Linear Analysis of Continuous Composite Concrete-Steel Beam with Partial Connection

Asst.Lect. Hesham Abd AL- Latef Numan
Civil Eng. Dept. College of Eng. AL- Mustansiriya University

Abstract

This work presents a theoretical study of the linear analysis of continuous composite concrete-steel beam with partial connection. The concrete and steel components are connected together by stud shear connectors.

Equilibrium and compatibility equations are derived for the forces and displacements at the assumed element. As a result, one simultaneous differential equation of second order in terms of slip is obtained. The role of concrete layer at negative moment is restricted to be a medium surrounding the steel bars and is not associated in the strength of the composite beam. A computer program is written in (Visual Basic) to apply the suggested theoretical model.

Keywords: Continuous beam, Composite beam, Finite differences, Partial connection, Shear connectors.

التحليل الخطي للعتبات المستمرة المركبة من كونكريت وحديد ذات الترابط الجزئي

الخـلاصـــــــة

يتضمن هذا البحث دراسة نظرية للتحليل الخطي للعتبات المركبة المستمرة ذات الترابط الجزئي .المكونات من كونكريت وحديد مرتبطة بروابط قص.

معادلات التوازن والتوافق أشتقت للقوى والإزاحات خلال عناصر الطبقتين ،حيث تم التوصل إلى معادلة تفاضلية أنية من الدرجة الثانية بدلالة الأنزلاق. وظيفة الطبقة الكونكريتية في منطقة العزم السالب أعتبرت طبقة مغطية لحديد التسليح ولاتساهم في المقاومة الكلية للعتبة المركبة. كتب البرنامج الحاسوبي بلغة(فيژيوال بيسك) لتطبيق النموذج النظري المفترض .

1. Introduction

1-1 General

The properties of any material differ from the properties of other; thus there is no material that can provide all the structural requirements. This is the reason of using two or more structural components of different materials and connecting them together in order to make full advantage of their properties in getting one structural composite element that uses the desirable properties of the materials.

Both components in a typical composite beam are usually connected together by shear connectors. If they are not connected, each component will act separately and thus the load carrying capacity of the composite beam is not greater than the sum of the capacities of the components. Otherwise, if provision is made for the horizontal shear force, transmitted between these two components, the total load capacity will be increased significantly.

Shear connectors have the main function of transferring longitudinal shear between the connected components in a composite beam. If the connectors are rigid enough, and their number is adequate then the connection can be considered as full. In this case, no differential movement between the connected components is allowed everywhere .However, in practice, this is not the case, shear connectors are usually flexible and this flexibility generates movement (slip), which is the sole basis of partial interaction behavior of composite beams.

Different types of shear connector are available in practice. Studs are the most widely used type of connectors with a diameter ranging from (13-25) mm, and length ranging from (65-100) mm. The studs are fixed to the steel component by an automatic stud-welding machine. There are two factors that influence the choice of stud diameter (d_S). One is the welding process, which becomes increasingly expensive and difficult for diameters exceeding (19mm), and the other is the thickness (t_f) of the plate or the flange to which the stud should be welded, especially when (d_S/t_f) is less than (2.7)[1]. The studs to be flexible are required to be made from steel with minimum elongation of (18%) and characteristic yield stress not less than (400N/mm^2) [2].

Continuous composite beams are used extensively in the construction of multistory buildings and bridges because of the great benefit that can be obtained by using this type of construction, such as reduced beam moments, considerable reduction in deflection, the simplification in joint details and increase erection facility due to self-supporting nature of the construction.

1-2 Literature review

In 1972, Yam and Chapman [3] presented a study for the inelastic behavior of continuous composite beams. It was assumed that the concrete component has no tensile strength and the horizontal forces are transmitted through the reinforcement at the negative moment region.

In 1975, Johnson [4] derived a similar differential equation of Newmark's approach. The equilibrium and compatibility equations were reduced to a single second order differential equation in terms of interface slip (u_{cs}) instead of axial forces. The solution lead to slip values at the interface along the beam span, after satisfying the suitable boundary conditions.

In 1985, Roberts [5] presented an elastic analysis for composite beams with partial interaction assuming a linear shear connector behavior while the normal stiffness of the connectors was taken infinity. The basic equilibrium and compatibility equations were expressed in terms of four independent displacements, which are the horizontal and vertical displacements in each component of the composite section. These equations were solved using the finite difference method of various derivatives.

In 1996, Jasim and Mohammed [6] studied the effect of partial shear connection on the mid-span deflection of simply supported composite beams under different loading. Ultimately, the results were presented as design charts from which the mid –span deflection can be determined. These charts can be used irrespective of the variation in type of loading, geometry of beam and properties of materials.

In 1997, Jasim [7] proposed a procedure to calculate the deflection of continuous composite beam. The reinforcement in the slab is neglected and the concrete is assumed to sustain tensile forces, ignoring the effect of high tension on the concrete. This model is based on prederived Newmark's differential equation, the solution of which will give the axial force and deflection along the composite beam. Design charts for various loading conditions were proposed to calculate the central deflection.

In 2000, Al-Shafi'i [8] presented a theoretical study to the behavior of continuous composite beams of reinforced concrete slab and steel shapes connected together by stud shear connectors. The nonlinear behavior of the three components (i.e. concrete, steel beam and shear connectors) is allowed for. Four basic differential equations in terms of four independent displacements are arrived after satisfying compatibility and equilibrium at the assumed element. Material nonlinearity has been introduced to the formulation using an increment-iterative procedure and a layered system is assumed for each component in which, the stress and strain are calculated at the layer center. The current model is applied to the typical continuous composite beams, tested previously, and shows close prediction with observed results.

