Behaviour of Non-Linear Partially Saturated Soils below Strip Footings

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Abstract

The effect of time dependent compression of consolidation settlement is modeled by using the finite element formulation of the coupled field equations for deformation and excess pore water pressure generated by an increment of loading applied to a saturated soil is studied. This formulation is extended based on Biot's theory to include the partially saturated soil; the formulation has been developed by noting that air and water pressure in partially saturated soils are approximately equal at high degrees of saturation. This led to a relatively simple formulation which can have a range of practical applications.

The result shows that there is an influence of partial soil saturation on strip footings behaviour. Both partially saturated and conventional saturated finite element analyses are performed with the Modified Cam Clay Model presented in this study.

الخلاصـــــة

تم دراسة تأثير التصرف الزمني للانضمام باستعمال صيغة العناصر المحددة للمعادلات المزدوجة أي التي تعنى بدراسة تأثير التشوهات وضغط الماء معاً بصورة عامة من خلال الزيادة في الحمل المسلط على التربة المشبعة هذا وقد تم في هذا البحث دراسة نظرية(بايوت) لتشمل التربة المشبعة جزئيا" ذلك بفرض إن ضغط الماء مساوي إلى ضغط الهواء في درجات التشبع العالية.

أظهرت النتائج وجود تأثير واضح للترب المشبعة جزئياً على تصرف الأسس الشريطية مثلاً وان النموذج الذي تم دراسته هو من نوع Cam clay المحسن.

1. Introduction

The mechanical behaviour of partially saturated soils is very different from that of fully saturated soils. It has long been established that for such soils, changes in degree of saturation do not have the same effect as changes in the applied stresses, and consequently the effective stress principle is not applicable. Conventional constitutive models, which are based on this principle, are therefore of limited use when analyzing geotechnical problems that involve the presence of partially saturated soil zones. Although the existing constitutive models for partially saturated soils can reproduce important features of the behaviour of such soils, such as collapse (failure of soil) under footing; they are less advanced than the conventional fully saturated soil models. In addition, only a limited number of applications of such models to boundary value problems have been performed in the past as reported by [1].

Conventional soil mechanics theories treat soil as either fully saturated (pores filled with water) or dry (pores filled with air). However, a large number of geotechnical problems involve the presence of partially saturated soil zones where the voids between the soil particles are filled with a mixture of air and water. These zones are usually ignored in practice and the soil is assumed to be either fully saturated or completely dry [2]. In reality, the pore air pressure and water pressure are not the same in partially saturated soil. However, at higher degrees of saturation, the pore air pressure and pore water pressure are approximately equal, especially when the changes of pore water pressures are positive [3]. The pore water pressures and pore air pressures are generally fairly close to each other for degrees of saturation above about 70 percent [1]. This degree of saturation depends on the type of soil, and needs to be established for a particular soil type. It has long been established however that the behaviour of partially saturated soils can be very different from that of fully saturated or completely dry soils.

Experimental and theoretical difficulties delay considerably the development of an understanding of the behaviour of partially saturated soils. It is only during the last few years that theoretical frameworks and constitutive models have been proposed to describe the mechanical behaviour of such soil as reported by [2].

Although the existing constitutive models are capable of reproducing important features of the behaviour of partially saturated soils, most models are basic, compared to those available for fully saturated soils, and often soil type specific.

There is; therefore, a need for improvement and an increasing number of researchers around the world are working on improving the understanding and constitutive modeling of the mechanical behaviour of partially saturated soils as reported by [2].

Constitutive models describe the mechanical behaviour of soils, but in most cases, especially for advanced models such as those for partially saturated soils, are only useful in practice when used in numerical analysis. Numerical analysis plays an important role in the investigation of the behaviour of partially saturated soils by highlighting aspects which are important in engineering practice as reported by [2].

2. Modeling of Clay

The Cam Clay theory is the basis for several more advanced theories. Cam-Clay models originated from the work of Roscoe and his co-workers at the University of Cambridge [4,5]. The original idea was further developed by [6] to the Modified Cam-Clay model; nowadays, the most widely used elasto-plastic model for the description of the mechanical behavior of clay.

The advantage of the Modified Cam-Clay model lies in its apparent simplicity and its capability to represent (at least qualitatively) the strength and deformation properties of clay realistically. Commonly observed properties such as an increasing stiffness as the material undergoes compression, hardening/softening and compaction/dilatancy behavior, and the tendency to eventually reach a state in which the strength and volume become constant are all captured by the Modified Cam-Clay model. Moreover, calibration of the model requires only a few conventional laboratory tests.

3. The Program of Finite Element Model

The finite element model in the present study assumes two dimensional plane strain conditions for linear elastic and Modified Cam Clay models. The program uses eight-node quadrilateral elements. All the analyses presented in this paper are made with the Modified Cam Clay Program (MCCP) which made in the Engineering College Al-Mustansiriya University.

4. Governing Equations

4-1 Stress-Strain Relationship for Elastoplastic Model

$$\Delta\sigma = D_{ep}\Delta\varepsilon^{ep} = D_{ep}\left(d\varepsilon^{e} + d\varepsilon^{p}\right) \qquad (1)$$

$$[D]_{ep} = [D]_{e} - \frac{[D]_{e}\left(\frac{\partial f}{\partial \sigma}\right)^{T}[D]_{e}\left(\frac{\partial f}{\partial \sigma}\right)}{[D]_{e}\left(\frac{\partial f}{\partial \sigma}\right)^{T}\left(\frac{\partial f}{\partial \sigma}\right) - A} \qquad (2)$$

where:

$A = \frac{1}{d\lambda}\frac{\partial F}{\partial p_c} dp_c$,

$d\lambda$: is the plastic multiplier,

D_{ep} : is the elastoplastic stiffness matrix,

$d\varepsilon^{e}$: is the elastic strain increment,

$d\varepsilon$: is the total strain increment, and

$d\varepsilon^{p}$: is the plastic strain increment.

