Theoretical Analysis of Coupling Constant, Longitudinal Modes, and Threshold Gain for Coupled-Cavity Semiconductor Lasers

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Abstract

A theoretical analysis of the coupled-cavity semiconductor laser requiers simultaneous consideration of the gain and loss in the two cavities after taking into account thier mutual optical feedback.

Is wavelength-independent (and hence the same for all Fapry-perot (FP)), in a coupled-cavity laser the effective facet loss becomes different.

الخـلاصـــــة

التحليل النظري لتزاوج التجويف لليزر أشباه الموصلات والمتضمن التزاوج في الربح والخسارة بين فجوتين بعد الأخذ بنظر الاعتبار للتغذية العكسية البصرية.

حيث إن هذا التزاوج لا يعتمد على الطول الموجي، في التزاوج للتجويف الليزري تكون الخسارة مؤثرة.

1. Introduction

The basic concept of using the coupled-cavity scheme for longitudinal-mode selection is known from the early work on gas lasers and has been widely used for mode selection [1]. In the case of semiconductor lasers, considerable work was carried out by coupling the semiconductor laser to an external cavity [2]. Monolithically integrated coupled-cavity device were considered occasionally in relation to thier specific properties such as optical disability, optical amplification, and longitudinal-mode selectivity [1,2]. Recently they have attracted wide attention in an attempt to obtain single-frequency semiconductor lasers that are useful for optical communications in the (1.55□m) wavelength region [3]. At the same time, the interest in semiconductor lasers coupled to a small-length external cavity has revived. To distinguish them from monolithic coupled-cavity devices such lasers are often reffferd to as external-cavity semiconductor lasers [2,3].

2. Theory

The mechanism of mode selectively in coupled-cavity semiconductor lasers can be understood by referring to **Fig.(1)** [4]. The effect of feedback from the external cavity can be modeled through an effective wavelength-dependent reflectivity of the facet facing the external cavity. As a result, the cavity loos are different for different Fabry-Perot (FP) modes of the laser cavity. In general the loss profile is periodic as shown schematically in the bottom part of **Fig.(1)**. The mode selected by the coupled-cavity device is the FP mode that has the lowest cavity loss and is closest to the peak of the gain profile. Because of the periodic nature of the loss profile, other FP modes with relatively low cavity losses may exist. Such modes discriminated by the gain roll-off because of their large seperation from each other [4]. The power carried by these side modes is typically 20-30 dB below that of the main mode occurring in the vicinity of the gain peak [5].
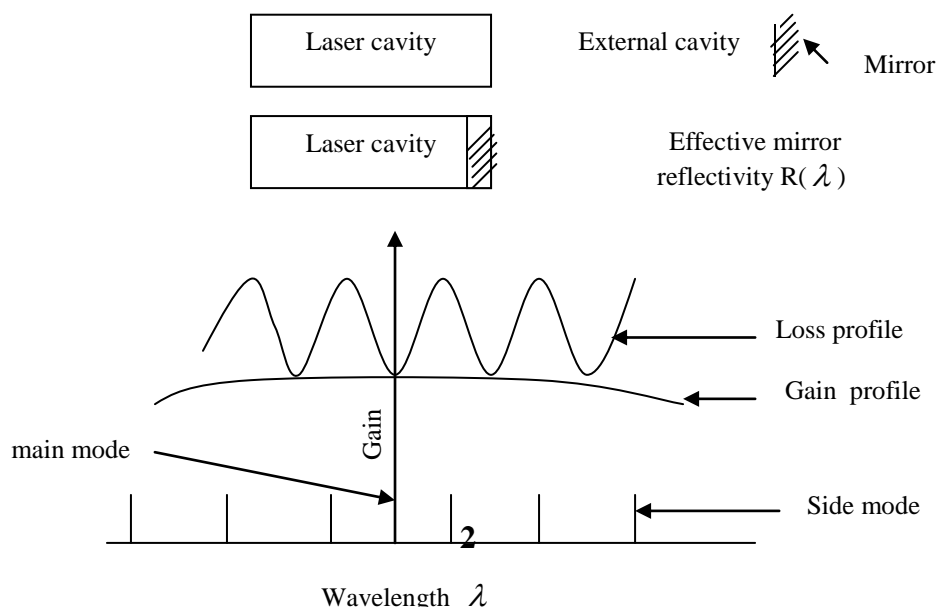
Figure (1) Schematic illustration of longitudinal-mode selectivity in a coupled-cavity laser to drawing shows the two cavities. The effect of external cavity is to make the effective reflectivity of thee facet wavelength dependent (middle drawing). Bottom drowing shows the resulting periodic loss profile superim posed on the gain profile. The main mode is the FP mode closest to the gain peak with the lowest loss, other FP mode become side modes suppressed because of their relatively higher losses

