Journal of Engineering and Development, Vol. 12, No. 1, March (2008) ISSN 1813-7822

Optimum Safe Hydraulic Design of Culverts

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Abstract

This research aims at obtaining the "Optimum safe hydraulic design of culverts", taking into consideration the cost of excavation, bedding, material, compacted fill, hunches, protection works, and additionally involved head loss.

Because of numerous shapes and materials of culverts, the wide-fame among them have been selected in this research, namely, reinforced concrete box culverts of both rectangular and squared shape, and pipe culverts of circular shape with materials of reinforced concrete, cast-iron, asbestos-cement, and ductile-steel.

To be close to field conditions, discharges of (0.5, 1.0, 1.5, 2.5, 5.0, 10.0, and 15.0m^3/s) have been selected to represent small, medium, and big discharges; culverts lengths of (5, 10, 15, 20, 25, 35, and 40m) have been selected to represent short, medium, and long structures. However, typical trapezoidal earthen irrigation channels have been considered to accommodate the respective culverts.

To prepare the optimization model, an objective function has been formulated to cover all aforementioned costs. The optimization model involved all structural and hydraulic design constraints. The established non-linear optimization model is solved by the modified Hooke and Jeeves direct search approach. A computer program is developed to handle the aimed solution.

The following categories of analyses have been considered:

1. *Cost as a function of discharge (for the different selected lengths).*
2. *Cost as a function of length (for the different selected discharges).*
3. *Number of vents as a function of discharge (for the different selected lengths).*
4. *Number of vents as a function of length (for the different selected discharges).*
5. *Dimensions of the culvert as a function of discharge (for the different selected lengths).*
6. *Dimensions of the culvert as a function of length (for the different selected discharges).*

The results showed the following:

1. *The computer program is efficient in giving the results.*
2. *The optimization process automatically excluded the pipe culverts, whereas the reinforced concrete box culverts are the optimum types for all considered discharges and lengths.*
3. *The discharge, rather than length, is the dominant factor controlling costs, number of vents, and dimensions of the culvert.*

الخلاصـــــــة

هذا البحث يهدف إلى "التصميم الهيدروليكي الأمين الأمثل للبرابخ"، اخذين بالاعتبار كلفة كل من الحفر والأرضية وألماده الأنشائية والدفن المرصوص والأكتاف (hunches) وأعمال الحماية والزيادة في الشحنة المفقودة.

بالنظر لتعدد أنواع وأشكال والمواد المصنوعة أو ألمنفذه منها البرابخ فقد تم اختيار الأوسع استعمالاً منها وهي البرابخ ألصندوقية المنفذة من الخرسانة المسلحة وبالأشكال ألمستطيلة وألمربعة والبرابخ الأنبوبية بالشكل الدائري المنفذة أو المصنوعة من الخرسانة ألمسلحة ومن حديد الصب و السمنت الأسبستي والفولاذ اللدن.

لربط الناحية ألعملية بالنظرية فقد انتخبت التصاريف (٠,٥ ، ١,٠ ، ١,٥، ٢,٥، ٥,٠، ١٠,٠و١٥,٠م^٣/ ثا) لتمثل التصاريف ألصغيرة والمتوسطة والكبيرة ، وتم انتخاب الأطوال (١٥،٥, ٢٥،١٠, ٣٥،٢٠و٤٠م) لتمثل منشات قصيرة ومتوسطة وطويلة. تم الأخذ بنظر الاعتبار قنوات ري نموذجية مرتبطة بتلك البرابخ.

ولغرض تهيئة نموذج الأمثلية فقد تم إعداد دالة الهدف لتغطي الكلف المذكورة سابقا. نموذج الأمثلية شمل كل المحددات ألتصميميه الأنشائيه والهيدروليكية.

نموذج الأمثلية اللاخطي تم حله باستخدام طريقة (Modified Hooke and Jeeves direct search). تم تطوير برنامج حاسوبي متضمنا كل ما تم ذكره لغرض الحصول على النتائج.

تم تحليل النتائج في ضوء ما يلي:

١ . الكلفة كدالة للتصريف لمختلف الأطوال المستعملة.
٢ . الكلفة كدالة للطول لمختلف التصاريف المستعملة.
٣ . عدد فتحات البربخ كدالة للتصريف لمختلف الأطوال المستعملة.
٤ . عدد فتحات البربخ كدالة للطول لمختلف التصاريف المستعملة.
٥ . أبعاد البربخ كدالة للتصريف لمختلف الأطوال المستعملة.
٦ . أبعاد البربخ كدالة للطول لمختلف التصاريف المستعملة.

لقد بينت النتائج ما يلي:

١ . برنامج الحاسوب كفوء في إعطاء النتائج.
٢ . استبعدت عملية الأمثلية البرابخ الأنبوبية تلقائيا، بينما جاءت البرابخ الخراسانية المسلحة الصندوقية كأفضل الأنواع تصميما للتصاريف و الأطوال المستعملة.
٣ . التصريف هو العامل المسيطر على الكلفة وعدد الفتحات والأبعاد للبربخ بشكل أوضح من الطول.

1. Introduction

Culverts, whether in road crossings or agricultural projects have a great practical and economical importance.

Culverts may be constructed with different shapes, such as circular pipe, ovalt, arch pipe, rectangular box, square box, and arch. Moreover, culverts may be made of a variety of materials such as metal (corrugated or plain), concrete (plain or reinforced), asbestos cement, and vitrified clay [1].

There are two major flow conditions that determine the hydraulics of culvert flow according to the location of the control section (i.e., the cross section which limits the maximum discharge through the culvert). These are ‛Inlet control' and ‛Outlet control' [2]. However, there are six types of flow through culverts [3].

The subject of this research is to attain an optimum safe hydraulic design of culverts through:

1. Building a general model that incorporates the assumptions and methods used in analysis and design of culverts, involving the basic parameters in the processes of construction and maintenance of such structures.
2. Of the possible alternative types of culverts, the most appropriate one (or ones) is (are) to be selected through an optimization approach.
3. Verifying the optimally-selected type (or types) through application to some selected practical case studies.

Figure (1) shows a typical longitudinal section of a culvert.

