to make it faster. Proposed conventional PID controllers are added to get a good steering and accurate conformer trajectory. The controller gains proposed for the X-directional channel are: $k_P = 30, k_i = 0.005$ and $k_D = 13$, and for the Y-directional channel are: $k_P = 5, k_i = 0.0002$ and $k_D = 15$. The resulting performance is depicted in **Fig.(4)**.

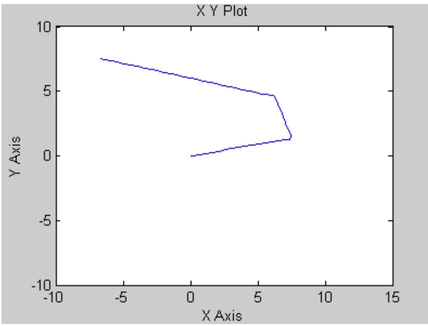


(a) **(b)**

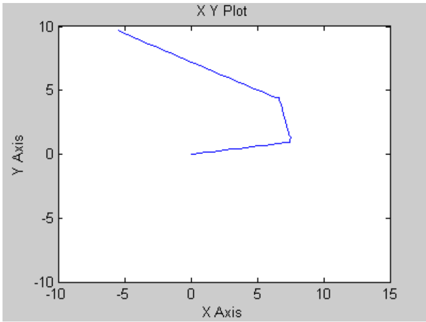
Figure (4) Stability responses a) when $\alpha = 0$ b) when $\alpha = 1.22 rad$

5. Simulation

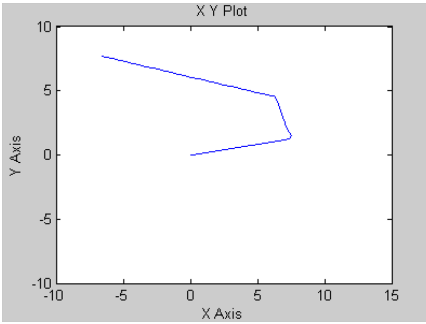
In this section, the proposed controllers in closed-loop with the WMR channels are simulated with Matlab7-Simulink environment. To test controllers' ability the WMR follow a path. It can be seen from **Fig.(5-c)** that the robot is following the paths and achieving a good accuracy. Mention here that the steering commands error was indicated by vision sensor and that is not the paper subject.



(a)



(b)



(c)

Figure (5) WMR following paths a) actual path b) following without controller c) following with controller

6. Conclusion

In this paper, a new kinematics model for wheeled mobile robot are developed, and derived. The kinematics model is derived under a nonholonomic constraint of pure rolling and nonslipping. A dynamic model with input saturations is considered. Linearization done and stability analysis tested and compensated by proposed PID controller to improving the system performance. Simulation and experimental results demonstrate the powerful of this develop, where the WMR is moved in the direction along the axis exactly like the actual path, and this demonstrate the important of adding the front steering wheels (groups 1) in addition with differential wheels (groups 2). The most important contribution of the paper is to use a front wheels as steering in addition with differential drive wheels supported by coaxial wheels.

7. References

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