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Effect of Partial Connection on the Behavior and Strength of Continuous Composite Beams

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Abstract

Composite construction can be defined as the usage of different materials in an optimum geometric configuration with the aim that only the desirable property of each material will be utilized by virtue of its designated position.

This study is concerned with the behavior of continuous composite beams of reinforced concrete and steel shapes taking into consideration the non-linear behavior of the components of the composite section, i.e. (concrete, steel shapes, reinforcing bar, and shear connectors). A theoretical model has been presented to cover the full range of non-linear behavior of such beams allowing for differential movement at the interface i.e., slip and separation.

The compatibility and equilibrium equations have been derived and arrived at four differential equations in terms of four independent displacements. Finite difference method has been used to solve these equations. Comparing with the available experimental data, the application of the current model gives close prediction.

الخلاصـــــة

يمكن تعريف المنشات المركبة بأنها تعتمد على الاستخدام الأمثل للمواد المختلفة بهدف الاستفادة من الخواص المرغوب بها نتيجة للميزة التي تمتلكها عند وضعها في مواقعها المتخصصة.

تختص الدراسة المقدمة بدراسة التصرف للجسور المركبة المستمرة المتكونة من خرسانة مسلحة وأشكال حديدية اخذين بنظر الاعتبار التصرف اللأخطي لعناصر المقطع المركب. في هذا البحث تم تقديم نموذج نظري ليغطي حدود التصرف اللأخطي لهكذا عتبات سامحة بحركة تفاضلية في الوجه الداخلي (الانفصال والانزلاق)

المعادلات التوافقية والتوازنية أعطت بالنتيجة أربع معادلات تفاضلية بدلالة أربع إزاحات مستقلة. استعملت طريقة الفروقات المحددة لحل هذه المعادلات وقورنت مع النتائج العملية حيث أعطت بالنتيجة توقعات متقاربة.

1. Introduction

The aim of using or selecting any material in construction is to make full use of its properties in order to get best performance for the structure being constructed keeping in mind the availability, strength, stiffness, workability, durability of the material and economy of construction.

The term composite construction generally means steel beams attached to concrete slab by means of (shear connectors). The functions of these connectors are to transfer horizontal and normal forces between the two components, by sustaining the composite action. Continuous composite beams are used extensively in recent years in the construction of multistory building and bridges because of the great benefit that are obtained from using this type of construction, such as reducing beam design moments, considerable reduction in deflection, the simplification in joint details and increased erection facility due to self-supportive nature of the construction.

Analysis of composite beams with partial interaction has, in general, been based on an approach which has been attributed to Roberts [1], in which the basic equilibrium and compatibility equations are formulated in terms of resulting differential equations and, then solved simultaneously by expressing the displacement derivatives in finite difference form. Al-Amery [2] developed Roberts's approach, taking into consideration non-linear material and shear connector behavior. The resulting non-linear differential equations are expressed in finite difference form and solved iteratively. This approach is prepared for simply supported composite beams only. Herein, the basic differential equations of non-linear behavior of materials and shear connectors, which was presented by Al-Amery, will be developed to cover the case of continuous beams, in which a negative moment existed in the region of the internal supports, and produced tensile forces on the concrete at this region. As tensile strength of the concrete is very low, the concrete will crack at the early stage of loading. This problem is taken in details and modeled accurately, while the region of positive moment was already solved and presented by Al-Amery's solution.

2. Assumptions

1. For each of the concrete slab and steel beam, the assumption of plane sections normal to neutral plane (or axis of the beam) before bending remain plane and normal after bending. This implies that the distribution of strain is linear over the depth of the concrete slab and the depth of the steel beam. Hence no transverse shear deformation exists in concrete slab or steel beam.
2. The shear connection between the two components of composite beam is continuous along the length, i.e. discrete deformable connectors are assumed to be replaced by a medium of negligible thickness having normal and tangential stifnesses.
3. At elastic stage the concrete was assumed to have tensile strength at early stage of loading but, it was assumed to have no tensile strength at subsequent stages of loading and the

forces are transmitted through the steel reinforcing bars and the steel beam at the region of negative moment.

4. Only longitudinal normal strain, slip (connector shear strain) and uplift (separation between concrete slab and steel beam) are taken into account.
5. Perfect bond between concrete and steel reinforcement in the slab is assumed.
6. Friction and bond effect between concrete slab and steel beam are neglected.

3. The Models

A composite beam that is continuous over two spans or more is different from a beam which is simply supported, where the continuous beam is subjected to positive and negative moments at the same span.

Therefore, the full length of the composite beam will be modeled by two different elements. The first element is applicable for the positive moment zone and the second element is applicable for the negative moment zone. The two elements are connected together by continuity property at the point of contra-flexural (i.e. the point of zero moment at the beam span), as shown in **Figs.(1-a)** and **(1-b)**.

