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Slantlet based Polynomial Cancellation Coding OFDM

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Abstract

In this paper a new technique is proposed to improve the performance of OFDM. This technique uses the slantlet transform (SLT) instead of fast Fourier transform (FFT) to reduce the level of interference. This also will remove the need for guard interval (GI) in the case of the FFT-OFDM and therefore improves the bandwidth efficiency of the OFDM. The effect of PCC has been studied in both the traditional and proposed OFDM systems on frequency selective fading channel, flat fading channel and additive white gaussian noise (AWGN) channel.

Results obtained indicate that the performance of proposed SLT based PCC-OFDM is better than FFT based PCC-OFDM in flat fading channels (bit error rate reduces from 0.4343 to 0.0046 at SNR equals 10 dB). For frequency selective fading channel the results show that the performance of the proposed SLT based PCC-OFDM is better than the FFT based PCC-OFDM at low SNR values, while the performance is approximately similar in both systems at moderate and higher SNR values.

الخلاصة

في هذا البحث تم اقتراح طريقة جديدة لتحسين منظومة الـ (OFDM) بتقليل مستوى التداخل وإزالة الحاجة إلى استعمال حزمة الأمان (Guard Interval)، أي تحسين كفاءة النطاق (Bandwidth) في المنظومة. وقد تم انجاز ذلك باستبدال تحويل الفوريير (FFT) بتحويل الانحدار المائل (Slantlet) وباستعمال شفرة الإلغاء متعددة الحدود (PCC) و من جهة أخرى فقد تم دراسة تأثير الـ PCC في قناة الإرسال ذات تردد الإضعاف الانتقائي (Selective fading) و قناة الإرسال ذات تردد الإضعاف اللاانتقائي (Flat fading) وقناة الإرسال ذات الضوضاء المضافة جاوسية الشكل (AWGN).

من أهم النتائج التي تم الحصول عليها هو إن أداء نظام (SLT based PCC-OFDM) يكون أفضل من أداء نظام (FFT based PCC-OFDM) في قناة الوهن المستوي (نسبة معدل الخطأ (BER) تقل من ٠,٤٣٤٣ إلى ٠,٠٠٤٦ عند نسبة ضوضاء للإشارة (SNR) مساوية إلى ١٠ ديسبل). في حالة القناة الوهن الانتقائي للتردد تبين النتائج بأن أداء نظام (SLT based PCC-OFDM) يكون أفضل فقط في قيم الـ SNR المنخفضة في حين يبقى الأداء مماثل تقريباً في كلا النظامين عند القيم العالية والمتوسطة لـ SNR.

1. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a special case of multicarrier transmission, where a single data stream is transmitted over a number of lower-rate subcarriers (SCs). It is worth mention that OFDM can be seen as either a modulation technique or a multiplexing [1].

OFDM as a modulation technique used in many new digital data transmission systems such as digital video broadcasting (DVB), digital audio broadcasting (DAB) and wireless local area networks (WLANs). However OFDM suffers from high sensitivity to frequency errors and high peak-to-average power ratio (PAPR) [2]. OFDM transmitters therefore require vary linear outputs amplifiers with wide dynamic range. These are expensive and inefficient. Although the PAPR is very large of OFDM, high magnitude peaks occur relatively rarely and most of the transmitted power is concentrated in signals of low amplitude. Moreover OFDM has a relatively large out-of-band (OOB) spectrum [3]. Any amplifier non-linearity causes intermodulation products resulting in unwanted out-of-band (OOB) power [4]. Therefore, the OOB power should be minimized to avoid interference between adjacent broadcast channels [2].

In OFDM, the subcarriers are implemented using an N-point inverse discrete Fourier transform (DFT). The spectrum of each subcarrier decreases according to a sinc function. The sinc functions have sidelobs that are relatively large and do not decay quickly with frequency. As a result, the spectral rolloff of OFDM signals is slow. The OOB power is not low enough for many OFDM application systems. One of the simple and effective ways to reduce the OOB power is to use windowing. This eliminates the sharp transitions at symbol boundaries in the time domain signal and results in more rapid spectral rolloff. However windowing reduces the delay spread tolerance. It is also possible to use filtering techniques, however filtering techniques are more complex to implement than windowing ones and may distort the wanted signal. Polynomial cancellation coding (PCC) is a technique that makes OFDM much less sensitivity to frequency errors and phase noise, and more tolerant to multipath with large delay spreads. An OFDM system with PCC is called a polynomial cancellation coded OFDM (PCC-OFDM) [2].

Recently, Selesnick has constructed the new orthogonal discrete wavelet transform called the slantlet (SLT) wavelet, with two zero moments and with improved time localization [5]. This transform method has played an important role in signal and image processing applications. The slantlet (SLT) has been successfully applied in compression and denoising. It is also retains the basic characteristic of the usual filterbank such as octave band characteristic, a scale dilation factor of two and efficient implementation. However, the SLT is based on the principle of designing different filters for different scales unlike iterated filterbank approaches for the DWT [6].

