Journal of Engineering and Development, Vol. 12, No. 4, December (2008) **ISSN 1813-7822**

The Performance of Wavelet Packet Division Multiplexing in Impulsive and Gaussian Noise

Asst. Lect. Fadel S. Hassen
Electrical Eng. Department, College of Eng.
Al-Mustansiriya University, Baghdad, Iraq

Asst. Lect. Mohanad E. Okab AL-Madi
Electrical Eng. Department, College of Eng.
Al-Mustansiriya University, Baghdad, Iraq

Abstract

The aim of this paper is to investigate the effect of impulsive noise on the performance of the WPDM (Wavelet Packet Division Multiplexing) system. The employed model of impulsive noise consists of Bernoulli distributed impulsive arrivals and Gaussian distributed amplitudes of the impulses. The effect of changing impulsivity of noise and different relative power of impulsive noise are investigated and are compared with Additive White Gaussian Noise (AWGN) case. Also, the performance of a WPDM is compared with single carrier system.

The results show that wavelet transform is sensitive to impulsive noise and increase number of subcarriers effect badly on the performance system. Two regions are founded comparing with single carrier system, the first region in which the performance of single carries system is better than WPDM system when SNR smaller than 15 dB. The second region in which the performance of WPDM system is better than single carrier when SNR higher than 15 dB.

الخلاصة

الهدف من هذا البحث هو دراسة تأثير الضوضاء المندفعة على منظومة (WPDM). النموذج المستخدم للضوضاء المندفعة يشمل توزيع برنولي للوصول الاندفاعي وتوزيع (Gaussian) للقيم الاندفاعية. تم تناول تأثير تغيير اندفاعية (impulsivity) الضوضاء والقدرة النسبية للضوضاء المندفعة ومقارنة النتائج مع (AWGN). كذلك تم مقارنة نتائج (WPDM) مع نظام أحادي الحامل (single carrier).

النتائج أثبتت تحسس محولة المويجة إلى الضوضاء المندفعة كما إن زيادة عدد الحاملات يؤثر بشكل سيئ على أداء المنظومة. منطقتان يمكن إيجادها مقارنة مع نظام أحادي الحامل، المنطقة الأولى يكون أداء أحادي الحامل أفضل من نظام (WPDM) عندما تكون SNR اقل من 15dB المنطقة الثانية يكون أداء WPDM أفضل من أحادي الحامل عندما تكون SNR اكبر من 15dB.

1. Introduction

Wavelet packet division multiplexing is a promising new technique which provides an alternative to well known scheme such as time division multiplexing (TDM) and frequency division multiplexing (FDM). A key to all multiplexing methods is that various message signals share a common channel without creating unmanageable interference at the receiver end. However, the recently developed wavelet packet decomposition generates a set of self and mutually orthogonally waveforms which could also be used for (synchronous) orthogonal multiplexing [1]. Whilst all synchronous orthogonal multiplexing schemes perform identically in additive white Gaussian noise (AWGN), they may perform differently in impulsive noise [2]. Impulsive noise is a primary source of performance degradation in severs applications, including data transmission over telephone networks, and its effects on various digital communication schemes have received considerable attention; e.g. [3-5].

The orthogonality of wavelet transform is not limited to either the frequency or the time domain, one can have a dual situation in which mutual orthogonal subspace and self-orthogonality is achieved by translation in time. Wavelets provide this kind of orthogonality by allowing orthogonal message waveforms to overlap in time and frequency domain [6].

In this paper a Wavelet Packet Division Multiplexing (WPDM) is studied over AWGN and impulsive noise channel.

2. Impulsive Noise Model

The model discussed in the following is a Bernoulli-Gaussian (BG) model of an Impulsive Noise (IN) process. The random time of occurrence of the impulsive is modelled by a Bernoulli process b(k), where k is the time point and b(k) is a binary-valued process that takes a value of “1” with a probability of α and a value of “0” with probability of $(1-\alpha)$. The amplitude of the impulsive is modelled by a Gaussian process g(k) with mean zero and variance σ^2. Each impulsive is shaped by a filter with the impulsive response h(k). The Bernoulli-Gaussian model of impulsive noise is illustrated in **Fig.(1)**. The IN can be expressed as [5]:

$$\mathbf{n(k)=\sum_{i=0}^{P-1} h(i)g(k-i)b(k-i)} \quad \text{...} \quad \mathbf{(1)}$$

where:

P: is the length of the impulsive response of the impulsive shaping filter.

In a Bernoulli-Gaussian model the probability density function (pdf) of impulsive noise n(k) is given by [1]:

$$\mathbf{pdf_N^{BG}(n(k))=(1-\alpha)\delta(n(k))+\alpha pdf_N(n(k))} \quad \text{...} \quad \mathbf{(2)}$$

where:

$\delta(n(k))$*: is the Kronecker delta function and*

$$\mathrm{pdf}_N(n(k)) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{n(k)}{\sigma}\right)^2} \quad \text{.........} (3)$$

is the probability density function of a zero Gaussian process.