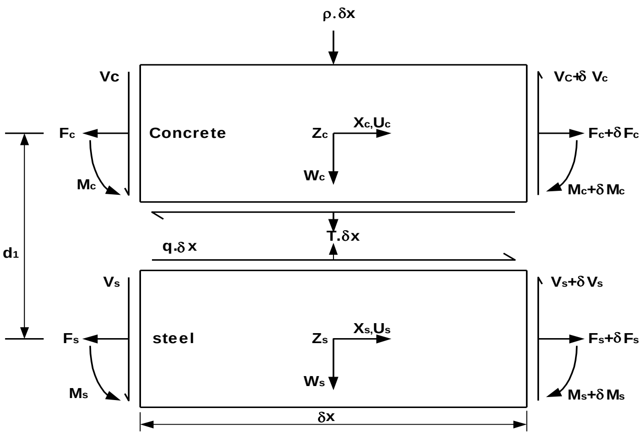


Figure (1-a) Element (1) Typical element of composite beam in the positive moment region

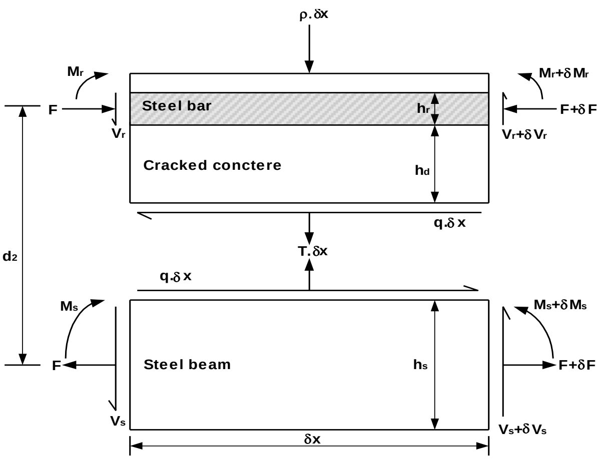


Figure (1-b) Element (2) Typical element of composite beam in the negative moment region

4. Displacements Strain and Stress Resultants

From full length of a composite beam, two elements of length δx, are shown in **Fig.(1-a)** and **(1-b)**. The beam is made from different materials, concrete and steel, for positive moment region and concrete, steel and reinforcing steel bars, for negative moment region jointed at interface by a medium of negligible thickness but having finite normal and tangential stifnesses.

The materials are subjected to moments (M), shear forces (V), and axial forces (F), while q and T denote the shear and normal forces per unit length at the interface.

Assuming that plane sections within each material remain plane, the axial strains ε can be expressed in term of displacements u and w relative to local x and z-axes, which are assumed to pass through the centroids of each material. Hence:

$$\varepsilon_c = u_{ct,x} = u_{c,x} - z_c . w_{c,xx} \quad \text{.......... (1)}$$

$$\varepsilon_s = u_{st,x} = u_{s.x} - z_s . w_{s,xx} \quad \text{.......... (2)}$$

$$\varepsilon_r = u_{rt,x} = u_{r,x} - z_r . w_{r,xx} \quad \text{.......... (3)}$$

in which subscript c, s and r denote the concrete, steel beam and steel reinforcing bars, subscripts, x and xx denote differentiation and z is the distance from the origin of co-ordinates.

Stresses σ can now be related to strains via the material properties (E_c), (E_s) and (E_r), which for a linear elastic material behavior are constants. However, for non-linear elastic and elasto-plastic material behavior, E_c, E_s and E_r are functions of strain. Hence the stress in the materials is given by:

$$\sigma_c = E_c \left(u_{c,x} - z_c . w_{c,xx} \right) \quad \text{.......... (4)}$$

$$\sigma_s = E_s \left(u_{s,x} - z_s . w_{s,xx} \right) \quad \text{.......... (5)}$$

$$\sigma_r = E_r \left(u_{r,x} - z_r . w_{r,xx} \right) \quad \text{.......... (6)}$$

The axial forces F and moments M can now be obtained by integrating the stresses, multiplied by the appropriate lever arms z, in the case of moments, over the cross section area of concrete, steel and reinforcing bar, denoted by $A_{c,}$, A_s and A_r. Hence:

$$F_c = \int \sigma_c . dA_c \, ; \; F_s = \int \sigma_s . dA_s \, ; \; F_r = \int \sigma_r . dA_r \quad \text{.......... (7)}$$

$$M_c = -\int \sigma_c . z_c . dA_c \, ; \; M_s = -\int \sigma_s . z_s . dA_s \, ; \; M_r = -\int \sigma_r . z_r . dA_r \quad \text{.......... (8)}$$

5. Equilibrium and Compatibility Equations

Since strains have been defined in terms of four independent variables, four independent equations for two moments region (i.e. negative and positive zone), are required to obtain a solution. These four equations can be obtained by considering the equilibrium of the elements of the composite beam and the compatibility at the interface between the two materials.

For equilibrium of the two elements shown in **Fig.(1-a)** and **(1-b)** in the x-direction.

$$\mathbf{F_{c,x} + F_{s,x} = 0} \quad \textbf{(9)}$$

$$\mathbf{F_{r,x} + F_{s,x} = 0} \quad \textbf{(10)}$$

For equilibrium in the z-direction

$$\mathbf{F_{c,x} + F_{s,x} = \rho = \rho_c + \rho_s + \rho_i} \quad \textbf{(11)}$$

$$\mathbf{F_{r,x} + F_{s,x} = \rho = \rho_c + \rho_s + \rho_i} \quad \textbf{(12)}$$

In which ρ is the total distributed load per unit length (superimposed load ρ_i plus dead loads ρ_c and ρ_s).

