Theoretical Formulation for using Outside Cold Air in Removing Stored Heat in Building (Free Cooling)

Asst. Lect. Mohammed Hadi Ali
M.Sc. in Mechanical Engineering
Al-Mustansiriya University, Baghdad, Iraq

Abstract

The using of Outside Cold Air at early morning or at night to get rid or to remove the Stored Heat content inside the Building when the temperature of the Outside Air at night or early morning is less than the temperature of the Conditioned Space can be considered as Free Cooling way.

This point was studied and analyzed theoretically in this papers, the target is to reach a General Equation that could be used directly with the required Data of the Building component (Furniture, Walls, Roof ……. etc) and the Air Conditioning Plant specification. The target was achieved to evolve the General Equation.

In this way, the estimation of Removed Heat content from the Building for a certain time during a summer season can be calculated depending on the Meteorology Database of each area which is considered as Energy Conservation.

The Free cooling way is widely and largely used now as a way to conserve the Energy consumption.

الخلاصـــــة

إن إستخدام الهواء الخارجي البارد عند الصباح أو أثناء الليل للتخلص أو إزالة المحتوى الحراري المخزون داخل الأبنية عندما تكون درجة حرارة الهواء الخارجي أقل من درجة حرارة الحيز أو الفضاء المكيف يمكن إعتباره طريقة من طرق التبريد المجاني.

هذه النقطة تمت دراستها وتحليلها نظريا في هذا البحث، والهدف هو الوصول إلى معادلة عامة رياضية يمكن إستخدامها بشكل مباشر وتحتاج فقط إلى البيانات المطلوبة لمكونات البناية (الأثاث، الجدران، السقف ...... إلخ) ومواصفات منظومة التكييف، ولقد أثمرت هذه الدراسة بالتوصل إلى مثل هذه المعادلة.

بهذه الطريقة يمكن حساب المحتوى الحراري المخزون داخل الأبنية والمزال بإستخدام الهواء الخارجي البارد ولفترة من الزمن خلال موسم الصيف وبالإعتماد على بيانات الأنواء الجوية لكل منطقة، وأن هذه الحرارة المزالة تعتبر ترشيدا في الطاقة.

إن طريقة التبريد المجاني تستخدم الآن بشكل واسع وكبير كطريقة من طرق ترشيد الإستهلاك في الطاقة.

1. Introduction

The Heat transfers by Conduction through the Wall and Roof of the Building is unsteady and it is difficult to obtain its value because of:

1. The change in Outside temperature during the 24 hour.
2. The change in the Solar Intensity falls on the walls and roofs during the 24 hours.
3. The Heat stores inside the walls and roofs.

The Solar radiation which passes through window glazing does not form an immediate load on the Air Conditioning System; the situation is similar to the Heat Gain through walls and due to:

1. The Air considers as transparent medium for radiation heat passing through it.
2. The cooling load imposes on the A/C plant changes according to the change in the Room Air temperature (the Return Air temperature).

The Solar radiation entering through the window and as well Heat liberated from Electrical Lighting and Equipments must first wok to warm up the solid surfaces of the Furniture, Floor slabs and Walls within the Conditioned spaces.

These surfaces are then in a position to liberate some of the Heat stored to Air by Convection (not all the heat will be liberated immediately, because some of the Energy is stored within the depth of the solid materials) [1,2].

Figure (1) below illustrates the Time lag between the Peak instantaneous heat gain to the conditioned space during the day and the peak load on the A/C Plant.

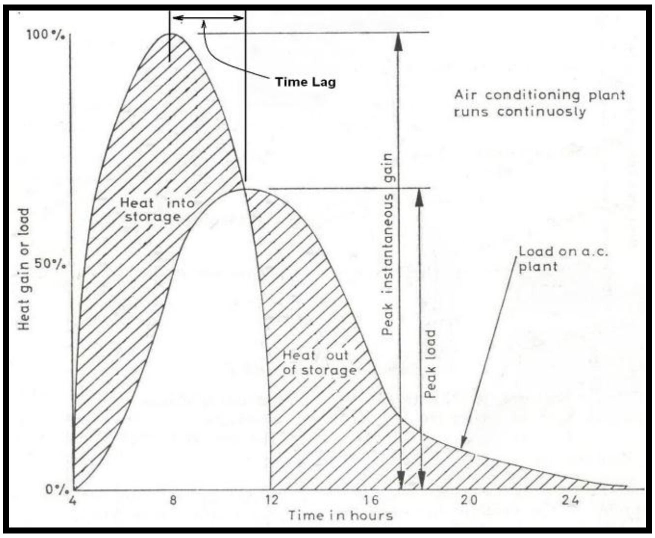


Figure (1) Actual cooling load and solar heat gain (a/c plant runs continuously for 24 hour operation) showing the time lag

In the long and continuous operating run (24 hour) for the A/C Plant, all the Heat or Energy of Cooling Load received by the Conditioned space is removed by the A/C Plant, under this case there is no what so called, Pull-Down Load.

If the A/C Plant operates for only partial time (8 hr, 10 hr or 12 hr) per a day in Commercial buildings, Schools, Libraries, Universities, Administrative offices ….. etc, then the heat stored in the building constructions and the furniture is released to the inside spaces during the night and on start up next morning the initial load on the A/C plant will be greater than expected. This surplus load is termed the Pull-Down Load, **Figure (2)** below illustrates the possible effect of such surplus load.