The value of α is a measure of impulsivity of the impulsive noise. By decreasing α the noise becomes more impulsive (α<1). The real world there is not the impulsive noise only but a mixture of impulsive noise and AWGN. In the simulation both IN and AWGN are considered. In this regard we also define a parameter that controls the power ratio of the AWGN part and the “impulsive” part of the total noise as [5]:

$$\gamma = \frac{\mathrm{power\ (impulsive\_component)}}{\mathrm{power\ (AWGN\_component)}} \quad \text{.........} (4)$$

with the definition of γ the noise impinging the system consists of IN and AWGN with a manageable ratio of power.

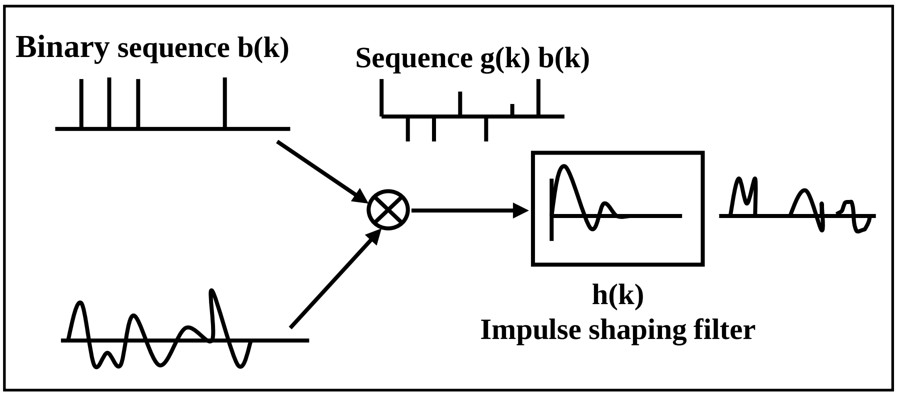

Figure (1) Impulsive noise model

3. Wavelet Packet Division Multiplexing (WPDM) System

Recently, a new class of multicarrier system based on wavelet transform has been proposed [7] and for simplicity, called wavelet Packet Division Multiplexing in this paper, (WPDM), stands for Wavelet Packet Division Multiplexing, shares all the benefit of Multicarrier (MC) technique and exhibits further benefit such as higher efficiency due to elimination of guard interval (GI). It is considering on wavelet transform which is well localized both in time and frequency domain, while sinusoid waveform, are only localized in frequency but not in time domain. Thus, time domain diversity of sinusoid waveform within one symbol period is difficult to achieve. Therefore, a guard interval (GI) is needed to eliminate residual Inter Symbol Interference (ISI). This addition of GI signal, then introduces overhead that decrease bandwidth efficiency.

The general form of an L-level WPT is written as follows [8]:

$$\mathbf{s(k)=\sum_{(\ell,m)}\sum_{n} f_{\ell m}(k-2^{\ell}n)\, a_{\ell m}(n)} \quad \text{.......} \quad \mathbf{(5)}$$

where:

$a_{\ell m}$ *(k)*= ±*1 is the binary messages,*

$f_{\ell m}(k)$ *the equivalent filter from the* (ℓ, m) *th terminal to the root of the tree.*

The original messages can be recovered from s(k) using [9]:

$$\mathbf{a_{\ell m}(k)=\sum_{k} f_{\ell m}(k-2^{\ell}n)s(k)} \quad \text{.......} \quad \mathbf{(6)}$$

In order to work directly with the wavelet transform coefficients, the relationship between the detailed coefficients at a given level in terms of those at previous level is used. In general, the discrete signal assumes the highest achievable approximation sequence, referred to as 0-th level scaling coefficients. The approximation and detail sequences at level j are given by [10]:

$$\mathbf{c_{j+1}(k)=\sum_{m} h_{o}(m-2k)c_{j}(m)} \quad \text{.......} \quad \mathbf{(7)}$$

and,

$$\mathbf{d_{j+1}(k)=\sum_{m} h_{i}(m-2k)c_{j}(m)} \quad \text{.......} \quad \mathbf{(8)}$$

Eqs.(7) and (8) state that approximation sequence at higher scale (lower level index), with the wavelet and scaling filters, $h_o(t)$ and $h_i(t)$ respectively, can be used to calculate the detail and approximation sequences at lower scales.

The scaling coefficients are related to wavelet coefficients by [10]:

$$\mathbf{h_{i}(m)=(-1)^{m} h_{o}(M-m)} \quad \text{.......} \quad \mathbf{(9)}$$

where,

M: is a finite odd length of quadrature mirror filter.

Figure (2) illustrates a generic block diagram of WPDM system, while the details are then shown in **Figs.(3)** and **(4)**. WPDM system employs two filter banks i.e. Inverse Wavelet Packet Transform (IWPT) placed at the transmitter side and Wavelet Packet Transform (WPT) placed at the receiver's side.

