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Frequency Hopped-Orthogonal Frequency Division Multiple Access (FH-OFDMA) System Performance with Extending Quadratic Congruence (EQC) Hoping Code patterns

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Abstract

In this paper Frequency-Hopped Orthogonal Frequency Division Multiple Access (FH-OFDMA) system is considered. The performance of different Extending Quadratic Congruence (EQC) orthogonal hopping code patterns is examined in relation to OFDM transmission.

The performance is evaluated in terms of Bit Error Rate (BER) in white Gaussian noise and Rayleigh fading. Multi-user interference (MUI) is eliminated and symbol recovery is guaranteed.

The performance of FH-OFDMA system shows low bit error rate and good correlation properties under the circumference of the assumed channel.

الخلاصـــــــة

في هذا البحث اخذ بالاعتبار النظام التقسيمي الترددي المتعامد المتعدد المداخل ذو القفز الترددي. تم فحص أداء هذا النظام بإستخدام نمط شفرة القفز المتعامد EQC مع نظام النقل التقسيمي الترددي المتعامد.

تم تقييم الأداء من ناحية نسبة الخطأ في قناة AWGN وقناة اخماد رايلي. التداخل بين المستخدمين تم ازالته وتحسين استرجاع العينات في هذا النظام .

أداء هذا النظام أظهر نسبة خطأ قليلة و خصائص ارتباط جيدة ضمن القنوات المفروضة.

1. Introduction

FH-OFDMA is one of the hybrid CDMA-OFDM systems, in which every user has a fixed set of sub-carriers. They are relatively easy changed to allow frequency hopping per timeslot. Hopping with different patterns for users actually transforms the OFDMA system to a Frequency Hopping CDMA system. This has the benefit of increasing the frequency diversity, because each user uses all of the available bandwidth as well as the interference averaging benefit that is common for all CDMA variants [1,2].

Using forward-error correction coding over multiple hops, the system has the ability to correct for sub-carriers in deep fades or sub-carriers that are interfered by other users. Because the interference and fading characteristics is changed for every hop, the system performance depends on the average received signal power and interference, rather than on the worst case fading and interference power.

Frequency hopping OFDMA system can eliminate multi-user interference (MUI) by careful code design [3,4]. Hopping Codes that are used in OFDMA system must be orthogonal in order to eliminate MUI.

To obtain OFDMA scheme, the Hadamard matrix is replaced by the identity matrix. Note that the spreading and dispreading multiplications do not need to be implemented in this scheme since only one position per codeword is nonzero. The OFDMA scheme can therefore be claimed to be of lower computational complexity compared to the schemes that use Hadamard codes [5].

This paper is organized as follows. In section 2 the FH-OFDMA system model is considered. Section 3 deals with EQC hopping sequence generator. Section 4 defines the FH-OFDMA modulation and demodulation. Simulation and results are presented in section 6.

2. FH-OFDMA System

The baseband block diagram of FH-OFDMA system is given in **Fig.(1)**. Data are converted from serial to parallel with block size M.

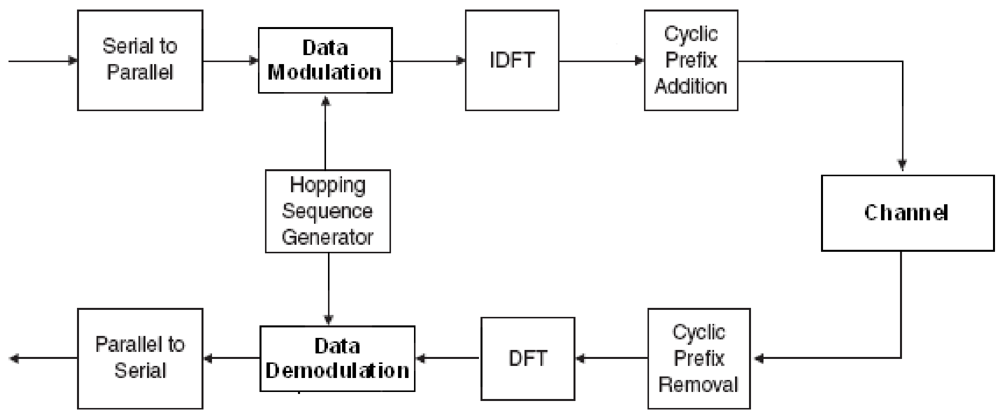


Figure (1) Frequency-hopping baseband OFDM transmitter and receiver structure

Let $X_k(i,n) := X(i, nM + k),\ k = 0,1,\cdots, M-1$, denote the kth symbol of the nth data block generated by the ith user.