This paper presents the design and implementation of a proposed SLT based PCC-OFDM. The usage of Slantlet transform instead FFT transform in conventional OFDM system would add new features like increasing the immunity of the system against interference and improving bandwidth efficiency. The performance of the proposed system would be compared with FFT based PCC-OFDM in both AWGN, flat and selective fading channels.

2. Traditional OFDM system (FFT-based OFDM)

The block diagram traditional OFDM system is depicted in **Fig.(1)**.

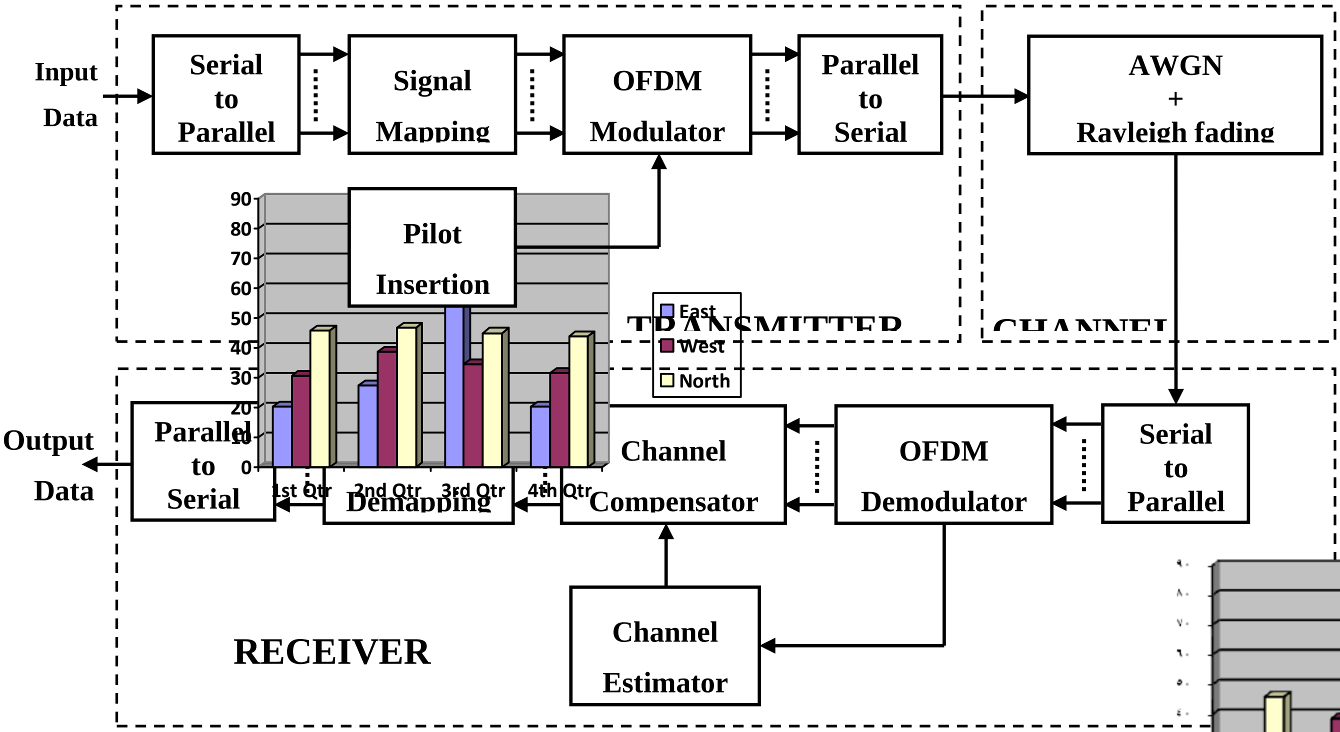
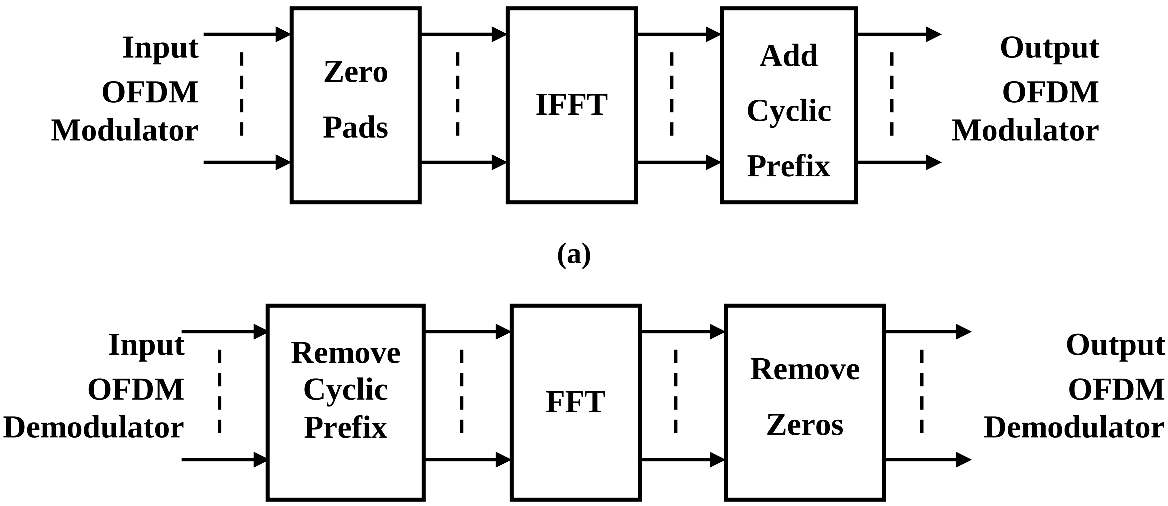