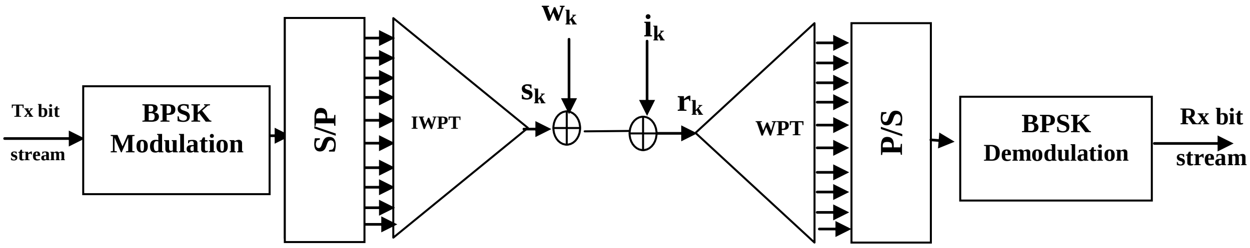

Figure (2) Block diagram of WPDM system

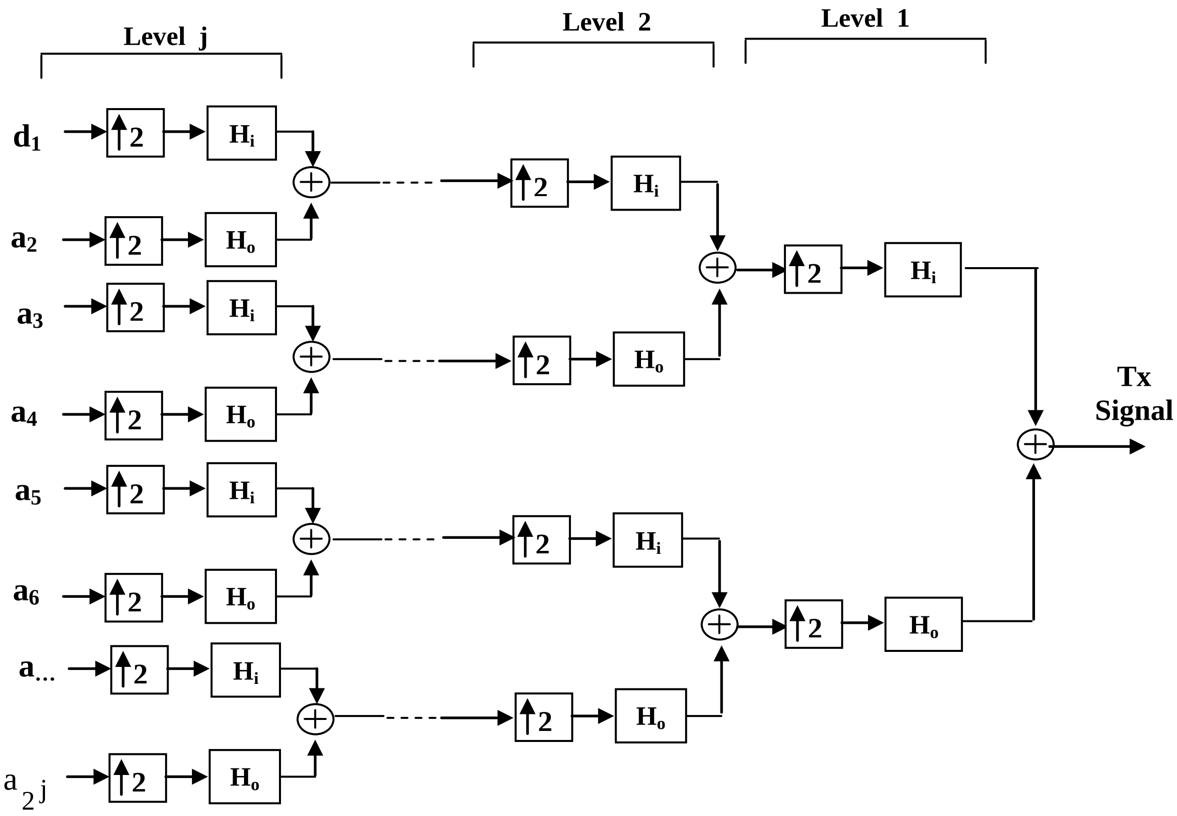

Figure (3) Inverse wavelet packet transform at the transmitter side

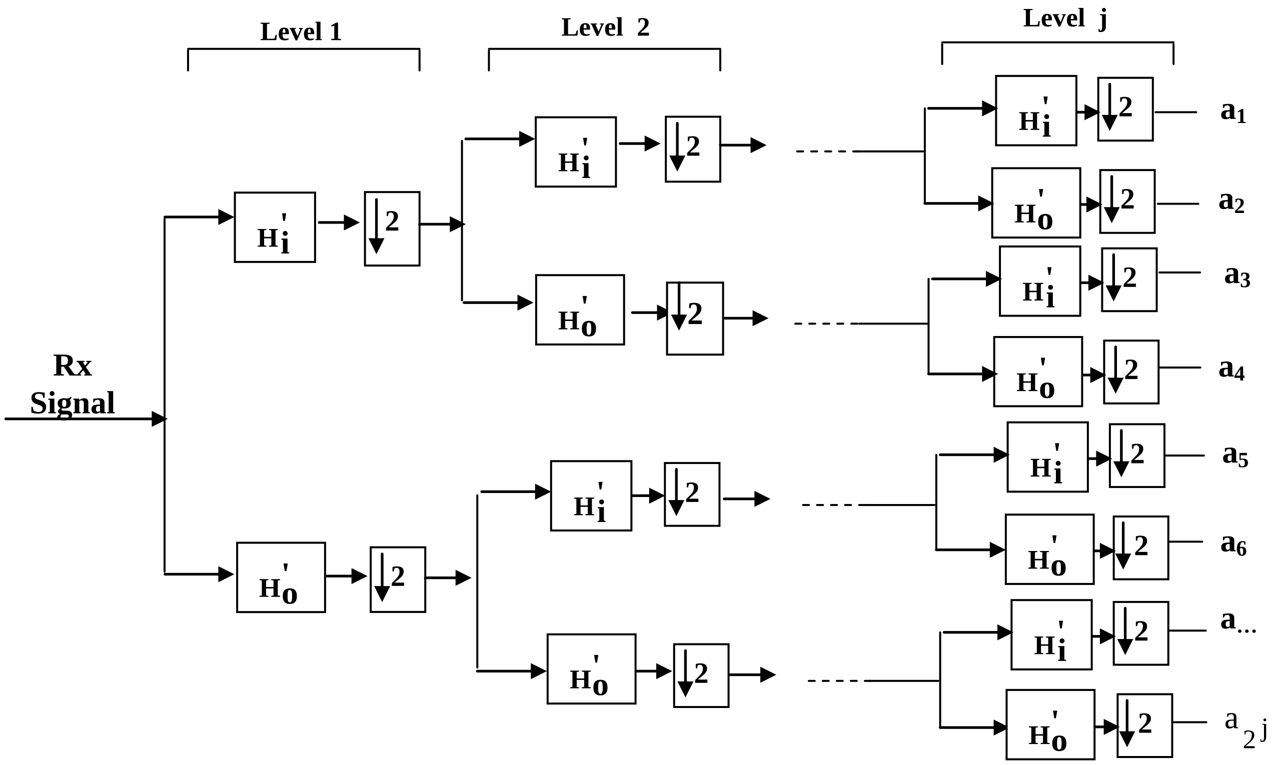

Figure (4) Wavelet packet transform (WPT) at the receiver side

As shown in **Fig.(2)**, modulated data in BPSK are imputed to the Inverse Wavelet Packet Transform (IWPT) to generate the multicarrier signal, and then transmitted over the transmission medium. The received signal corrupted by additive white Gaussian noise (AWGN) and impulsive noise is:

$$\mathbf{r_k = s_k + w_k + i_k}, \quad k=0,1,\ldots,N-1 \text{ .... } \mathbf{(10)}$$

where:

w_k: is the additive white Gaussian noise (AWGN), and

i_k: is the impulsive noise (s_k, w_k, i_k are assumed to be mutually independent) and

N: is the number of subcarriers.

The received signal is received by Wavelet Packet Transform (WPT), demodulated and converted back to serial-sequence for recovering the transmitted data.

Figures (3) and **(4)** show the detail structure of a transmultiplexer with the number of input and output are a power of two (2^j) where j is the number of stages. Wavelet filters H_i is high pass filter (HPF) and H_o as low pass filter (LPF) at the transmitter while H'_i and H'_o are as HPF and LPF respectively at receiver. All filters should be designed so that the occurred aliasing are exactly cancelled out. This condition leads to the