In 2001, Al-Sa'ady [9] presented the behavior of multi-layer beam system with partial connection. An assumed element consists of three layers or more, each of which is connected with the other by so-called "Shear Connectors". Equilibrium and compatibility are satisfied for the forces and displacements at the assumed element. As a result, two simultaneous differential equations of the second order in terms of slip at lower and upper interface are obtained; another pair of basic differential equations in terms of axial forces is obtained as an alternative method.

2. Simply supported composite concrete - steel beam with interlayer slip

The basic model, developed by Johnson [4] is presented herein, and the basic differential equation for this approach is derived. The basic assumption of the conventional beam theory was employed, in which plane sections are assumed to remain plane after bending. Also, the connection was assumed to have negligible thickness and possess finite normal and tangential stiffness. Frictional and uplift forces between the two layers are neglected.

2-1 Formulation

An element of a composite beam (concrete slab and steel beam),of length (δx), is shown in Fig. (1).

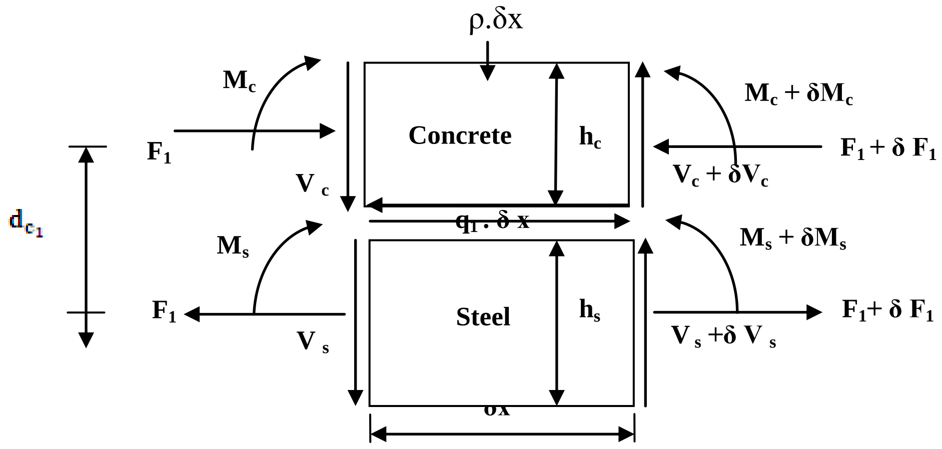


Fig. (1): Composite finite element.

i. Equilibrium

Longitudinal equilibrium of either the concrete or steel beam gives:

$$F_{1,x} = q_1 \quad \ldots(1)$$

Taking moments about the center of the concrete component alone gives:

$$M_{c,x} + V_c = q_1 \cdot \left(\frac{h_c}{2}\right) \quad \ldots(2)$$

Similarly , for steel component:

$$M_{s,x} + V_s = q_1 \cdot \left(\frac{h_s}{2}\right) \quad \ldots(3)$$

in which (q_1) is the shear flow (shear force per unit length) at interface between components, (M) is the bending moment, (V) is shear force, (h) is the thickness of component and subscripts (c) and (s) denote the concrete slab and the steel beam respectively .

The vertical shear at a section, distance (x) from the support, is denoted by (N).Hence.

$$V_c + V_s = N \quad \ldots(4)$$

From equations (2), (3) and (4):

$$M_{c,x} + M_{s,x} + N = q_1 \cdot d_{c_1} \quad \ldots(5)$$

where:

ii. Compatibility

Assuming equal curvatures for the concrete and the steel components gives:

$$W_{c,xx} = W_{s,xx} = W_{,xx} \qquad \dots (6)$$

From elastic beam theory:

$$W_{,xx} = W_{c,xx} = \frac{M_c}{E_c \cdot I_c} \qquad \dots (7)$$

$$W_{,xx} = W_{s,xx} = \frac{M_s}{E_s \cdot I_s} \qquad \dots (8)$$

in which (E) is the modulus of elasticity, (I) is the moment of inertia.

Differentiating equations (7) and (8) once with respect to (x) and rearranging give:

$$M_{c,x} = E_c \cdot I_c \cdot W_{c,xxx} = E_c \cdot I_c \cdot W_{,xxx} \qquad \dots (9)$$

$$M_{s,x} = E_s \cdot I_s \cdot W_{s,xxx} = E_s \cdot I_s \cdot W_{,xxx} \qquad \dots (10)$$

Substituting for ($M_{c,x}$, $M_{s,x}$) into equation (5) gives:

$$W_{,xxx} \cdot E_c \cdot (I_c + m_1 \cdot I_s) + N = q_1 \cdot d_{c_1} \qquad \dots (11)$$

where : $m_1 = \frac{E_s}{E_c}$

The shear flow (q_1) is related to the slip (U_{cs}) by the equation:

$$q_1 = \left(\frac{K}{S}\right) \cdot U_{cs} \qquad \dots (12)$$

in which (K) is the shear stiffness of connectors in one row of interface and (S) is the spacing between the rows of shear connectors. Substituting for (q_1) into equation (11) gives:

$$W_{,xxx} = \frac{1}{E_c \cdot I_{0_1}} \cdot \left(\frac{K \cdot U_{cs}}{S} \cdot d_{c_1} - N\right) \qquad \dots (13)$$

where: $I_{0_1} = (I_c + m_1 \cdot I_s)$

Strains (ε) at the interface can be expressed as:

$$\varepsilon_c^+ = \frac{1}{2}\,.\,h_c\,.\,W_{,xx} - \frac{F_1}{E_c\,.\,A_c} \quad \ldots(14)$$