4-2 The Yield Function

The Yield Function, F, determines the boundary between purely elastic and elastic-plastic deformation. At stress states below the yield surface, only elastic deformation occurs. At the yield surface, elastic and plastic deformation occurs. The yield function F({σ'}, {κ}), where {σ'} is the stress state and {κ} are state parameters. The yield function for the Modified Cam Clay model is given by the following equation [7]:

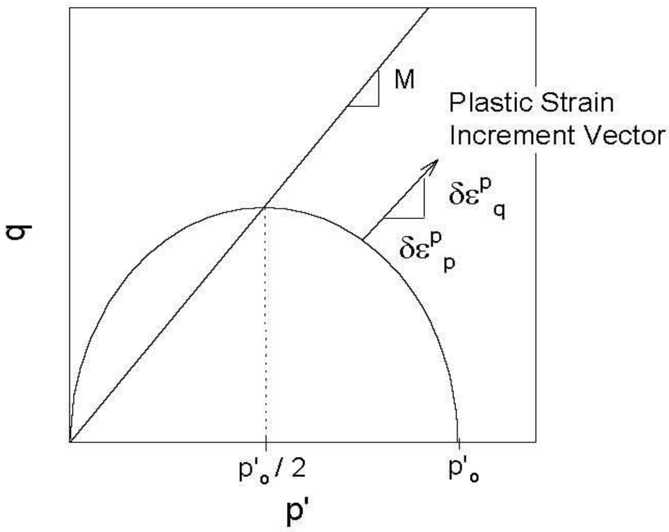
$$\mathbf{F(\{\sigma'\},\{\kappa\}) = \left(\frac{q}{p'M}\right)^2 - \left(\frac{p_o'}{p'} - 1\right) = 0} \quad \text{.. (3)}$$

where:

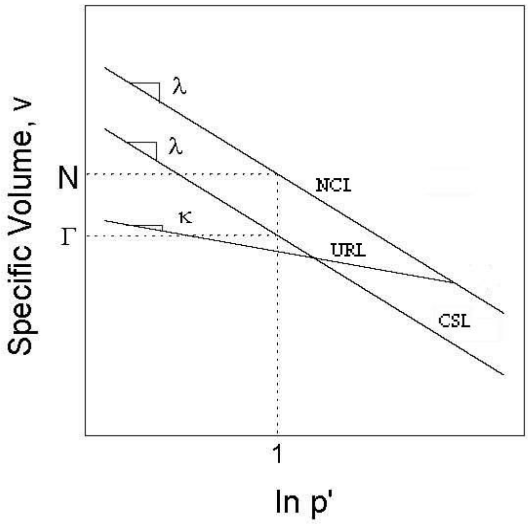
M: is the slope of the critical state line in p-q plane,

p'_o : is the hardening parameter for the Modified Cam Clay model.

The yield surface intersects the M line at: $\mathbf{p' = p'_o/2}$ for the Modified Cam Clay model. The shape of the yield function for the Modified Cam Clay model is an ellipse. The shapes of the yield functions are shown in **Fig.(1)**.



(a): Compressibility relationships
NCL= Isotropic consolidation line
CSL= Critical state line
URL= Unload-Reload line



(b): Modified cam clay model yield surface and plastic

Figure (1) Properties of modified cam clay model

4-3 Flow Rule

The flow rules may be specified as a relationship between a plastic potential and yield curve. Plastic potential function, g, describes the relative magnitudes of the plastic shear strain and the plastic volumetric strain when the soil is yielding. If the plastic potential and the yield curve coincide, the material is said to have an associated flow rule and the normality condition applies in the sense that the vectors of strain increment are normal to a yield curve, as shown in **Fig.(1)**. Both models assume the condition of normality. The ratio of the plastic shear strain to the plastic volumetric strain is given by the direction of the plastic strain increment vector. The plastic strain increment vector is perpendicular to the plastic potential function, as shown in **Fig.(1)** [8, 9].

The associated flow rule is adopted in the present study; and the flow vector can be determined by using chain rule:

$$\frac{\partial F}{\partial \sigma} = \frac{\partial F}{\partial p} \cdot \frac{\partial p}{\partial \sigma} + \frac{\partial F}{\partial q} \cdot \frac{\partial q}{\partial \sigma} \quad \text{..........} \quad (4)$$

For modified Cam Clay where the yield functions is given in Eq.(3):

$$\frac{\partial F}{\partial \sigma} = \left\{\frac{\partial p}{\partial \sigma}\right\}^T \left(-2\frac{q^2}{M^2 p^3} + \frac{p_o}{p^2}\right) + \left\{\frac{\partial q}{\partial \sigma}\right\}^T \left(2\frac{q}{M^2 p^2}\right) \quad \text{..........} \quad (5)$$

4-4 Hardening Law

The Hardening Law relates the magnitude of a plastic strain to the magnitude of an increment of stress as the state of stress traverses the yield surface and the material strain hardens. Both models can be described by the following equation [7, 10, and 11]:

$$\delta p'_o = \frac{v p'_o}{\lambda - \kappa} \delta \varepsilon_v{}^p \quad \text{......... (6)}$$

where:

p'_o: would be used for the Modified Cam Clay model.