construction of perfectly reconstruction (PR) filter bank, which is called quadrature mirror filter (QMF). Considering the carrier allocation which is divided into 2^j subcarriers, the symbol a_1 is the symbol with the highest frequency, while symbol a_{2^j} is the symbol with the lowest frequency among all of the transmultiplexer subbands. Upsampling is performed at the transmitter before entering filter of its stage, and downsampling is held at the receiver after each data passed a filter of its stage.

4. Simulation Results

Simulations of the WPDM system with various impulsivities (α) and power of impulsive noise with respect to AWGN (γ) are considered. Typical values for α are 0.1, 0.01, and 0.001 and that for γ are 1, 5 and 10. The impulse response of the filter h (t) is 9th order low pass filter with cutoff frequency 5MHz (these values of h(t) are taken from [5] for practical values of impulsive noise). In this simulation the WPDM signal is transmitted in AWGN and impulsive noise channel with number of transmission bits is about 20000 bits.

Figures (5) and **(6)** show the influence of α on the BER performance of a WPDM system with subcarrier numbers (N=128) and γ=1 and 5 respectively. From these figures it can be noticed that, any increase of α will increase BER for given γ. When α increases more and more the distribution of impulsive noise becomes close to Gaussian noise until the two distribution become exactly the same for α =1.