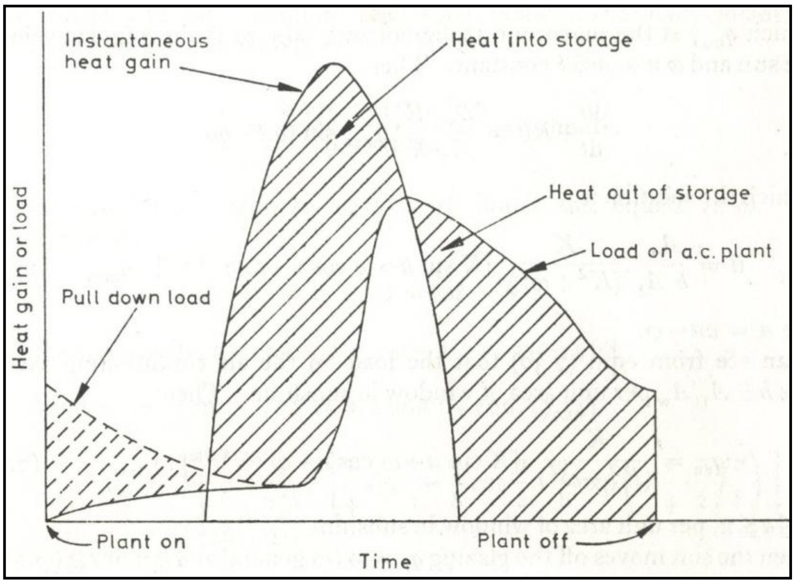


Figure (2) Actual cooling load and solar heat gain (a/c plant runs for partial time of 16 hour operation per a day) showing the pull-down load on a/c plant

The importance of the Pull-Down Load is open to question depending on the operating time factor of the A/C Plant and the Outside dry-bulb temperatures that fall at night to be less than the design room temperature (the return air temperature).

Then using of 100% of Outside Air and introducing it directly to the Conditioned space through the A/C Plant by operating its Fans only, will cause a reduction in Cooling Load imposes upon the A/C Plant, and if the Fans have more than one speed it is recommended to operate the Fans at the lower speed to insure better operation and more less Energy that will be consumed when the Fans operate at high speed [3].

It is desirable to use the Outside Cold Air at early morning to cool the Building, this way of Free Cooling is capable to reduce the Heat stored in the Building, a long cooling period with Outside Cold Air may reduce the temperature of the Building by one to two degree Centigrade which represents a big cooling load that the A/C plant should removes it [3].

2. The Theoretical Analysis

In this article, the theoretical analysis is presented and focused to find or evolve a General Formula for the Air Room temperature change with time during the cooling period with Outside Cold Air at night or early morning which can be used for all models of residences and other buildings.

To study this case in details, **Figure (3)** represents the thermal interaction between the conditioned room components (Furniture, Walls and the Air inside the conditioned room) and the Outside Cold Air for a simple room with the following assumptions:

1. The conditioned space has only unique mixing area (the conditioned space has only one temperature through out the room).
2. The heat transfer coefficient between the room components (Furniture, Walls … etc) with the inside Air is constant.
3. The conduction heat flow through the walls is perpendicular to wall surfaces.
4. There is an active thickness for the inside walls have the same and uniform temperature during the cooling period.
5. The external circumstances around the room is identical (the Air temperature is not changed during the cooling period).

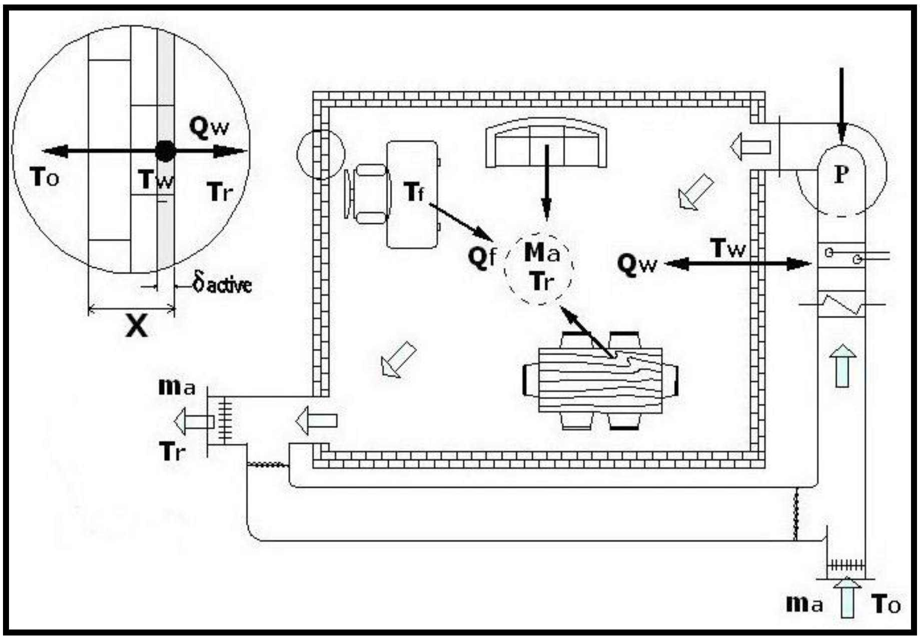


Figure (3) The thermal interaction model between the conditioned room components and the outside cold air

2-1 The Heat Balance for the Furniture inside the Room

The Furniture inside the room possesses a stored heat due to the solar radiation entering through the window, when the A/C plant is off; the Furniture will lose the stored heat to Air Room during the cooling with the Outside Cold Air, the energy balance equation therefore for the Furniture is as follows:

$$-\mathbf{M_f \cdot C_{pf} \cdot \frac{dT_f}{dt} = h_f \cdot A_f \cdot (T_f - T_r)}$$

By using the operator notation (D) and rearranging the above equation becomes:

$$K_1 \cdot T_r - (D + K_1) T_f = 0 \quad \text{.. (1)}$$

where:

M_f = *Mass of the Furniture (kg)*
C_{pf} = *Specific heat for the Furniture (W. sec / kg. ^oC)*
T_f = *Temperature of the Furniture (^oC)*
t = *Time (sec)*
h_f = *Heat transfer Coefficient of the Furniture (W / m^2. ^oC)*
A_f = *Area of the Furniture (m^2)*
T_r = *Temperature of the Room Air (^oC)*

$$K_1 = \frac{h_f \cdot A_f}{M_f \cdot C_{pf}}$$

K_1 = Constant (1/sec) = Constant * 60 (1/min)

2-2 The Heat Balance for the Walls, Floor and Ceiling of the Room

During the cooling period with the outside cold air and for a certain time, only specific thickness of each wall, floor and ceiling will be affected with the cooling operation [4], and the thickness can be considered to have a uniform temperature.