Figure (1) Block diagram of traditional OFDM system

The input serial data is formatted into the word size required for transmission, for example two bits/word for QPSK (or 4QAM) and four bits per word for 16QAM, then converted into parallel form. Then, the data is transmitted in parallel by assigning each word to one subcarrier. After that each word on each subcarrier is mapped by any constellation format available [as M-ary PSK, M-ary QAM, and differential-PSK (DPSK) and DQAM constellation]. The training frame (pilot subcarriers) will be inserted and sent prior to information frame. This pilot frame used to make channel estimation compensate the channel effects on the signal. After that, the complex word and pilot frame passed to IFFT to generate an OFDM symbol. Zeros inserted in some bins of the IFFT in order to make the transmitted spectrum compacts and reduce the adjacent carrier interference. The FFT based OFDM modulator and demodulator is shown in **Fig.(2)**.



(a)

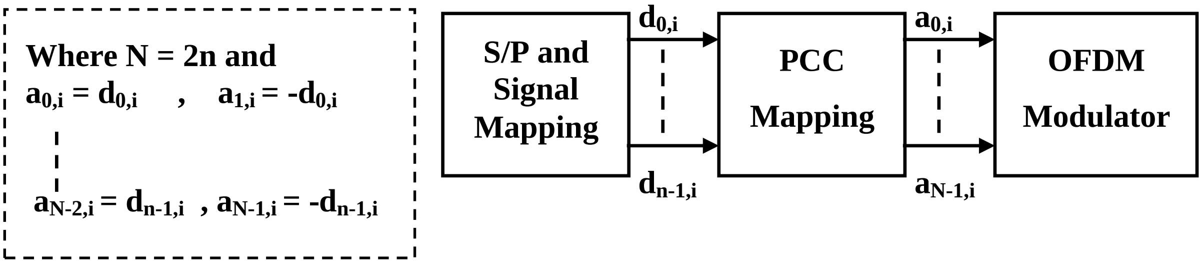
(b)

Figure (2) (a) FFT-based OFDM modulator
(b) FFT-based OFDM demodulator

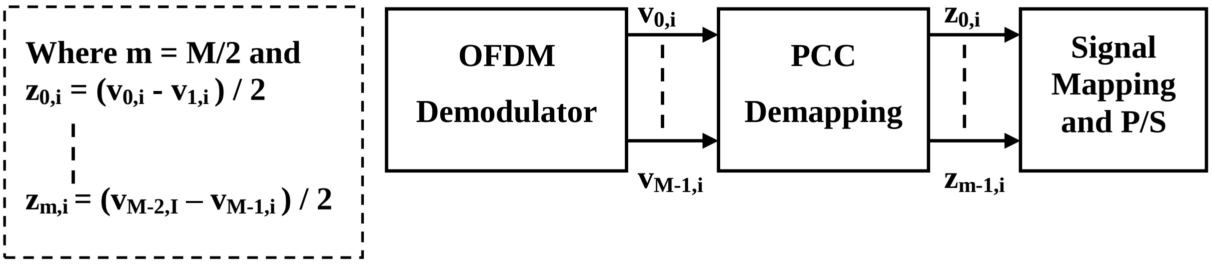
3. Polynomial cancellation coded OFDM (PCC-OFDM)

Polynomial cancellation coding (PCC) is a coding method for OFDM in which the information to be transmitted is modulated onto weighted groups of subcarriers rather than onto individual subcarriers.