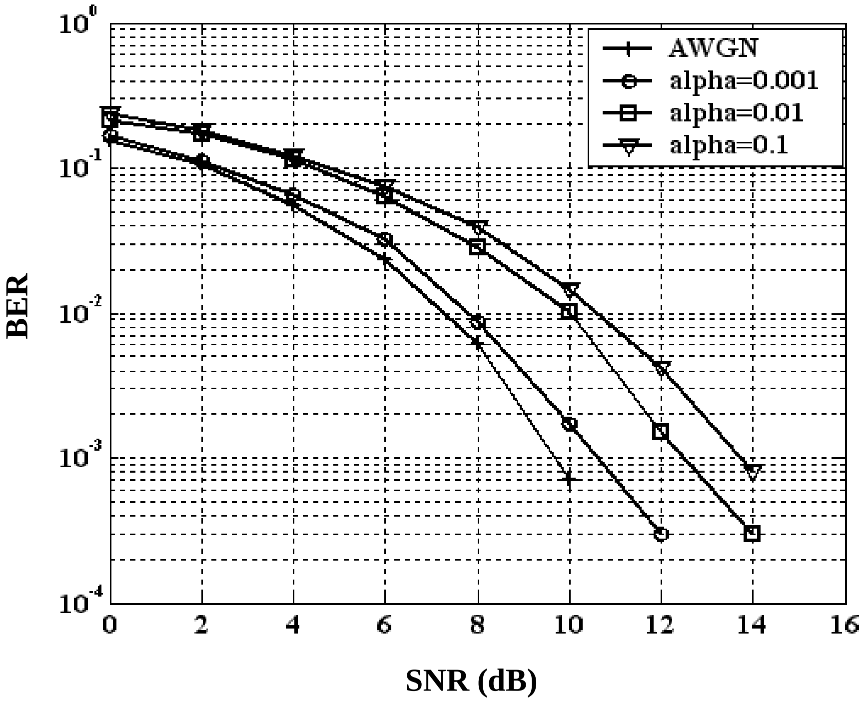

Figure (5) The influence of α on BER performance of a WPDM system with γ =1

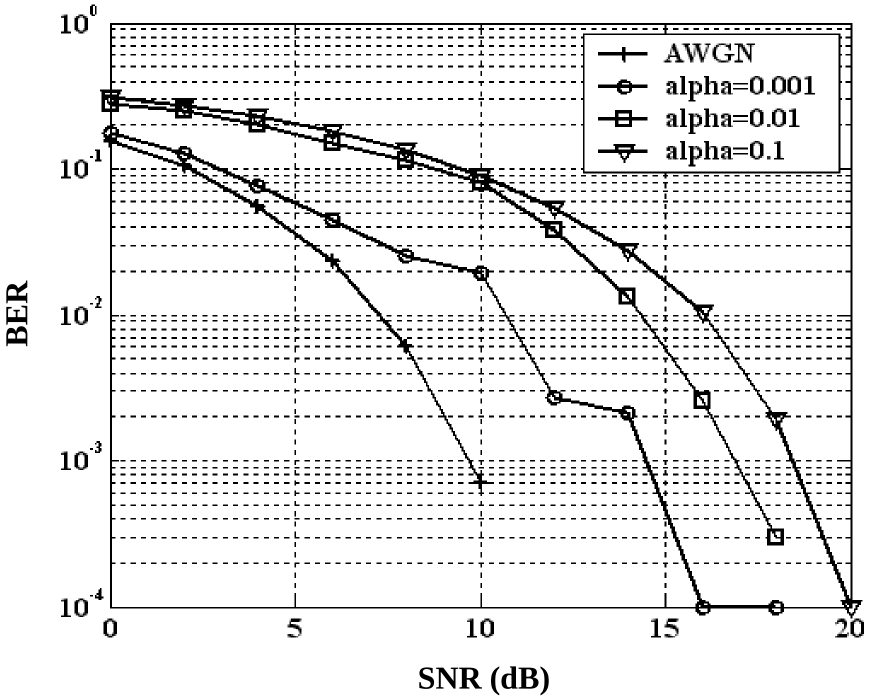

Figure (6) The influence of α on BER performance of a WPDM system with γ =5

Figure (7) shows the effect of γ on the BER performance of a WPDM system with α =0.01 and subcarrier numbers (N=128). It is seen that as γ increases the performance is degraded.

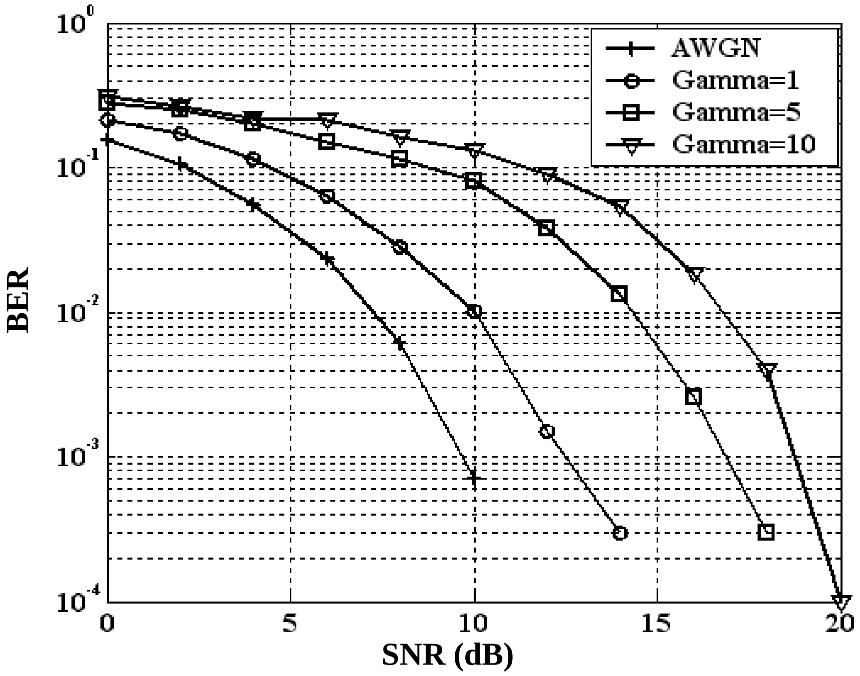

Figure (7) The influence of γ on BER performance of a WPDM system with α =0.01

Figures (8) and **(9)** show the effect of impulsive noise on the performance of a WPDM system with different number of subcarriers (N) with α=0.01 and γ=1 and 5 respectively. It can be seen that increase number of N will effect badly on BER.