This thickness is called the active thickness (δ_{act}), the value of the active thickness according to [4] is equal to:

$$\delta_{act} = \sqrt{\frac{2\alpha}{w}} , \qquad w = \frac{2 * \pi}{t_c}$$

where:

δ_{act} = *The active thickness (mm)*
α = *The Coefficient of thermal Diffusion (mm^2/sec)*
w = *The frequency (cycle/sec)*
t_c = *The time for the cooling cycle (sec)*

On this basis, assuming that the temperature of the active thickness of the walls has a uniform temperature which is (T_w) during the cooling period with outside cold temperature as shown in the above **Fig.(3)**, when the A/C plant is off, the walls will start to lose the stored heat to air inside the room and to outside when the outside temperature becomes less than the wall temperature, the energy balance for the walls can be represented by the following equation:

$$-M_w \cdot C_{pw} \cdot \frac{dT_w}{dt} = h_w \cdot A_w \cdot (T_w - T_r) + \left\{ \frac{1}{U_w \cdot A_w} - \frac{1}{h_w \cdot A_w} \right\}^{-1} \cdot (T_w - T_o)$$

By using the operator notation (D) and rearranging the above equation becomes:

$$K_2 \cdot T_r - (D + K_4)\,T_w = -K_3 \cdot T_o \quad \text{...} (2)$$

where:

M_w = *Mass of the walls (kg)*
C_{pw} = *Specific heat of the walls (W. sec / kg. °C)*
T_w = *The walls temperature (°C)*
h_w = *Heat transfer Coefficient of the Walls (W / m^2. °C)*
A_w = *Area of the Walls (m^2)*
U_w = *The overall Heat transfer Coefficient for the Walls (W / m^2. °C)*
T_o = *The Outside Cold Air temperature (°C)*

$$K_2 = \frac{h_w \cdot A_w}{M_w \cdot C_{pw}}, \qquad K_3 = \frac{A_w \cdot U_w \cdot h_w}{M_w \cdot C_{pw} \cdot (h_w - U_w)}, \qquad K_4 = K_2 + K_3$$

K_2, K_3, K_4 = Constants (1/sec) = Constants * 60 (1/min)

2-3 The Heat Balance for the Air inside the Room

The air inside the room and during the cooling with outside cold air will gain heat from the A/C plant's Fan, furniture and the walls and loses heat as well to outside cold air which is entering the room through the A/C plant as shown in **Fig.(3)**, thus the temperature of the air inside the room (T_r) will change and decrease during the cooling cycle, the energy balance for the air inside the room can be represented by the following equation:

$$-M_a \cdot C_{pa} \cdot \frac{dT_r}{dt} = m_a \cdot C_{pa} \cdot (T_r - T_o) - h_w \cdot A_w \cdot (T_w - T_r) - h_f \cdot A_f \cdot (T_f - T_r) - P_{fan}$$

By using the operator notation (D) and rearranging, the above equation becomes:

$$(D + K_9)\,T_r - K_6 \cdot T_w - K_7 \cdot T_f = K_8 \cdot T_o \quad \text{...................................} (3)$$

where:

M_a = *Mass of the Air inside the room (kg)*
C_{pa} = *Specific heat of the Air (W. sec / kg. °C)*
T_r = *Temperature of the Room Air (°C)*
m_a = *Mass flow rate of the Outside Cold Air (kg/sec)*
T_o = *The Outside Cold Air temperature (°C)*
P_{fan} = *The A/C plant's Power (W)*

$$K_5 = \frac{m_a}{M_a}, \qquad K_6 = \frac{h_w \cdot A_w}{M_a \cdot C_{pa}}, \qquad K_7 = \frac{h_f \cdot A_f}{M_a \cdot C_{pa}}, \qquad K_8 = \frac{P_{fan}}{M_a \cdot C_{pa} \cdot T_o} + K_5,$$

$$K_9 = K_5 + K_6 + K_7$$

K_5, K_6, K_7, K_8, K_9 = Constants (1/sec) = Constants * 60 (1/min)

2-4 The Equation of Air Room Temperature Change During Cooling

Equations (1, 2 and 3) are simultaneous linear differential equations of several dependent variables (T_r, T_w and T_f), each of these equations is function of a single independent variable usually the time (t).

Thus, equations (1, 2 and 3) can be written and satisfied by the air room temperature (T_r) in the form:

$$\begin{vmatrix} K_1 & 0 & -(D+K_1) \\ K_2 & -(D+K_4) & 0 \\ (D+K_9) & -K_6 & -K_7 \end{vmatrix} T_r = \begin{vmatrix} 0 & 0 & -(D+K_1) \\ -K_3 \cdot T_o & -(D+K_4) & 0 \\ K_8 \cdot T_o & -K_6 & -K_7 \end{vmatrix}$$

Expanding the determinant and operating on the known function on the right hand side, it will yields to:

$$\left(D^3 + L\,D^2 + J\,D + N\right) \cdot T_r = E \quad \text{.. (4)}$$

where:

$$L = K_1 + K_4 + K_9, \qquad J = K_1 \cdot K_4 + K_1 \cdot K_9 + K_4 \cdot K_9 - K_1 \cdot K_7 - K_2 \cdot K_6$$

$$N = K_1 \cdot \left(K_4 \cdot K_9 - K_4 \cdot K_7 - K_2 \cdot K_6\right), \qquad E = K_1 \cdot \left(K_4 \cdot K_8 + K_3 \cdot K_6\right) \cdot T_o$$

Equation (4) has two solutions, the Complementary function and the Particular Integral [7], it easy to see that the Particular Integral solution is:

$$\left(T_r\right)_p = E / N$$

Simplifying, the above equation for $(T_r)_p$, it yields to:

$$\left(T_r\right)_p = T_o + \frac{\left(K_2 + K_3\right) \cdot P_{fan}}{M_a \cdot C_{pa} \cdot \left(K_2 \cdot K_5 + K_3 \cdot K_5 + K_3 \cdot K_6\right)}$$