Tacking moments about the origin of co-ordinates in the concrete for positive moment and reinforcement steel bars for negative moment respectively gives:

$$\mathbf{M_{c,x} + M_{s,x} = V_c + V_s + F_{s,x}.d_1} \quad \textbf{(13)}$$

$$\mathbf{M_{r,x} + M_{s,x} = V_r + V_s + F_{s,x}.d_2} \quad \textbf{(14)}$$

in which d_1 and d_2 are the distances between the centroids of the concrete and the steel beams, and the steel bars and the steel beam cross-sections, respectively.

Combining equations (11) and (13) gives:

$$\mathbf{M_{c,xx} + M_{s,xx} - F_{s,xx}.d_1 = \rho} \quad \textbf{(15)}$$

Similarly, equation (12) and (14) gives

$$\mathbf{M_{r,xx} + M_{s,xx} - F_{s,xx}.d_2 = \rho} \quad \textbf{(16)}$$

The slip, u_{cs} at the interface between the two materials is defined as the relative displacement in the x-direction of initially adjacent particle. If z_{ci}, z_{si}, and z_{ri} denote the z-coordinates of the interface in the materials (z_{si} being negative), u_{cs} for positive and negative moment zones are given by:

$$u_{cs} = (u_c - z_{ci} . w_{c,x}) - (u_s - z_{si} . w_{s,x}) \quad (17)$$

$$u_{cs} = (u_r - z_{ri} . w_{r,x}) - (u_s - z_{si} . w_{s.x}) \quad (18)$$

If the shear stiffness of the joint per unit length is denoted by K_s, then for element (1)

$$q = K_s . u_{cs} = F_{c,x} \quad (19)$$

and for element (2):

$$q = K_s . u_{cs} = F_{r,x} \quad (20)$$

Hence, from equations (17) and (19):

$$F_{c,x} - K_s \{(u_c - z_{ci} . w_{c,x}) - (u_s - z_{si} . w_{s,x})\} = 0 \quad (21)$$

and from equations (18) and (20):

$$F_{r,x} - K_s \{(u_r - z_{ri} . w_{r,x}) - (u_s - z_{si} . w_{s,x})\} = 0 \quad (22)$$

The separation w_{sc} at the interface between the concrete and the steel is the relative displacement in the z-direction i.e.,

$$w_{sc} = w_s - w_c \quad (23)$$

and

$$w_{sc} = w_s - w_r \quad (24)$$

If the normal stiffness of the joint per unit length is denoted by K_n, then:

$$T = K_n . w_{sc} = K_n (w_s - w_c) \quad (25)$$

$$T = K_n . w_{sc} = K_n (w_s - w_r) \quad (26)$$

For equilibrium of the two elements (concrete-steel and reinforcement-steel) in the z-direction:

$$V_{c,x} = \rho_i + \rho_c + T \quad \text{......} (27)$$

$$V_{r,x} = \rho_i + \rho_c + T \quad \text{......} (28)$$

For moment equilibrium of the two elements (concrete-steel and reinforcement-steel) about the interface:

$$V_c = M_{c,x} + q.z_{ci} \quad \text{......} (29)$$

$$V_r = M_{r,x} + q.z_{ri} \quad \text{......} (30)$$

Hence, combining equations (27), (29), and (23) gives:

$$M_{c,xx} + F_{c,xx}.z_{ci} - K_n(w_s - w_c) = \rho_i + \rho_c \quad \text{......} (31)$$

and equations (28), (30), and (24) gives:

$$M_{r,xx} + F_{r,xx}.z_{ri} - K_n(w_s - w_r) = \rho_i + \rho_c \quad \text{......} (32)$$

Equations (9), (15), (21), and (31) (for positive moment zone) and equations (10), (16), (22), and (32) (for negative moment zone) are the equilibrium and compatibility equations required for a complete solution, which can be expressed in terms of displacement derivatives, after substitution from equations (4)-(8). These equations cover the whole length of the continuous beam.

6. Material Properties

Integration of equations (7) and (8) to determine the axial forces and moments in the materials requires specification of the material properties E_c ,E_s, and E_r in equations (4), (5), and (6). Solution of the four equilibrium and compatibility equations also requires specification of the shear and normal stiffness of the connector, K_s and K_n. In general, E_c, E_s, E_r, K_s, and K_n are all functions of strain or displacement and solutions had to be obtained iteratively.

The assumed uniaxial stress-strain curves, neglecting the influence of coexistent shear stress, are as shown in **Fig.(2)** [3] and **Fig.(3)** [4] in which the appropriate units, are Newton and millimeters. The concrete is assumed to have no tensile strength and the ultimate compressive strain is limited to 0.0035. The curved portion of the stress-strain curve of concrete is defined by the equation.

$$\sigma = 5500\sqrt{f_{cu}}\varepsilon - 11.3*10^6\varepsilon^2 \quad \text{......} (33)$$

in which f_{cu} is the concrete cube strength. For simplicity, a bilinear stress-strain curve was assumed for the steel beams and reinforcing steel bars, with equal yield stresses σ_y in tension and compression.