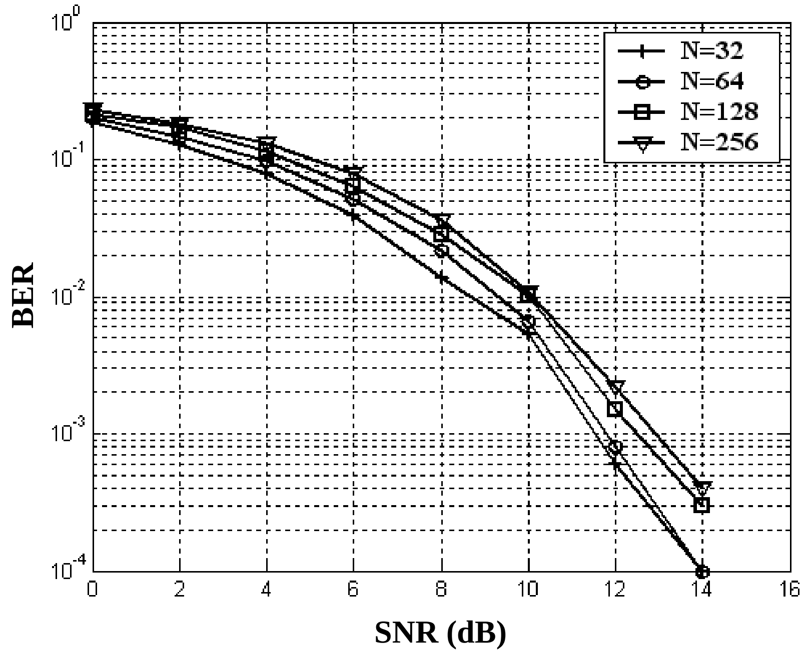

Figure (8) The influence of number of subcarrier (N) on BER performance of a WPDM system for α=0.01 γ=1

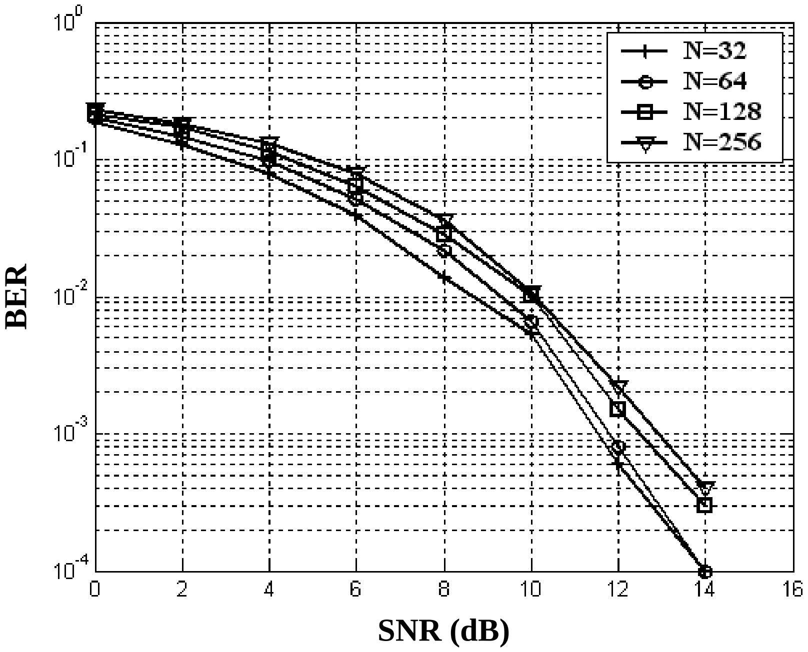

Figure (9) The influence of number of subcarriers (N) on BER performance of a WPDM system for α=0.01 and γ=5

Figure (10) shows the influence of α on the BER performance of a single carrier system with γ =5.