A general analysis of coupled-cavity semiconductor lasers is extermely complicated, so it is necessary to make several simplifying assumptions. We assume that the field distribution associated with foundemental lateral and transerves mode of the wave guide is uneffected by intercavity coupling and that only axial propagation in each cavity needs to be considered. This reduces the problem to one dimension. However, that the coupling depends upon the mode width and other related parameters, and mode-conversion losses occur as to optical mode leavs cavity 1, diffracts in the air inside the gas, and then reenters cavity 2. Such losses would be incoprated through an effective gap loss (α_g) [6].

In this paper we Interdev the concept of coupling constant and then obtain the longitudinal modes and their respective threshold gains for the coupled system.

2-1 Coupling Constant

The first step in the analysis is to determine the extent of coupling between the two cavity. We consider the genral case applicable for both active-active and active-passive devices [7]. **Figure (2)** shows the geometry and notion .the coupling between the cavities is goverend by an airgap of width Lg. The air gap itself froms a third FP cavity, and the intercavity coupling is affected by the loss and phase shift experienced by the optical field while traversing the gap. In the scattering matrix approach the fields in the two cavities are related by:

$$\begin{pmatrix} E_1' \\ E_2' \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \end{pmatrix} \quad \text{.. (1)}$$

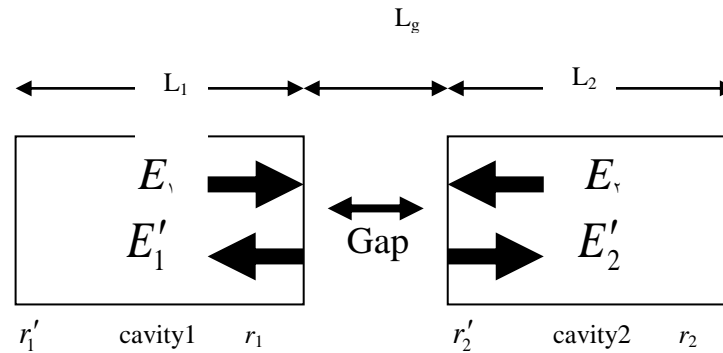The scattering matrix elemnts can be obtained by considring multiple reflections inside the gap.

Figure (2) Geometry and nation used in the theoretical description of a coupled-cavity semiconductor laser. In the general case, reflectivity's rate the four facets may differ from each other

More explicity, the gap is treated as a FP cavity on which the field E1 is incident from left (**Fig.(2)**) and the reflected field ($\mathbf{E'_1}$) and the transmitted field ($\mathbf{E'_2}$) are obtained in terms E1 after considering multiple round trips inside the cavity .the procedure yields $\mathbf{S_{11} = E'_1 / E_1}$ and $\mathbf{S_{21} = E'_2 / E_1}$ the same method is used to obtain S22 and S12 after assuming that the only the field E2 is incident on the FP cavity from the rigth .

The result is:

$$\mathbf{S_{11} = r_1 - \frac{r_2\left(1 - r_1^2\right)t_g}{1 - r_1 r_2 t_g}} \quad \text{...} (2)$$

$$\mathbf{S_{22} = r_2 - \frac{r_1\left(1 - r_2^2\right)t_g}{1 - r_1 r_2 t_g}} \quad \text{..} (3)$$

$$\mathbf{S_{12} = S_{21} = \frac{\left[t_g\left(1 - r_1^2\right)\left(1 - r_2^2\right)\right]^{1/2}}{1 - r_1 r_2 t_g}} \quad \text{...} (4)$$

where:

$$\mathbf{t_g = \exp\left(2i\beta_g L_g\right) = \exp\left(2iK_o L_g\right)\exp\left(-\alpha_g L_g\right)} \quad \text{....................................} (5)$$