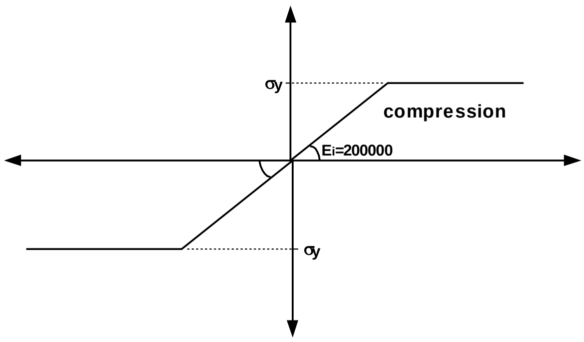


Figure (2) Stress-Strain curve for steel

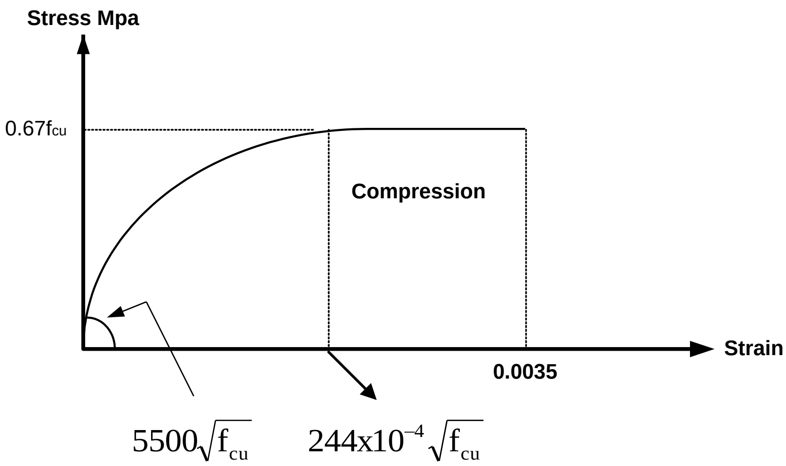


Figure (3) Stress-strain curve for concrete

The assumed shear force Q vs. slip u_{cs} curve for the shear connectors can be represented by equation [4]:

$$\mathbf{Q = Q_u\left(1 - e^{-\phi ucs}\right)} \quad \text{.. (34)}$$

in which Q_u is the ultimate shear strength of a connector and Φ is a constant, which can be determined from test result.

If, for example, the slip at load $\overline{Q}$ is equal to $\overline{u}_{cs}$, then from equation (34).

$$\mathbf{\phi = \frac{1}{\overline{u}_{cs}} \ln\left\{\frac{Q_u}{Q_u - \overline{Q}}\right\}} \quad \text{.. (35)}$$

The tangent stiffness, K_s is given by differentiating equation (34) once with respect to slip value:

$$K_s = dQ/du_{cs} = Q_u.\phi\left(e^{-\phi u_{cs}}\right) \qquad (36)$$

The tangent value of the material properties E was used in the iterative analysis.

For most composite construction, separation of the two materials is negligible since K_n is relatively large. Therefore, for simplicity, a relatively large constant value of K_n is assumed.

7. Layered Model

For non-linear analysis, the modulus of elasticity for a composite beam is a function of strain value at the point under consideration. But the strains vary across the depth of the beam. This difficulty can be overcome by using the layered system in which, the cross sectional area of each material is divided into a number of layers as shown in **Fig.(4)**, so that:

$$EA = \int_A E\,dA = \sum_{i=1}^{n} E_{i.}.A_i \qquad (37)$$

$$EI = \int_A E\,z^2\,dA = \sum_{i=1}^{n} E_{i.}.(z_i)^2 A_i \qquad (38)$$

where,

n: the number of layers in the material under consideration.

Ei: A layer modulus of elasticity.

Z_i: the distance from the layer center to the origin of co-ordinates.

A_i: the cross-sectional area of the layer.

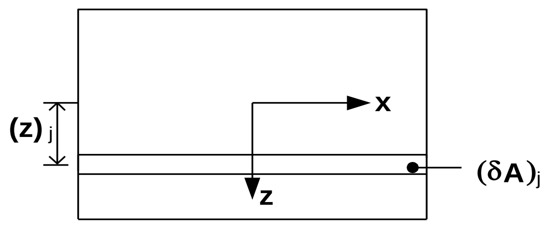


Figure (4) Subdivision of cross-section into elemental area

The values of tangential modulus, K_S, and normal modulus, K_n, of the shear connector layer, elastic modulus of concrete slab steel reinforcement, E_r, and steel beam modulus of elasticity, E_S, and concrete slab modulus of elasticity, E_C, are obtained from the corresponding constitutive relationship as discussed previously.