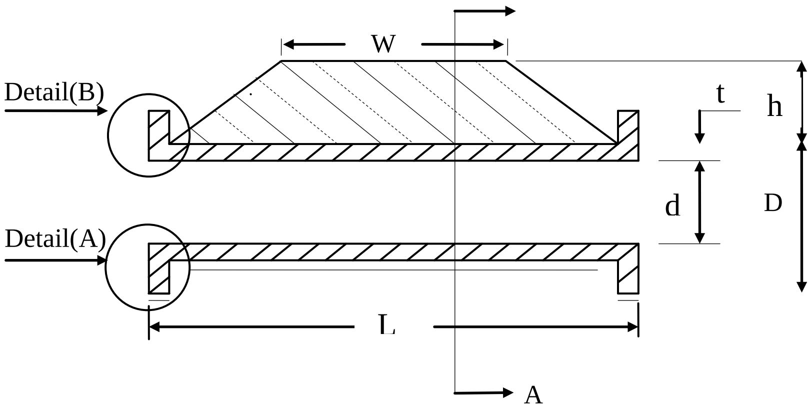


Figure (1) Typical longitudinal section of culverts [4]

2. The Case Study

The research considers two basic types of culverts, namely, the reinforced-concrete box culvert and the circular pipe culvert. The box culvert involves rectangular and squared shape. For the pipe culvert, the research considers different materials which are reinforced concrete, cast-iron, ductile-steel, and asbestos-cement.

As a case study, actual data from an actual land reclamation project, namely, Hilla-Kifl Project, have been considered in order to make the research as close as possible to reality. With reference to **Fig.(1)**, the basic controlling factors in deciding on the size and shape of a culvert are: the discharge (Q), culvert length (L), height of compacted fill (h), top width of embankment (W), section of the channel accommodating the culvert, depth of headwater (HW), characteristics of hunches (s, q, e, f), thickness of bedding (a), Manning roughness of the culvert (n), culvert inlet and outlet loss coefficients, (K_1) and (K_2), and inlet and outlet transition loss coefficients, (K_i) and (K_o). Values considered in the research for the aforementioned parameters are:

Q= 0.5, 1.0, 1.5, 2.5, 5.0, 10.0, and, 15.0 m^3/s.

L= 5, 10, 15, 20, 25, 35, and, 40 m.

h= 1m {Constant} [4].

W= as given by:

$$\mathbf{W = L - 3h} \quad \text{.......... (1)}$$

However, the typically–considered channel is earthen, trapezoidal, with best hydraulic section of [5]:

$$\mathbf{P_C = 2\sqrt{3}y_\circ,\ b_C = \frac{2\sqrt{3}y_\circ}{3},\ A_C = \sqrt{3}y_\circ^{\,2}} \quad \text{.......... (2)}$$

where:

P_C, b_C, and A_C: Are the wetted perimeter, width, and area of the channel respectively.

On substituting $\left(R_C = A_C / P_C\right)$ in Manning equation, the result will be:

$$\mathbf{Q = \frac{1}{n_c}\sqrt{3}y_\circ^{\,2}\left(\frac{y_\circ}{2}\right)^{2/3} S^{1/2}} \quad \text{.......... (3)}$$

where:

Manning roughness coefficient of the channel (n_c) = 0.025

Channel bed slope (S_c) = 0.00005

Depth of headwater (HW) = as calculated from Eq.(3), where(HW = y_o).

Characteristics of hunches: {Typical values}[6]: s=0.50m; q=0.30m; e=0.20m; f=0.30m;

where:

s and q: Are height and width of the lower hunches, respectively; e and f are height and width of the upper hunches, respectively

Thickness of bedding (a) = 0.10m {Typical} [4,6].

Manning roughness coefficient of the culvert (n) = {Typical values for the considered shapes and materials} [2].

K_1, K_2, K_i, K_o: *{Typical values}* [2].

3. The Optimization Problem

The purpose of optimization is to find the best possible solution among the many potential solutions satisfying the chosen criteria. In this research, the optimum design is based on the minimum cost as the objective, taking into account mainly the costs of the structure itself, safety, and serviceability. The following criteria are considered in the optimization problem:

3-1 Basics

The design of a culvert is based on the hydraulic provisions and then the structural provisions. However, a 'good' design should take into consideration the overall cost of the designed structure. For this, a reasonable and practical survey in this respect would delineate the following constituents of a cost objective function: Excavation, bedding (blinding) layer, the material the culvert will be constructed from, compacted fill, hunches, protection works, and additionally-involved head loss.

3-2 The Design Variables

The design variables (which are virtually the decision variables in the optimization model) represent the dimensions that characterize the respective sectional shape. The design variables of a rectangular box culvert are $(X_1 = b)$, $(X_2 = d)$. The optimum solution will give the optimum shape. If ($b > d$), then the optimum shape shall be denoted as a horizontal rectangle; the optimum shape of the reverse is a vertical rectangle. A special case of the rectangular shape is the squared shape (i.e., $b = d$). In such a case, there would be a single design variable, $(X_1 = d)$. The shape of a pipe culvert is circular. In this case, the design variable will be $(X_1 = d)$.

3-3 The Objective Function

The cost objective function (ZT) is the sum of the costs of excavation, bedding, and material of the culvert, compacted fill, hunches, protection works and additional head loss. The respective unit costs are denoted (C_1) through (C_7) for box culverts, respectively; the respective partial cost functions are denoted (Z1) through (Z7) , respectively; subscripts (P), (Pc), (Pa), and (Pd) are added to denote circular pipe culverts of reinforced concrete, cast iron, asbestos cement, and ductile steel, respectively.

3-3-1 The Objective Function of the R.C. Box-Culvert (Rectangular Shape)

According to the details shown in **Fig.(2)**, the total cost objective function (ZT_1) of the reinforced-concrete, rectangular box-culvert is as summarized in **Table (1)**.

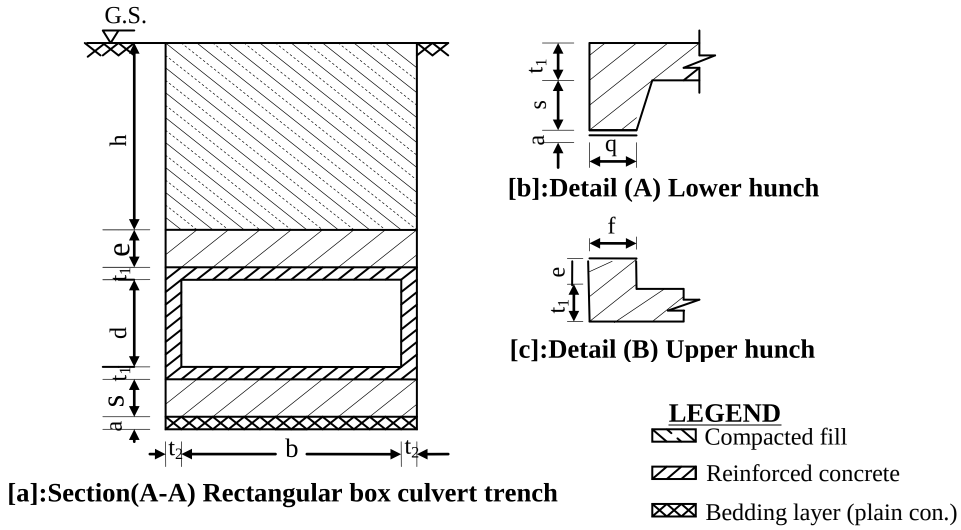


Figure (2) Typical works of box culverts [4]