$$\varepsilon_s^- = -\frac{1}{2}\,.\,h_s\,.\,W_{,xx} + \frac{F_1}{E_s\,.\,A_s} \quad \ldots(15)$$

in which (F_1) is the axial force and (A) is the cross sectional area.
The interface slip strain ($U_{cs,x}$) is given by:

$$U_{cs,x} = \varepsilon_c^+ - \varepsilon_s^- \quad \ldots(16)$$

Substituting for strains (ε_c^+ and ε_s^-) into equation (16) gives:

$$U_{cs,x} = W_{,xx}\,.\,d_{c_1} - F_1\,.\,\alpha_1 \quad \ldots(17)$$

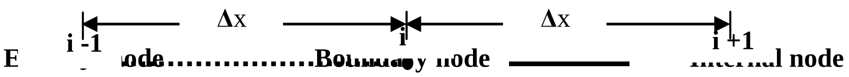
where: $\alpha_1 = \frac{1}{E_c\,.\,A_c} + \frac{1}{E_s\,.\,A_s}$

After differentiating equations (17) once with respect to (x) and substituting for (W,xxx) and ($F_{1,x}$) from equations (13) and (1) , then simplifying and rearranging , equation (17) becomes:

$$U_{cs,xx} = \frac{K\,.\,U_{cs}}{S}\,.\left(\frac{d_{c_1}^2}{E_c\,.\,I_{0_1}} + \alpha_1\right) - \frac{N\,.\,d_{c1}}{E_c\,.\,I_{0_1}} \quad \ldots(18)$$

2-2 Numerical solution and boundary conditions

Equation (18) contains derivative of second order in terms of slip (U_{cs}), which can be expressed in central finite difference form, using three node points, as given below:



$$U_{i,xx} = \frac{U_{i-1} - 2.U_i + U_{i+1}}{\Delta x^2} \quad \text{(Central)} \quad \ldots(19)$$

in which, (Δx) is the spacing between nodes,(i) is the node number.

One external node must be specified at each end of the beam to verify the substitution of the differential equation until last node at the beam. At each interior node along the beam, there is one finite difference equation, therefore; two additional equations are needed to complete the system of algebraic equations, as illustrated below:

$U_{cs} = 0$ when x=L/2 … (20)

$U_{cs,x} = 0$ when x=0 and x=L … (21)

3. Linear Analysis of continuous composite concrete-steel beam with partial connection

3-1 Introduction

The case of a continuous composite steel-concrete beam is understood to differ form that of simply supported beam, as a negative moment exists in the region of the internal supports, which produce tensile forces on the concrete at this region. As the tensile strength of the concrete is very low, the concrete will crack at early stage of loading. This problem will be discussed in details and modeled accurately.

3-2 Assumptions

1. For each of the concrete slab and steel beam, the plane sections before bending is assumed to remain plane after bending. This implies that the distribution of strain is linear over the depth of the concrete slab and the depth of the steel beam. Hence, no transverse shear deformation exists in concrete slab or in steel beam.

2. The shear connection between the two components of the composite beam is continuous along the length (i.e. discrete deformable connectors are assumed to be replaced by a medium of negligible thickness).

3. In negative moment region, concrete has no tensile strength, therefore; the strength of the cracked region will then be based on the steel beam, together with the slab reinforcement, as contribution by the concrete is ignored.

4. Friction and bond effect between concrete slab and steel beam are neglected.

5. It is assumed that uplift forces are resisted by shear connectors without separation, and do not affect of the behavior of the composite beam.

At the cracked section, the role of the concrete slab is restricted to be a medium surrounding the steel bars and is not associated in the strength of the composite beam. The cracked concrete is allowed to have the same curvatures of the steel beam and steel bars along the negative moment region.

3-3 Formulation

An element of a composite beam (reinforced concrete and steel beam),of length (δx), is shown in Fig. (2), which is connected by shear connectors distributed along the length of the span.

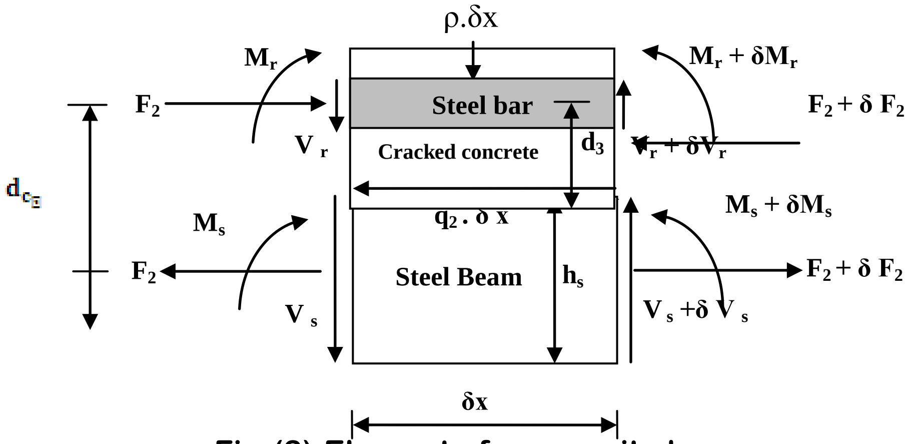


Fig.(2):Element of composite beam.

The basic differential equation derived in section (2.1), is convenient to simulate the behavior of the composite element (steel beam and concrete slab) at positive moment region, while a new formulation is needed to simulate the behavior of the composite element at the negative moment region. The basic differential equation for positive and negative moment regions are superimposed at the point of contra- flexure by using special arrangement of finite difference form.

