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High performance technique FIR Filter Using Least Squares Method

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Abstract

The FIR filter is the one from the most DSP components used. There are two main approaches to design of FIR, first using the windows that have simple mathematics calculations but its results are not optimal. The second method that uses Least Squares error approach to get an optimal design but it required to large number of calculations.

A new algorithm for weighted least squares linear-phase FIR filter design has been presented, this algorithm used to solve quadratic programming problems. This paper shows how to rearrange this algorithm to reduce the number of flouting point operations (FPOs), that will results in a faster method for constrained least squares FIR filter design. The proposed algorithm is fast, stable and suitable for high order filter design.

الخـلاصــــــة

يعد مرشح النبضة المنتهية (FIR) من أهم وحدات بناء دوائر معالجة الإشارة الرقمية. هناك إسلوبان أساسيان لبناء مرشح (FIR)، الأول يستخدم إسلوب النوافذ والذي يكون مبسط رياضيا إلا انه لايعطي أفضل النتائج. أما الإسلوب الثاني الذي يعتمد على تقليل مربع الخطأ للحصول على أفضل النتائج إلا انه معقد رياضيا. تعتمد الخوارزمية الجديدة على إستخدام تقليل مربعات الأوزان لمرشح (FIR) الخطي الطور، وهذه الخوارزمية تعتمد أسلوب حل الرباعيات برمجيا (quadratic programming).

يقوم هذا البحث بإعادة ترتيب هذه الخوارزمية ليقلل عدة عمليات الضرب الحر (FPO) ، وهذا سيعطي طريقة أسرع من الإسلوب الثاني المستخدم. وهذه الخوارزمية ستكون أسرع ومستقرة ومناسبة للمرشحات الكبيرة.

1. Introduction

The constrained least squares (CLS) design of FIR filters was presented in [1] for the first time. Subsequently, it has also been considered in a number of references [2,3,4]. However, the algorithms proposed in those papers are neither guaranteed to converge nor can be used in every case. On the other hand the Goldfarb-Idnani (GI) algorithm, which was presented in [5] (see also [6]) is one of the best algorithms available to solve positive definite quadratic programming problems subject to linear constraints.

This algorithm is very efficient, it is guaranteed to find a solution in a finite number of steps and it can be implemented in a numerically stable way. It has been successfully applied to the CLS design of FIR filters in [6].

The rest of this paper is organized as follows: in section II, the original algorithm is developed, section III, the proposed algorithm is developed. Section IV present some filter design examples that show the applicability of the proposed algorithm. In section V, the final remarks are presented.

2. Algorithm formulation [5]

When designing linear phase FIR filters using the least squares method will minimize the mean square error given by

$$\varepsilon = \sum_{i=1}^{L-1} W(w_i)\left|F(e^{jw_t}) - D(e^{jw_t})\right|^2 \quad \text{.......... (1)}$$

where:

$F(e^{jw})$: is the frequency response of the FIR filter and,
$D(e^{jw})$: is the desired frequency response.

Let us define an error function given by:

$$E(w) = F(e^{jw_t}) - D(e^{jw_t}) \quad \text{.......... (2)}$$

This equation can be written in vector form as:

$$e = Ax - d \quad \text{.......... (3)}$$

where:

x: is the vector whose elements are the filter coefficients, and

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & \cdots & 2 \\ 1 & 2\cos(w_1) & \cdots & 2(\cos(nw_1) \\ \vdots & \vdots & \ddots & \cdots \\ 1 & 2\cos(wl) & \cdots & 2\cos(nwl) \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots (4)$$

$$\mathbf{d} = \begin{bmatrix} D(e^{jw0}) \\ D(e^{jw1}) \\ \vdots \\ D(e^{jwl}) \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots (5)$$

where:

n: is the Filter order coefficients, and l is the frequency grid.

Minimizing $\varepsilon = e^T$ will equivalent to minimizing the following function

$$\mathbf{f(x) = a^T x + \frac{1}{2} x^T G x} \quad \ldots\ldots\ldots\ldots\ldots (6)$$

where:

$$\mathbf{G = A^T\ W\ A} \quad \ldots\ldots\ldots\ldots\ldots (7)$$

and,

$$\mathbf{a = A^T\ W\ d} \quad \ldots\ldots\ldots\ldots\ldots (8)$$

In CLS filter design, the minimization of the above cost function (6) is subject to the following constraints:

$$\mathbf{|Ax - d| \le \delta} \quad \ldots\ldots\ldots\ldots\ldots (9)$$

where:

δ: is a column vector whose elements are all δ > 0.