Table (1) Summary of final cost objective function, (ZT1), for the R.C. rectangular box culvert

Cost function	Constant	Terms of the decision variables						
		b	b^2	d	d^2	$(bd)^{-2}$	bd	$(b+d)^{4/3}(bd)^{-10/3}$
Z_1	----	$C_1L(h+s+e+a)$	$0.2C_1L$	$0.2C_1L(h+s+e+a)$	$0.04C_1L$	----	$1.04C_1L$	----
Z_2	----	C_2aL	----	$0.2C_2aL$	----	----	----	----
Z_3	----	----	$0.2C_3L$	----	$0.2C_3L$	----	$0.04C_3L$	----
Z_4	----	$0.5C_4h(L+W)$	----	$0.1C_4h(L+W)$	----	----	----	----
Z_5	----	$2C_5(qs+ef)$	----	$0.4C_5(qs+ef)$	----	----	----	----
Z_6	$Z_{61}, Z_{62}, Z_{63}, Z_{64}$	----	----	----	----	----	----	----
Z_7	$-\frac{C_7kQ^2}{2gA_C^2}$	----	----	----	----	$\frac{C_7KQ^2}{2g}$	----	$2.52C_7Q^2n^2L$

3-3-2 The Objective Function of the R.C. Box-Culvert (Squared Shape)

The square is a rectangular with ($b = d$). The final cost objective function $(ZT_2)$of the reinforced-concrete, square box-culvert is as summarized in **Table (2)**.

Table (2) Summary of the final cost objective function,(ZT2), for the R.C. square box culvert

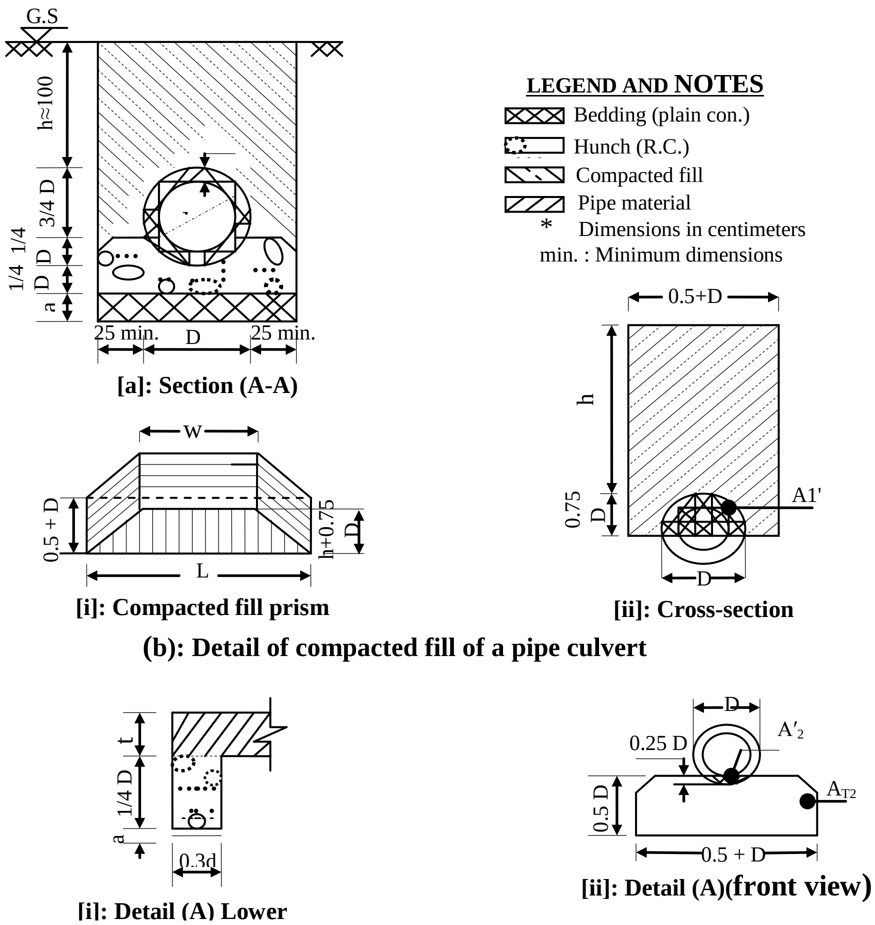
Cost function	Constant	Terms of the decision variables			
		$d^{-16/3}$	d^{-4}	d	d^2
Z_1	----	----	----	$1.2C_1L(h+s+e+a)$	$1.44C_1L$
Z_2	----	----	----	$1.2C_2aL$	----
Z_3	----	----	----	----	$0.44C_3L$
Z_4	----	----	----	$0.6C_4h(L+W)$	----
Z_5	----	----	----	$2.4C_5(qs+ef)$	----
Z_6	$z_{61}, z_{62}, z_{63}, z_{64}$	----	----	----	----
Z_7	$-\frac{C_7kQ^2}{2gA_C^2}$	$6.35C_7Q^2n^2L$	$\frac{C_7KQ^2}{2g}$	----	----

3-3-3 The Objective Function of the R.C. Circular Pipe-Culvert

According to the details shown in **Fig.(3)**, the total cost objective function (ZT_3) of the reinforced-concrete, circular pipe-culvert is as summarized in **Table (3)**.