Figure (3) illustrates the PCC mapping and PCC demapping. For normal OFDM N = n and $a_{k,i} = d_{k,i}$; one data value is used to modulate each subcarrier. With PCC, the data to be transmitted is mapped onto weighted groups of subcarriers. For example, to apply PCC to pairs of subcarriers, the subcarriers in each pair must have relative weightings +1 and -1. In this case N = 2n. The first data value in each symbol period is used to modulate the first two subcarriers: $a_{0,i} = d_{0,i}$, $a_{1,i} = -d_{0,i}$. The second data value modulates the third and fourth subcarrier and so on [7,8].



(a)



(b)

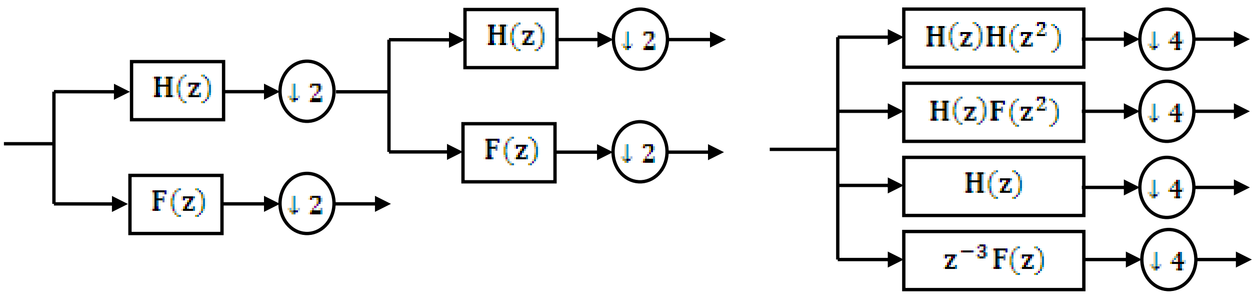
Figure (3) Structure of PCC in OFDM communication system
(a) PCC mapping
(b) PCC demapping

Typically in OFDM system the bandwidth and the average power of the transmitted signal is fixed. The number of subcarriers can be chosen by the designer. The spectrum of a subcarrier falls off as $1/(f^2N^2)$. The overall spectrum being the sum of spectrum of N subcarriers, the OOB spectrum of the complete OFDM signal falls off as $1/(f^2N)$. Even for large N, the OOB power is high and a relatively large guard band must be left between different OFDM signals. However a large N results in high PAPR and increased sensitivity to frequency errors. In the case of pairs of weighted subcarriers in a group, the Fourier transform of each weighted pair of subcarriers falls off with an envelope that depends on $1/(f^2N^2)^2$ which is faster than the case of OFDM. The overall OOB spectrum of the complete PCC-OFDM signals falls off as $1/(f^4N^3)$ [2].

One of the major disadvantages of PCC is that it causes the spectral efficiency to be reduced by half. However some of it will be compensated by elimination of cyclic prefix. In addition the coding redundancy required for PCC-OFDM will be a less than in OFDM to achieve the same error performance [2].

4. Slantlet Filterbank

It is useful to consider first the conventional iterated DWT filterbank and an equivalent form, which is shown in **Fig.(4)**. The slantlet filterbank, based on the second structure, will be described here. But it will be occupied by different filters that are not products. With the extra degrees of freedom obtained by giving up the product form, it is possible to design filters of shorter length, while satisfying orthogonal and zero moment conditions [5].



(a) **(b)**

Figure (4) Two-scale (level) DWT filterbank and an equivalent structure
(a) Two-scale filterbank structure
(b) An equivalent structure for two-scale filterbank