The new differential equation that governs the behavior of the composite beam at the negative moment region, can be derived as given follows:

i. Equilibrium

Longitudinal equilibrium of either the steel bar or the steel beam gives:

$$F_{2,x} = -q_2 \quad \ldots(22)$$

Taking moments about the center of the steel bar element alone gives:

$$M_{r,x} + V_r = q_2 \, . d_3 \quad \ldots(23)$$

Similarly , for steel beam element:

$$M_{s,x} + V_s = q_2 \cdot \left(\frac{h_s}{2}\right) \quad \ldots(24)$$

in which (q_2) is the shear flow at interface between components, (M) is the bending moment, (V) is the shear force , (h_s) is the thickness steel beam , (d_3) is the distance from bottom fiber of concrete to the center of steel bar and subscripts (r) and (s) denote the steel bar and steel beam respectively.

The vertical shear at a section, distance (x) from the support, is denoted by (N).Hence.

$$V_r + V_s = N \quad \ldots(25)$$

From equations (23), (24) and (25):

$$M_{r,x} + M_{s,x} + N = q_2 \cdot d_{c_2} \quad \ldots (26)$$

where :

ii. Compatibility

Assuming equal curvatures for the steel bar and steel beam then:

$$W_{r,xx} = W_{s,xx} = W_{,xx} \quad \ldots (27)$$

From elastic beam theory:

$$W_{,xx} = W_{r,xx} = \frac{M_r}{E_r \cdot I_r} \quad \ldots (28)$$

$$W_{,xx} = W_{s,xx} = \frac{M_s}{E_s \cdot I_s} \quad \ldots (29)$$

in which (E) is the modulus of elasticity, (I) is the moment of inertia.
Differentiating equation (28) and (29) once with respect to (x) and rearranging gives:

$$M_{r,x} = E_r \cdot I_r \cdot W_{r,xxx} = E_r \cdot I_r \cdot W_{,xxx} \quad \ldots (30)$$

$$M_{s,x} = E_s \cdot I_s \cdot W_{s,xxx} = E_s \cdot I_s \cdot W_{,xxx} \quad \ldots (31)$$

Substituting for ($M_{r,x}$, $M_{s,x}$) into equation (26) gives:

$$W_{,xxx} \cdot E_r \cdot (I_r + m_2 \cdot I_s) + N = q_2 \cdot d_{c_2} \quad \ldots (32)$$

where: $m_2 = \frac{E_s}{E_r}$

The shear flow (q_2) is related to the slip (U_{rs}) by the equations:

$$q_2 = \left(\frac{K}{S}\right) \cdot U_{rs} \quad \ldots (33)$$

in which (K) is the shear stiffness of connector in one row of interface and (S) is the spacing between rows of shear connectors. Substituting for (q_2) into equation (32) gives:

$$W_{,xxx} = \frac{1}{E_r . I_{0_2}} . \left(\frac{K . U_{rs}}{S} . d_{c_2} - N \right) \quad ... (34)$$

where: $I_{0_2} = (I_r + m_2 . I_s)$

Strains (ε) at the interface can be expressed as:

$$\varepsilon_r^+ = d_3 . W_{,xx} - \frac{F_2}{E_r . A_r} \quad ...(35)$$

$$\varepsilon_s^- = -\frac{1}{2} . h_s . W_{,xx} + \frac{F_2}{E_s . A_s} \quad ...(36)$$

in which (F_2) is the axial force and (A) is the cross sectional area.

The interface slip strain ($U_{rs,x}$) is given by:

$$U_{cs,x} = \varepsilon_r^+ - \varepsilon_s^- \quad ...(37)$$

Substituting for strains (ε_r^+ and ε_s^-) into equation (37) gives:

$$U_{rs,x} = W_{,xx} . d_{c_2} - F_2 . \alpha_2 \quad ...(38)$$

where:

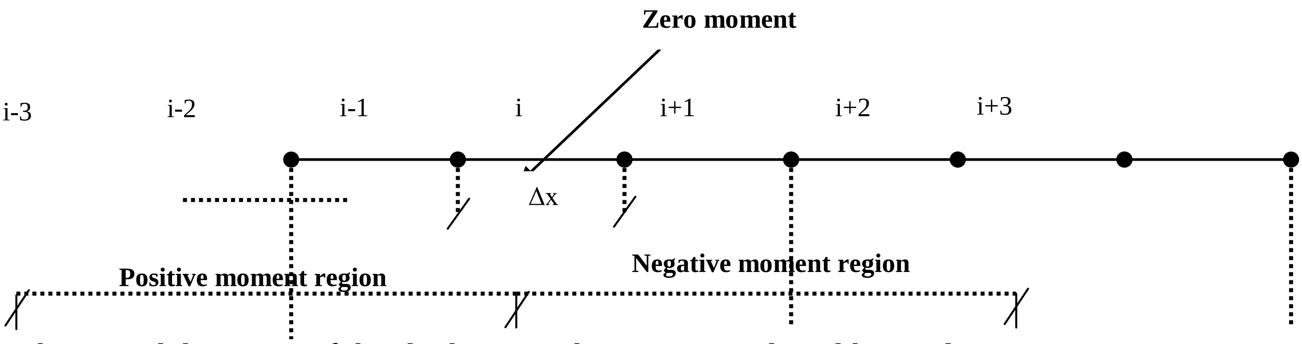
After differentiating equations (38), once with respect to (x) and substituting for (W,xxx) and ($F_{2,x}$) from equations (34) and (22) , then simplifying and rearranging , equation (38) becomes:

... (39)

3-4 Numerical solution and boundary conditions

Equation (39) contains second order derivative in terms of slip (Urs), so that three nodes are required to represent it in finite difference form. One additional node is required at each end of the beam to verify the differential equation at the ends of the beam.