Table (3) Summary of the final cost objective function,(ZT_3), for R.C. circular pipe culvert

Cost function	Constant	Terms of the decision variables			
		$d^{-16/3}$	d^{-4}	d	d^2
Zp_1	$0.5C_1L(a+h)$	----	----	$C_1L(0.75+1.2(a+h))$	$1.8C_1L$
Zp_2	$0.5C_2La$	----	----	$1.2C_2La$	----
Zp_3	----	----	----	----	$0.12\pi C_3L$
Zp_4	$0.25C_4h(L+W)$	----	----	$C_4(0.6h+0.225)(L+W)$	$C_4(0.54(L+W)-0.91L)$
Zp_5	----	----	----	$0.3C_5L$	$0.5C_5L$
Zp_6	$z_{61}, z_{62}, z_{63}, z_{64}$	----	----	----	----
Zp_7	$-\frac{kQ^2C_7}{2gA_C^2}$	$10.30C_7Q^2n^2L$	$\frac{1.621C_7KQ^2}{2g}$	----	----



[a]: Section (A-A)

[i]: Compacted fill prism

[ii]: Cross-section

(b): Detail of compacted fill of a pipe culvert

[i]: Detail (A) Lower

[ii]: Detail (A)(front view)

(c): The lower hunches

Figure (3) Typical works of circular pipe culverts [4]

3-3-4 The Objective Function of Pipe Culverts of Materials other than Reinforced Concrete

With respect to the material of a circular pipe culvert other than the reinforced concrete, the most common types in use in Iraq are cast iron, asbestos-cement, and ductile steel. These types are considered in this research. The unit prices of the aforementioned types for some useful standard sizes are given in **Table (4)** [7].

The best function obtained to express the cost of pipe per unit of the installed length as a function of its size (diameter) is [8]:

$$\mathbf{Y = K'L + K^{*}Ld^{N}} \qquad \text{.......... (4)}$$

in which:

Y : Cost of pipe furnishing in U.S. Dollars per unit length;

L : Length of pipe, (m);

d : Diameter of pipe, (m);

K', K^{*}, N : Fitting parameters.

Table (4) Commercial prices of pipes [7]

Pipe type	Inner diameter(d), (m)	Unit price, ($/m)
Cast iron	0.50	46.67
	0.60	50.00
	0.70	60.00
	0.80	66.67
	1.00	106.67
	1.20	116.67
Asbestos-Cement	0.50	16.67
	0.60	26.67
	0.70	30.00
	0.80	33.33
Ductile-Steel	0.50	33.33
	0.60	43.89
	0.70	83.33
	0.80	111.11
	0.90	133.33
	1.00	155.55

The final cost of the cast-iron, asbestos-cement, and ductile-steel, circular pipe culverts, (ZT_4), (ZT_5),and (ZT_6) are as summarized in **Tables (5)**, **(6)**, and **(7)**.

Table (5) Summary of the final cost objective function,(ZT_4), for cast-iron circular pipe culvert

Cost function	Constant	Terms of the decision variables				
		$d^{-16/3}$	d^{-4}	d	$d^{1.3}$	d^2
Zpc_1	$0.5C_1L(a+h)$	----	----	$C_1L(0.75+1.2(a+h))$	----	$1.8C_1L$
Zpc_2	$0.5C_2La$	----	----	$1.2C_2La$	----	----
Zpc_3	$5.325459L$	----	----	----	$90.86784L$	----
Zpc_4	$0.25C_4h(L+W)$	----	----	$C_4(0.6h+0.225)(L+W)$	----	$C_4(0.54(L+W)-0.91L)$
Zpc_5	----	----	----	$0.3C_5L$	----	$0.5C_5L$
Zpc_6	$z_{61}, z_{62}, z_{63}, z_{64}$	----	----	----	----	----
Zpc_7	$-\frac{kQ^2C_7}{2gA_C^2}$	$10.30C_7Q^2n^2L$	$\frac{1.621C_7KQ^2}{2g}$	----	----	----

Table (6) Summary of the final cost objective function,(ZT_5), for asbestos-cement circular pipe culvert

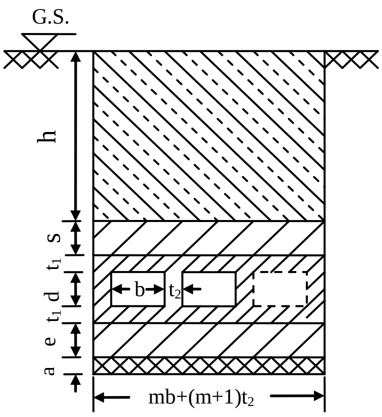
Cost function	Constant	Terms of the decision variables				
		$d^{-16/3}$	d^{-4}	d	$d^{1.535724}$	d^2
Zpa_1	$0.5C_1L(a+h)$	----	----	$C_1L(0.75+1.2(a+h))$	----	$1.8C_1L$
Zpa_2	$0.5C_2La$	----	----	$1.2C_2La$	----	----
Zpa_3	$-0.83457L$	----	----	----	$51.02211L$	----
Zpa_4	$0.25C_4h(L+W)$	----	----	$C_4(0.6h+0.225)(L+W)$	----	$C_4(0.54(L+W)-0.91L)$
Zpa_5	----	----	----	$0.3C_5L$	----	$0.5C_5L$
Zpa_6	$z_{61}, z_{62}, z_{63}, z_{64}$	----	----	----	----	----
Zpa_7	$-\dfrac{kQ^2C_7}{2gA_C^{\ 2}}$	$10.30C_7Q^2n^2L$	$\dfrac{1.621C_7KQ^2}{2g}$	----	----	----

Table (7) Summary of the final cost objective function,(ZT_6), for ductile-steel circular pipe culvert

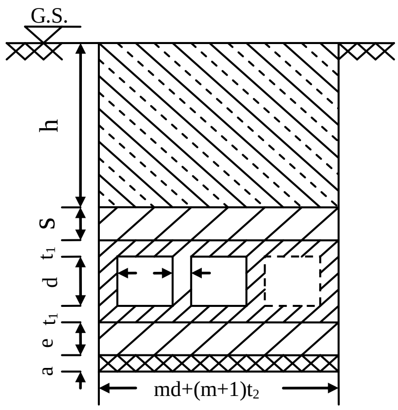
Cost function	Constant	Terms of the decision variables				
		$d^{-16/3}$	d^{-4}	d	d^2	$d^{2.053063}$
Zpd_1	$0.5C_1L(a+h)$	----	----	$C_1L(0.75+1.2(a+h))$	$1.8C_1L$	----
Zpd_2	$0.5C_2La$	----	----	$1.2C_2La$	----	----
Zpd_3	$-0.83457L$	----	----	----	----	$165.232L$
Zpd_4	$0.25C_4h(L+W)$	----	----	$C_4(0.6h+0.225)(L+W)$	$C_4(0.54(L+W)-0.91L)$	----
Zpd_5	----	----	----	$0.3C_5L$	$0.5C_5L$	----
Zpd_6	$z_{61}, z_{62}, z_{63}, z_{64}$	----	----	----	----	----
Zpd_7	$-\dfrac{kQ^2C_7}{2gA_C^{\ 2}}$	$10.30C_7Q^2n^2L$	$\dfrac{1.621C_7KQ^2}{2g}$	----	----	----

