The Performance of Turbo Coded Orthogonal Frequency Division Multiplexing System over Flat Rayleigh Fading Channel

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Abstract

This work investigates the effectiveness of Turbo Codes and Orthogonal Frequency Division Multiplexing (OFDM) system, for communication over the flat Rayleigh fading additive white Gaussian noise (AWGN) channel. The Fast Fourier Transform (FFT) is used in the transmitter and receiver to satisfy the orthogonality between sub-carriers and reduce the implementation complexity. The system performance is investigated for various Doppler frequencies for channel. To reduce the interference at receiver, a guard time with cyclic extension is used. The simulation results show that the turbo code orthogonal frequency division multiplexing (TCOFDM) system achieves large coding gain with lower BER and reduced decoding iterations, where the Turbo code improves the performance of OFDM system by a factor of 5-8 dB, therefore offering higher data rate in wireless mobile communications.

الخلاصية

يتناول هذا البحث دراسة فعالية المرمز Turbo ومنظومة التعدد التقسيمي الترددي المتعامد (OFDM) للإتصال عبر قناة خفوت رابلي المستوي (Flat Rayliegh Fading) و (AWGN). منظومة التعدد التقسيمي الترددي المتعامد (OFDM) مصممة للعمل على معدل إرسال بيانات عالية بإستخدام تقنية التضمين BPSK. يستخدم تحويل فورير السريع (FFT) في المرسل والمستلم لتحقيق التعامد بين الحاملات الثانوية وتقليل التعقيد في بناء منظومة التعدد التقسيمي الترددي المتعامد. تناول هذا البحث أيضاً أداء المنظومة بقيم مختلفة من ترددات (Doppler) للقناة. تم إستخدام الفترة التردي المتعامد. تناول هذا البحث أيضاً أداء المنظومة بقيم مختلفة من ترددات (Doppler) للقناة. تم إستخدام الفترة الحارسة (Guard Time) مع التوسيع الدوري (Cyclic Extension) لتقليل هذا التداخل بين الرموز والحاملات. أثبتت المارسة (Guard Time) مع التوسيع الدوري (Cyclic Extension) لتقليل هذا التداخل بين الرموز والحاملات. أثبتت المارسة (BER مع منظومة التعدد التقسيمي الترددي المتعامد أبين الرموز والحاملات. أثبتت ويالتائج إن إستخدام المرمز OTDD مع منظومة التعدد التقسيمي الترددي المتعامد أعطى مكسب ترميز (Berd ويالتائج إن إستخدام هذه المزمز Ourbar مع منظومة التعدد التقسيمي الترددي المتعامد أعطى مكسب ترميز الما BER ويالتائج إن إستخدام هذه المنظومة في إستادي التوسيمي التردي المتعامد أعطى مكسب ترميز (Berd ويالتائي يمكن إستخدام هذه المنظومة في إتصالات اللاسلكي النائة.

1. Introduction

Orthogonal Frequency Division Multiplexing (OFDM), a popular type of multicarrier transmission ^[1], is an effective modulation technique for high-data-rate wireless and wire line applications, including Digital Subscriber Line (DSL), Digital Audio Broadcasting (DAB), Digital Video Broadcasting (DVB), and Wireless Local Area Network (WLAN). The main advantage of OFDM is its ability to encounter multipath fading without requiring complex equalizer ^[2]. Moreover, OFDM is a bandwidth efficient transmission system and can be easily implemented by the FFT ^[3].

To eliminate intercarrier interference (ICI) arising from intersymbol interference (ISI) in multipath channel, OFDM employs a cyclic prefix, which results in the loss of spectral efficiency. This gives rise to another problem, which is the fact that in a multipath-fading channel, all subcarriers will be received with different amplitudes. In fact, some subcarriers may be completely lost because of deep fades. Hence, even though most subcarrier may be deleted without errors, the overall bit error ratio (BER) will be largely dominated by a few subcarriers with the smallest amplitudes, for which the bit error probability is increased. To avoid this domination by the weakest subcarrier, forward error correction (FEC) coding is essential. In 1993, Turbo codes were shown to have a astonishing performance close to the theoretical Shannon capacity limit in AWGN channel with relatively simple iterative decoding technique ^[4].