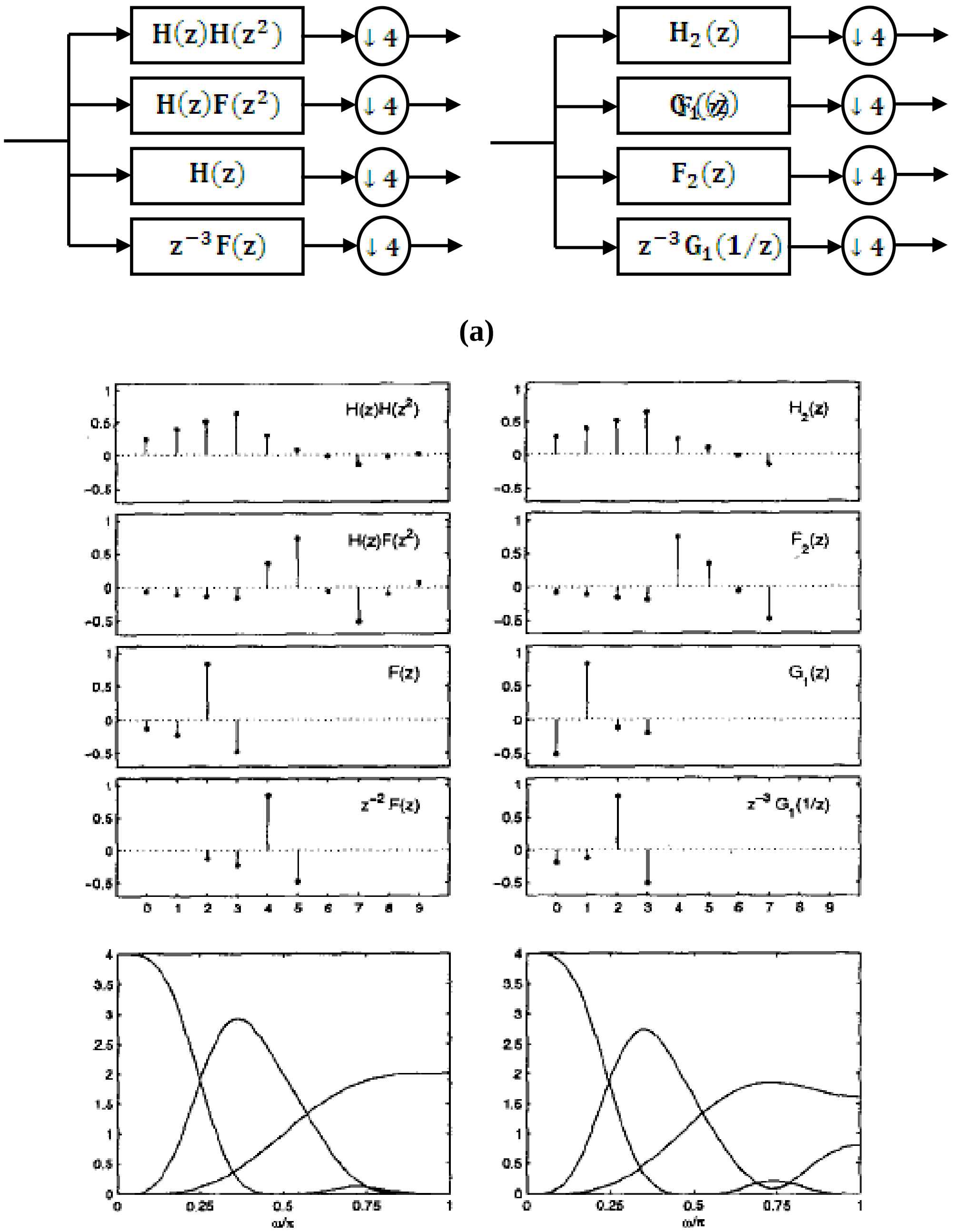
For the two-channel case, the shortest filters for which the filter bank is orthogonal and having K zero moments are the well known filters described by Daubechies [9]. For K = 2 zero moments, those filters H(z) and F(z) are of length 4. For this system, designated D_2, the iterated filters in **Fig.(4)** are of length 10 and 4. Without the constraint that the filters are products, an orthogonal filterbank with K = 2 zero moments can be obtained where the filter lengths are 8 and 4, as shown in **Fig.(5)**, side by side with the iterated D_2 system. That is a reduction by two samples grows with the number of stages. The filters shown on right-hand side of **Fig.(5)** are:

$$G_1(z)=\left(-\frac{\sqrt{10}}{20}-\frac{\sqrt{2}}{4}\right)+\left(\frac{3\sqrt{10}}{20}+\frac{\sqrt{2}}{4}\right)z^{-1}+\left(-\frac{3\sqrt{10}}{20}+\frac{\sqrt{2}}{4}\right)z^{-2}+\left(\frac{\sqrt{10}}{20}-\frac{\sqrt{2}}{4}\right)z^{-3} \quad \text{......................} \ (1)$$

$$F_2(z)=\left(\frac{7\sqrt{5}}{80}-\frac{3\sqrt{55}}{80}\right)+\left(-\frac{\sqrt{5}}{80}-\frac{\sqrt{55}}{80}\right)z^{-1}+\left(-\frac{9\sqrt{5}}{80}+\frac{\sqrt{55}}{80}\right)z^{-2}+\left(-\frac{17\sqrt{5}}{80}+\frac{3\sqrt{55}}{80}\right)z^{-3}+\left(\frac{17\sqrt{5}}{80}+\frac{3\sqrt{55}}{80}\right)z^{-4}+\left(\frac{9\sqrt{5}}{80}+\frac{\sqrt{55}}{80}\right)z^{-5}+\left(\frac{\sqrt{5}}{80}-\frac{\sqrt{55}}{80}\right)z^{-6}+\left(-\frac{7\sqrt{5}}{80}-\frac{3\sqrt{55}}{80}\right)z^{-7} \quad \text{.............} \ (2)$$

$$H_2(z)=\left(\frac{1}{16}+\frac{\sqrt{11}}{16}\right)+\left(\frac{3}{16}+\frac{\sqrt{11}}{16}\right)z^{-1}+\left(\frac{3}{16}+\frac{\sqrt{11}}{16}\right)z^{-2}+\left(\frac{7}{16}+\frac{\sqrt{11}}{16}\right)z^{-3}+\left(\frac{7}{16}-\frac{\sqrt{11}}{16}\right)z^{-4}+\left(\frac{5}{16}-\frac{\sqrt{11}}{16}\right)z^{-5}+\left(\frac{3}{16}-\frac{\sqrt{11}}{16}\right)z^{-6}+\left(\frac{1}{16}-\frac{\sqrt{11}}{16}\right)z^{-7} \quad \text{..........................} \ (3)$$