Letting $(T_r)_p$ to be equal to (T_e) which is representing the room Air temperature at the end of the cooling cycle:

$$T_e = T_o + \frac{\left(K_2 + K_3\right) \cdot P_{fan}}{M_a \cdot C_{pa} \cdot \left(K_2 \cdot K_5 + K_3 \cdot K_5 + K_3 \cdot K_6\right)}$$

Then, to find a complete solution for the homogeneous equation of equation (4) by letting it equal to zero:

$$\left(D^3 + L\,D^2 + J\,D + N\right) \cdot T_r = 0 \quad \text{.. (5)}$$

It is natural to try a solution of:

$$T_r = e^{mt}$$

where, (m) is a constant to be determined. Substituting into equation (5), to obtain a purely algebraic equation which is known as the Characteristic equation:

$$\mathbf{m^3 + L\,m^2 + J\,m + N = 0} \quad \textbf{(6)}$$

To find the roots of equation (6) which is a general cubic equation, the algebraic formulas are used as follows:

Letting, $m = m_o - L/3$. Substituting for (m) in Equation (6) to obtain new equation in term of (m_o):

$$\mathbf{m_o^3 = A\,m_o + B} \quad \textbf{(7)}$$

where:

$$\mathbf{A = \left(1/3 \cdot L^2 - J\right),\; B = \left(1/3 \cdot J\,L - 2/27 \cdot L^3 - N\right)}$$

letting,

$$\mathbf{p = A/3,\;\; q = B/2}$$

to have three roots real and distinct, the following condition should be satisfied:

$$\mathbf{q^2 - p^3 < zero}$$

the roots for equation (7) are:

$$\mathbf{m_{o1} = 2 \cdot \sqrt{p} \cdot \cos(u/3)}$$

$$\mathbf{m_{o2} = 2 \cdot \sqrt{p} \cdot \cos(u/3 + 120)}$$

$$\mathbf{m_{o3} = 2 \cdot \sqrt{p} \cdot \cos(u/3 + 240)}$$

where:

$$\mathbf{\cos u = q/\left(p \cdot \sqrt{p}\right) \qquad 0 < u < 180^o}$$

Thus, the complete solution for equation (6) the characteristic equation after substituting for ($m = m_o - L/3$) becomes:

$$\boldsymbol{m_1 = 2 \cdot \sqrt{p} \cdot \cos(u/3) - L/3}$$

$$\boldsymbol{m_2 = 2 \cdot \sqrt{p} \cdot \cos(u/3 + 120) - L/3}$$

$$\boldsymbol{m_3 = 2 \cdot \sqrt{p} \cdot \cos(u/3 + 240) - L/3}$$

Thus the general solution for equation (4) is:

$$\mathbf{T_r = T_o + \frac{(K_2 + K_3) \cdot P_{fan}}{M_a \cdot C_{pa} \cdot (K_2 \cdot K_5 + K_3 \cdot K_5 + K_3 \cdot K_6)} + C_1 \cdot e^{m_1 t} + C_2 \cdot e^{m_2 t} + C_3 \cdot e^{m_3 t}}$$

Substituting for (T_e), then the general solution becomes:

$$T_r = T_e + C_1 \cdot e^{m_1 t} + C_2 \cdot e^{m_2 t} + C_3 \cdot e^{m_3 t} \quad \ldots\ldots (8)$$

The arbitrary constants (C_1, C_2 and C_3) may be obtained by applying the initial conditions at the beginning of the cooling by the outside cold air which are:

$$T_r = T_{ri} = T_{wi} = T_{fi} \qquad \text{at} \quad t = 0$$

This implies that:

$$\left.\begin{aligned} \frac{dT_r}{dt} &= 0 \\ \frac{dT_r^2}{dt^2} &= 0 \end{aligned}\right\} \qquad \text{at} \quad t = 0$$

Substituting in equation (8) the general equation and solving for (C_1, C_2 and C_3) which yields to:

$$C_1 = \left[\frac{(T_{ri} - T_e)\cdot m_2 \cdot m_3 \cdot (m_3 - m_2)^2}{m_1 \cdot m_2 \cdot m_3 (m_3 - 2m_1 + m_2) + m_2^2 (m_1^2 + m_2 \cdot m_3 - m_1 \cdot m_2) + m_3^2 (m_1^2 + m_2 \cdot m_3 - m_1 \cdot m_3 - 2m_2^2)}\right]$$

$$C_2 = -\left[\frac{(T_{ri} - T_e)\cdot m_1 \cdot m_3 \cdot (m_3 - m_1)\cdot (m_3 - m_2)}{m_1 \cdot m_2 \cdot m_3 (m_3 - 2m_1 + m_2) + m_2^2 (m_1^2 + m_2 \cdot m_3 - m_1 \cdot m_2) + m_3^2 (m_1^2 + m_2 \cdot m_3 - m_1 \cdot m_3 - 2m_2^2)}\right]$$

$$C_3 = -\left[\frac{(T_{ri} - T_e)\cdot m_1 \cdot m_2 \cdot (m_1 - m_2)\cdot (m_3 - m_2)}{m_1 \cdot m_2 \cdot m_3 (m_3 - 2m_1 + m_2) + m_2^2 (m_1^2 + m_2 \cdot m_3 - m_1 \cdot m_2) + m_3^2 (m_1^2 + m_2 \cdot m_3 - m_1 \cdot m_3 - 2m_2^2)}\right]$$