Here $\mathbf{\beta_g = K_o + i\alpha_g / 2}$ and accounts for the phase shift and the loss inside the gap. Further, r1 and r2 are the reflection coefficients at the two facets forming the gap; i.e, $\mathbf{r_n = \left(\mu_n - 1\right)/\left(\mu_n + 1\right)}$ where μ_1 and μ_2 are the mode indices in the two effective reflection

coefficients are S11 and S22are the effective-transmission coefficients from cavity 1 to cavity 2 and vice versa .It is useful to defined a complex coupling parameter:

$$\overline{\mathbf{C}} = \mathbf{C}\exp(\mathbf{i}\theta) = \left(\frac{\mathbf{S}_{12}\mathbf{S}_{21}}{\mathbf{S}_{11}\mathbf{S}_{22}}\right)^{1/2} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (6)$$

where:

   C: governs the strength of mutual coupling and θ *is the coupling phase.*

The magnitudes of C and θ depend on a large number of device parameters. The simplest case occurs for a semiconductor laser coupled to an external mirror.In this case the coupling element is just the laser-air interface. Since Lg=0, tg=1. Furthermore $\mu_2 = 1$, and therefore r2=0. Using these values in eqs.(2)-(6), we find that $\mathbf{C} = \left(\mathbf{1} - \mathbf{r}_1^2\right)^{1/2} / \mathbf{r}_1$ and $\theta = \pi/2$. For a C3 laser, the two cavities have nearly equal indices of refraction, and therefore r1=r2=r. Using eqs.(2)-(6), we now obtain:

$$\mathbf{C}\exp(\mathbf{i}\theta) = \left(\frac{\mathbf{1}-\mathbf{r}^2}{\mathbf{r}}\right)\left(\frac{\mathbf{t}_g^{1/2}}{\mathbf{1}-\mathbf{t}_g}\right) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (7)$$

2-2 Longitudinal Modes and Threshold Gain

In this section we obtain an eigenvalue equation whose solutions yield the wavelength and threshold gain associated with the longitudinal modes of the coupled system [8]. Asimple way to do this is to consider the relationship between E1 and $\mathbf{E}_1'$ using **Fig.(2)**. The field $\mathbf{E}_1'$ results from reflection of E1 and transmission of E2 and given by (see eq.(1)).

$$\mathbf{E}_1' = \mathbf{S}_{11}\mathbf{E}_1 + \mathbf{S}_{12}\mathbf{E}_2 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (8)$$

On the other hand, the round trip through cavity 1 results in the relation

$$\mathbf{E}_1 = \mathbf{r}_1'\exp(2\mathbf{i}\beta_1\mathbf{L}_1)\mathbf{E}_1' = \mathbf{r}_1' + \mathbf{E}_1' \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (9)$$

where, the complex propagation constant:

$$\beta_n = \mu_n\mathbf{K}_o - \mathbf{i}\overline{\alpha}_n / 2 \qquad (\mathbf{n} = 1,2) \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (10)$$

Governs wave propagation in nth cavity, $\mathbf{K}_o = 2\pi/\lambda$ where λ is the device wavelength, and α_n is the mode gain.the mode gain is related to the material gain gn by:

$$\overline{\alpha}_n = \Gamma\mathbf{g}_n - \alpha_n^{\mathbf{int}} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (11)$$

where:

Γ *: is the confinement factor and* α_n^{int} *is the internal loss. For an active-active device, g2 =0 and*

$\overline{\alpha}_2$ *: accounts for the absorption loss in the passive cavity.*

Eqs.(8) and (9) can be combined to obtain the relation:

$$\left(1 - r_1' t_1 S_{11}\right) E_1 = r_1' t_1 S_{12} E_2 \quad .... \text{ (12)}$$

Similar considerations for cavity 2 lead to:

$$\left(1 - r_2' t_2 S_{22}\right) E_2 = r_2' t_2 S_{21} E_1 \quad .... \text{ (13)}$$

These two homogeneous equations have nontrivial solutions only if the secular condition:

$$\left(1 - r_1' t_1 S_{11}\right)\left(1 - r_2' t_2 S_{22}\right) = r_1' r_2' t_1 t_2 S_{12} S_{21} \quad .... \text{ (14)}$$

Is satisfied Eq.(14) is desired eigen value equation for the coupled system and has been extensively. In the absence of coupling, S12=0, Snn=rn, and we recover the threshold condition for uncoupled cavities. Note that:

$$t_n = exp(2i\beta_n L_n) = exp(2i\mu_n K_o L_n) exp(-\overline{\alpha}_n L_n) \quad .... \text{ (15)}$$

Incorporates the phase shift and gain (or loss) experienced by the optical field during a round trip in each cavity. The loss and phase shift inside the gap are included through tg and Sij as given by eqs.(2)-(5). The eigen value eq.(14) is applicable for all kinds of coupled-cavity device with arbitrary reflectivities at four interfaces (see **Fig.(2)**).