Since one finite difference expression is applied at each node, two boundary conditions are required. For a complete solution, it is required to define the interface between the two systems of differential equations near the point of contra- flexure, in which the material properties has to be separated for the two moment regions at that point. This can be illustrated by the following:



The second derivative of the slip between the concrete and steel beam element at positive moment region (for example at the node number (i-2)) can be expressed as follows:

$$U_{cs,xx} = \frac{U_{cs_{i-3}} - 2\,.\,U_{cs_{i-2}} + U_{cs_{i-1}}}{\Delta x^2} \qquad \ldots (40)$$

The same derivative can be expressed at negative region (for example at node number (i+2)) as follows:

$$U_{rs,xx} = \frac{U_{rs_{i+3}} - 2\,.\,U_{rs_{i+2}} + U_{rs_{i+1}}}{\Delta x^2} \qquad \ldots (41)$$

The second order derivative of the same variable at the point of contra flexural can be expressed as follows:

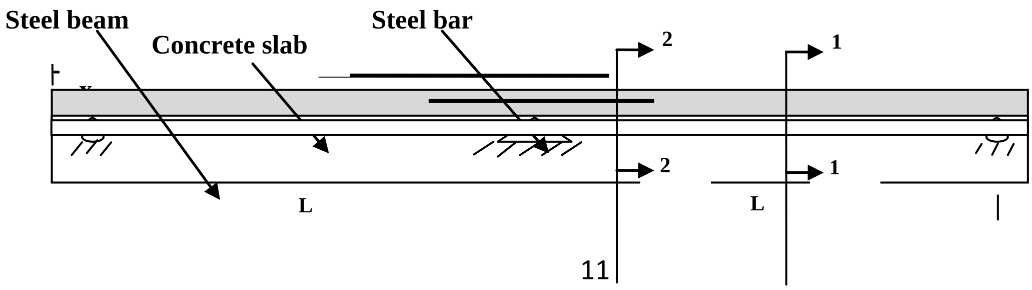
$$U_{,xx} = \frac{U_{cs_{i-1}} - 2\,.\,U_{cs_i} + U_{rs_{i+1}}}{\Delta x^2} \qquad \ldots (42)$$

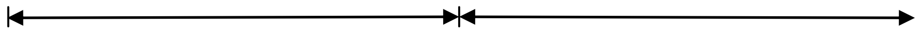
Different types of finite difference representations are used near the point of contra-flexure to verify the continuity conditions at this region.

To complete the set of algebraic equations, for a continuous beam of length (L) (shown in Fig. (3)), in terms of interface slip, the following boundary condition are used:

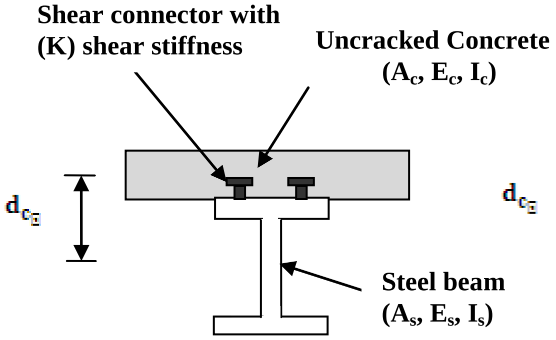
$U_{rs,xx} = 0$ when x=0 … (43)

$U_{rs,x} = 0$ when x=L … (44)

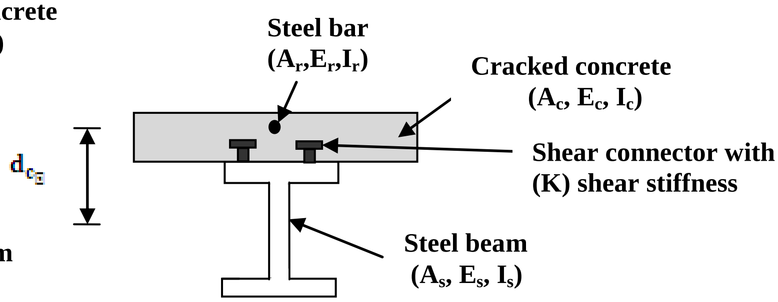




(a)



(b)

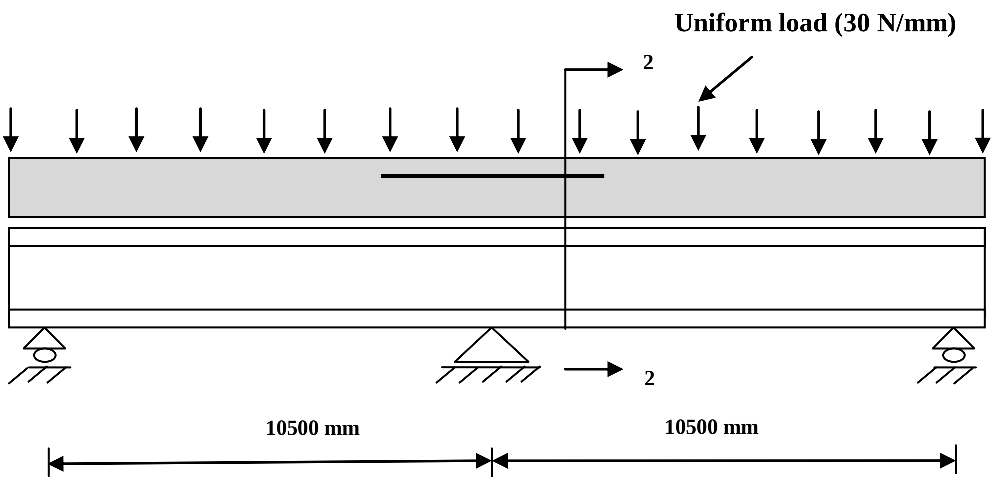


(c)