These filters have the following features: each filterbank (equivalently, discrete-time basis) is orthogonal, has two zero moments, the slantlet filterbank is less frequency selective than the traditional DWT filterbank due to the shorter length of the filters, the time localization is improved with a degradation of frequency selectivity and the slantlet filters are piecewise linear [5].



(a)

(b)

Figure (5) Comparison of two-scale iterated D_2 filterbank (left-hand side) and two-scale slantlet filterbank (right-hand side):

- **(a) Equivalent two-scale iterated D_2 (LHS) filterbank and two-scale slantlet filterbank (RHS) structure.**
- **(b) Magnitude of the coefficients and frequency response magnitude for each filter**

5. Proposed Slantlet based PCC-OFDM system

The block diagram of the proposed Slantlet based PCC-OFDM system is depicted in **Fig.(6)**. The overall system of OFDM is the same as in **Fig.(1)**. The difference is adding the PCC mapping and PCC demapping. The SLT-based OFDM modulator and demodulator that used are shown in **Fig.(7)**.

The processes of the S/P converter, the signal demapper and the insertion of training sequence are the same as in the FFT-OFDM system. Also the zeros will be added as in the FFT based case and for the same reasons. Then the PCC mapping and the inverse slantlet transform (ISLT) will be applied to the signal.

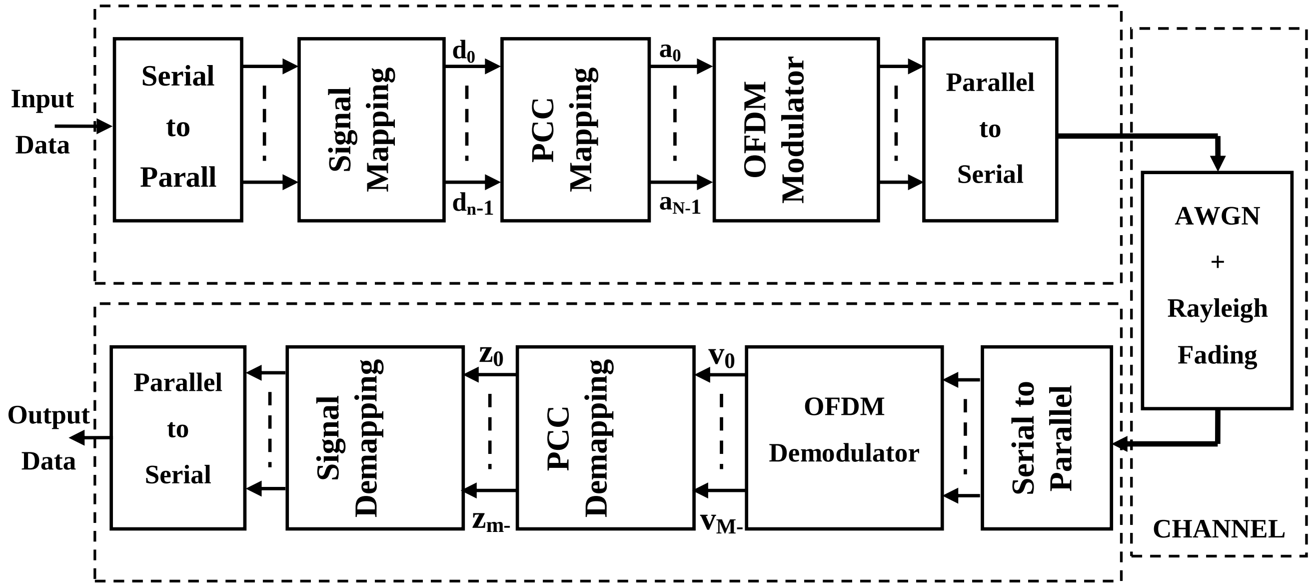
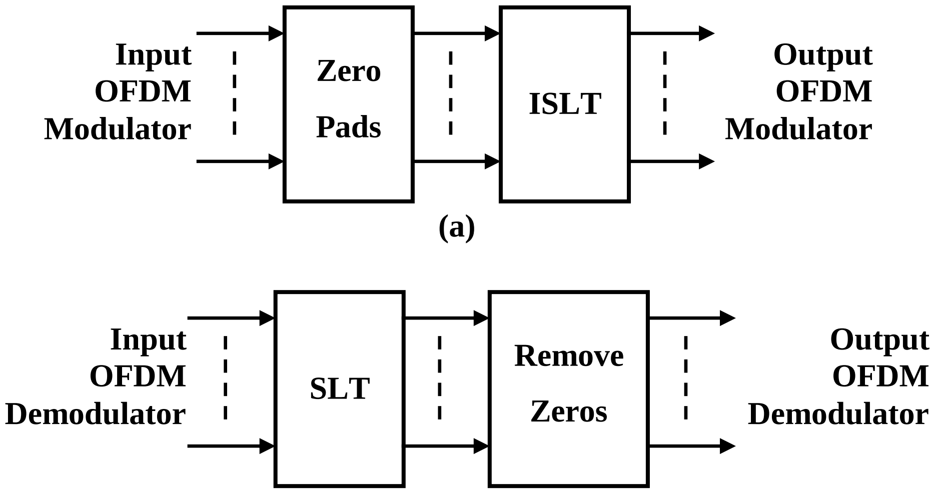