8. Numerical Integration of Force-Displacement Equations

When the material properties E are constants, equations (7) and (8) can be integrated analytically to give the axial forces F and moment M in terms of displacements u and w. Alternatively, if the material properties are non-linear, the layered approach is used to overcome these problems. In the layered approach, the cross-sectional area of the concrete, steel shape and reinforcing bars are divided into a chosen number of layers having areas (A_C^i), (A_S^j), (A_r^k) from the origin of co-ordinates, respectively. Hence, from equations (4) to (8).

$$F_c = \sum_j (E_c)_j \{u_{c,x} - (z_c)_j w_{c,xx}\}(\delta A_c)_j \quad \text{......} \quad (39)$$

$$F_s = \sum_j (E_s)_j \{u_{s,x} - (z_s)_j w_{s,xx}\}(\delta A_s)_j \quad \text{......} \quad (40)$$

$$F_r = \sum_j (E_r)_j \{u_{r,x} - (z_r)_j w_{r,xx}\}(\delta A_r)_j \quad \text{......} \quad (41)$$

$$M_c = \sum_j (E_c)_j \{u_{c,x} - (z_c)_j w_{c,xx}\}(z_c \delta A_c)_j \quad \text{......} \quad (42)$$

$$M_s = \sum_j (E_s)_j \{u_{s,x} - (z_s)_j w_{s,xx}\}(z_s \delta A_s)_j \quad \text{......} \quad (43)$$

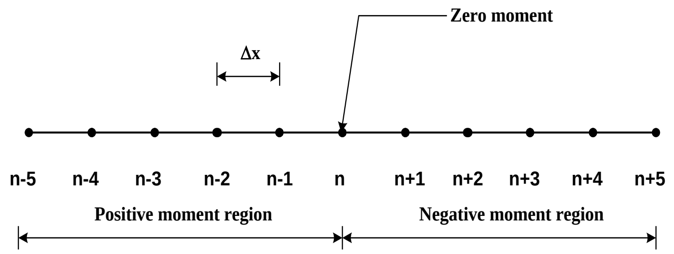
$$M_r = \sum_j (E_r)_j \{u_{r,x} - (z_r)_j w_{r,xx}\}(z_r \delta A_r)_j \quad \text{......} \quad (44)$$

The appropriate values of $(E)_j$ for the strips $(\delta A)_j$ are the tangent value determined from the assumed stress-strain curve, as discussed in the previous section, corresponding to the total strain $(\varepsilon)_j$ in strips (δA_j). For example, for concrete section, $(E_c)_j$ the corresponding strain $(\varepsilon_c)_j$, is given by:

$$(\varepsilon_c)_j = u_{c,x} - (z_c)_j w_{c,xx} \quad \text{......} \quad (45)$$

9. Finite Difference Analysis

Substituting equations (39)-(44) into equations (9), (15), (21), and (31) and equations (10), (16), (22), and (32) provides a set of four simultaneous differential equations for each moment zone in terms of the displacements u and w in each material. These equations can be solved by expressing the displacement derivatives in finite difference form and solving the resulting set of algebraic equations. These equations contain mixed derivatives of different type of variables with different order, which must be expressed in finite (central) difference form. These equations contain derivatives of fourth order in (w), so that five nodes are required to represent them in the finite difference form and this leads to define two external nodes at each end of the beam to verify the substitution of the differential equations at the ends of the beam. Since each node is assigned with four degrees of freedom, eight boundary conditions are requiring at each end. It is required for a complete solution, to define the interface between the two systems of differential equations near the point of contra flexure, in which the material properties has to be separated for the two moment a region at that point. This can be illustrated by the following example:



The fourth derivative of the deflection of the upper component of the composite element at positive moment region (for example at node number (n-3)) can be expressed as follows:

$$\mathbf{W_{xxxx}=\frac{W_{c,n-5}-4W_{c,n-4}+6W_{c,n-3}-4W_{c,n-2}+W_{c,n-1}}{\Delta x^4}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \mathbf{(46)}$$

in which (w_c) represent the displacement variable at the concrete. The same derivative can be expressed at negative region (for example at node number (n+3)) as follows:

$$\mathbf{W_{xxxx}=\frac{W_{r,n+1}-4W_{r,n+2}+6W_{r,n+3}-4W_{r,n+4}+W_{r,n+5}}{\Delta x^4}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \mathbf{(47)}$$

in which (w_r) represent the displacement variable at the reinforcing bar.

The fourth order derivative of the same variable at the point of contra flexural can be expressed as follows:

$$\mathbf{W_{xxxx}=\frac{W_{c,n-2}-4W_{c,n-1}+6W_{c,n}-4W_{r,n+1}+W_{r,n+2}}{\Delta x^4}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \mathbf{(48)}$$

Different types of finite difference stencil are used near the point of contra flexural to verify the continuity conditions at this region.