Then, the vector $x(i,n) = [X_0(i,n)\ X_1(i,n) \cdots X_{M-1}(i,n)]^T$ represents the ith user's nth data block [4].

Every user transmits over M sub-carriers, which are assigned or selected according to EQC hopping code. Let $C_i(n) = \{ f_{i,n,0}, f_{i,n,1}, \cdots, f_{i,n,M-1} \}$ be the set of sub-carriers which user i transmits its nth block where the ordering of sub-carriers is according to Y_l. Thus $C_i(n) \subset \{1, e^{j2\pi/N}, \cdots, e^{j2\pi(N-1)/N}\}$ has cardinality M for all i and n. The transmission of sub-carriers in $C_i(n)$ are altered for all i at each symbol block time as dictated by a hopping sequence generator.

Define the $M \times 1$ vector as:

$$\mathbf{f}_{i,n}^{m} = [\mathbf{e}^{j2\pi f_{i,n,0} m} \mathbf{e}^{j2\pi f_{i,n,1} m} \cdots \mathbf{e}^{j2\pi f_{i,n,M-1} m}]^T \quad \text{.......... (1)}$$

For $m = 0,1,\cdots, M-1$, then the modulation of the sub-carriers set produces the OFDM symbol.

$$\mathbf{s}(\mathbf{i},\mathbf{n}) = [\mathbf{S}_0(\mathbf{i},\mathbf{n})\mathbf{S}_1(\mathbf{i},\mathbf{n}) \cdots \mathbf{S}_{M-1}(\mathbf{i},\mathbf{n})]^T \quad \text{.......... (2)}$$

where:

$$\mathbf{S}_m(\mathbf{i},\mathbf{n}) = \frac{1}{M}\mathbf{x}^T(\mathbf{i},\mathbf{n})\mathbf{f}_{i,n}^{m} = \frac{1}{M}\sum_{k=0}^{M-1}\mathbf{X}_k(\mathbf{i},\mathbf{n})\mathbf{e}^{j2\pi f_{i,n,k} m} \quad \text{.......... (3)}$$

A cyclic prefix (CP) is appended to each block before transmission, where the last P symbols of the OFDM stream are added to the beginning in order to combat the inter-carrier interference, creating:

$$\mathbf{s}_{CP}(\mathbf{i},\mathbf{n}) = [\mathbf{S}_{M-P}(\mathbf{i},\mathbf{n}) \cdots \mathbf{S}_{M-1}(\mathbf{i},\mathbf{n})\ \mathbf{S}_0(\mathbf{i},\mathbf{n})\mathbf{S}_1(\mathbf{i},\mathbf{n}) \cdots \mathbf{S}_{M-1}(\mathbf{i},\mathbf{n})]^T \quad \text{.......... (4)}$$

The channel output for user (i) is:

$$\mathbf{y}(\mathbf{i},\mathbf{n}) = \mathbf{H}(\mathbf{i},\mathbf{n})\mathbf{s}_{CP}(\mathbf{i},\mathbf{n}) + \mathbf{z}(\mathbf{i},\mathbf{n}) \quad \text{.......... (5)}$$

where: *the discrete-time Fourier transform of the complex Gaussian channel coefficients* $\{h_k(i,n)\}_{k=0}^{M-1}$ *is from the elements of the* $M \times M$ *diagonal matrix* $H(i,n)$ *for user i. The vector* $z(i,n)$ *represents the aggregate of AWGN as well as, possible MUI to user i.*

3. Hopping Sequence Generator

The frequencies which are determined by EQC construction can be given by [6]:

$$\mathbf{Y_k} = \begin{cases} \mathbf{0} & \mathbf{if} \quad \mathbf{k = 0} \\ [\mathbf{Y_{k-1} + ak}]\mathbf{mod}(\mathbf{N}) & \mathbf{if} \quad \mathbf{1 \le k \le (N-1)/2} \\ [\mathbf{Y_{k-1} + bk}]\mathbf{mod}(\mathbf{N}) & \mathbf{if} \quad \mathbf{(N+1)/2 \le k \le N-1} \end{cases} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \mathbf{(6)}$$

where:

a and b: are some integers members of the set $J_N = [1,2, \ldots, N-1]$ and,

N: is assumed to be an odd prime.