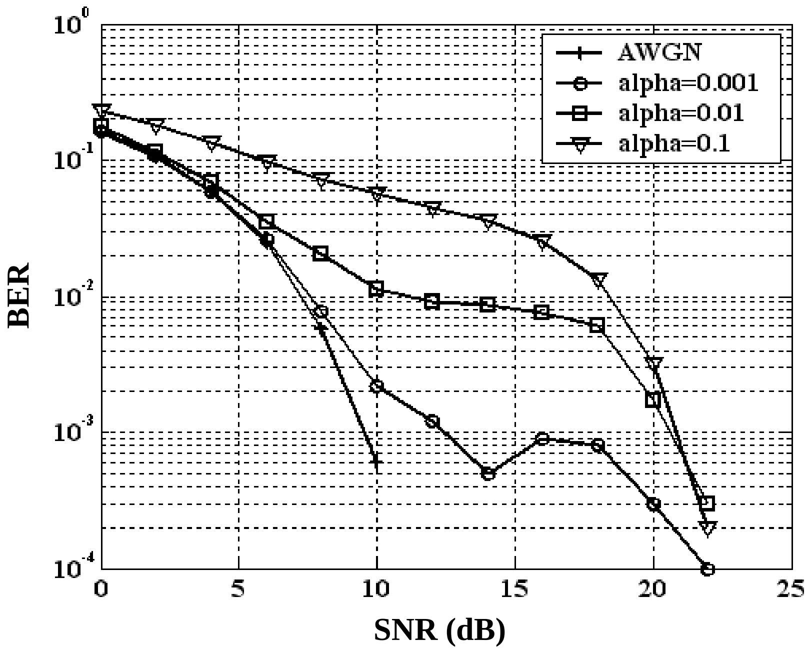

Figure (10) The influence of α on BER performance of a WPDM system with γ =5

Figures (11), **(12)** and **(13)** show comparison results between WPDM and single carrier system for different values of α and γ. From these figures, two regions can be indicated, the first region in which the performance of single carries system is better than WPDM when SNR less than 15 dB. The second region in which the performance of WPDM is better than single carrier when SNR higher than 15 dB. This is because the WPDM waveform overlap in time, and hence the energy of an impulsive noise burst is dispersed over several bits at each terminal. At very low SNR, the performance of WPDM degrades with respect to that single carrier because a strong noise burst may induce more than one bit error in WPD.

The difference in BER performance between the single carrier system and a WPDM system is obvious. For example, given a fixed BER (say 10^{-4}) and an impulsiveness of α =0.01 and γ =5 there is a difference of about 6 dB in the SNR.