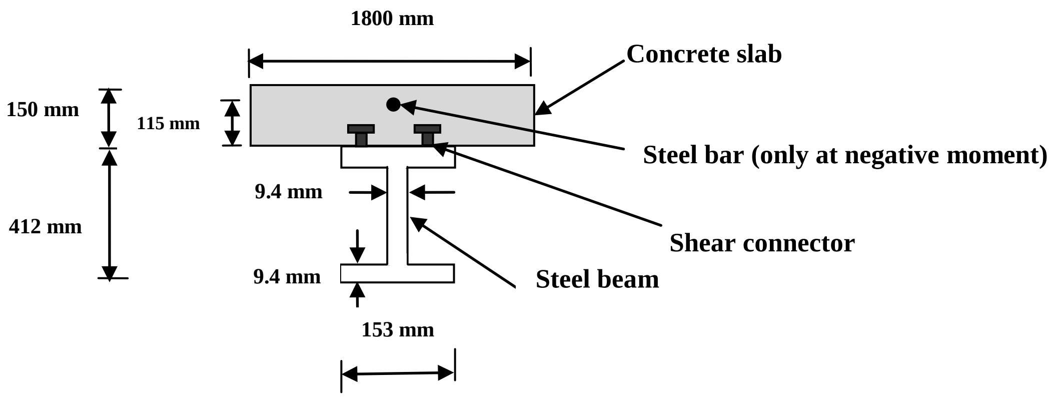
Fig.(3): (a)Typical continuous composite beam.
(b) Section (1-1) at positive region.
(c) Section (2-2) at negative region.

4. Example (1) Continuous composite beam

A single continuous composite beam of two equal spans (each 10500 mm) is subjected to uniformly distributed load of (30 N/mm), as shown in Fig. (4).The applied load is about (43%) of the calculated ultimate capacity of the beam, therefore, the behavior of the beam is within the elastic range. The other properties are given in Table (1), and shown in Fig. (4-b).Superscript (*) indicates assumed information as they are missing from the reference.



(a)



(b)

Fig (4): (a) Continuous composite beam subjected to uniformly distributed load.
(b) Section (2-2) at the beam.

Table (1): Composite beam material properties [8].

Material	Property	Value
Concrete Slab	Characteristic Cube Strength f_{cu} (N/mm^2).	30
	Modulus of Elasticity E_c (N/mm^2).	26700
Steel Beam (I-section)	Characteristic Yield Strength f_y (N/mm^2).	250*
	Modulus of Elasticity Es (N/mm^2).	200000
Steel Bar (six-Φ16mm)	Area (mm^2).	1206*
	Moment of Inertia (mm^4).	19300*
	Modulus of Elasticity E_r(N/mm^2).	200000

Shear Connectors (headed stud)	Diameter (mm) x Height(mm).	19x100
	Spacing (mm).	250
	Number in Row.	2
	Shear Stiffness of Connector (N/mm).	$150x10^3$

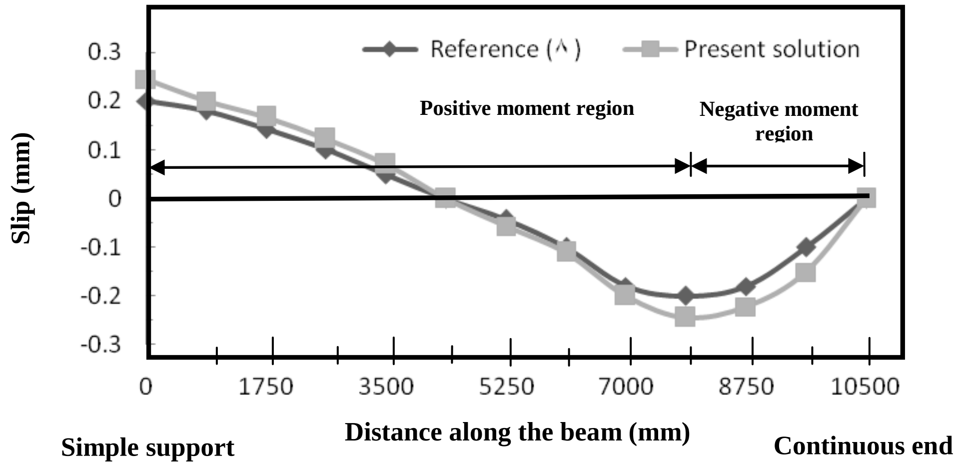


Fig.(5): Variation of interlayer slip along beam

Fig.(5) shows a comparison between Al-Shafi'i [8] and the numerical solution presented in this study. The general behavior for two curves, that maximum slip occurs at simply supported ends of the continuous beam (zero moment) (0.20 from Ref.8 and 0.2442 from present solution) and decreases slowly along the length of the beam until it approaches zero at (0.416 L from left support), near the point of maximum positive moment. Then, the slip is increased in apposite direction until it reaches the maximum value at this region at (0.75L from left support).At the region of negative moment, slip decrease rapidly until it reaches to zero at interior support. Due to symmetry, half of the continuous beam is considered.