4-5 Continuity Equations for Saturated Soils

The continuity eq.(7) derived below can be solved using the virtual work principle within the finite element frameworks for saturated porous media.

$$\nabla^T \left[-\frac{K_p}{\gamma_w} \nabla (p + \gamma_w z) \right] = m^T \frac{\partial \varepsilon}{\partial t} \quad \text{......... (7)}$$

This finite element Equation when coupled with the finite element equation for equilibrium can be expressed as:

$$\begin{bmatrix} K & L \\ L^T & -0.5\Delta t.H \end{bmatrix} \begin{Bmatrix} du^n \\ dp^n \end{Bmatrix} = \begin{Bmatrix} dF(\Delta t) \\ \Delta t.H.p^n(t) \end{Bmatrix} \quad \text{......... (8)}$$

where:

K: is the tangential stiffness matrix given as:

$$K = \int_\Omega B^T \ D_t \ B \, d\Omega \quad \text{......... (9)}$$

L: is the coupling matrix given as:

$$L = \int_\Omega B^T \ m \ N_p^T \ d\Omega \quad \text{......... (10)}$$

H: is the flow matrix given as:

$$H = \int_\Omega (\nabla . N_p^T)^T \frac{K_p}{\gamma_w} \nabla N_p^T \ d\Omega \quad \text{......... (11)}$$

4-6 Continuity Equations for Partially Saturated Soils

The continuity eq.(12) derived below can be solved using the virtual work principle within the finite element frameworks for saturated porous media.

$$\nabla^T \mathbf{v} - S_r \frac{\partial \varphi}{\partial t} = n\gamma_w \frac{\partial S_r}{\partial u_w} \frac{\partial h}{\partial t} \quad \text{.......... (12)}$$

where:

v : is the vector of superficial velocities of water,

∇ : is an operator vector for directions x,y and z.

$\partial\varphi / \partial t$: is the rate of change of volumetric strain,

u_w : is the pore water pressure,

S_r : is the degree of saturation at a pore water pressure u_w in kPa and

h: is total water head.

This finite element Equation when coupled with the finite element equation for equilibrium can be expressed as:

$$\begin{bmatrix} K & L \\ L^T & 0.5\Delta t.H_s - S \end{bmatrix} \begin{Bmatrix} du^n \\ du_w^n \end{Bmatrix} = \begin{Bmatrix} dF(\Delta t) \\ \Delta t.H_s.u_w^n(t) \end{Bmatrix} \quad \text{.......... (13)}$$

where:

u_w^n : is unknown incremental of pore pressure,

S: is storage matrix in partially saturated soil.

Matrix S represents the change of storage of moisture with respect to change in water head. For identification purposes S is called the storage matrix here,

$$S = \int_\Omega \frac{n\gamma_w}{S_r} \frac{\partial S_r}{\partial u_w} N_{u_w} N_{u_w}^T \; d\Omega \quad \text{.......... (14)}$$

and H_s is the flow matrix which is also a function of degree of saturation.

$$H_s = \int_\Omega \frac{1}{S_r} (\nabla.N_{u_w}^T)^T \frac{K_{u_w}}{\gamma_w} \nabla N_{u_w}^T \; d\Omega \quad \text{.......... (15)}$$

5. Plane Strain Consolidations under Strip Footing

The problem considered is a classical plane strain problem involving a strip footing. This example is useful for studying consolidation behaviour of partially saturated soils with different initial degrees of saturation and studying the relationship between degree of saturation and positive pore pressure, as defined by this Eq.(16).

$$\mathbf{S_r = \frac{0.0099u_w + 0.98S_{ro}}{0.98 + 0.0097u_w}} \quad \text{...} \quad \mathbf{(16)}$$

This example presents the influence of partially soil saturation on the behaviour of footings which is investigated and also presents comparison between the predictions of partially saturated and conventional analysis for Linear Elastic soil and Elastoplastic Modified Cam Clay soil.

A finite element mesh comprising 180 eight-nodded quadrilateral elements is employed for this example. The finite element mesh and the boundary conditions can be seen in **Fig.(2)**. The unit weight of pore water is assumed to be $\gamma_w = 10\, kN/m^3$ and the saturated unit weight of the soil is to be $\gamma_{sat} = 20\, kN/m^3$. A uniform normal pressure of 300kN/m² is applied over a strip footing 4m on the surface of the soil.

The following parameters are used to present the mechanical behavior of the soil material in this program as shown in **Table (1)**.

Table (1) Soil properties and parameters for modified cam clay model

Strength parameter M	0.888	**Plastic consolidation parameter κ**	0.062
Plastic consolidation parameter λ	0.161	**Critical void ratio**	1.789
Poisson's ratio v'	0.3	**Young's modulus E**	$1x10^4$ kN/m2
Unit weight of the soil γ_{sat}	20 kN/m^3	**Permeability k**	$1x10^{-5}$ m/day