Rearranging the general solution to have the final form which becomes:

$$\frac{T_r - T_e}{T_{ri} - T_e} = C_1 \cdot e^{m_1 t} + C_2 \cdot e^{m_2 t} + C_3 \cdot e^{m_3 t} \quad \ldots\ldots (9)$$

where: (C_1, C_2 and C_3) become:

$$C_1 = \left[\frac{m_2 \cdot m_3 \cdot (m_3 - m_2)^2}{m_1 \cdot m_2 \cdot m_3 (m_3 - 2m_1 + m_2) + m_2^2 (m_1^2 + m_2 \cdot m_3 - m_1 \cdot m_2) + m_3^2 (m_1^2 + m_2 \cdot m_3 - m_1 \cdot m_3 - 2m_2^2)}\right]$$

$$C_2 = -\left[\frac{m_1 \cdot m_3 \cdot (m_3 - m_1)\cdot (m_3 - m_2)}{m_1 \cdot m_2 \cdot m_3 (m_3 - 2m_1 + m_2) + m_2^2 (m_1^2 + m_2 \cdot m_3 - m_1 \cdot m_2) + m_3^2 (m_1^2 + m_2 \cdot m_3 - m_1 \cdot m_3 - 2m_2^2)}\right]$$

$$C_3 = -\left[\frac{m_1 \cdot m_2 \cdot (m_1 - m_2)\cdot (m_3 - m_2)}{m_1 \cdot m_2 \cdot m_3 (m_3 - 2m_1 + m_2) + m_2^2 (m_1^2 + m_2 \cdot m_3 - m_1 \cdot m_2) + m_3^2 (m_1^2 + m_2 \cdot m_3 - m_1 \cdot m_3 - 2m_2^2)}\right]$$

3. Sample of Calculation

In this paragraph a sample of calculation will put into practice for a small building with the following data and as shown in **Fig.(4)** below:

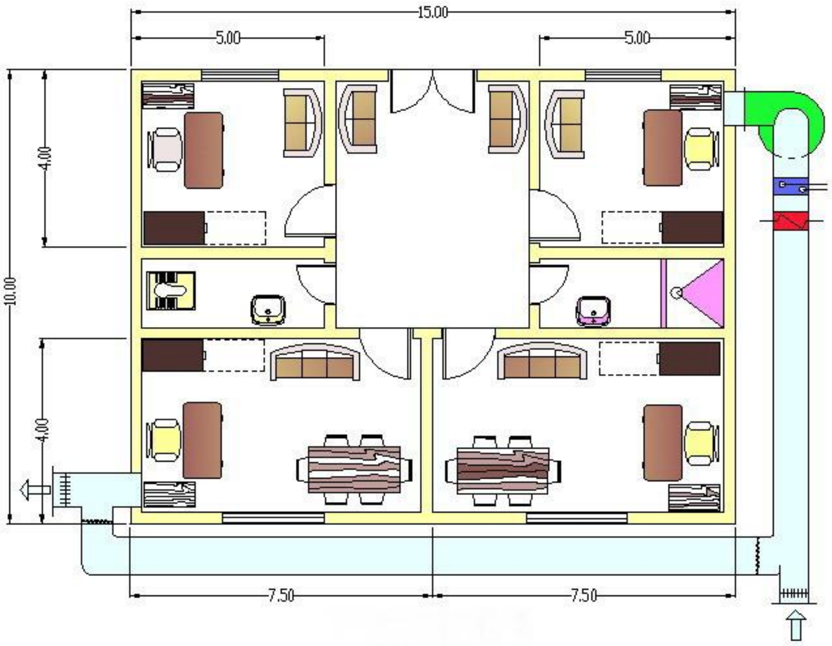


Figure (4) Building of one story with walls, ceiling and floor made of bricks contains furniture

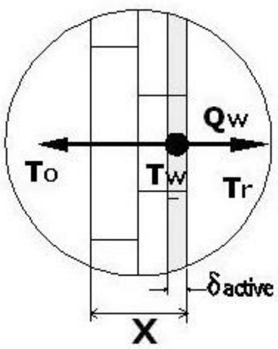
For simplification, the building is of one story and with walls; ceiling and floor are all made of Bricks of thickness (25 cm).

The Active thickness for these walls, ceiling and floor will be calculated first to have a complete data for this building and then all other factors will be found.

Using the formula for the Active thickness after finding out the Thermal diffusion coefficient for the Brick as shown below:

$$\delta_{act} = \sqrt{\frac{2\alpha}{w}}$$

$$w = \frac{2 * \pi}{t_c}$$



where:

α = The Coefficient of thermal Diffusion (m^2/sec) = 0.3 X 10^{-6}

t_c = The time for the cooling cycle (sec) = 1 hour = 3600 sec (this time is recommended for free cooling because the removing of storage heat become too slow).

w = The frequency (cycle/sec), will be calculated as follows:

$$\mathbf{w} = \frac{\mathbf{2 * \pi}}{\mathbf{t_c}} \quad \Rightarrow \quad w = 1.745 \text{ X } 10^{-3} \text{ (rad/sec.)}$$

Then, the Active thickness will be:

$\delta_{act} = 0.0185$ m

The mass (M_w) for exterior walls, floor and roof which are exposed to outside temperature will be used to calculate the constant (K_3), and the mass (M_w) for all walls, floor and roof with the partitions inside the building should be used to calculate the constant (K_2) because the partitions not lose heat to the outside air during the cooling period. Data for the furniture, walls, roof and floor are shown in the below **Tables (1)** and **(2)**:

Table (1) Furniture data used in the building

No.	Description	Dimension	Qty.	Unit area (m2)	Total Area (m2)	Unit mass (kg)	Total mass (kg)
1	Desk	1.8x1.2x0.75	1	10	10	85	85
2	Desk	1.2x1.0x0.75	3	5	15	45	135
3	Meeting table	2.4x1.2x0.75	2	7	14	80	160
4	Computer table	1.2x0.5x0.75	4	3	12	25	100
5	Chair-Desk & Chair	0.5x0.5x1.0	16	1.5	24	20	320
6	File-Cabinet	2.0x1.0x0.5	4	8	32	50	200
7	Sofa- 3 seat	1.5x0.6x1.0	2	3.5	7	40	80
8	Sofa- 3 seat	1.2x0.6x1.0	4	2.5	10	30	120
Total					**124**		**1200**