Equation (6) can be rewritten in closed form as [6]:

$$\mathbf{Y_k} = \begin{cases} [\mathbf{a.k(k+1)/2}]\mathbf{mod}(\mathbf{N}) & \mathbf{if} \quad \mathbf{0 \le k \le (N-1)/2} \\ [\mathbf{b.k(k+1)/2 - (a-b).(N^2-1)/8}] & \mathbf{if} \quad \mathbf{(N-1)/2 \le k \le N-1} \end{cases} \quad \ldots\ldots\ \mathbf{(7)}$$

The sequence of the integers Y_k defined in Eq.(7) is permutation for the set $J_N = [1,2, \ldots, N-1]$ if and only if a and b are not both, quadratic residues (QR) or quadratic non-residues (QNR) for the odd prime N, each permutation is uniquely defined by the ordered pair (a, b).

In the case of odd primes, Gauss show [6] that α is QR if and only if $[\alpha^{(N-1)/2}]$ mod(N)=1 and that β is QNR if and only if $[\beta^{(N-1)/2}]$ mod(N)=-1, where α=a and β=b. The number of QRs is equal (N-1)/2 and the number of QNRs is also equal to (N-1)/2, therefore the numbers of extended quadratic code word are exactly equal to $2((N-1)/2)^2$ [6].

4. FH-OFDMA Modulator

FH-OFDMA system can be considered as a frequency hopped multi-carrier on-off keying FH-MC-OOK [7], Modulation and demodulation processes of FH-OFDMA depending on those operations in FH-MC-OOK will be explained. **Figure (2)** shows the block diagram of FH-OFDMA (FH-MC-OOK) modulator.

The tone frequencies are chosen such that modulator outputs are orthogonal. The minimum tone spacing which makes the signal non-coherently orthogonal is $1/T$, where T is the symbol (M-ary) duration [8, 9]. Let:

$$\mathbf{d_l^{(k)} = \left(d_{0,l}^{(k)}, d_{1,l}^{(k)}, \cdots, d_{M-1,l}^{(k)}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \mathbf{(8)}$$

Be the binary representation of the transmitted symbol *l* of user k, where $l \in \{0,1,\cdots,M-1\}$ and $k \in \{0,1,\cdots,K-1\}$.

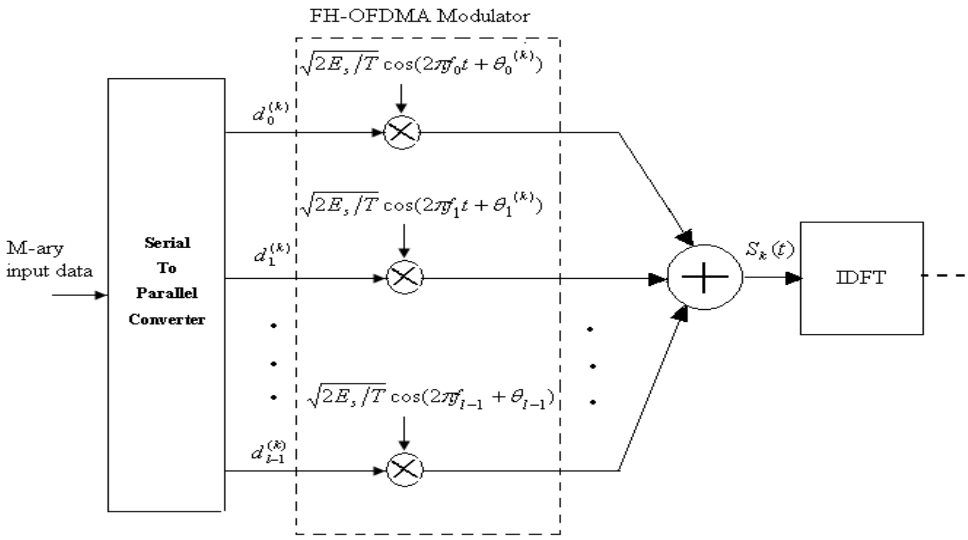


Figure (2) FH-OFDMA modulator

The output signal from modulator of user *k* is:

$$s_k(t)=\sqrt{\frac{2E_s}{T}}\sum_{i=0}^{M-1} d_{i,l}^{(k)}\cos\left[2\pi\left(f_h^{(k)}+f_i\right)t+\theta_i^{(k)}\right],\qquad 0\le t\le T \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (9)$$

where:

E_s: is the energy per tone,

$f_h^{(k)}$: is the hopping frequency of user k,

fi: is the tone frequency, and

$\theta_i^{(k)}$: is the phase in ith tone of user k.

We assume that $\{\theta_i^{(k)}\}$, $i=0,1,\cdots,M-1$ and $k=01,\cdots,K-1$, are independent and identically (uniformly distributed over [0,2π]), and the tone spacing $|f_i - f_{i+1}|$ is *1/T*, which makes the signal non-coherently orthogonal [8,9].

The channel is assumed to be a slow frequency nonselective Rayliegh fading, which is appropriate when the signal bandwidth is much smaller than the coherence bandwidth of the channel. For all except very slow hopping cases, detection is normally performed non-coherently because of the difficulty in maintaining carrier phase reference under changing frequency condition [10].

5. FH-OFDMA Demodulator

Figure (3) shows FH-OFDMA demodulator, where the output signal from DFT is non-coherent demodulated, and decision is made independently by comparing the energy detector output $r_i = r_{ic}^2 + r_{is}^2$ with a threshold γ.

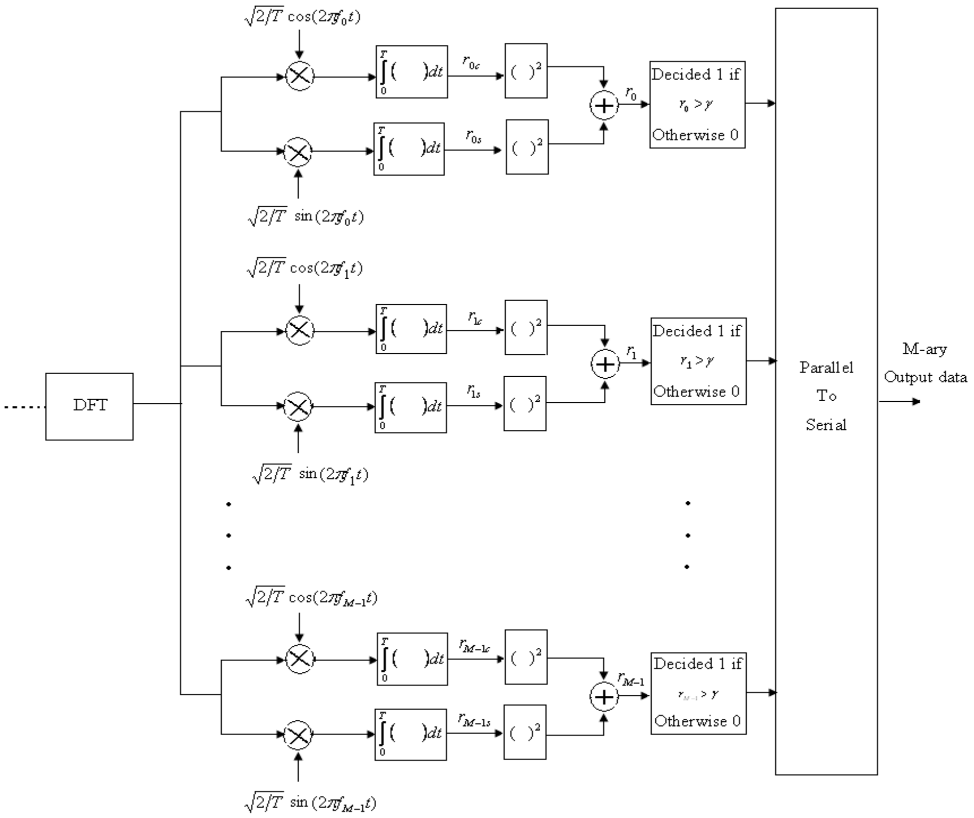


Figure (3) FH-OFDMA demodulator

If the zeroth user is assumed to be the reference user, then the correlator output r_{ic} and r_{is} for ith tone, i=0, 1,…, M-1, is given as [8]:

$$\begin{aligned} \mathbf{r_{ic}} &= \int_0^T \mathbf{r'(t)}\sqrt{\frac{2}{T}}\cos(2\pi f_i t)\,dt \\ &= d_{i,l}^{(o)}\sqrt{E_s}\beta_0\cos\theta_i^0 + \sum_{k=1}^{m} d_{i,l}^{(k)}\sqrt{E_s}\beta_k\cos\theta_i^k + n_{ic} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10) \\ \mathbf{r_{is}} &= \int_0^T \mathbf{r'(t)}\sqrt{\frac{2}{T}}\sin(2\pi f_i t)\,dt \\ &= -\left[d_{i,l}^{(0)}\sqrt{E_s}\beta_0\sin\theta_i^0 + \sum_{k=0}^{m} d_{i,l}^{(k)}\sqrt{E_s}\beta_k\sin\theta_i^k + n_{is}\right] \end{aligned}$$

where:

$r'(t)$ *: is the output signal from DFT and*

n_{ic} *and* n_{is}*: are independent Gaussian random variables with mean zero and variance N0/2. If* r_i *is greater than* γ *we decide logic "1" is transmitted in the ith tone, otherwise, we decide logic "0" is transmitted in the ith tone.*

6. Simulation and Results

FH-OFDMA system was modeled using Matlab 7 to allow various parameters of the system to be varied and tested. The aim of simulation is to evaluate the performance of FH-OFDMA under different channel conditions, and to allow for different FH-OFDMA configurations to be tested. Three main criteria were used to assess the performance of the FH-OFDMA system, system under AWGN, under AWGN and one path fading, under AWGN and one path fading with Compensation [10]. All tests were obtained with fast and slow hopping with two different word lengths EQC code.

6-1 FH-OFDMA under AWGN

BER of FH-OFDMA system four users under AWGN is shown in **Fig.(4)**.

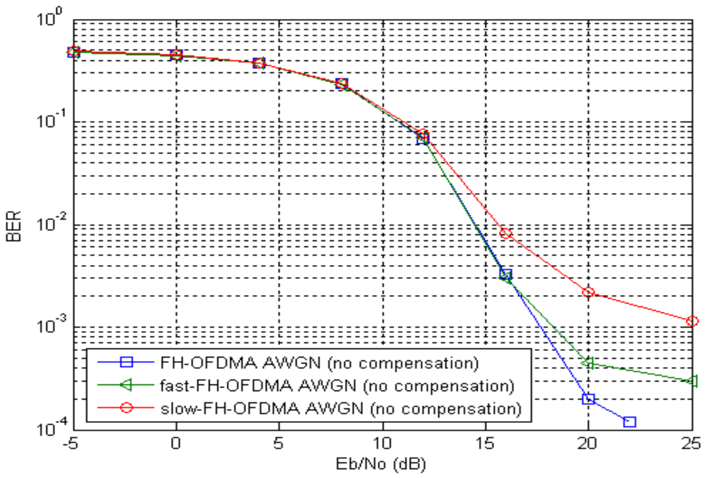
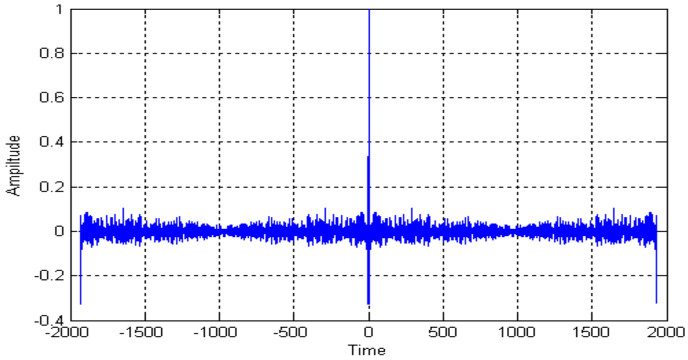
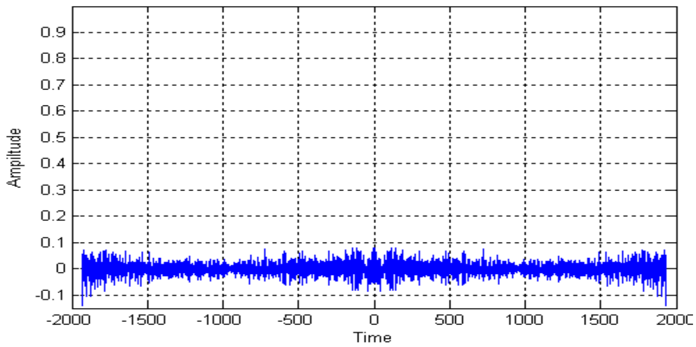