3-3-5 The Objective Function of Multiple-Vents of Culverts

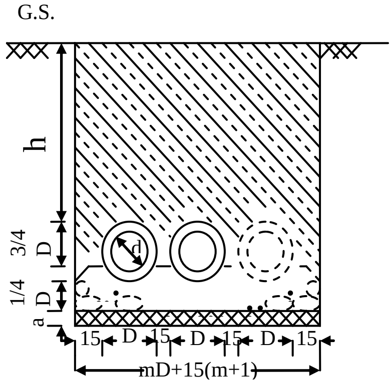
A single vent for a certain discharge may be insufficient or impractical; therefore, multiple-vents of culverts is considered to cover the respective discharge [according to the details shown in **Fig.(4)**]. The final total cost objective functions (ZT_1'), (ZT_2'), (ZT_3'), (ZT_4'), (ZT_5'), and (ZT_6') of reinforced-concrete box culverts (rectangular and square) and circular pipe culverts (reinforced concrete, cast-iron, asbestos-cement, and ductile-steel) are summarized in **Tables (8)**, **(9)**, **(10)**, **(11)**, **(12)**, and **(13)**, respectively.



(a): Section of multiple-vents, reinforced concrete, rectangular box culvert



(b): Section of multiple-vents, reinforced concrete, square box culvert



(c): Section of multiple-vents circular pipe culverts

Figure (4) Typical multiple-vents culverts

Table (8) Summary of final cost objective function,(ZT'_1)
for multiple-vents, reinforced concrete rectangular box culverts

Cost function	Constant	Terms of the decision variables						
		b	b^2	d	d^2	$(bd)^{-2}$	bd	$(b+d)^{4/3}$ $(bd)^{-10/3}$
Z_1	----	mC_1L $(h+s+e+a)$	$0.2m\ C_1L$	$0.1C_1L$ $(m+1)$ $(h+s+e+a)$	$0.1C_1L$ $(m+1)$	----	$0.02\ C_1L$ $(m+1)$	----
Z_2	----	C_2aLm	----	$0.1C_2a$ $L(m+1)$	----	----	----	----
Z_3	----	----	$0.2m\ C_3L$	----	$0.1C_3L$ $(m+1)$	----	$0.02\ C_3L$ $(m+1)$	----
Z_4	----	$0.5C_4mh$ $(L+W)$	----	$0.05C_4h$ $(m+1)$ $(L+W)$	----	----	----	----
Z_5	----	$2C_5m(qs+ef)$	----	$0.2C_5$ $(m+1)$ $(qs+ef)$	----	----	----	----
Z_6	$Z_{61}, Z_{62}, Z_{63}, Z_{64}$	----	----	----	----	----	----	----
Z_7	$-\frac{C_7kQ^2}{2gA_C^{\ 2}}$	----	----	----	----	$\frac{C_7K\left(\frac{Q}{m}\right)^2}{2g}$	----	$2.52C_7$ $\left(Q/m\right)^2$ n^2L

Table (9) Summary of final cost objective function, (ZT'_2) for multiple-vents, reinforced concrete square box culverts

Cost function	Constant	Terms of the decision variables			
		$d^{-16/3}$	d^{-4}	d	d^2
Z_1	----	----	----	$0.1C_1L(11m+1)(h+s+e+a)$	$C_1L(1.23m+0.12)$
Z_2	----	----	----	$0.1C_2aL(11m+1)$	----
Z_3	----	----	----	----	$C_3L(0.32m+0.12)$
Z_4	----	----	----	$0.05C_4h(L+W)(11m+1)$	----
Z_5	----	----	----	$0.2C_5(11m+1)(qs+ef)$	----
Z_6	$z_{61}, z_{62}, z_{63}, z_{64}$	----	----	----	----
Z_7	$-\frac{C_7kQ^2}{2gA_C^{\ 2}}$	$6.35C_7\left(\frac{Q}{m}\right)^2 n^2L$	$\frac{C_7K\left(\frac{Q}{m}\right)^2}{2g}$	----	----

Table (10) Summary of the final cost objective function,($ZT'3$) for multiple-vents, reinforced concrete circular pipe culverts

Cost function	Constant	Terms of the decision variables			
		$d^{-16/3}$	d^{-4}	d	d^2
Zp_1	$0.15C_1L(m+1)(a+h)$	----	----	$1.2C_1L(m(a+h)+0.1875(m+1))$	$1.8C_1Lm$
Zp_2	$0.15C_2La(m+1)$	----	----	$1.2C_2Lam$	----
Zp_3	----	----	----	----	$0.6\pi C_3Lm$
Zp_4	$0.075C_4h(L+W)(m+1)$	----	----	$C_4(L+W)(0.6hm+0.0675(m+1))$	$C_4(0.54(L+W)-0.91L)m$
Zp_5	----	----	----	$0.18C_5L(m+1)$	$0.5C_5Lm$
Zp_6	$z_{61}, z_{62}, z_{63}, z_{64}$	----	----	----	----
Zp_7	$-\frac{kQ^2C_7}{2gA_C^{\ 2}}$	$10.30C_7\left(Q/m\right)^2 n^2L$	$\frac{1.621C_7K\left(Q/m\right)^2}{2g}$	----	----

Table (11) Summary of the final cost objective function, (ZT'_4) for multiple-vents cast-iron circular pipe culverts

Cost function	Constant	Terms of the decision variables				
		$d^{-16/3}$	d^{-4}	d	$d^{1.3}$	d^2
Zpc_1	$0.15C_1L(m+1)(a+h)$	----	----	$1.2C_1L(m(a+h)+0.186(m+1))$	----	$1.8C_1Lm$
Zpc_2	$0.15C_2La(m+1)$	----	----	$1.2C_2Lam$	----	----
Zpc_3	$5.325459Lm$	----	----	----	$90.86784Lm$	----
Zpc_4	$0.075C_4h(L+W)(m+1)$	----	----	$C_4(L+W)(0.6hm+0.0675(m+1))$	----	$C_4m(0.54(L+W)=-0.91L)$
Zpc_5	----	----	----	$0.18C_5L(m+1)$	----	$0.5C_5Lm$
Zpc_6	$z_{61}, z_{62}, z_{63}, z_{64}$	----	----	----	----	----
Zpc_7	$-\frac{kQ^2C_7}{2gA_C^2}$	$10.30C_7\left(Q/m\right)^2 n^2L$	$\frac{1.621C_7K\left(\frac{Q}{m}\right)^2}{2g}$	----	----	----