As a powerful coding technique, Turbo codes are a prime candidate for wireless applications and being considered for future mobile radio communications ^[4,5]. Turbo code can achieve a remarkably low bit rate (BER) with iterative decoding at an SNR close to the Shannon capacity limit on additive white Gaussian noise channels. Turbo codes are often chosen over Serial concatenated codes in practice because they are less computationally complex given the same constituent codes; they also have lower BERs than seria concatenated codes at low SNRs. To obtain high coding gains with moderate decoding complexity, concatenation of codes has proved to be an attractive scheme.

In this paper the performance of Turbo coded OFDM is improving.

2. Basic Principles of OFDM

OFDM is a parallel transmission scheme, where a high-rate serial data stream is split up into a set of low-rate sub streams, each of which is modulated on a separate subcarrier (SC) (frequency division multiplexing). Thereby, the bandwidth of the subcarriers becomes small compared with the bandwidth of the channel ^[6].

The orthogonal part of the OFDM name indicates that there is a precise mathematical relationship between the frequencies of the carriers in the system. In a normal FDM system, carriers are spaced apart in such way that the signals can be received using conventional filters and demodulators. In such receivers, guard bands have to be introduced between the different carriers, and the introduction of these guard bands in the frequency domain results in a lowering of this efficiency. It is possible, however, to arrange the carriers in an OFDM

signal so that the sidebands of the individual carriers overlap and the signals can still be received without adjacent carrier interference as shown in **Fig.(1**).



The transmitted spectral shape is chosen so that InterCarrier Interference (ICI) does not occur; that is, the spectra of the individual subcarriers are maximum at their frequency and zero at other subcarrier frequencies. The FFT serial data elements (spaced by T=1/R where R is the symbol rate) modulate FFT subcarrier frequencies, which are then frequency division multiplexed. The symbol duration (T_s) has been increased to (NT), which makes the system less susceptible to delay spread impairments ^[6].

The subcarrier frequencies are separated by the multiples of (1/NT) so that, with no signal distortion in transmission, the coherent detection of a signal element in any subcarrier of OFDM system gives no output for a received element in any other subcarriers. Hence, the FFT received signal elements, corresponding to the N subcarriers of OFDM system, are said to be orthogonal. So, no further filtering is needed to separate the different subcarriers. In other words, the power density spectrum has a central positive peak at an individual carrier frequency, and zeros at all other subcarrier frequency ^[7].

Using digital modulation format, the transmitted OFDM symbol waveform can be represented as ^[3]:

$$\mathbf{S}(t) = \mathbf{R}\mathbf{e}\left\{\sum_{k=0}^{N-1} \mathbf{d}(k) \exp(\mathbf{j}2\pi \mathbf{f}_k t)\right\} \qquad (1)$$

where:

d(k): is the modulated data symbol

 f_k : is subcarrier frequency of k^{th} subcarrier which is equal to ($f_c + k\Delta f$).

 Δf : is subcarrier spacing (bandwidth) equal to (1/NT).

f_c : is the carrier frequency.

This expression represents the passband OFDM signal, if the equivalent complex baseband notation is used which is given by ^[3]:

$$\mathbf{S}(t) = \sum_{k=0}^{N-1} \mathbf{d}(k) \exp\left(j2\pi k\Delta \mathbf{f} t\right) \dots (2)$$

Equation (1) represents the general form of complex baseband OFDM signal. At the receiver, all operations at the transmitter are reversed.

It is shown that an OFDM signal is effectively the inverse Fourier Transform of original data stream, and the bank of coherent demodulators is effectively the Fourier Transform.

If the signal is sampled at a rate of (T), then (t=nT), and for orthogonality ($\Delta f = 1/NT$), then Equation (2) can be rewritten as ^[6]:

$$S(n) = \sum_{k=0}^{N-1} d(k) \exp(j2\pi kn/N)$$
 (3)

Equation (3) is exactly the Inverse Discrete Fourier Transform (IDFT) of the data sequence d(k). Further reductions in complexity are possible by using the Fast Fourier Transform algorithm to implement the DFT.