Table (2) Walls, roof and floor data used in the building

No.	Description	Total Area (m2)	Active thickness	Volume (m3)	Density (kg/m3)	Total mass (kg)	Exposed only (kg)
1	Exposed walls	150	0.0185	2.775	1500	4162.5	4162.5
2	Exposed roof	150	0.0185	2.775	1500	4162.5	4162.5
3	Exposed floor	150	0.0185	2.775	1500	4162.5	4162.5
4	Partitions	160	0.0185	2.96	1500	4440	
Total		**610**				**16927.5**	**12487.5**

For such Building shown in **Fig.(4)**, the required air conditioning system to cover the cooling load for this building has the following specification:

- Actual cooling capacity (with ambient temp. equal 46 ^{o}C) = 31 kw
- Air Conditioning Compressor power input = 10.5 kw
- Fan flow rate = 125 m^3 / min
- Fan power input = 1.76 kw

With the above data, the equation for (T_r) with (T_o = 24 ^{o}C & T_{ri} = 30 ^{o}C) will be obtained for two different air flow rate:

- Mass flow rate for cold air (m_a = 150 kg/min.), the equation is:

$$\mathbf{T_r = 24.007 + 11.13 \cdot e^{-0.015\,t} + 0.003 \cdot e^{-0.983\,t} - 5.14 \cdot e^{-0.033\,t}}$$

- Mass flow rate for cold air (m_a = 75 kg/min.), the equation is:

$$\mathbf{T_r = 24.001 + 9.66 \cdot e^{-0.012\,t} + 0.003 \cdot e^{-0.847\,t} - 3.66 \cdot e^{-0.032\,t}}$$

Figure (5) below shows room air temp. change with the time and the difference between each of the above equations:

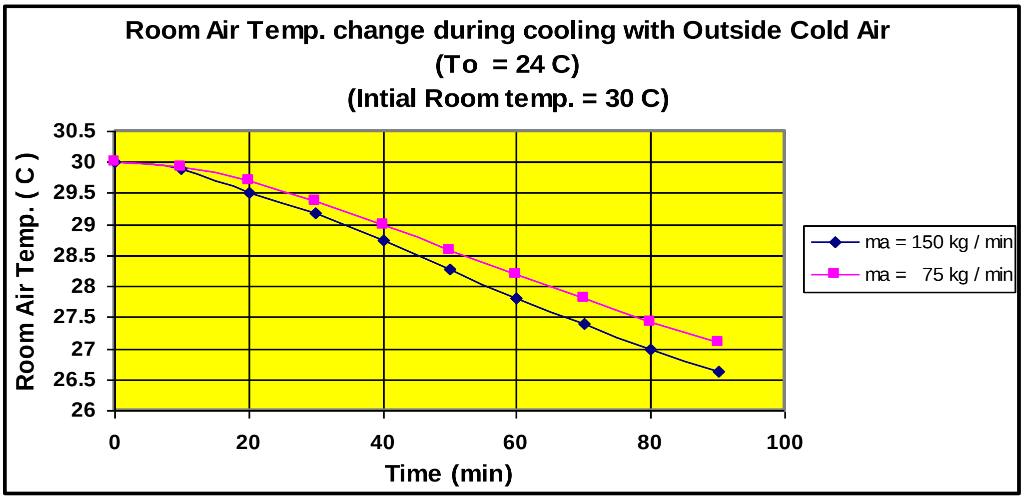


Figure (5) Room air temp. change during cooling with outside cold air (T_o = 24 °C) & (T_{ri} = 30 °C) for different air mass flow

The equation for (T_r) with (T_o = 24 ^{o}C and T_{ri} = 32 ^{o}C) will be obtained for two different air flow rate:

- Mass flow rate for cold air (m_a = 150 kg/min.), the equation is:

$$\mathbf{T_r = 24.007 + 14.84 \cdot e^{-0.015\,t} + 0.004 \cdot e^{-0.983\,t} - 6.85 \cdot e^{-0.033\,t}}$$

- Mass flow rate for cold air (m_a = 75 kg/min.), the equation is:

$$\mathbf{T_r = 24.001 + 12.87 \cdot e^{-0.012\,t} + 0.005 \cdot e^{-0.847\,t} - 4.88 \cdot e^{-0.032\,t}}$$

Figure (6) below shows room air temp. change with the time and the difference between each of the above equations:

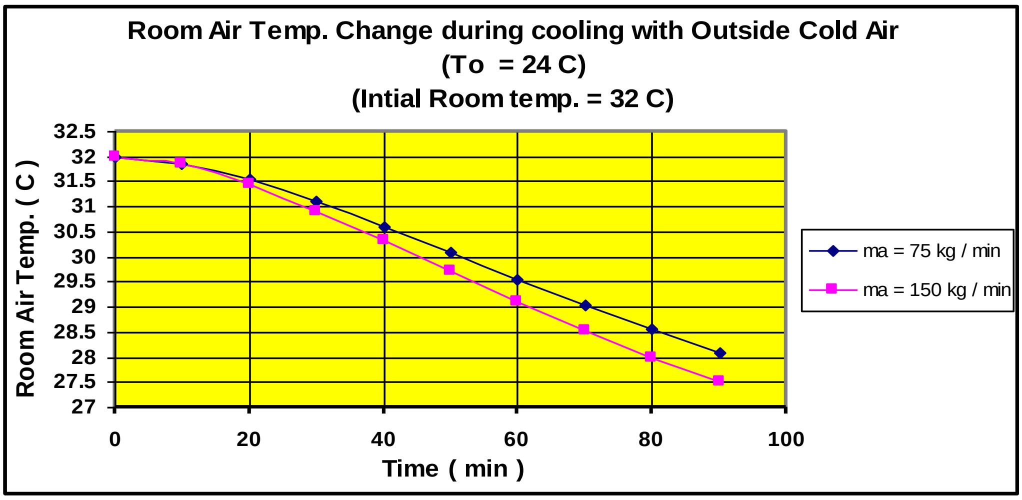


Figure (6) Room air temp. change during cooling with outside cold air (To = 24 °C) & (Tri = 32 °C) for different air mass flow

As the outside cold air passes through the building, it will work to remove the stored heat in the building components (furniture, walls, floor and ceiling) at any time during the cooling period.