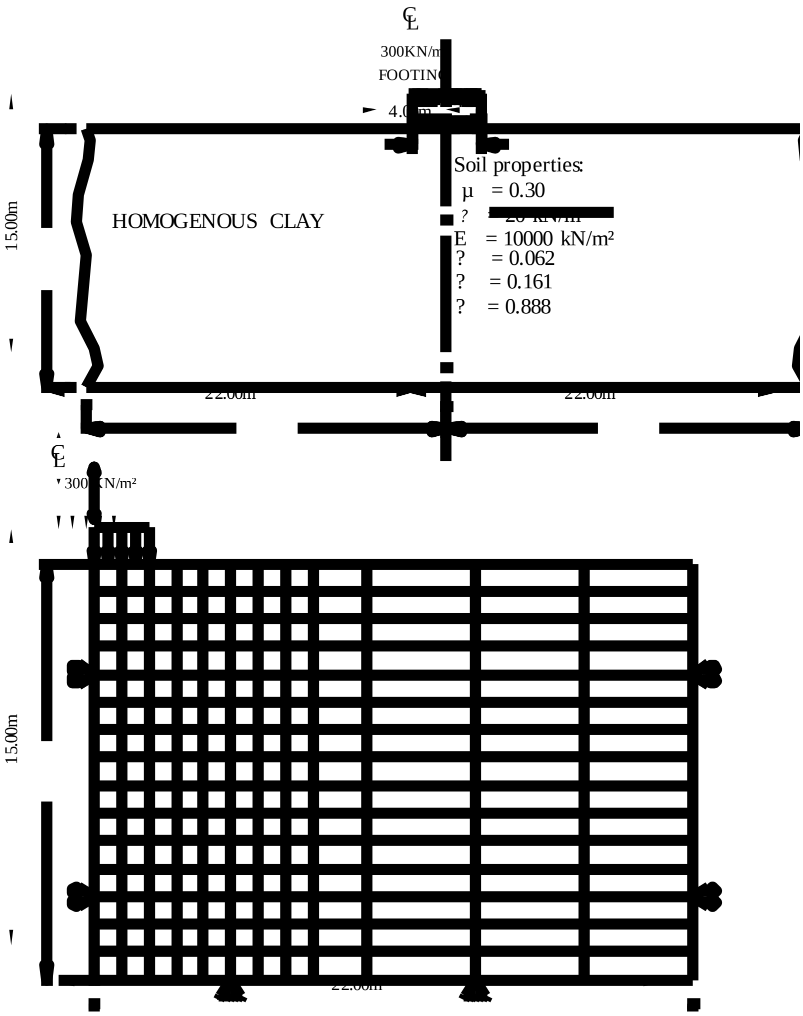
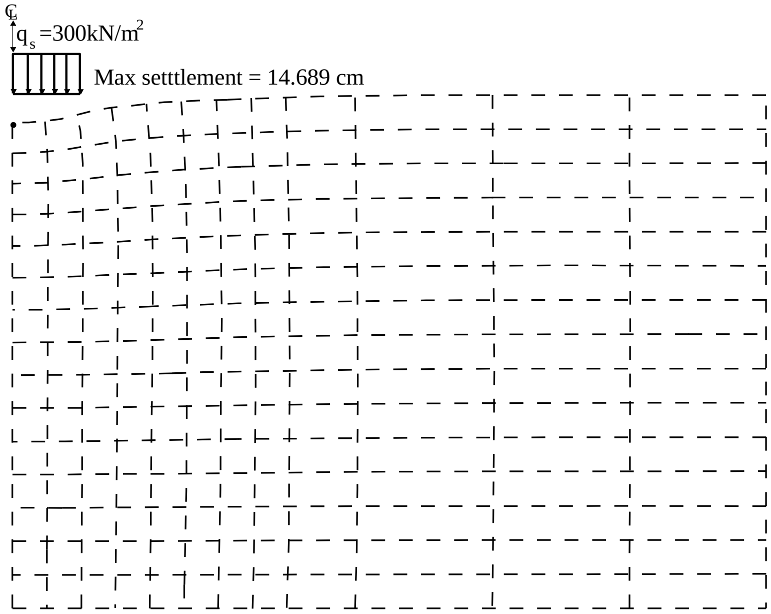


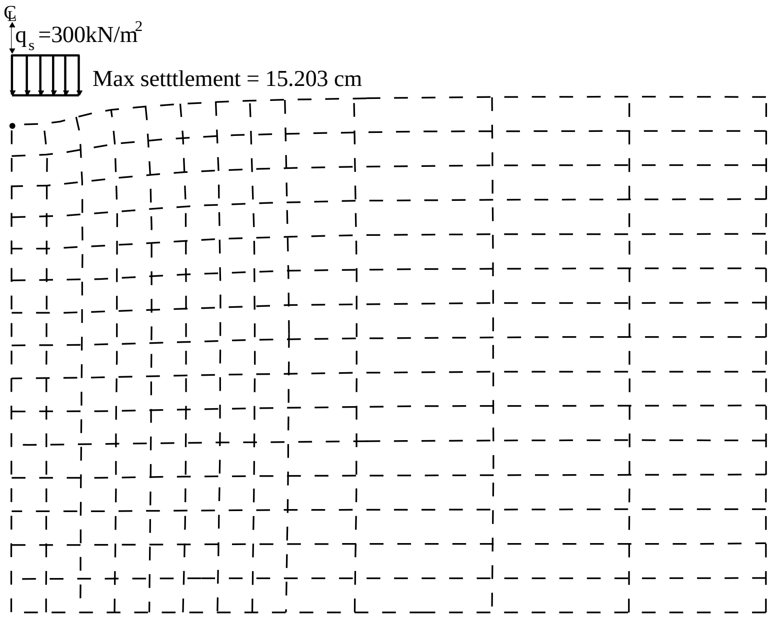
Figure (2) Finite element mesh and geometry of problem used in two dimensional analysis

6. Results and Discussions

In general, the deformed shapes for linear elastic soil and nonlinear elastoplastic (Modified Cam Clay Model) soil behaviour are similar as shown in **Figs.(3,4)**. It can be seen that the consolidation settlements occurring in the partially saturated soils are considerably higher than these in the saturated soil. This is expected as the volume of the partially saturated soils can reduce by compressing the air voids, as compared to saturated soils, where the volume can only be reduced by driving the pore water gradually out of the soil. In general, the settlement increases with the decrease of degree of saturated for elastic and elastoplastic soil.

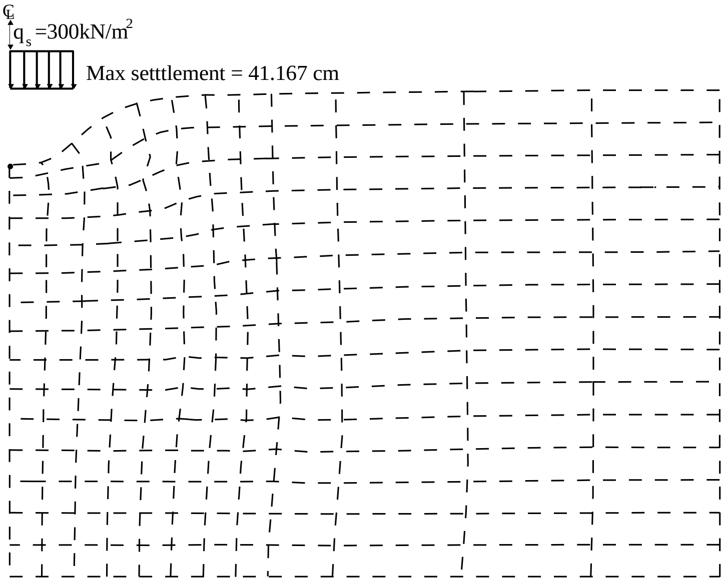


fully saturated soil-SR=100% (deformed exaggerated by a factorof 5)