To complete the set of algebraic equations, the boundary conditions for a continuous beam of length (L), used in this analysis, are:

$\mathbf{W_s = 0}$ when x= 0 and L ……………………………………………... **(49)**

$$W_{s,xx} = 0 \quad \text{when } x= 0 \quad (50)$$

$$W_{s,x} = 0 \quad \text{when } x= L \quad (51)$$

$$W_{c,xx} = 0 \quad \text{when } x=0 \quad (52)$$

$$W_{r,x} = 0 \quad \text{when } x= L \quad (53)$$

$$u_{s,x} = 0 \quad \text{when } x= 0 \quad (54)$$

$$u_{s} = 0 \quad \text{when } x= L \quad (55)$$

$$u_{c,x} = 0 \quad \text{when } x= 0 \quad (56)$$

$$u_{r} = 0 \quad \text{when } x= L \quad (57)$$

$$V_c + V_s = R_O \quad \text{when } x= 0 \quad (58)$$

$$V_r + V_s = R_L \quad \text{when } x= L \quad (59)$$

$$u_{c,xxxx} = 0 \quad \text{when } x= 0 \quad (60)$$

$$u_{r,xxxx} = 0 \quad \text{when } x= L \quad (61)$$

$$u_{s,xxxx} = 0 \quad \text{when } x= 0 \text{ and } L \quad (62)$$

To express the shear forces carried by the reinforcing steel bar in terms of displacement derivatives, moments are taken about the origin of co-ordinates of the reinforced concrete element, as:

$$V_r = M_{r,x} + F_{r,x}.z_{ri} \quad (63)$$

After introducing the boundary conditions, the non-linear algebraic equations can be solved iteratively. Initially, all materials are assumed to have constant properties, and a set of nodal displacements corresponding to a specified applied loading, is determined. From these displacements, slip at the interface and strains throughout the composite beam can be determined, which are used to define the tangent values of the material properties for the second stage of the solution. The process is repeated until the calculated displacements have converged, according to a prescribed criterion. For subsequent values of the applied loading, the iterative procedure is commenced with tangent values of the material properties corresponding to the previously converged solution, which reduces the required number of iterations.

10. Illustrative Example

To illustrate the application of the theory presented herein, the example which is presented by Yam and Chapman [5] and tested by Teraskiewicz [6] was used to carry out a convergence study and to examine the effect of some properties on the behavior of a continuous composite beam. A single continuous composite beam of two equal spans (336 cm) are considered to be subjected to a concentrated load at the middle of each span. Because of symmetry, half of the continuous beam is considered. The calculated ultimate load based on plastic theory is (130 kN).

At each load increment a convergence limit is introduced on slip values, since the slip is more sensitive than other parameters to the change in loading. The properties of the beam are given below. All dimensions in the original reference were in imperial units and have to be presented herein in SI-units.

Concrete Slab

Width of concrete slab	=92 cm.
Depth of concrete slab	=6 cm.
Cross sectional area	=552 cm^2.
d_1	=10.6 cm.
Initial modulus of elasticity	=3790 kN/cm^2.
Ultimate compressive strength	=47.5 MPa.
Ultimate compressive strain	=0.0035.

Steel Beam

I-beam (6 in * 3 in * 12 Ib/ft, rolled steel joist)

Overall depth (h_s)	=15 cm.
Flange width	=7.5 cm.
Cross sectional area	=22.77 cm^2.
Moment of inertia	=874 cm^4.
Initial modulus of elasticity	=20700 kN/cm^2.
Yield stress	=300 MPa.
Yield strain	=0.0014.

Steel Reinforcing Bar

Total area	=4.45 cm^2.
Moment of inertia	=0.52 cm^4.
Initial modulus of elasticity	=20700 kN/cm^2.
Yield stress	=320 MPa.
d_2	=12.3 cm.

Shear Connector

The connection between the concrete slab and the steel beam was assumed to be provided by pairs of (10mm.) diameter, (50 mm.) length, headed studs, with spacing of (14.6 cm) and initial shear stiffness (K) of (1935 kN/cm). The ultimate shear strength of a single stud, (Qu) was taken as (100 kN), while the slip ($\overline{U}_{cs}$) corresponding to shear force ($\overline{Q}$=62 kN) was taken as (0.5 mm). The value of (Φ) can be determined after substituting the values of $\overline{Q}$, Q_u and $\overline{U}_{cs}$ in equation (35), and was found to be equal to (1.935).

It should be noted that the concrete slab was divided into ten equal strips. Each flange of the steel beam was divided into four equal strips and the web was divided into ten strips. The reinforcing bars were treated as one strip.

The following results were obtained using 25 nodes along the length of the beam, including the four external nodes required to specify the boundary conditions. Solutions were considered to have converged when the change in the maximum slip at the ends of the beam was less than 0.002mm.

The variation of the central deflection wmax with the concentrated load p is shown graphically in **Fig.(5)**. The numerical results appear to be converging to a value pu =140 kN, which is the same concentrated load corresponding to the ultimate flexural strength of the composite beam, based on assumed rectangular plastic stress blocks.

