# A solution of Nonlinear Rate Equations of Directly Modulated Semiconductor Laser to Chose Optimum Frequency Modulation

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## Abstract

The purpose of this paper is to solve the nonlinear rate equations of directly modulated semiconductor laser by direct simulation (numerical solution) where the rate equations are described the dynamic behavior of semiconductor laser because of optical field is depended on the time. The other side, the rate equations are described the interactions among photon density ,carrier density ,and the phase of photon .However ,these equations are used to predict the nonlinear distortions which it is appeared in the performance of directly modulated semiconductor laser .

الغلاصيية

الغرض من هذه المقاله هو حل معادلات المعدل اللاخطية لليزر اشباه الموصلات ذات التضمين المباشر مـن خـلال المحاكـاة حيث ان هـذه المعادلات تصف السلوك الديناميكي لليزر اشباه الموصلات معتمداً علـى الـتغير الحاصـل بالمجـال البصـري نسـبة الى الـزمن ومـن جانـب اخـر هـذه المعادلات تصف كثافة الفوتونات وحاملات الشحنة وطور الفوتون .

من خلال هذه المعادلات يمكن ملاحظة التشوهات اللاخطية التي تظهر في اداء ليزر اشباه الموصلات ذات التضمين المباشر.

## 1. Introduction

The main application of semiconductor laser is as a source for optical communication systems, where the output of the semiconductor lasers is modulated by applying the electrical signal either directly to the optical source or to the external modulator <sup>[1]</sup>.

In direct modulation of the semiconductor laser, the injection current is composed of DC component plus number of frequency modulation signals occupying separate frequency band <sup>[1,2]</sup>. Due to inherent laser nonlinearity ,energy is transferred to frequencies which are combinations of the original frequencies .these are nonlinearity distortions .

#### 2. Theory

To predict the significance of the nonlinear distortion ,laser operation can be modeled by the rate equations, which must then be solved for situations where the drive current consist of bias term together with sinusoidal components representing the wanted multi- carrier modulation .the single mode laser rate equations may be written by symbolically as <sup>[3]</sup>:

$$\frac{dp}{dt} = \Gamma g_o (n - n_s) (1 - \xi p) p - \frac{p}{\tau_p} + \beta \Gamma_n \left( \frac{1}{r_n} + \beta_n + Cn^2 \right) \qquad \dots (1)$$

$$\frac{dn}{dt} = \frac{I}{qv} - g_o \left(n - n_s\right) \left(1 - \xi p\right) p - n \left(\frac{1}{\tau_n} + \beta_n + Cn^2\right) \qquad \dots (2)$$

$$\frac{d\phi}{dt} = \frac{\alpha}{2} g_o \left( n - n_{th} \right) \qquad \dots (3)$$

Where P is photon density and n the carrier density the gain term is modeled by :

(4)

Where  $g_0$  is the optical gain coefficient  $n_s$  the carrier density for which the gain is zero and  $\xi$  is the gain compression (or saturation) factor .the photon decay rate is considered in the terms of the photon lifetime  $\tau_p$ , the modal confinement factor  $\Gamma$  is used in defining the photon density ,whilst the spontaneous recombination terms take into account both radiative and nonradiative recombination processes . Thus  $\tau_n$  represents the nonradiative recombination lifetime , $\beta$  the bimolecular recombination and C the anger recombination processes. Carrier injection is given by the current I, flowing through the active region ,divided by the electronic charge q and the active volume V .  $\phi$  is the phase of the photon ,  $\alpha$  is the line width enhancement factor and n<sub>th</sub> is the carrier threshold <sup>[3]</sup>.

Under direct modulation with a multi - carrier signal, the injection current is :

$$I(t) = I_b + \sum_{m=0}^{s-1} I_m(t)$$
 ...(5)

Where  $I_b$  is the bias current and  $I_m(t)$  is th  $m_{th}$  component of the modulating current

$$I_m(t) = I_m Exp(2\pi f_m t + \phi_m) \qquad \dots (6)$$

#### 3. Result

To determined the nonlinear rate equations in eqs. (1) and (2) are used in eq.(3), the rate equation about the phase of the photon will not be used here because the distortions affected by lasing frequency modulation (chirping) are out of concern .By using eqs. (1) and (2) and the table (1) which its appears the parameter values for InGaAsP laser <sup>[4]</sup>. And the results as illustrated in figs. (1) and (2) represent the input current of (2.7 GHz and phase =0 rad/sec ) and (2.7 GHz and 2.9 GHz and phase =0 and  $2\pi$  rad/sec) respectively .By using direct simulation ,we can get on first ,second ,and third orders of nonlinear transfer function of semiconductor laser as shown in fig.(3).

	Table (1)		
Parameter	Description	Units	Value
I <sub>th</sub>	Threshold Current	mA	10
V	Active volume	m <sup>3</sup>	90 <i>×</i> 10 <sup>-8</sup>
Q	Electronic charge	С	1.6×10 <sup>-19</sup>
Γ	Optical confinement factor		0.44
g <sub>o</sub>	Optical gain coefficient	S <sup>-1</sup> m <sup>3</sup>	3 <i>×</i> 10 <sup>-12</sup>
n <sub>s</sub>	Carrier density at g=0	m <sup>-3</sup>	1.2×10 <sup>24</sup>
$ au_{S}$	Carrier Lifetime	ns	3
$ au_p$	Photon lifetime	Ps	1
β	Spontaneous emission factor		4 <i>×</i> 10 <sup>-4</sup>
ξ	Gain compression	M <sup>3</sup>	3.4×10 <sup>-17</sup>
В	Bimolecular	cm³/s	2×10 <sup>-10</sup>
С	Auger recombination	cm⁵/s	3 <i>×</i> 10 <sup>-29</sup>
α	Line width enhancement factor		5
n <sub>th</sub>	Carrier threshold	cm <sup>-3</sup>	2.14×10 <sup>8</sup>

Table (1)



Fig. (1-a) Input current of 2.7 GHz and phase=0 rad/sec



Fig. (1-b) S(t) Vs time for 2.7 GHz rad/sec input



Fig. (1-c) N(t) Vs time for 2.7 GHz rad/sec input





Fig. (2-a) Input current of 2.7 GHz and 2.9 GHz &phase=0 and 2  $^{\pi}$  rad/sec



Fig. (2-b) S(t) Vs time for 2.7 GHz and 2.9 GHz &phase=0 and 2  $\pi$  rad/sec input



Fig. (2-c) N(t) Vs time for 2.7 GHz and 2.9 GHz &phase=0 and 2  $\pi$  rad/sec input



Fig. (3) first and second order laser nonlinear transfer function f2=f1+0.2GHz

## 4. Conclusion

By using the nonlinear rate equations ,an approximate analysis has been presented for predicting the nonlinear distortion in performance of semiconductor laser where this distortions will effect on the photon density and carrier density .as shown in figs(1-b),(1-c), (2-b),and (2-c).

Fig.(3) shows the optimum frequency modulation we can used to modulator the semiconductor laser to get frequency response having reduced distortions.

## 5. References

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