5. Conclusions

Based on the results obtained in this investigation, the following can be concluded:

1. Theoretical model for the analysis of continuous composite beam with interlayer slip has been presented in which, the basic equilibrium and compatibility equations are reduced to a single second order differential equation in terms of the interlayer slip instead of axial forces. The suggested approach gives reasonable prediction and can be used for any type of loading and boundary conditions.
2. Solutions of the basic equations can be obtained by expressing the derivatives in finite difference form. Numerical solution obtained in this way shows close agreement with the existing analytical solution, which assumes linear material and shear connector behavior and that the assumption of neglecting concrete strength at the region of negative moment is valid.

3. The numerical solution (finite difference method) can be used even at small intervals with acceptable tolerance since the basic differential equations are of the second order.
4. A computer program is written in Visual Basic Language to do computation, and it is found adequate, saving time and effort.
5. The proposed method can be used to investigate the effect of many parameters on the behavior of composite beam.

6. References

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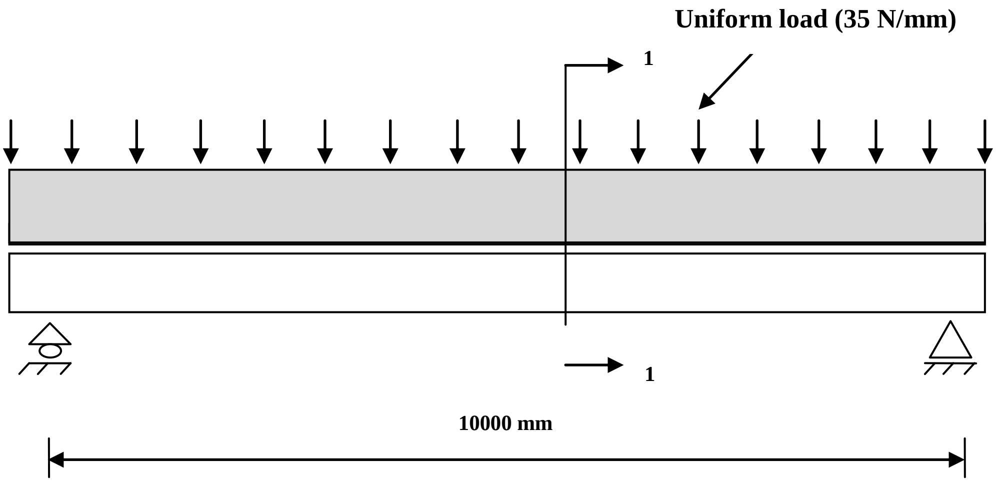
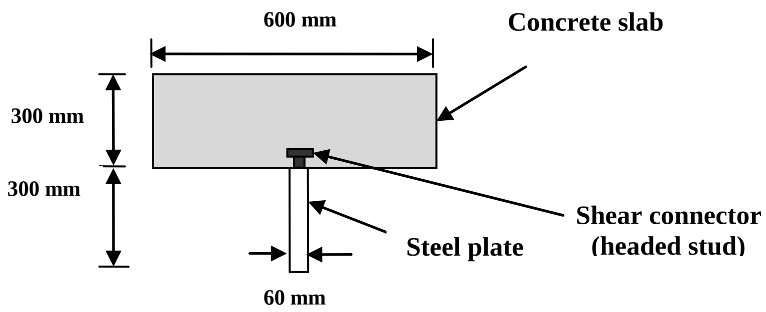
Appendix-1

Example .2 Johnson's example (4)

A typical composite beam is assumed with material properties as used by Johnson's [4]. A convergence study has been carried out on this beam in order to indicate the right number of nodes to be taken for the numerical solution from which an acceptable result can be obtained. The beam of span (10000mm) is subjected to a uniformly distributed load (35 N/mm), as shown in Fig. (6-a).The higher stress is less than (43%) of the cube strength, so assumption of elastic behavior is reasonable. The other properties are given in Table (2), and shown in Fig. (6-b).



(a)



(b)

Fig (6): (a) Simply supported beam subjected to uniformly distributed load.
(b) Section (1-1) at the beam.

Table (2):Material properties of Johnson's example [(4)] .

Material	Property	Value
Concrete Slab	Characteristic Cube Strength f_{cu} (N/mm^2).	30
	Modulus of Elasticity E_c (N/mm^2).	20000
Steel Plate	Characteristic Yield Strength f_y (N/mm^2).	250
	Modulus of Elasticity E_s (N/mm^2).	200000
Shear Connectors (headed stud)	Diameter (mm) x Height (mm).	19x100
	Spacing (mm).	180
	Number in Row.	1
	Shear Stiffness of Connector (N/mm).	$150x10^3$

Table (3): Comparison between numerical solution for the suggested model and Johnson's solution

Number of nodes	Numerical Solution for Suggested Models				Johnson's Solution (for partial Interaction)
	45	35	15	9	
Max. Slip at left hand support (mm)	0.49	0.5	0.52	0.54	0.45

Table (3) shows a comparison between Johnson's solution and the numerical solution presented in this study. It can be seen that, the difference between these solutions is within (20%) when the number of nodes is (9) (including external nodes).This difference is reduced to (15.56%) when the number of nodes becomes (15) and it becomes (11.11%) at

(35) nodes. When the number of nodes becomes (45) the difference is reduced to (8.89%). This illustrates that the numerical solution can be used at small intervals between adjacent nodes with acceptable tolerance. Therefore, the total number of nodes, used to apply the current numerical solution, is (35).This is obtained by dividing the beam into thirty-three equal elements. Hence, close agreement at both ends of the beam has been obtained.