3. Turbo Codes

Turbo Codes is a recently proposed technique that has been shown to achieve very low bit errors at even low SNR values. It essentially combines the concepts of Interleaving and parallel concatenated convolutional encoding. Berrou et. al. proposed Turbo Codes in 1993 in ^[7]. In the following sections, the principles behind turbo-encoding and turbo-decoding are introduced.

3-1 Turbo Encoding

Turbo codes are parallel concatenation of two or more systematic codes. Systematic codes are those for which one of the outputs is the input bits itself. **Figure (2)** shows the block diagram of a rate (1/3) turbo encoder ^[8]. The encoders, encoder 1 and encoder 2, encode the same input information bit u_k but in different order due to the interleaver in front of encoder 2. Appropriate puncturing of parity bits from two encoders can create a Turbo code of desired rate. The sequence x_0 is directly passed to the first encoder, but it's interleaved before being sent to the second encoder. The encoders are usually identical in all other aspects. The sequence x_0 is concatenated with the outputs x_1 and x_2 to form the code word. This is then transmitted over the channel.



Figure (2) Rate (1/3) turbo encoder

The coded bit rate can be found as:

$$\mathbf{R}_{\rm c} = \frac{\mathbf{k}}{3\mathbf{n}} \tag{4}$$

Usually the tail bits are less compared to the length of the input bit sequence and hence the approximation $(k \approx n)$ holds good. Thus the coded bit rate can be approximated as:

$$R_c \approx \frac{1}{3}$$
(5)

The number of encoders is taken to be 2 in **Fig.(2**). More encoders can be used, but the coded bit rate will be reduced significantly.

3-1-1 Encoders in Turbo Encoder

Usually the encoders used in turbo encoder are the Recursive Systematic Convolutional Encoders (RSC). Convolutional encoders are used so that a modified version of the Viterbi Algorithm, known as Soft-Output Viterbi Algorithm (SOVA) can be applied in the decoding process. It can be shown that non-recursive systematic encoders have poor distance properties compared to their recursive counterpart ^[7]. Hence recursive systematic encoders are used.

Figure (3) shows a recursive systematic convolutional encoder. It's systematic since one of the outputs x_k is the same as the input d_k . The constraint length of the encoder is K=3. It can be seen that since the output is fed-back, it is "recursive". In recursive encoders, even an input sequence containing just a single '1' bit will result in a large weight code word.



Figure (3) Recursive systematic convolutional encoder

3-1-2 Interleaver

As shown in **Fig.(2)**, an Interleaver is introduced before the input bit sequence is passed onto the second encoder. By proper interleaving, high weight code word is achieved at the output of the second encoder if the output of the first encoder turns out to be low weight code word ^[4]. Thus in turbo codes, the goal is to make the number of low weight code words less and increase the number of high weight code words. By doing this, the bit error probability is reduced.

An interleaver π is a device characterized by fixed permutation of the time axis ρ_{π} . Interleaver, π , maps input sequence $x=(X(i))_{i=-\infty}^{\infty}$ to obtain permutated output sequence $y=(Y(i))_{i=-\infty}^{\infty}$ with ^[9]:

Y(**i**)=**X**(ρ_{π} (**i**))(6)

Delay function $d_{\pi}(i)$ is defined as:

 $d_{\pi}(i) = i - \rho_{\pi}(i)$ (7)

The characteristic latency is equal to:

where:

 $d_{\pi} \max$ and $d_{\pi} \min$: are maximum and minimum delays.

To obtain the original sequence X(i) from permuted sequence Y(i), deinterleaver is used which has time axis $\rho_{\pi}^{-1}(i)$ obtained from $\rho_{\pi}^{-1}(\rho_{\pi}(i))=i$, then ^[9]:

$$X(i)=Y(\rho_{\pi}^{-1}(i))$$
(9)

<u>Row by Column Interleaver</u>

It is a subclass of block interleaver. Input bits are written as row wise into a matrix with N_R rows and N_C columns ($N_{\pi}=N_R\times N_C$) and read column wise. The corresponding permutation time axis is given by ^[9]:

An (N_R, N_C) row by column interleaver has the following parameter:

$$d_{\pi} \max = -d_{\pi} \min = (N_R - 1)(N_C - 1)$$
(11)

$$D_{\pi} = 2(N_R - 1)(N_C - 1)$$
(12)

3-1-3 Puncturing

Puncturing is defined as the selective deletion of some of the parity bits ^[10]. It can be used to increase the coded data rate. Referring to Fig.(2), puncturing can be accomplished by taking the even bits of the first encoder and the odd bits of the second encoder. Thus the odd bits of first encoder and even bits of second encoder are punctured (deleted). In this case, the coded bit rate is increased from (1/3) to (1/2). The puncturing operation can be accomplished using a multiplexer, which switches itself between the outputs of the two encoders.

In ^[8], the use of puncturing depending on the quality of the channel is proposed. If the channel does not undergo much of fading, then puncturing can be done (coded data rate becomes (1/2)) since a clean version of the user bit sequence will be available at the receiver at low coding itself. In case the channel undergoes severe fading, the puncturing operation can be removed (coded data rate becomes (1/3)) so that more error protection is provided to counteract the fading.

3-2 Turbo Decoding

The Turbo decoding process is done in an iterative manner. A modified BCJR is used to account for the recursive nature of iterative turbo decoding ^[7]. The turbo code decoder consists of two soft input/ soft output constituent convolutional decoders that work together in an iterative fashion. Soft input soft output decoding of convolutional sub codes is done with the use of a priori information of previous decoding steps.

3-2-1 Initialization Stage

The decoding process consists of two stages-Initialization and Iteration stage. **Figure (4)** is the block diagram of the Initialization stage. Let rate (1/2) coding is done at the transmitter. So multiplexer is used at the transmitter to select the even bits of first encoder and odd bits of the second encoder. Let the received sequence corresponding to the information sequence x_0 and that corresponding to multiplexed x_1 , x_2 be r_0 and r_1 respectively. The parity sequence r_1 is de-multiplexed and the even bits are sent to the first decoder. The first decoder also receives the information sequence r_0 . Using these, it produces a soft output of the transmitted bit sequence. This soft output is then interleaved (the interleaver used must be the same as the one used at the transmitter) and passed onto the second decoder. The second decoder also receives its parity sequence from the de-multiplexer. Using these inputs, it produces a soft output. Now the Iteration stage comes into play.



Figure (4) Turbo decoding-initialization stage

3-2-2 Iteration Stage

In this stage, the soft outputs produced by the second decoder are properly de-interleaved so that it can be fed into the first decoder. The first decoder works on this improved bit sequence and produces a soft output. This is, then, interleaved, and passed onto the second decoder. As the number of iterations tends to infinity, the output of the second decoder approaches the MAP estimate ^[8]. Even if the number of iterations is restricted to around 6-8, good performance can be achieved. The block diagram is shown in **Fig.(5**).



Figure (5) Turbo decoding-iteration stage

4. Rayleigh Fading Channel

The characteristics of the Rayleigh fading process will affect the turbo decoder performance. To generate Rayleigh fading channel gain by using mathematical functions, which is called Jake fading generator ^[11]. a_c and a_s are given as the following:

$$\mathbf{A}_{1} = \frac{\left[\left(\mathbf{a}_{c}\right)^{2} + \left(\mathbf{a}_{s}\right)^{2}\right]^{\frac{1}{2}}}{\sqrt{2}} \dots (15)$$

and $\beta_n = \frac{\pi \cdot n}{M_0}$, $\zeta = \pi/4$, and $\omega_n = \omega_m \cos(\frac{2 \cdot \pi \cdot n}{4M_0 + 2})$ where M_0 is the number of low frequency oscillators with frequencies equal to ω_n . a_s and a_c are approximately Gaussian random processes with zero means and unit variances. Then $A_1 = \sqrt{\frac{(a_s)^2 + (a_c)^2}{2}}$ is Rayleigh distributed. In the simulations, M0 is 8 (in practice the ranges of Mo are between 8 and 20) [11].