Figure (4) BER of FH-OFDMA under AWGN for 4-users

From figure 4 it is clear that slow-FH-OFDMA is the worst between all three categories. FH-OFDMA with EQC code show a good auto-and cross correlation properties as shown in **Fig.(5)**. The normalized sidelobe level and the peak of cross correlation are less than 0.1.



(a)



(b)

Figure (5) Correlation properties of FH-OFDMA signal; (a) auto-correlation, (b) cross- correlation

6-2 FH-OFDMA under one Path Fading and Perfect Compensation

BER performance of FH-OFDMA system under one path fading is shown in **Fig.(6)** while the perfect compensation for one path fading is shown in **Fig.(7)**.

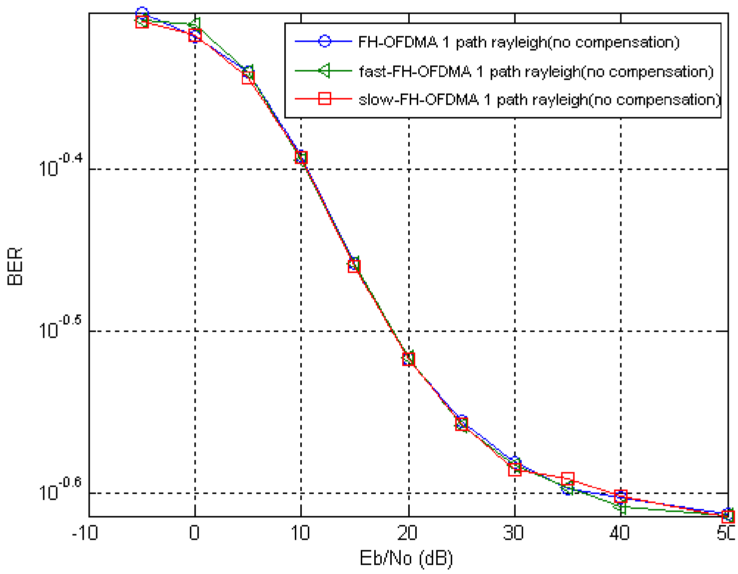


Figure (6) BER performance of FH-OFDMA under one path fading

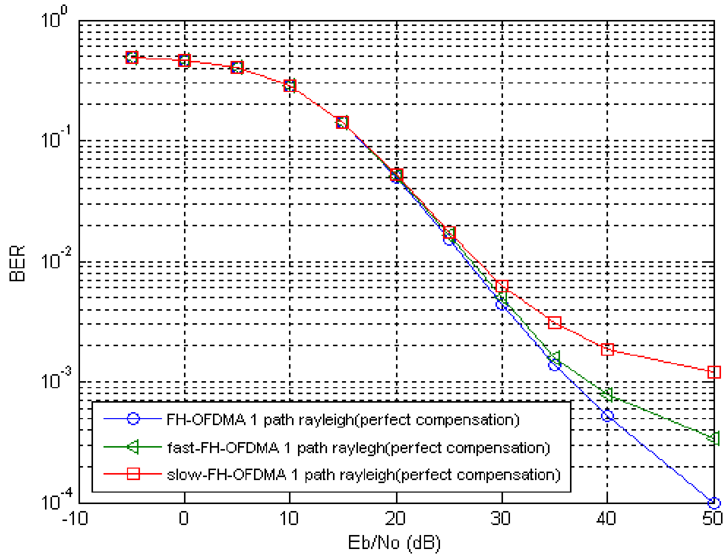


Figure (7) BER performance of FH-OFDMA under one path fading and perfect compensation

From **Fig.(6)**, it is noted that, fading in FH-OFDMA that caused high BER could be overcome by using perfect compensation.