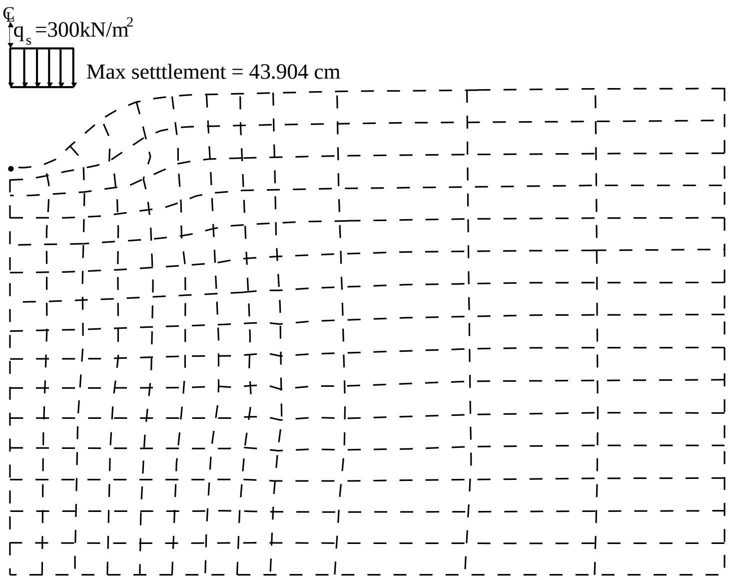


partially saturated soil-SR=80% (deformed exaggerated by a factorof 5)

Figure (3) Predicted deformed shape under footing in linear elastic model



fully saturated soil-SR=100% (deformed exaggerated by a factorof 5)



partialy saturated soil-SR=80% (deformed exaggerated by a factorof 5)

Figure (4) Predicted deformed shape under footing in modified cam clay model

The relationship between max settlement of footing and time in elastic soil (fully, partially saturated soil) and the relationship between max settlement of footing and time in elastoplastic soil Modified Cam Clay (fully, partially saturated soil) are illustrated in **Fig.(5)**. It seems that all the three cases in elastic and elastoplastic indicate the same settlement at a long time and this is because the soil skeleton is treated as an elastic medium, the ultimate settlement is stress path independent and all three cases indicate the same ultimate settlement when all the excess pore water pressures have dissipated at a long time.

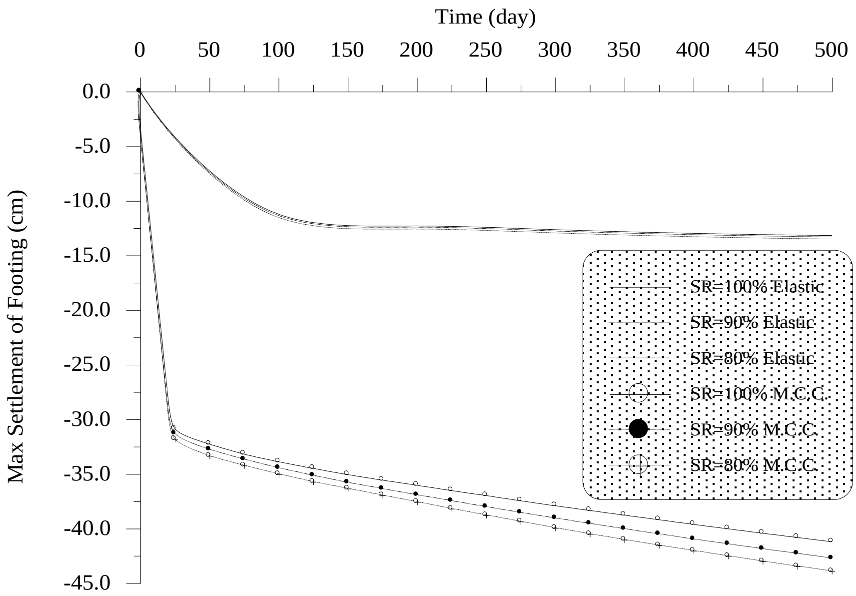


Figure (5) Relationship between max settlement of footing and time

Figure (6) demonstrates the displacement profile under the footing in elastic and elastoplastic soil (fully and partially saturated soil) with bearing pressure 300 kN/m^2 similar to Terzaghi schematic failure mechanism.

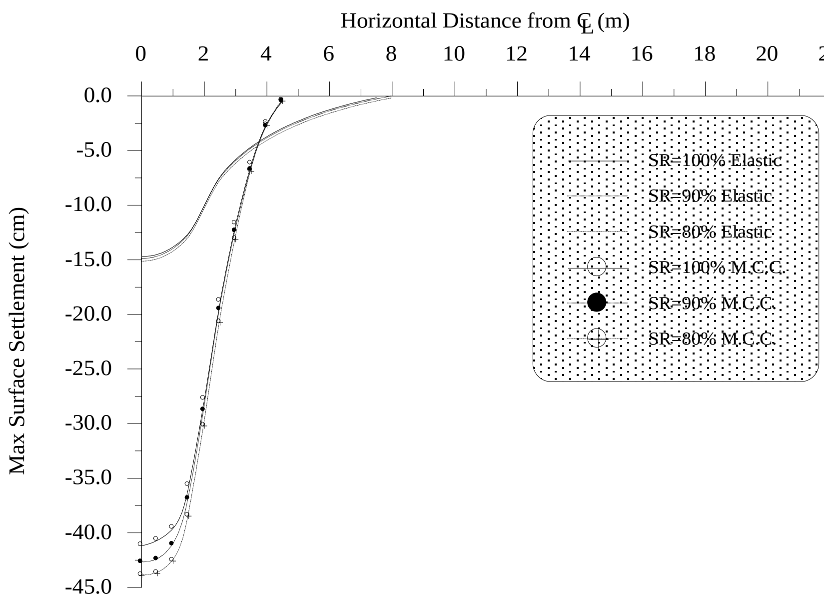


Figure (6) Displacement profile under thr footing (qs=300kN/m^2)

Figures (7) and **(8)** shows the contour lines for the shear stresses under footing in elastic fully and partially saturated soil with bearing pressure300 kN/m^2. It seems that the shear stresses are centered under beneath corner of a strip footing.