The instantaneous heat (Q_i) which is removed can be calculated with the following equation:

$$\mathbf{Q_i = m_a \cdot C_{pa} \cdot (T_r - T_o) - P_{fan}}$$

As a practice to obtain (Q_i) at a specified time (t = 40 min.), (m_a = 150 kg/min.) and (T_o = 24 ^{o}C), room air temp. (T_r = 30.3 ^{o}C) during the cooling period, then by using the above equation:

$$\mathbf{Q_i = (150/60) \times 1.00 \times (30.3 - 24) - 1.76}$$
$$\mathbf{Q_i = 13.99\ kw}$$

For (m_a = 75 kg/min.), and fan power (P_{fan} = 0.2 kw (obtained by using fan third law)), then by using the above equation:

$$\mathbf{Q_i = (75/60) \times 1.00 \times (30.3 - 24) - 0.2}$$
$$\mathbf{Q_i = 7.675\ kw}$$

Figure (7) below shows instantaneous heat (Q_i) change with the time and the difference between (Q_i) for two flow rates used during the cooling cycle:

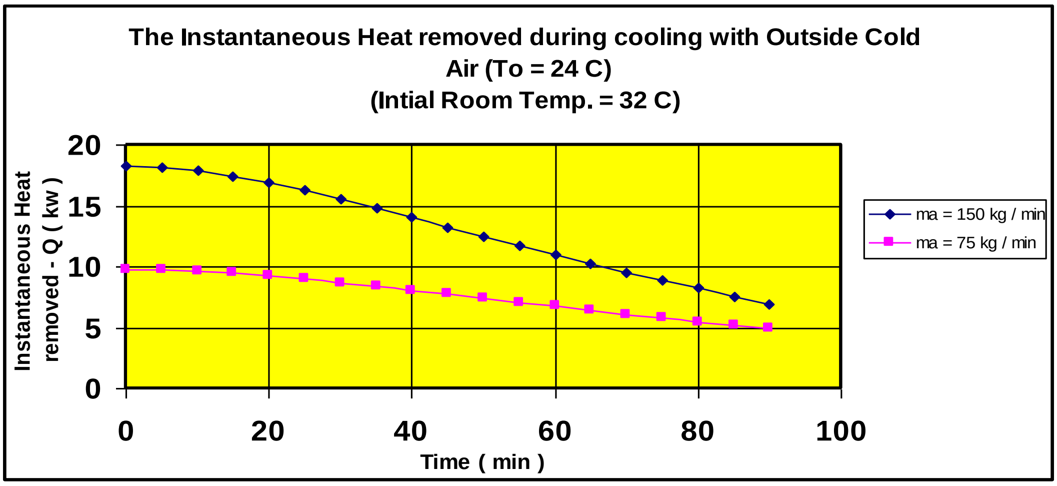


Figure (7) Instantaneous heat removed during cooling with outside cold air (To = 24 °C) & (Tri = 32 °C) for different air mass flow

and to calculate the average of instantaneous heat removed (Q_{av}) during any time of the cooling cycle with outside cold air, the Trapezoidal rule can be used to obtain it as states below:

$$Q_{av} = \frac{\Delta t}{t_c} \cdot \left[\frac{Q_{i \cdot 1} + Q_{i \cdot n}}{2} + Q_{i \cdot 2} + Q_{i \cdot 3} + \dots\dots\dots + Q_{i \cdot n-1} \right]$$

For (25 min.) period and with time interval of (5 min.), the Average Heat will be:

$$Q_{av} = \frac{5}{25} \cdot \left[\frac{18.24 + 16.25}{2} + 18.15 + 17.88 + 17.44 + 16.89 \right]$$

Q_{av} = **17.52 kW**

Figure (8) below shows the average heat (Q_{av}) change with the time and the difference between (Q_{av}) for two flow rates used during the cooling cycle:

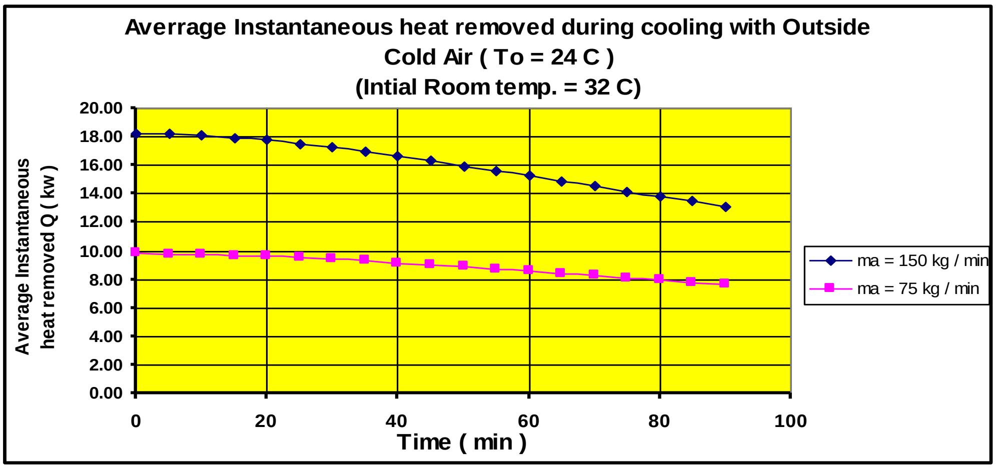


Figure (8) Average heat removed during cooling with outside cold air (To = 24 °C) & (Tri = 32 °C) for different air mass flow

To see the effectiveness of cooling with outside cold air, the coefficient of performance (C.O.P) should be found out and compare it with the (C.O.P) for air conditioning system used for cooling the building.