Figure (6) Proposed slantlet based PCC-OFDM system



(a)

(b)

Figure (7) (a) SLT-based OFDM modulator
(b) SLT-based OFDM demodulator

The use of slantlet transform (SLT) instead of fast transform (FFT) in OFDM system would improve the performance because the SLT does not need cyclic prefix; leading to reduce the frame length and increasing the speed of the system. Feathers more the filters in the SLT depend on the wavelet functions (namely Daubechies basis functions) which are simple and fast to compute with respect to the sine and cosine functions that are used in FFT. Therefore the data rates in SLT based OFDM can surpass those of the FFT implementation. After that the P/S converter will convert the OFDM symbol to its serial version and will be sent through the channel.

At the receiver, also assuming synchronization conditions are satisfied, first S/P converts the OFDM symbol to parallel version. After that the SLT will be performed. The zero pads will remove and the other operations of the channel estimation, channel compensation, PCC demapper, signal demapper and P/S will be performed in a similar manner to that of the FFT based OFDM.

6. Performance Evaluation and Simulation Results

First the FFT and SLT based OFDM system have been tested. Next, the FFT, and SLT based PCC-OFDM system have been also tested in three types of channels. The channel types considered are: additive white Gaussian noise (AWGN), flat fading and frequency selective. Modulation type is taken to be 4-ary QAM.

6-1 AWGN Channel

Figure (8) illustrates the performance of the FFT-based OFDM (FFT-OFDM) and the SLT -based OFDM (SLT-OFDM) systems for 4QAM modulation type in AWGN channel with and without using the PCC. It can be easily noticed that the SLT-OFDM system performance better than the FFT-OFDM system since the filters in the SLT depend on the wavelet function.

For example at SNR=10 dB, the bit error rate (BER) is reduced to 0.0036 in SLT-based PCC-OFDM as compared with 0.4290 in FFT based PCC-OFDM. However, the performance of SLT based PCC-OFDM is more improved as SNR increases. It could be also noticed from **Fig.(8)** that the performance of both FFT and SLT-based OFDM is improved when adding PCC. This improvements starts at SNR values exceeds 15 dB in FFT-based OFDM while starts early at about 0 dB SLT-based OFDM which is another positive feature added to the proposed SLT based PCC-OFDM.

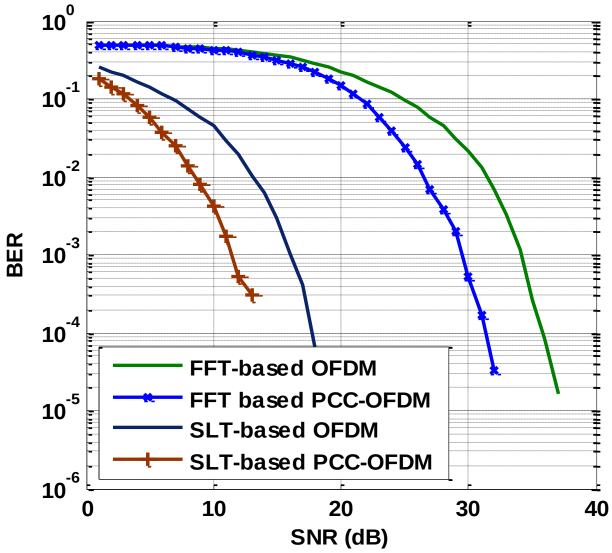


Figure (8) Performance of OFDM system in AWGN channel

6-2 Flat Fading Channel

In this type of channel, the signal would be affected by the flat fading with addition to AWGN. In this case all the frequency components in the signal will be affected by a constant attenuation and linear phase distortion of the channel, which has been chosen to have a Rayleigh's distribution. A Doppler frequency of 50 Hz is used in this simulation. **Figure (9)** illustrates the BER performance of FFT-OFDM, SLT-OFDM, FFT based PCC-OFDM and SLT based PCC-OFDM in flat fading channel.