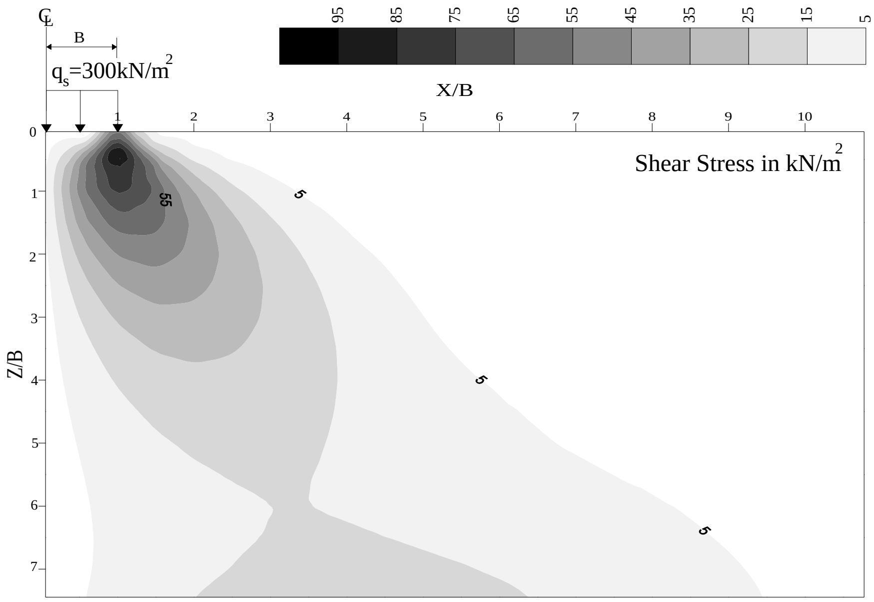


Figure (7) Contour lines for the shear stress under footing in linear elastic fully saturated soil (qs=300kN/m2, SR=100%)

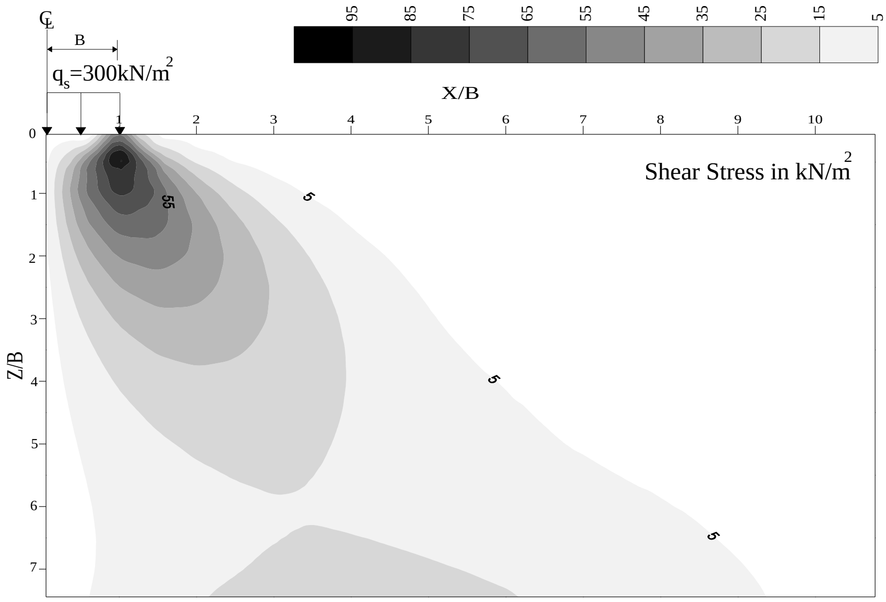


Figure (8) Contour lines for the shear stress under footing in linear elastic partially saturated soil (qs=300kN/m2, SR=80%)

Figures (9) and **(10)** demonstrate the contour lines for the shear stresses under footing in elastoplastic soil with bearing pressure 300 kN/m^2. In general, the shear stresses increase with the decrease in degree of saturation for elastic and elastoplastic soil

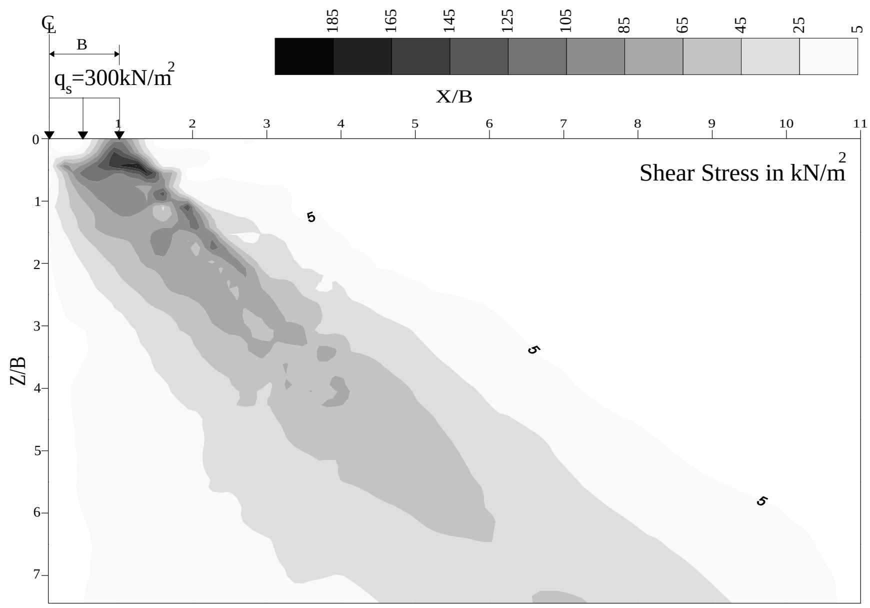


Figure (9) Contour lines for the shear stress under footing in modifed cam clay fully saturated soil (qs=300kN/m2, SR=100%)