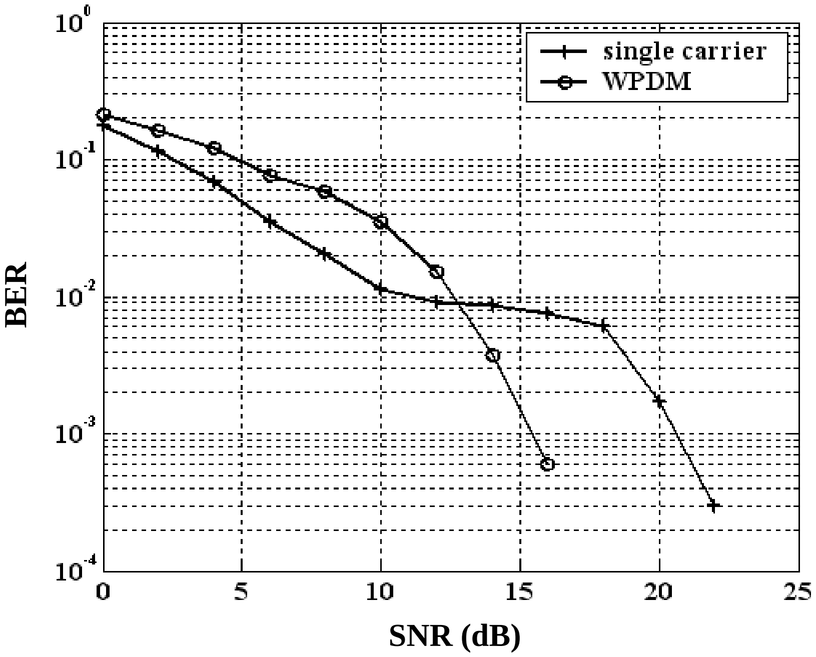

Figure (11) Comparison between single carrier and WPDM for α=0.01, γ=5 and N=32

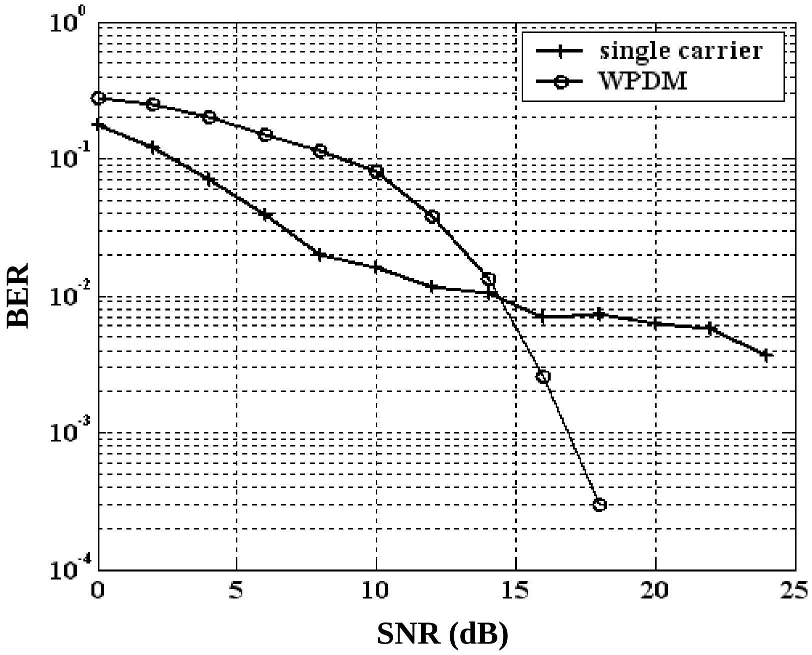

Figure (12) Comparison between single carrier and WPDM for α=0.01,γ=5 and N=128

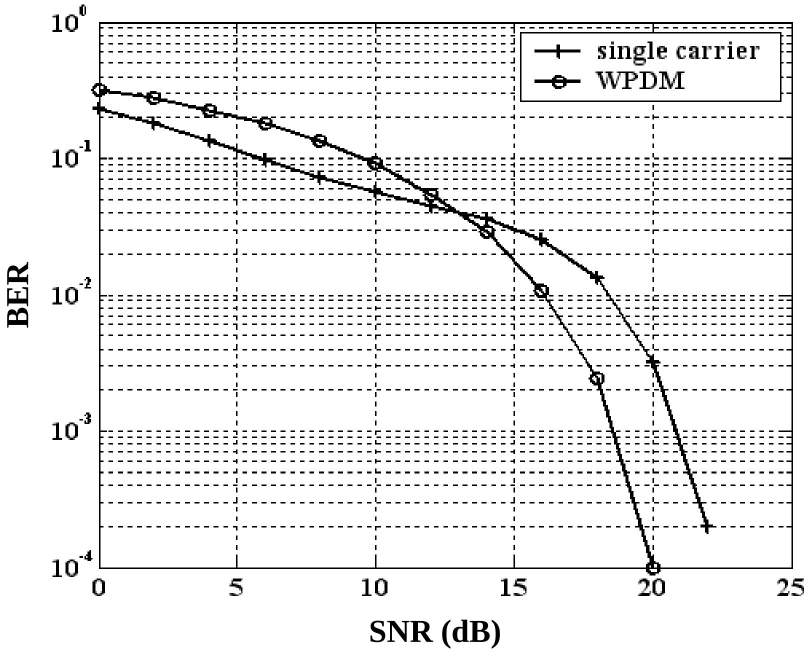

Figure (13)
Comparison between single carrier and WPDM for α=0.1, γ=5 and N=32

5. Conclusions

The important points were during simulation and discussions of the results are given below:

1. The performance of a WPDM system in the impulsive noisy environment depends on the impulsivity of the noise (α) and its power relative to the AWGN (γ).
2. Any increase of α will increase BER for given γ, also when γ increases the performance of system is degraded.
3. The wavelet transform is sensitive to impulsive noise and increase number of subcarriers effect badly on the performance system.
4. Two regions are founded comparing with single carrier system, the first region in which the performance of single carries system is better than WPDM system when SNR smaller than 15 dB. The second region in which the performance of WPDM system is better than single carrier when SNR higher than 15 dB.

6. References

1. G. W., Wornell, ***“Emerging Applications of Multirate Signal Processing and Wavelets in Digital Communications”***, Proc. IEEE, Vol. 84, Apr. 1996, pp. 586-603.

2. D., Middleton, ***"Statistical-Physical Models of Electromagnetic Interference"***, IEEE Trans. Electromag. Compat., Vol. EMC-19, Aug.1977, pp. 106-127.

3. P. A., Bello, and R., Espsito, ***"A New Method for Calculating Probabilities of Errors Due to Impulsive Noise"***, IEEE Trans. Commun. Tech., Vol. COM-17, June 1969, pp. 368-379.

4. S., Oshita, and K., Feher, ***"Performance of Coherent PSK and DPSK Systems in Impulsive and Gaussian Noise Environment"***, IEEE Transaction Commun., Vol. COM-30, Dec. 1982, pp. 2540-2546.

5. Homayoun Nikookar, and Danesh Nathoeni, ***"Performance Evaluation of OFDM Transmission over Impulsive Noise Channel"***, IEEE International Research Center for Telecommunications-Transmission and Radar, 2002.

6. S., Mallat, ***"A Wavelet Tour of Signal Processing"***, Academic Press., New York, 1998.

7. Khoirul Anwar, ***"Peak-to-Average Power Ratio Reduction of OFDM Signals Using Carrier Inter Interferometry Codes and Iterative Processing"***, M.Sc. Thesis, Nara Institute of Science and Technology, 2005.

8. Datul, R. V., ***"Orthogonal Wavelet Division Multiplexing (OWDM) for Broadband Wireless Communication"***, M.Sc. Thesis, South Florida University, August 1999.

9. K., Max Wong, Jiangfeng Wu, Timothy N. Davidson, Qu Jin, and P. C., Ching, ***"Performance of Wavelet Packet Division Multiplexing in Impulsive and Gaussian Noise"***, IEEE Transactions on Communications, Vol. 48, No. 7, July 2000.

10. Fadel S. Hassen, ***"Performance of Discrete Wavelet Transform (DWT) Based Speech Denoising in Impulsive and Gaussian Noise"***, Journal of Engineering and Development, Al-Mustansiriya University, Vol. 10, No. 2, June 2006.