This Figure shows the robustness of SLT-based OFDM and SLT based PCC-OFDM against flat fading as compared with FFT based ones. For example at 10 dB SNR, the bit error rate is about 0.4748 for FFT-OFDM and 0.4343 for FFT based PCC-OFDM, while bit error rates of 0.06 and 0.005 are achieved in SLT-OFDM and SLT based PCC-OFDM respectively. Once again the usage of PCC would increase the performance of FFT and SLT based OFDM systems.

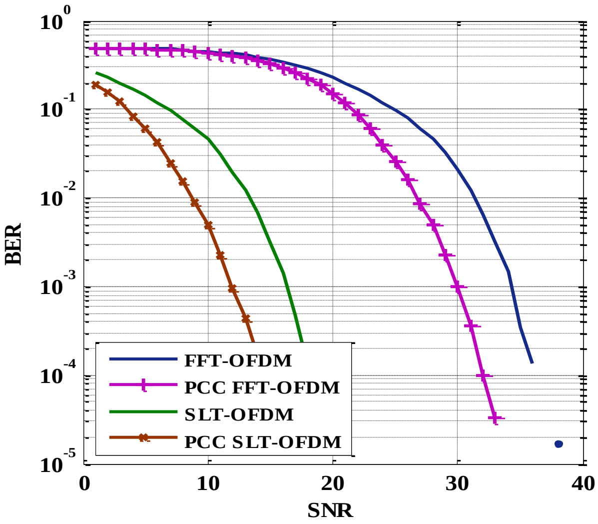


Figure (9) Performance of OFDM system in flat fading channel assuming doppler frequency of 50 HZ

6-3 Frequency Selective Fading (Multipath) Channel

The last case study is selective fading channel. The parameters of the considered channel are corresponding to multipath where two paths have been chosed; the line of sight (LOS) path and a second path. The LOS path has average path gain equals 0 dB and path delay of 0 sample, while the second path has average path gain -8 dB and path delay of 8 samples.

Figure (10) shows the simulation result at Doppler frequency of 50 Hz in a multipath frequency selective Rayleigh distributed channel with AWGN. It is clearly shown in **Fig.(10)** that the BER performance of SLT-OFDM becomes constant after certain SNR values. Also, one can see that the BER curves of FFT-OFDM decrease with the increase of the SNR values. The SLT-OFDM system in frequency selective channel has better performance than FFT-OFDM until some SNRs then falls down to have a constant behavior with SNR. This is attributed to the eliminating of the cyclic prefix. As a consequence the filter bank becomes unable to separate the required signal. An enhanced performance occurred when using SLT based PCC-OFDM system because mapping into adjacent pairs is like a cyclic prefix with each subcarrier. The performance is good until some point then falls down (up to SNR values of 30 dB). This is an evidance that the SLT filterbank is no longer able to distinguish the required signal.

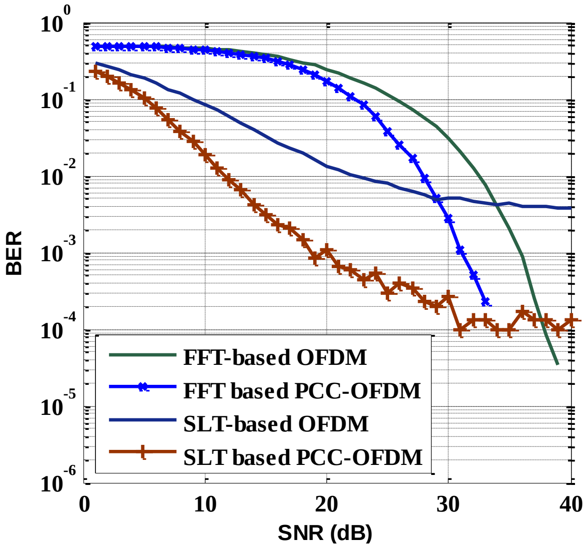


Figure (10) Performance of OFDM system in selective fading channel

7. Conclusions

In the proposed system there is no need for using CP because of the good orthogonality that is offered by slantlet, and subsequently, reduces the system complexity, increase the transmission rate and finally increase spectral efficiency.

The proposed OFDM system (SLT based PCC-OFDM) have good BER performance as compared to FFT-OFDM, FFT based PCC-OFDM and SLT -based OFDM in all types of channels as shown in Figures 8,9,10.

An enhanced performance occurred when using SLT based PCC-OFDM system because mapping into adjacent pairs is like a cyclic prefix with each subcarrier.

8. References

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