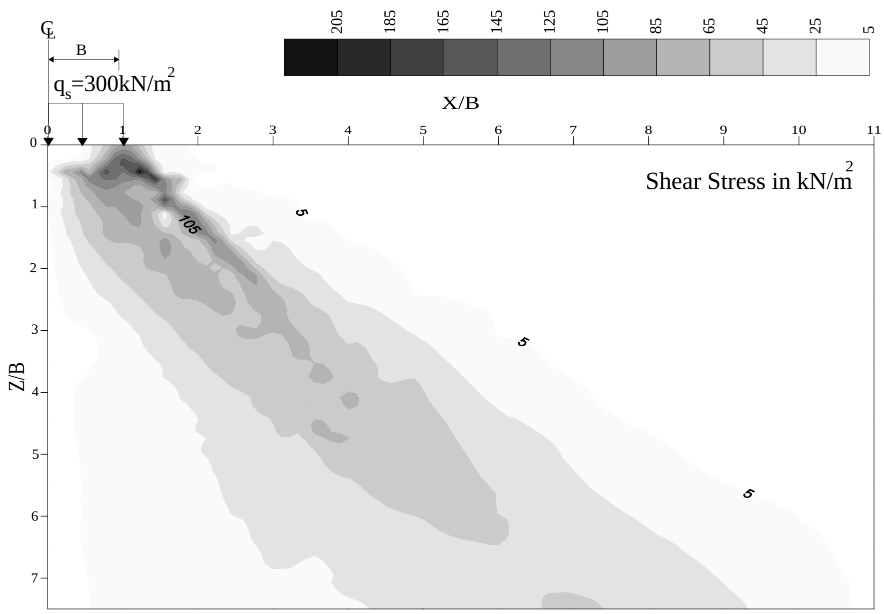


Figure (10) Contour lines for the shear stress under footing in modifed cam clay partially saturated soil (qs=300kN/m2, SR=80%)

Figures (11) and **(12)** show the contour lines for the distribution of excess pore pressure under footing in elastic soil. **Figures (13)** and **(14)** show the contour lines for the distribution of excess pore pressure under footing in elastoplastic soil. In general, the excess pore pressure decreases with the reduction in the degree of saturation for elastic and elastoplastic and it dissipates at the end of consolidation.

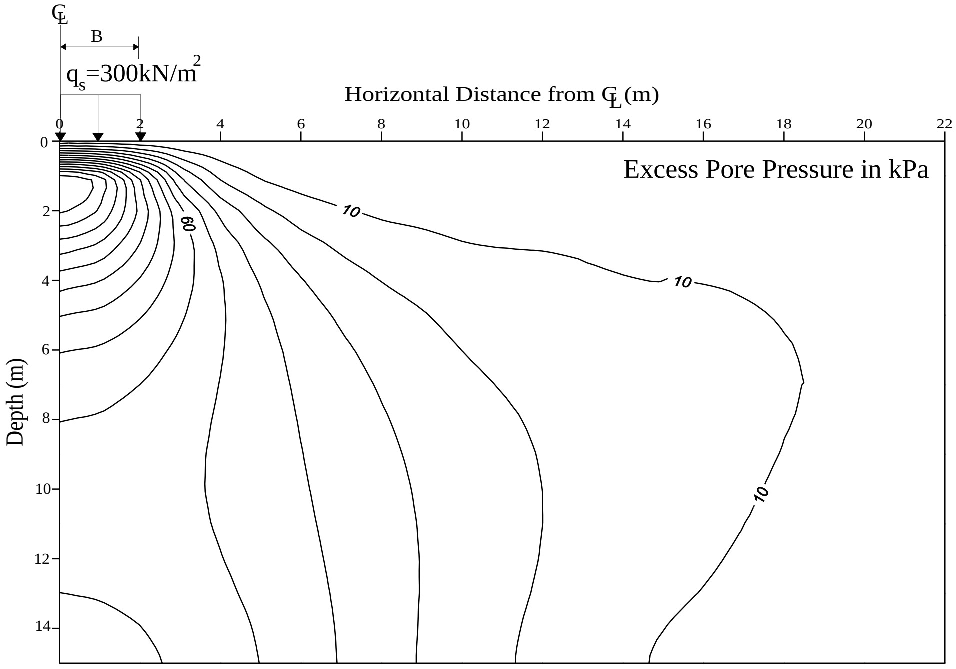


Figure (11) Distribution of excess pore pressure under footing in linear elastic fully saturated soil (qs=300kN/m2, SR=100%)

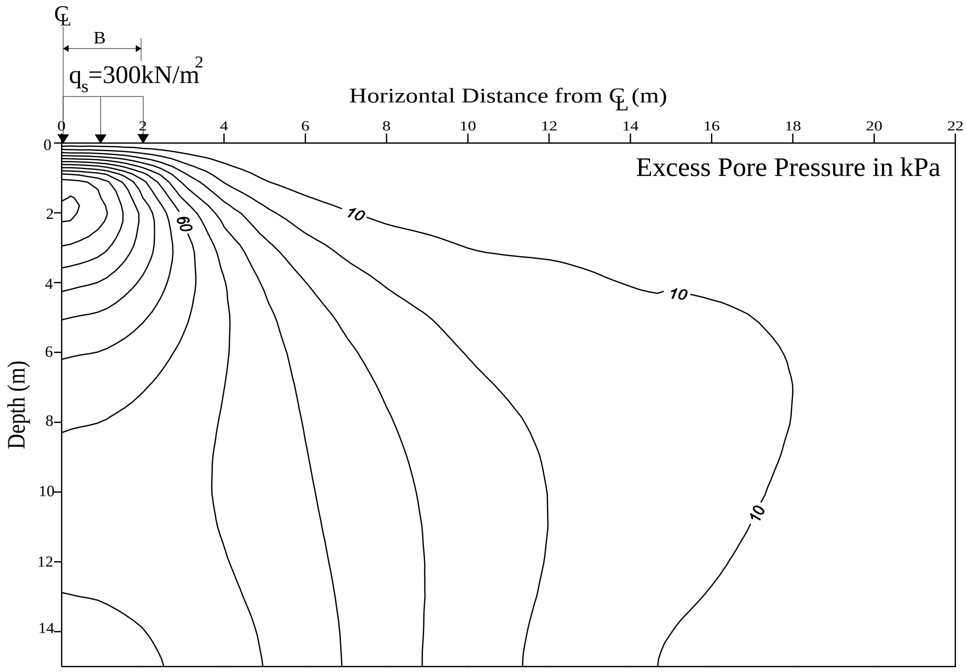


Figure (12) Distribution of excess pore pressure under footing in linear elastic patially saturated soil (qs=300kN/m2, SR=80%)