Table (12) Summary of the final cost objective function, (ZT′5) for multiple-vents, asbestos-cement circular pipe culverts

Cost function	Constant	Terms of the decision variables				
		$d^{-16/3}$	d^{-4}	d	$d^{1.535724}$	d^2
Zpa_1	$0.15C_1L(m+1)(a+h)$	----	----	$1.2C_1L(m(a+h)+0.1875(m+1))$	----	$1.8C_1Lm$
Zpa_2	$0.15C_2La(m+1)$	----	----	$1.2C_2Lam$	----	----
Zpa_3	$-0.83457Lm$	----	----	----	$51.02211Lm$	----
Zpa_4	$0.075C_4h(L+W)(m+1)$	----	----	$C_4(L+W)(0.6hm+0.0675(m+1))$	----	$C_4m(0.54(L+W)-0.91L)$
Zpa_5	----	----	----	$0.18C_5L(m+1)$	----	$0.5C_5Lm$
Zpa_6	$z_{61}, z_{62}, z_{63}, z_{64}$	----	----	----	----	----
Zpa_7	$-\frac{kQ^2C_7}{2gA_C^2}$	$10.30C_7\left(\frac{Q}{m}\right)^2 n^2L$	$\frac{1.621C_7K\left(\frac{Q}{m}\right)^2}{2g}$	----	----	----

Table (13) Summary of the final cost objective function, (ZT'_6) for multiple-vents, ductile-steel circular pipe culvert

Cost function	Constant	Terms of the decision variables				
		$d^{-16/3}$	d^{-4}	d	d^2	$d^{2.053063}$
Zpd_1	$0.15C_1L(m+1)(a+h)$	----	----	$1.2C_1L(m(a+h)+0.1875(m+1))$	$1.8C_1Lm$	----
Zpd_2	$0.15C_2La(m+1)$	----	----	$1.2C_2Lam$	----	----
Zpd_3	$-3.03502Lm$	----	----	----	----	$165.232Lm$
Zpd_4	$0.075C_4h(L+W)(m+1)$	----	----	$C_4(L+W)(0.6hm+0.0675(m+1))$	$C_4m(0.54(L+W)-0.91L)$	----
Zpd_5	----	----	----	$0.18C_5L(m+1)$	$0.5C_5Lm$	----
Zpd_6	$z_{61}, z_{62}, z_{63}, z_{64}$	----	----	----	----	----
Zpd_7	$-\frac{kQ^2C_7}{2gA_C^2}$	$10.30C_7\left(\frac{Q}{m}\right)^2 n^2L$	$\frac{1.621C_7K\left(\frac{Q}{m}\right)^2}{2g}$	----	----	----

4. The Constraints

The cost objective function is minimized subject to a set of constraints. The basic controlling constraints are:

4-1 Dimensions

i) The minimum vent dimensions of a box culvert are (0.75m) [9]. This constraint can be written as:

$$b \geq 0.75m \ ; \ d \geq 0.75m \ \ldots\ldots (5)$$

ii) The maximum span length of (4m) and a maximum height of (3m) of a box culvert have been adopted in this research [6]. This condition can be written as:

$$b \leq 4.0m \ ; \ d \leq 3.0m \ \ldots\ldots (6)$$

iii) For a pipe culvert, the minimum diameter is (0.60m) [10]. The constraint that covers this limit can be written as:

$$d \geq 0.60m \ \ldots\ldots (7)$$

iv) To ensure a closed-conduit flow (full flow), the height of culverts must be less than (HW/1.5), where (HW) is the headwater [3]; that is:

$$\mathbf{d \le HW/1.5} \quad \text{.......... (8)}$$

4-2 Head Loss

The minimum net submergence (h_N) is (0.05m) [2] as shown in **Fig.(5)**. The constraint covering this case is:

$$\mathbf{h_N \ge 0.05m} \quad \text{.......... (9)}$$

4-3 Limiting Velocity

The minimum velocity is related to the slope of the culvert; the maximum velocity is dictated by the channel conditions at the outlet. To ensure minimum velocity and for preventing sedimentation (non–silting), a minimum slope of (0.005) has been considered [10], that is:

$$\mathbf{S_b \ge 0.005} \quad \text{.......... (10)}$$

This, with Manning formula, gives:

$$\mathbf{\frac{V^2n^2}{R^{4/3}} \ge 0.005} \quad \text{.......... (11)}$$

With (V=Q/A), A (for a fully-flowing box culvert)= bd, P(=2(b+d)), and (R=A/P), then Eq.(11) could be written as:

$$\mathbf{\frac{(bd)^{10/3}}{(b+d)^{4/3}} \le 503.96Q^2n^2} \quad \text{.......... (12)}$$

Equation (11) for a square box-culvert will read:

$$\mathbf{d^{16/3} \le 1270Q^2n^2} \quad \text{.......... (13)}$$

and, for a circular pipe culvert will be:

$$\mathbf{d^{16/3} \le 2058.72Q^2n^2} \quad \text{.......... (14)}$$

The formulation of the objective function of the optimization problem is non-linear. There are several methods for solving a non–linear optimization problem. The modified direct search method of Hooke and Jeeves [11] has been adopted for use in the research.

5. The Results

A computer program has been developed to handle the optimization process. The final optimum results are as given in **Table (14)**.

The behavior of the results could be viewed as shown in **Figs.(6)** through **(11)**.

Table (14) Final optimum results

Q (m^3/s)	L (m)	The optimum design				
		Material	Shape	No. of vents	Dimensions (mm)	Overall cost($)
0.5	5	R.C.	□*	1	(750×750)	820
	10	R.C.	□*	1	(750×750)	1400
	15	R.C.	□*	1	(750×750)	1980
	20	R.C.	□*	1	(750×750)	2560
	25	R.C.	□*	1	(750×750)	3140
	35	R.C.	□*	1	(750×750)	4300
	40	R.C.	□*	1	(750×750)	4880
1.0	5	R.C.	□	1	(750×750)	1010
	10	R.C.	□	1	(750×750)	1590
	15	R.C.	□	1	(750×750)	2170
	20	R.C.	□	1	(750×750)	2750
	25	R.C.	□	1	(750×750)	3330
	35	R.C.	□	1	(750×750)	4490
	40	R.C.	□	1	(750×750)	5070