3. Fast Algorithm Formulation

The most disadvantages for CLS linear phase FIR filter design algorithm is the large number of floating point operations. This paper analyzes the components of the FPOs then rearranges this algorithm to reduce the number of FPOs and in result design a fast algorithm for CLS linear phase FIR filter design. It combines the powerful GI algorithm with a fast method used to compute orthogonal cosine basis.

The result of this combination is a fast algorithm which also has the desirable property of numerical stability and that can be applied to high order filter design.

The key to reduce the numbers of FPOs is depending on the god selection of the initial value of the algorithm. However, the eqs.(7) and (8) have three matrix variables G, W and with two equations, that's mean there is a one free selection variable.

The god selection will give a fast algorithm. However, the selection of matrix a in equation 8 will capture the value of desired matrix d and matrix W while give a small reduction ratio in FPOs, therefore the available choice is a selection of matrix G or matrix W in eq.(7).

The deep analysis for algorithm equations shows the computation of G is at the core of the GI algorithm is a part of matrix computations [5] that have to be performed at every step of that algorithm. It is mentioned that efficient computational methods can be devised to reduce the computational burden.

There is infinity selection choices for G matrix, the triangular G matrix is a suitable choice but not efficient. The diagonalization of matrix G will reduce all the computational effort associated with G without any effect on the algorithm efficiency.

Because G has been replaced with the identity matrix. That is leads to the simplified eq.(6) to present as:

$$\mathbf{f(x) = a^T x + \frac{1}{2} x^T x} \qquad \textbf{(10)}$$

The solution of eq.(7) for G=I will leads to W=I and in result the eq.(8) will presented as:

$$\mathbf{a = A^T d} \qquad \textbf{(11)}$$

Now, the number of FPOs will reduce to 50% ideally, or to about 50-60% practically as shown in the next section.

4. Examples

In order to demonstrate the applicability of the proposed algorithm, a couple of filters have been designed using a direct implementation of the GI algorithm (CLS algorithm) and the algorithm developed in this paper. The implementation of the GI algorithm follows the one presented in [5].

In implementing the CLS algorithm, the symmetric structure of G has been taken into account in order to reduce the number of operations.

Example (1): The first example is design a linear phase lowpass filter of order 50 according to the following specifications:

$$|D(e^{jw})| = \begin{cases} 1 & \text{if } 0 \le w \le 0.45\pi \\ 0 & \text{if } 0.45\pi \le w \le \pi \end{cases} \qquad (12)$$

The frequency response of this filter is depicted in **Fig.(1)**. The value of δ, the number of frequency points and the number of floating point operations required to get the coefficients are shown in **Table (1)**. This table shows results of different filters differ in order, frequency grid and δ. This table shows results of different filters differ in order, frequency grid and δ**.** This table shows the experimental number of FPOs for the original and modified algorithms with the experimental percentage ratio of them.

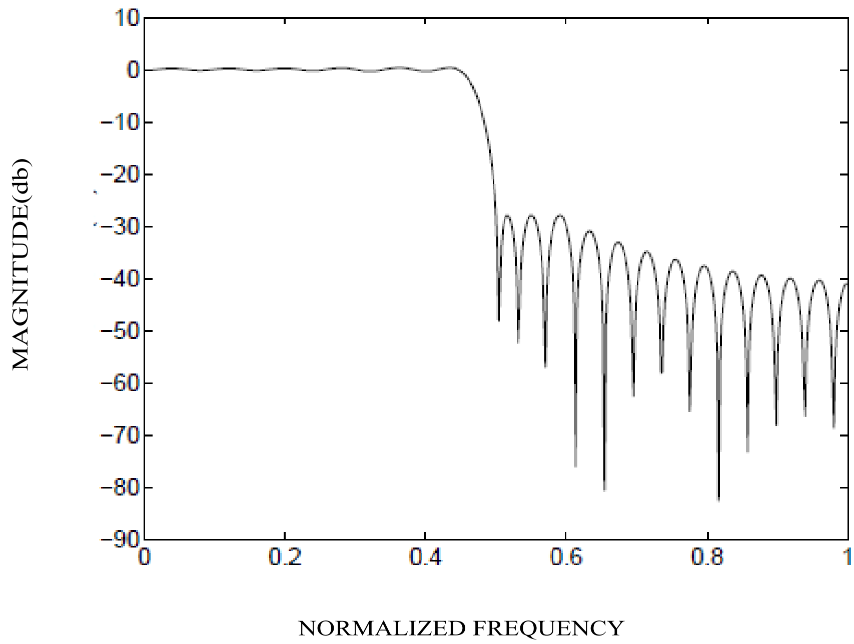

Figure (1) Frequency response for example (1)