5. System Model

Figure (6) shows the block diagram of a typical Turbo coded OFDM (TCOFDM) transceiver. In the transmitter section, binary input a systematic Turbo encoder first encodes data then the encoded data is mapped into baseband symbols using BPSK scheme. The symbols are then transmitted in parallel by assigning each symbol to one carrier in the

transmission. An inverse fast Fourier transform (IFFT) is used to convert this signal to the time domain, and to produce the orthogonality between subcarriers, allowing it to be transmitted. The guard time is inserted to OFDM symbol. Using cyclic prefix that is copying the end of OFDM symbol and appending this to start of OFDM symbol makes guard time insertion. The transmitted signal is corrupted by AWGN and Flat Fading Rayleigh channel.

The receiver section performs the reverse operation of the transmitted; remove the guard band, then using a fast Fourier transform to analyze the signal in the frequency domain. The subcarriers are then picked out and converted back to digital data, then decoded by using MAP decoder to produce binary output data.



Figure (6) Block diagram of turbo coded OFDM (TCOFDM) transceiver

6. Simulation and Results

The simulation parameters for the TCOFDM system are shown in **Table** (1). The BER performance of TCOFDM system is compared with the respective uncoded system under the Rayleigh correlated fading and AWGN channel.

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Modulation Type	BPSK
Number of sub-carriers	64,512
Number of N points	64,512
Guard interval	16,128
Channel model	AWGN / Flat fading + AWGN
Turbo code frame length	128, 1024
Interleaver	128, 1024
RSC1 encoder	Recursive Convolutional $(1,5/7)_{8}$
RSC2 encoder	Recursive Convolutional $(1,5/7)_{8}$
Bit Rate	2Mbps
Over all Code Rate	1/2
Iterative Decoding	Log-Map algorithm
Interleaver Type	Block interleaver [(8,16),(16,64)]
f _c	2GHz
f _d	25Hz,100Hz,250Hz

Table (1) Simulation parameters

6-1 Performance of TCOFDM System in AWGN Channel

Figures (7) and **(8)** illustrate the performance of TCOFDM system in BPSK modulation type over AWGN channel compared with OFDM and uncoded systems. The number of subcarriers N=64 and 512 respectively and 5 iterations for TCOFDM system. (notice that "uncoded" in the figures refers to use only BPSK modulation while "FFT OFDM" refers to use OFDM system without turbo codes and the iterations refer to use TCOFDM system).



Figure (7) Comparison of TCOFDM, OFDM and uncoded systems over AWGN channel, N=64



Figure (8) Comparison of TCOFDM, OFDM and uncoded systems over AWGN channel, N=512

From **Fig.(7)** at BER= 10^{-3} the SNR of TCOFDM system outperforms the uncoded system by 12dB and the OFDM system by 7.3dB. The TCOFDM system outperforms OFDM system about 8dB coding gain from iteration 5 in TCOFDM system. When the number of iterations is increased from 1 to 5 iterations the performance enhanced about 1dB. The effect of TCOFDM in AWGN channel is not sharp like the case in fading channel.

From **Fig.(8)** above, it is clear that at BER $=10^{-3}$ the SNR of TCOFDM system with frame length =1024 and 5 iterations, system us outperforms both of uncoded system and OFDM system about 10dB for the first and 8 dB for the last. Comparable results are obtained between the fifth iteration and OFDM system. A coding gain of 0.57dB is obtained by increasing the number of iterations from 3 to 5 iterations. From **Figures (7 and 8)** the OFDM system is not changed when subcarrier numbers is increased because this channel is Gaussian channel and no effect of changes N on this channel (subcarrier numbers effect appears at fading channel).

6-2 Performance of TCOFDM System in the Flat Fading Channel

Figures (9-14) illustrate the resultant BER performance of TCOFDM and uncoded systems versus signal to noise ratio in flat fading channel for BPSK modulation. The following parameters are applied as an input to the OFDM signal:

- 1. Sub-carriers number N=64, 512.
- 2. Flat Rayleigh fading channel with different Doppler frequencies ($v_c=3.75$ mph ($f_d=25$ Hz), 37.5 mph ($f_d=250$ Hz), 15 mph ($f_d=100$ Hz) and 150 mph ($f_d=1000$ Hz)) and $f_c=2$ GHz.
- 3. Generator polynomial g=[1,5/7].
- 4. Five decoding iterations.
- 5. Block interleaver with length=128 and 1024 respectively.