Table (14) Continued

Q (m^3/s)	L (m)	The optimum design				
		Material	Shape	No. of vents	Dimensions (mm)	Overall cost($)
1.5	5	R.C.	□	1	(750×750)	1070
	10	R.C.	□	1	(750×750)	1650
	15	R.C.	□	1	(750×750)	2230
	20	R.C.	□	1	(750×750)	2810
	25	R.C.	□	1	(750×750)	3390
	35	R.C.	□	1	(760×760)	4650
	40	R.C.	□	1	(780×780)	5450
2.5	5	R.C.	□	1	(830×830)	2130
	10	R.C.	□	1	(870×870)	3010
	15	R.C.	□	1	(900×900)	3970
	20	R.C.	□	1	(930×930)	5000
	25	R.C.	▭	1	(1110×800)	6060
	35	R.C.	▭	1	(1150×810)	8250
	40	R.C.	▭	1	(1160×810)	9390

Table (14) Continued

Q (m^3/s)	L (m)	The optimum design				
		Material	Shape	No. of vents	Dimensions (mm)	Overall cost($)
5.0	5	R.C.	□	2	(750×750)	2920
	10	R.C.	□	2	(750×50)	4090
	15	R.C.	□	2	(760×760)	5320
	20	R.C.	□	2	(780×780)	6830
	25	R.C.	□	2	(800×800)	8430
	35	R.C.	□	2	(840×840)	11850
	40	R.C.	□	2	(860×860)	13660
10.0	5	R.C.	□	2	(940×940)	4450
	10	R.C.	□	2	(980×980)	6510
	15	R.C.	□	2	(1010×1010)	8730
	20	R.C.	□	2	(1040×1040)	11090
	25	R.C.	□	2	(1060×1060)	13570
	35	R.C.	□	2	(1100×1100)	18840
	40	R.C.	□	2	(1120×1120)	21620
15.0	5	R.C.	▭	1	(2510×1000)	9590
	10	R.C.	▭	1	(2740×930)	13220
	15	R.C.	▭	1	(1840×1540)	17370
	20	R.C.	▭	1	(2100×1330)	19390
	25	R.C.	▭	1	(2120×1340)	22480
	35	R.C.	▭	1	(2100×1410)	28860
	40	R.C.	▭	1	(2120×1420)	32140

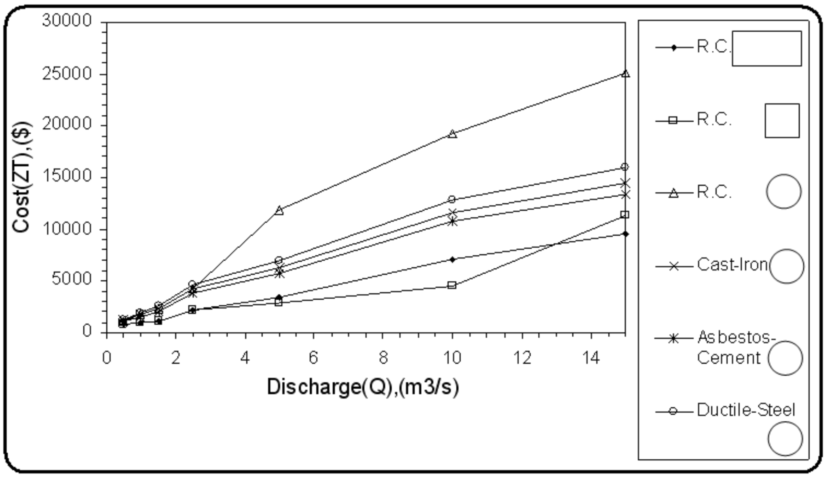


Figure (6) Optimum cost of different types of culverts (L=5m)

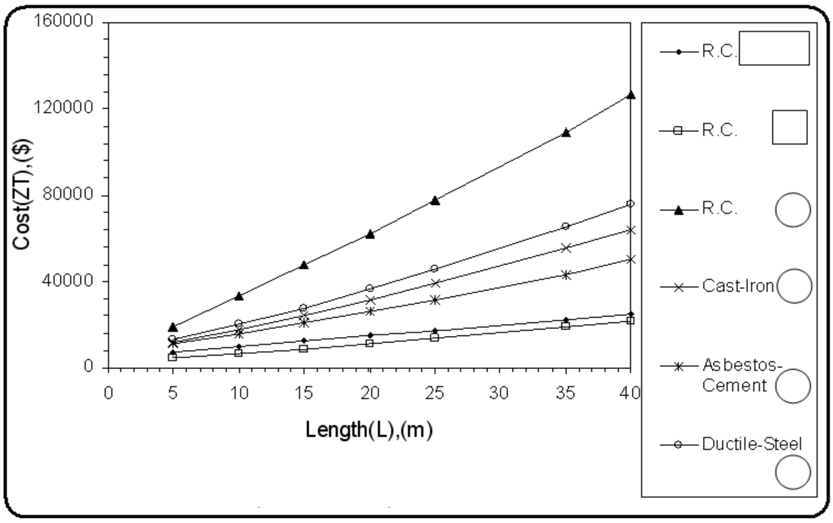


Figure (7) Optimum cost of different types of culverts (Q=10.0m^3/s)

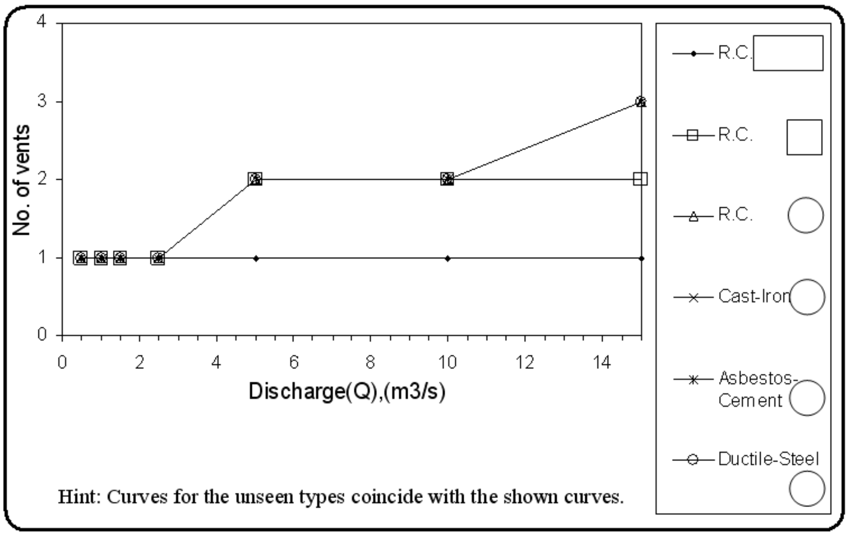


Figure (8) Optimum cost of different types of culverts (L=5m)