Table (1) Number of floating point operations for filters of example (1)

Freq. grid	δ	Filter order	Number of FPOs for CLS algorithm	Number of FPOs for proposed CLS algorithm	Experiment Ratio of FOPs %
1000	0.04	50	2,675100	1,672156	62.5
2000	0.004	100	35,266965	19,995324	56.7
3000	0.00046	150	58,063193	29,332337	50.5

Example 2: The second example is design a linear-phase bandpass filter of order 50 according to the following specifications:

$$\left|D(e^{jw})\right| = \begin{cases} 0 & \text{if } 0 \le w \le 0.45\pi \\ 1 & \text{if } 0.5\pi \le w \le 0.7\pi \\ 0 & \text{if } 0.75\pi \le w \le \pi \end{cases} \quad \text{.....................................} \quad (13)$$

The frequency response of this filter is depicted in **Fig.(2)**. The value of δ, the number of frequency points and the number of floating point operations required to get the coefficients is shown in **Table (2)**. This table shows results of different filters differ in order, frequency grid and δ**.** This table shows the experimental number of FPOs for the original and modified algorithms with the experimental percentage ratio of them.

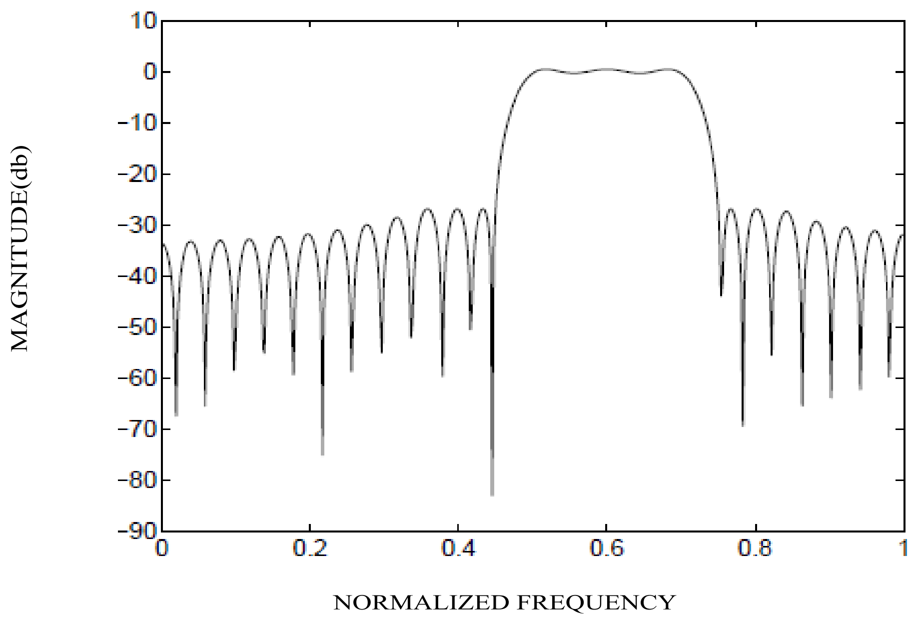

Figure (2) Frequency response for example (2)

Table (2) Number of floating point operations for filters of example (2)

Freq. grid	δ	Filter order	Number of FPOs for CLS algorithm	Number of FPOs for proposed CLS algorithm	Experiment Ratio of FOPs %
1000	0.045	50	4752006	3013473	63.4
2000	0.0046	100	26208253	14805181	56.5
3000	0.00049	150	151218001	82116668	54.3

5. Conclusions

This paper presented a fast and numerically stable algorithm that takes the advantage of a powerful for constrained least squares filter design, this method has an optimal design but it required to large number of calculations.

This paper rearranged this algorithm to solve quadratic programming problems. This rearranges results to reduce the number of FPOs to 50%. However, it gives a faster method for constrained least squares FIR filter design. The proposed algorithm in result is a fast, stable and suitable for high order filter design.

6. References

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