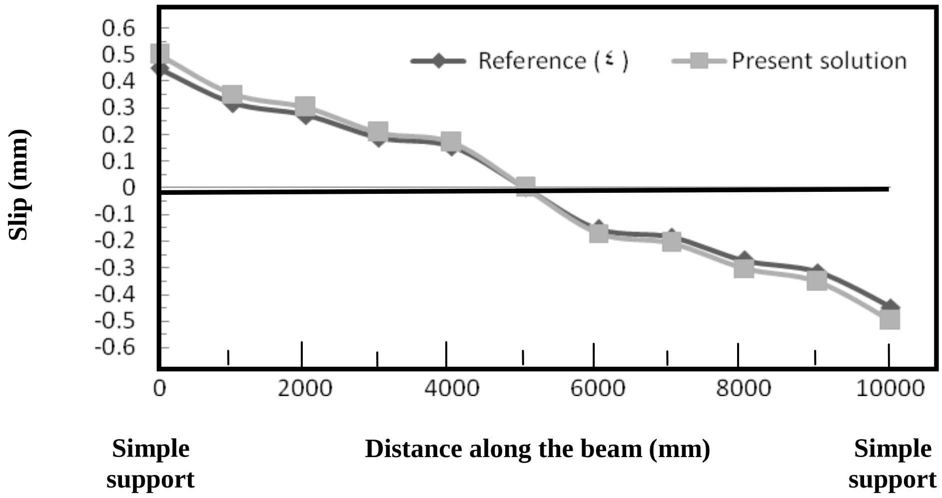


Fig. (7): Variation of interlayer slip along simply supported beam

Appendix-2

A computer program is prepared in this study to solve the numerical models produced in sections (2) and (3) for material properties and boundary conditions. The generated matrices (by applying the finite difference stencil at each node) are solved by using the inverse of matrix as solver subroutine. The program is written in Visual Basic. The program has a wide capacity of application on different examples.

Input data required in this program:

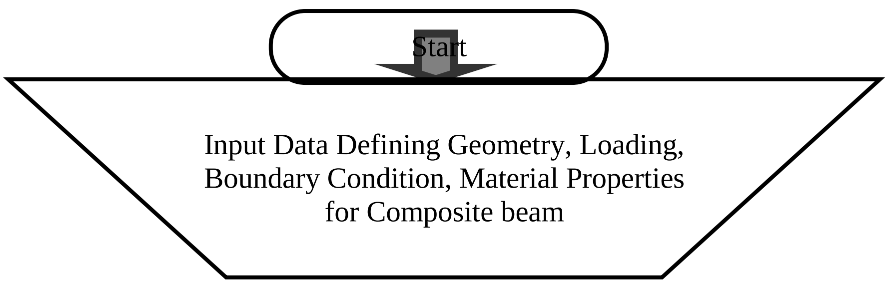
1) Material properties.

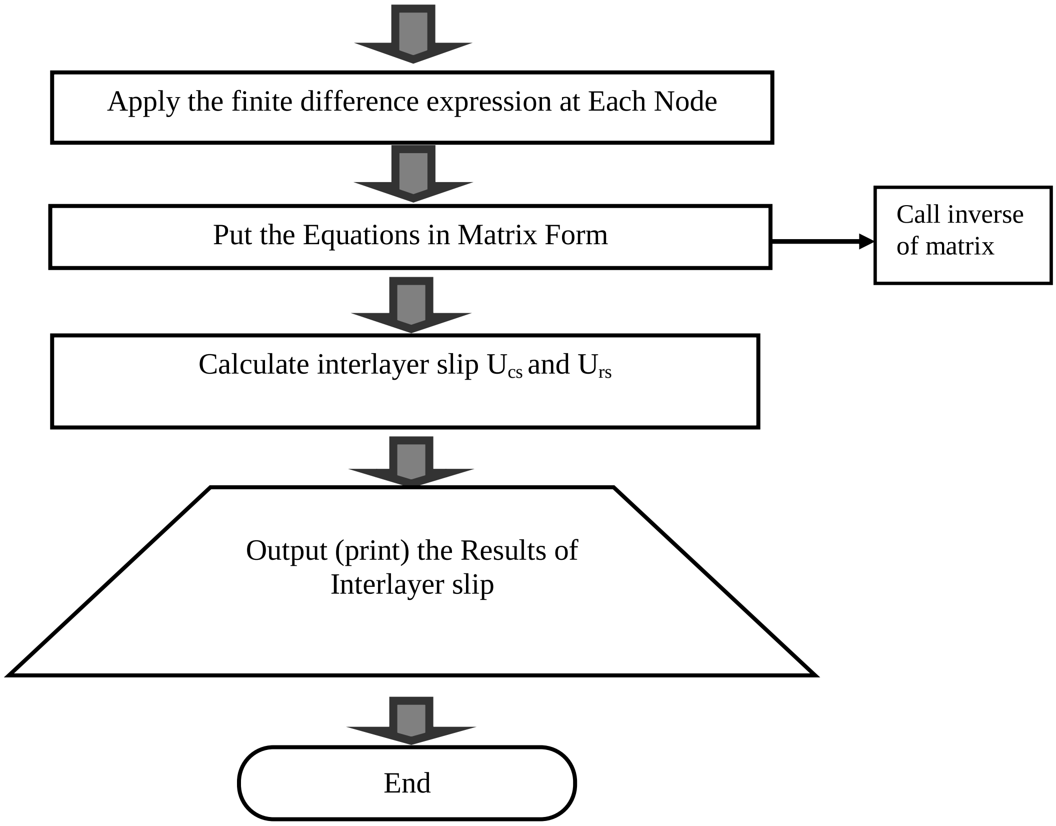
2) Beam geometry like dimensions.

3) Boundary conditions.

4) Finite difference parameter like number of nodes.

Output data produced from this program (for each node) is interlayer slip between layers.





Flow chart showing the linear solution of continuous composite beam