6-3 FH-OFDMA under AWGN with Two Different Word Lengths and Different Number of Users

In this section EQC of lengths 4 and 10 have been used. **Figure (8)** show good BER performances. Increasing the word length of EQC code enhances the BER performance. Since the number of code words for each length is $2[(N-1)/2]^2$ so it is applicable for multiple access systems.

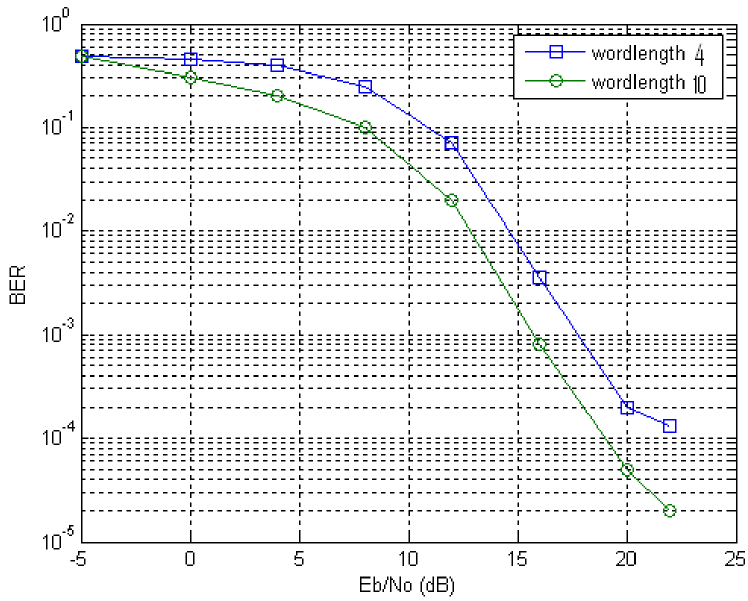


Figure (8) BER of FH-OFDMA under AWGN with two word length

BER of FH-OFDMA system with various numbers of users under AWGN is shown in **Fig.(9)**.

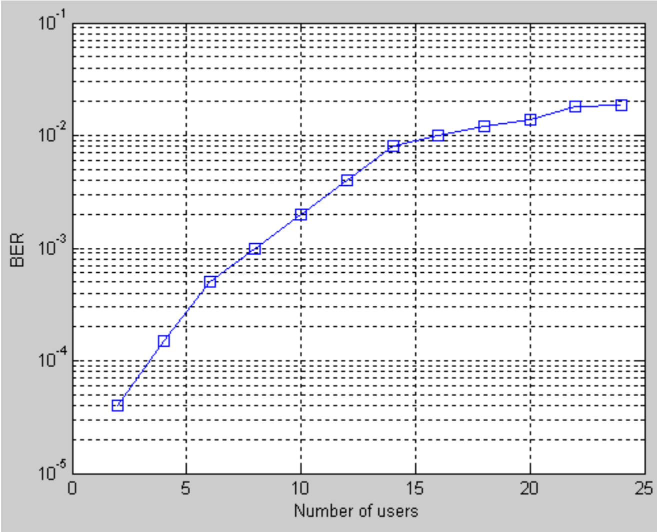


Figure (9) BER of FH-OFDMA under AWGN with various number of users

From **Fig.(9)** it is clear that BER increased with the increasing of number of users. We did not use this number of users that reached to 25 users in the other tests because of delay time that occurs in running Matlap program, running time reaches to two hours and half for one reading, so we take 4 users to make the running time as short as possible. This reason is the same for way that we take 4 and 10 word length of EQC code. As long as word length of the hopping code is long enough reaches to 1000 hop per user [11] BER will be decreased.

7. Conclusion

This paper focuses on the performance of FH-OFDMA system. The system is tested under different channel conditions using different EQC hopping code patterns. It was shown that EQC code gives high orthogonality and perfect low interference between users. Perfect compensation is used in the test and it is capable to overcome the selective fading in FH-OFDMA system. According to the results FH-OFDMA system can be considered as a high bit rate system with optimal BER performance, MUI elimination and high symbol recovery, making this system capable for uplink transmission.

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