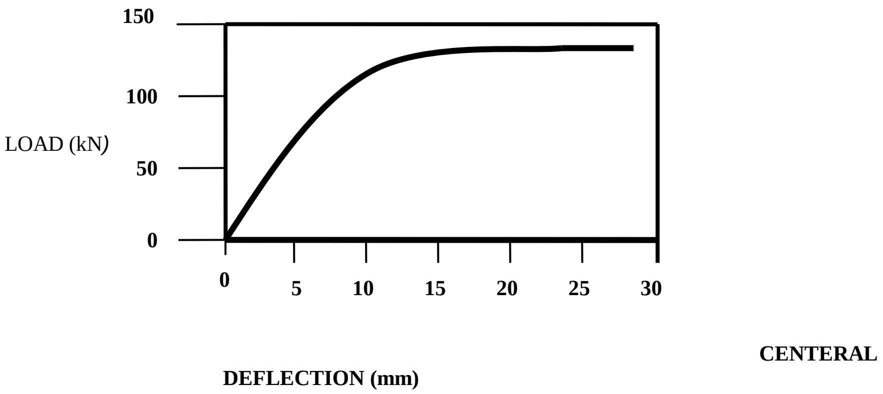


Figure (5) Load deflection curve

In order to assess the convergence of the non-linear (iterative) finite difference solution, results were obtained by using different numbers of nodes along the beam (including the two external nodes required to specify the boundary conditions) and different convergence limits on the slip at the simply supported end of the continuous beam and at the different applied load levels, as can be seen in **Table (1)**.

Table (1) Convergence of iterative solution

No. of Node	45			25			10		
δu_{cs}	$1*10^{-06}$mm			0.002mm			0.002mm		
Load (%)	It	u_{cso}	W_{cl}	It	u_{cso}	W_{cl}	It	u_{cso}	W_{cl}
30	7	0.028	2.06	5	0.028	2.09	2	0.021	1.8
55	11	0.07	5.8	7	0.069	5.71	5	0.05	5.07
85	54	0.161	12.3	20	0.140	11.3	16	0.102	10.9
95	32	0.22	18.2	15	0.196	17.1	13	0.167	17.03

It: number of iterations; u_{cso}: slip at support
w_{cl}: central deflection; δu_{cs} convergence limit

The variation of the interface slip values has been calculated along the beam span for different load levels up to failure as shown in **Fig.(6)**. As the load is increased, values of interface slip along the beam have been increased, with zero value at the position of external load and the middle continuous support. Also, it can be seen that slip values are unsymmetrical on both sides of load as different boundary conditions exist, with higher values at the middle continuous due to the high shear values. It should be noted that, zero slip values were located at the points of contra flexural (i.e. points of high shear, which are near the concentrated load position for positive moment and at interior support for negative moment). This is due to change in the direction of the longitudinal shear force at the interface, on both sides of these points.

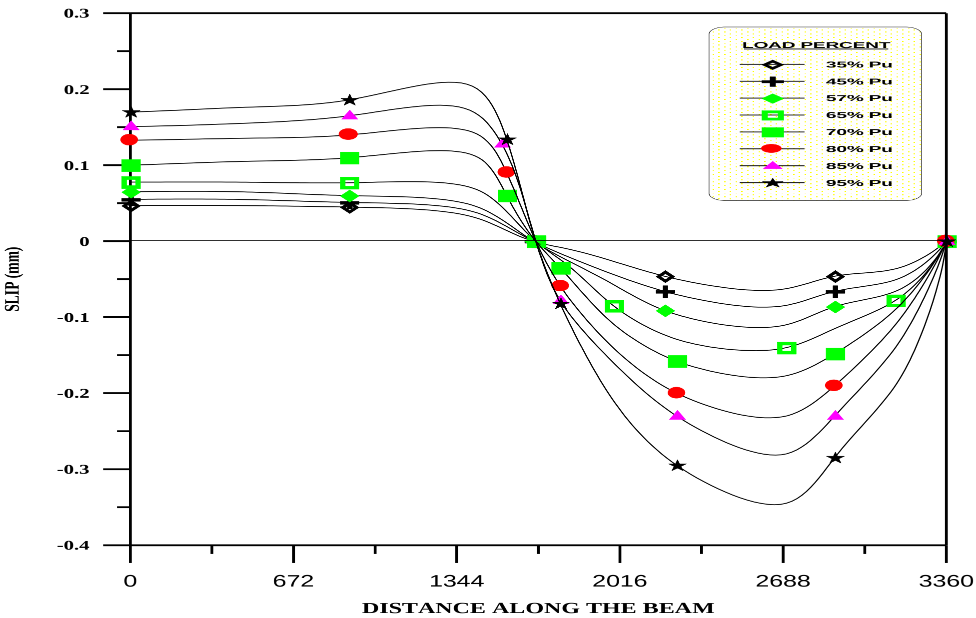


Figure (6) Slip distribution for different load levels

Figure (7) shows the variation of deflection values along the span for different load levels up to failure. As the load is increased, values of deflection along the beam have been increased with maximum value being at the span center region.