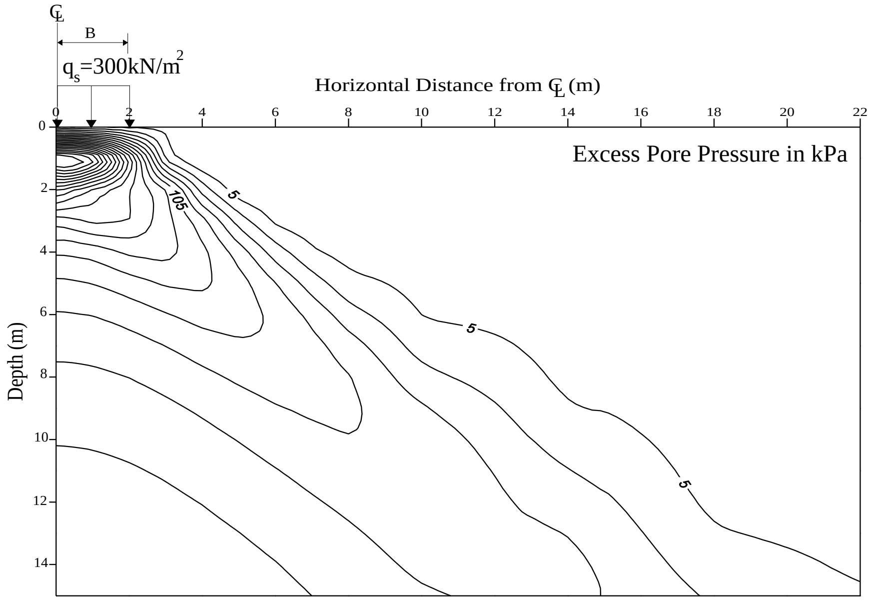


Figure (13) Distribution of excess pore pressure under footing in modifed cam clay fully saturated soil (qs=300kN/m2, SR=100%)

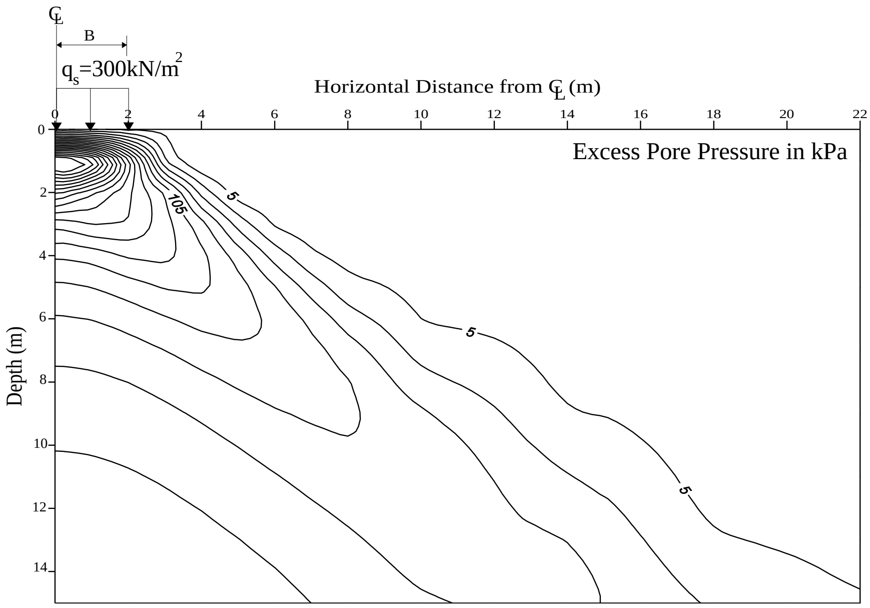


Figure (14) Distribution of excess pore pressure under footing in modifed cam clay partially saturated soil (qs=300kN/m2, SR=80%)

7. Conclusions

1. Comparisons between fully saturated soil and partially saturated soil in Modified Cam Clay prove that the behaviour is the same but the values of the stresses and the deformations in the Modified Cam Clay are larger because the strain in the modified cam clay is larger than the linear elastic model. Also, in the modified cam clay model, the points which approach critical state based on the value of (q/p) can be limited. The points which approach to critical state are concentrated in the regions which have maximum shear stresses. These regions are concentrated under and around the footing.
2. The deformations in case of partially saturated soils are larger than the fully saturated soils. It means that the deformation increases with the decrease in degree of saturation in linear elastic soil and nonlinear elastoplastic soil. In general, Linear Elastic soil and Nonlinear Elastoplastic (Modified Cam Clay) soil have the same behaviour but the deformation in the elastoplastic soil (M.C.C.) is larger than that of the elastic soil because the strain in the elastoplastic is larger than elastic soil.
3. Settlement increases with the increase of loading and the shear stresses increase with the decrease in the degree of saturation in elastic and elastoplastic soils, but the values of the shear stresses in elastoplastic soils are larger than the values of elastic soil because the excess pore water pressure in the fully saturated soil is more than partially saturated soil.
4. In general, the shear stresses are concentrated beneath the corners of a loaded area of a strip footing.

8. References

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