Figure (9) Comparison of TCOFDM, OFDM and uncoded systems over flat Rayleigh fading channel N=64, f_d =25



Figure (10) Comparison of TCOFDM, OFDM and uncoded systems over flat Rayleigh fading channel N=64, fd=100



Figure (11) Comparison of TCOFDM, OFDM and uncoded systems over flat Rayleigh fading channel N=64, fd=250



Figure (12) Comparison of TCOFDM, OFDM and uncoded systems over flat Rayleigh fading channel N=512, f_d =25



Figure (13) Comparison of TCOFDM, OFDM and uncoded systems over flat Rayleigh fading channel N= 512, f_d = 100



Figure (14) Comparison of TCOFDM, OFDM and uncoded systems over flat Rayleigh fading channel N=512, fd= 250

From **Fig.(9)** below, it is clear that at BER $=10^{-3}$ the SNR of TCOFDM system over flat fading channel with f_d=25, N=64, frame size=128 and 5 iterations outperforms both of uncoded system and OFDM system by 16dB for the first and about 5.2dB for the last.

From **Fig.(10)** it is found that at BER $=10^{-3}$, the SNR of TCOFDM system over flat fading channel with $f_d=100$, N=64, frame size=128 and 5 iterations outperforms both of uncoded system (by more than 10dB) and OFDM system by about 5dB. A comparable result is obtained by increasing the number of iterations from 3 to 5 iteration where the coding gain is about 0.2 dB.

From **Fig.(11**) it is clear that the SNR of TCOFDM system over flat fading channel at BER = 10^{-3} with f_d=250, N=64, frame size=128 and 5 iterations outperforms both of uncoded system (by more than 12dB) and OFDM system by about 7dB. No significant improvement is obtained by increasing the number of iterations from 4 to 5 iteration while there is improvement at about 0.6dB when the number of iterations increased from 3 to 5 iteration.

From **Fig.(12)** it is found that at BER $=10^{-3}$, the SNR of TCOFDM system over flat fading channel with f_d=25, N=512, frame size=1024 and 5 iterations outperforms the uncoded system about 9.5dB and OFDM system with the same parameters by 4dB at high SNRs more than 4dB, but at low SNRs the results between both systems is comparable. A coding gain of about 1dB is obtained by increasing the number of iterations from 3 to 5 iteration.

From **Fig.(13**) it is found that at BER $=10^{-3}$, the SNR of TCOFDM system over flat fading channel with $f_d=100$, N=512, frame size=1024 and 5 iterations outperforms the uncoded system about 16 dB and OFDM system by 9dB. A comparable results is obtained between the two presented system (OFDM and TCOFDM) at low SNRs less than 4dB, about 0.6 dB coding gain is obtained by increasing the number of iterations from 3 to 5 iteration.

The coding gain of system performance obtained here as compared proposal with the case of OFDM is greater than the gain that is obtained in case of $f_d=25$ by about 5dB.

From **Fig.(14**), at BER $=10^{-2}$ the SNR of TCOFDM system over flat fading channel with $f_d=250$, N=512, frame size=1024 and 5 iterations outperforms the uncoded system about 20 dB and OFDM system by 8dB. As a comparison between the two systems OFDM and TCOFDM it is found that at low SNRs less than 6dB, about 0.2 dB coding gain is obtained by increasing the number of iterations from 3 to 5 iteration.

7. Conclusion

The simulation results show that increasing Doppler frequency (f_d) for flat fading channel leads to large SNR at the same BER. The Turbo code improves the performance of OFDM system with improving factor of about 5-8 dB even though f_d is increased.

Also the results show that the turbo coded orthogonal frequency division multiplexing (TCOFDM) system achieves large coding gain with lower BER at reduced decoding iterations.

The performance of the coded system is enhanced with the increased the number of iterations, while the performance is degraded as the number of bits per frame is decreased.

Furthermore, increasing the number of subcarrier leads to performance enhancement of the present TCOFDM system.

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