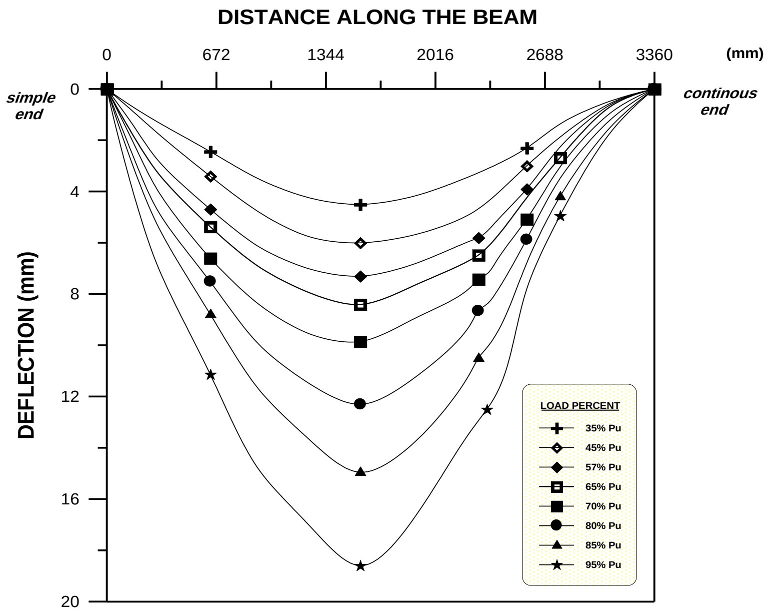
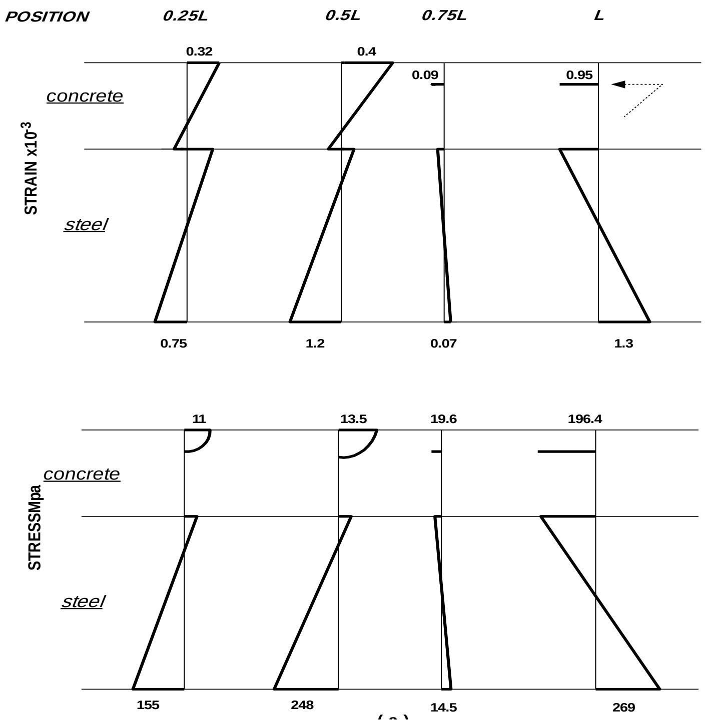
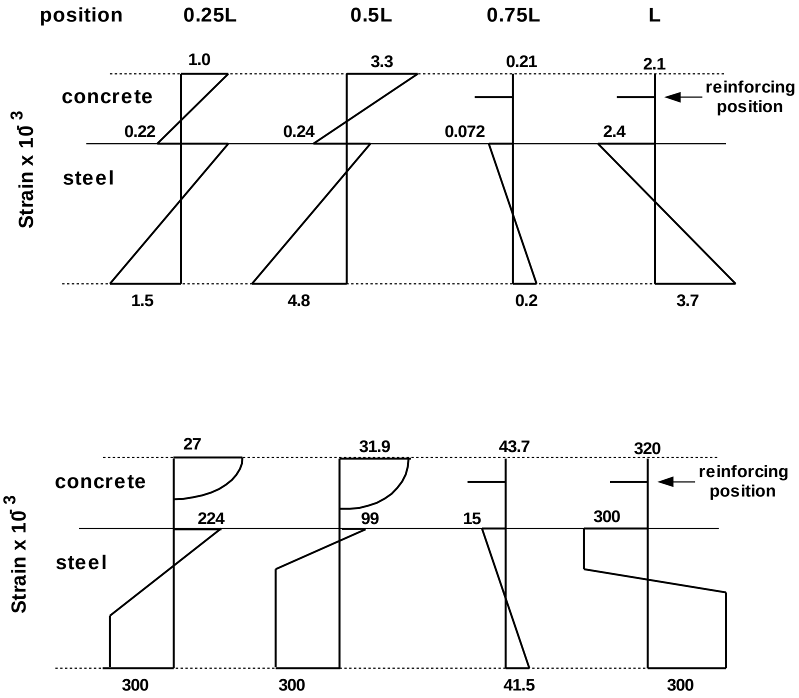


Figure (7) Deflection distribution for different load levels

Strain and stress profiles throughout the depth of the composite beam, corresponding to service and ultimate loads are shown in **Fig.(8)**. The discontinuous strain profiles indicate the existence of slip at the interface between the concrete and steel, while the stress profiles indicate the spread of plasticity.



(a)



(b)

Figure (9) Stress and strain profiles
(a) For services load
(b) For ultimate load

11. Discussion and Conclusions

A general formulation for the analysis of continuous composite beams with partial interaction, which incorporates the influence of slip and separation at the interface between the materials and non-linear material and shear connector behavior, has been developed. Solutions of the four basic equilibrium and compatibility equations are obtained by expressing the displacement derivatives in finite difference. This method with incremental-iterative solution technique is efficient for the non-linear analysis of continuous composite beams and gives a good saving in computer time and effort.

12. References

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