The instantaneous $(C.O.P)_i$ during the cooling cycle at any time (t = 20 min.), from which (Q_i = 16.89 kw) and with (P_{fan} = 1.76 kw), then it will be:

$$(C.O.P)_i = \frac{Q_i}{P_{fan}} \quad \Rightarrow \quad (C.O.P)_i = \frac{16.89}{1.76} = 9.6$$

The chart below shows the instantaneous $(C.O.P)_i$ change with the time and the difference between $(C.O.P)_i$ for two flow rates used during the cooling cycle:

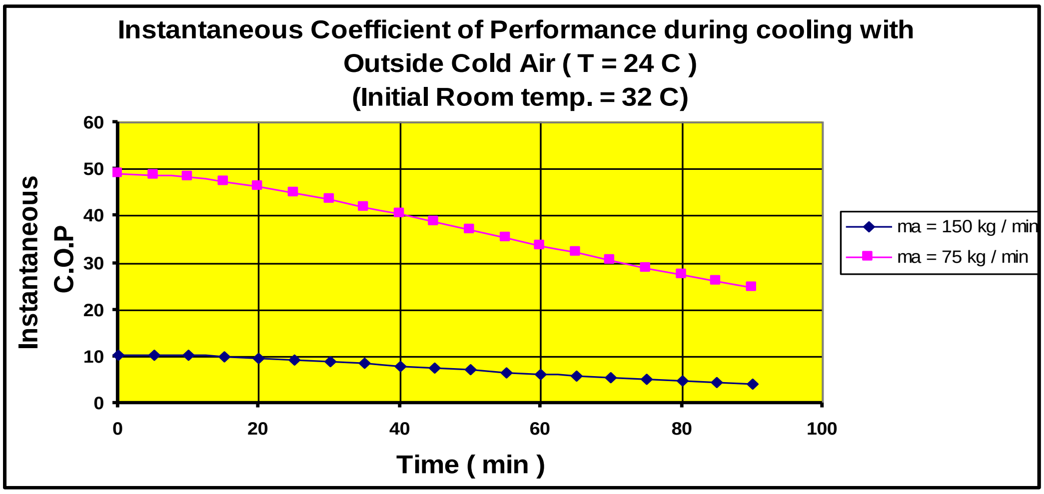


Figure (9) Instantaneous coefficient of performance during cooling with outside cold air (To = 24 °C) & (Tri = 32 °C) for different air mass flow

4. Results and Discussion

The analysis for this research shows the possibility to use the outside cold air for free cooling to remove the stored heat inside the air-conditioned spaces, and as well can be used for a limited time depend mainly on the meteorology database of certain area.

At the beginning of cooling process with outside cold air, the room air temperature will not change immediately in a fast way, **Figs.(5)** and **(6)** show that and representing the change in room air temperature for different air flow rate (150 kg/min.) and (75 kg/min.), and as well for different initial room air (t_{ri} = 30 and 32 ^oC). It appears from these figures that the change in room air temperature for air flow rate of (150 kg/min.) is faster than air flow rate of (75 kg/min.), which indicate that the change depends on:

- The ratio of (m_a/M_a), increasing this value causing faster change of air temperature inside the building and also increasing the air velocity inside the space which causing to increase the heat transfer coefficient between the air and the building components.

- The temperature difference between the initial room temperature and outside cold air ($T_{ri} - T_o$) causing faster heat exchange between the air and the building components.

Similarly, the instantaneous heat and the average heat (Q_i and Q_{av}) removed from the building is at its highest value at the beginning of cooling process as shown in **Figs.(7)** and **(8)**, this is because of the temperature difference is at its greatest value which is equal to ($T_{ri} - T_o$), it is the most important factor to increase the heat exchange between the air and the building components, then it begin to decrease as soon as the room air temperature changes and becomes lower and lower, and in addition to that the building components (furniture, walls …. etc) start to lose its stored heat.

The last term to be discussed is the coefficient of performance (C.O.P), which is shown in **Fig.(9)** as the Instantaneous $(C.O.P)_i$.

From the figure and the notes above, it is clear that C.O.P increases as:

- The outside cold air temperature (T_o) is decreased or increasing in the initial room temperature (T_{ri}).
- The C.O.P will increase as the air flow rate is decreased, this is because as the flow rate is decreased the fan consumes less power according to the fan third law explained below:

$$\frac{\mathbf{P_1}}{\mathbf{P_2}} = \left[\frac{\mathbf{V_1}}{\mathbf{V_2}}\right]^3$$

where: (V) is the air volume flow rate in (m^3/min.) and (P) is the fan power, thus when the volume flow rate is decreased to the half (1/2) of the initial flow rate then the power will become (1/8) of the initial power, and for research case the fan power will change from (1.76 kw) to (0.2 kw).

5. Conclusions

1. The using of the outside cold air to remove the stored heat in building even in small quantity will lead to energy conservation for specified time, after that the cooling process of free cooling become un-useful.
2. The C.O.P will increase when the air flow rate decreases, but the heat removed (Q) will be less and in this case it needs more time to remove or absorb the stored heat in the building, this point is open for discussion and depends on the meteorology database for each area which can be scheduled to use the outside cold air for free cooling.
3. The time required for the free cooling where after that the cooling process will not be economical can be calculated or obtained, this value is equal to the (C.O.P) for air conditioning plant used to cool the building, and in this case it is equal to:

$$\mathbf{(C.O.P)_{sys}} = \frac{\mathbf{Capacity}}{\mathbf{Work}} \quad \Rightarrow \quad \mathbf{(C.O.P)_{sys}} = \frac{\mathbf{31\,kW}}{\mathbf{10.5\,kW}} = \mathbf{2.95}$$

This is because as the (C.O.P) of the free cooling process reaches the value of the (C.O.P) of the air conditioned plant, then it is better to use air conditioned plant to cool the building because it needs less time to remove the heat of the building with the same power consumed if free cooling is used which needs more time to cool the building.

6. References

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