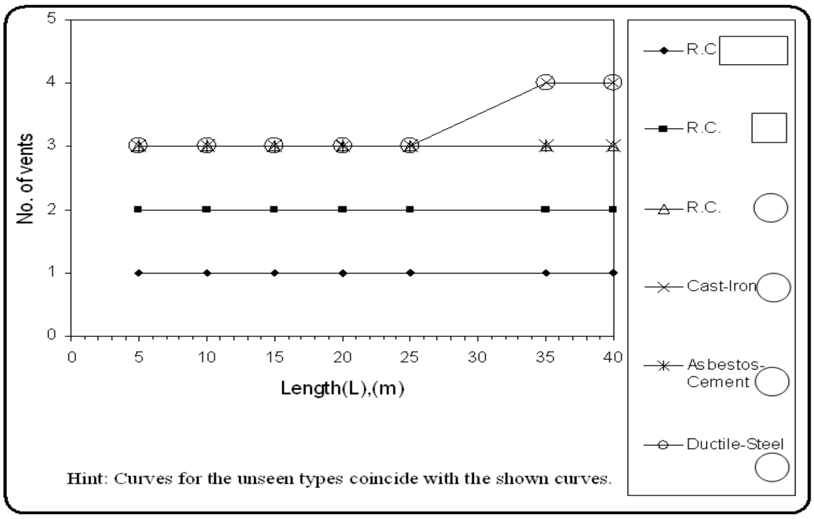


Figure (9) Optimum number of vents of different types of culverts (Q=15.0m^3/s)

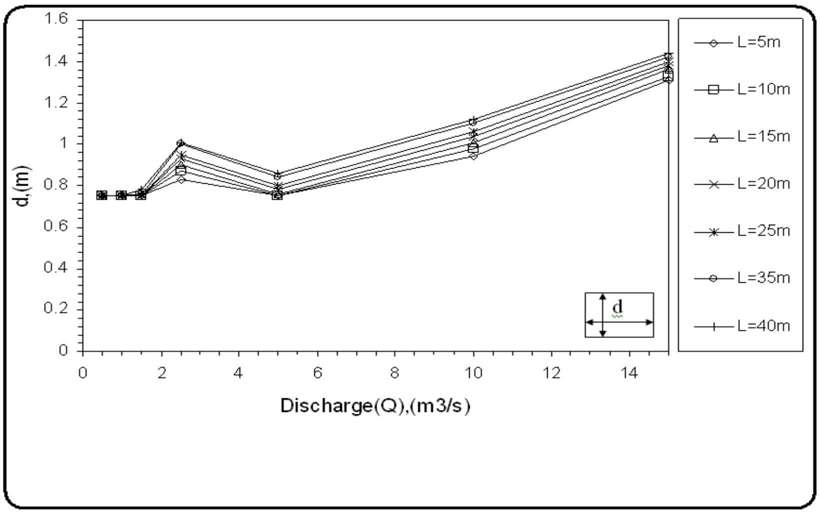


Figure (10) Optimum dimension (d) of R.C., square box culverts

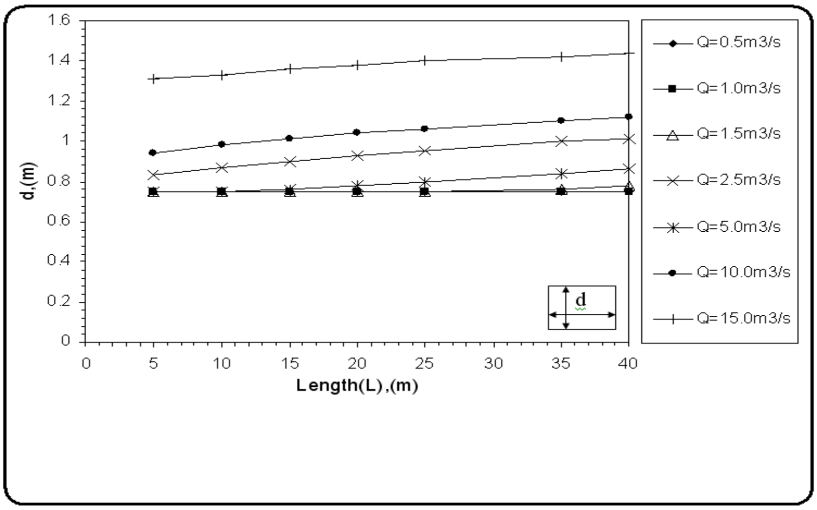


Figure (11) Optimum dimension (d) of R.C., square box culverts

6. Conclusions

Based on the obtained results, the following conclusions are abstracted:

1. The computer program is efficient in giving the results.
2. The optimization process automatically excluded the pipe culverts.
3. The discharge, rather than length, is the dominant factor controlling costs, number of vents, and dimensions of the culvert.
4. Reinforced concrete box culverts are the optimum types for all considered discharges and lengths as follows:
   a) For (Q=0.5, 1.0, and1.5m^3/s) and for all considered lengths, the optimum types are single-vent, reinforced concrete square box culverts.
   b) For (Q=2.5m^3/s) and (L=5 through 20m), the optimum type is a single-vent, reinforced concrete square box culvert, whereas for (L=25 through 40m), the optimum type is a single-vent, reinforced concrete rectangular box culvert.
   c) For (Q=5.0 and 10.0m^3/s) and for all considered lengths, the optimum types are two-vents, reinforced concrete square box culverts.
   d) For (Q=15.0 m^3/s) and for all considered lengths, the optimum type is a single-vent, reinforced concrete rectangular box culvert.

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List of Symbols

A:	Cross-sectional area of the culvert, m^2
A_c:	Cross-sectional area of the channel, m^2
d:	Height of the culvert or diameter of a pipe culvert, m
D:	Outer diameter of pipe culverts, m
g:	Gravitational acceleration, m/s^2
G.S.:	Ground surface
h:	Height of compacted fill, m
h_A:	Head on the culvert, m
h_L:	Head loss on the culvert, m
h_N:	Head loss on the culvert, m
H or HW:	Head water depth, m
k:	K_i+K_o
K:	$K_i+K_o+K_1+K_2$
L:	Length of the culvert, m
m:	Number of vents of a culvert
n:	Manning roughness coefficient of the culvert
P:	Wetted perimeter of the culvert, m
Q:	Discharge through the structure (culvert and channel), m^3/s
R:	Hydraulic radius of the culvert, m
R_C:	Hydraulic radius of the channel, m
R.C.:	Reinforced concrete
S_c:	Bed slope of the channel, m/m
t:	Thickness of the culvert, m
V:	Velocity within the culvert, m/s
y_o:	Water depth of the channel, m