Before proceeding it is useful to interduce the concept of an effective-mirror reflectivity. In many cases the role of one cavity (say cavity 2) is provide a control through which a signal FP mode of cavity 1 is selected. The effect of cavity 2 on mode selectivity can be treated by an effective reflectivity for the laser facet facing cavity 2. Eq.(14) can be written in the equivalent form of:

$$\left(1 - r_1' R_{eff} t_1\right) = 0 \quad .... \text{ (16)}$$

where, the effective reflectivity:

$$R_{eff} = S_{11} + \frac{r_2' t_2 S_{12} S_{21}}{1 - r_2' t_2 S_{22}} = S_{11}\left(1 + \frac{\overline{C}f_2}{1-f_2}\right) \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(17)}$$

and $f_2 = r_2' t_2 S_{22}$ and is the fraction of the amplitude coupled back into the laser cavity after a round trip in cavity 2. Eq.(16) suggests that as far as mode selectivity is concerned, the couple-cavity laser is equivalent to a single-cavity laser with facet reflection coefficients r_1' and Reff. Although eq. (16) is forwally exact, coupling so that a change in t1 (through operating conditions of cavity 1) does not affect Reff through a change in the feedback fraction f2. This is often the case for active-passive devices. In the case of active-active devices, the effective reflectivity concept is valid when cavity 2 is based below threshold.

3. Results

The evolutions of C and θ requires Knowledge of the gap loss α_g. These losses rise mainly from diffraction-spreading of the beam inside the gap and have been estimated using a simple diffraction-spreading model perpendicular to the junction plane [2]. **Figure (3)** shows the variation of C and θ with gap width using this model after taking $|r|^2 = 0.31$ as the cleaved-facet reflectivity. Both C and θ vary considerably with small changes in Lg. The in-phase coupling ($\theta = 0$) occurs when ever $L_g = \frac{m\lambda}{2}$ (m is an integer), and C also gose through a maximum for that value of Lg.

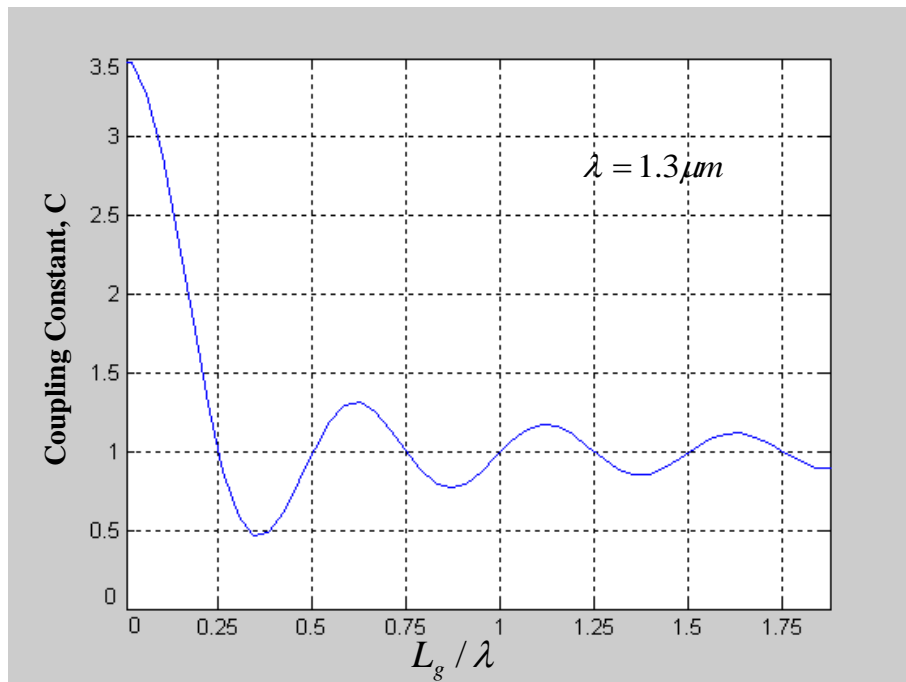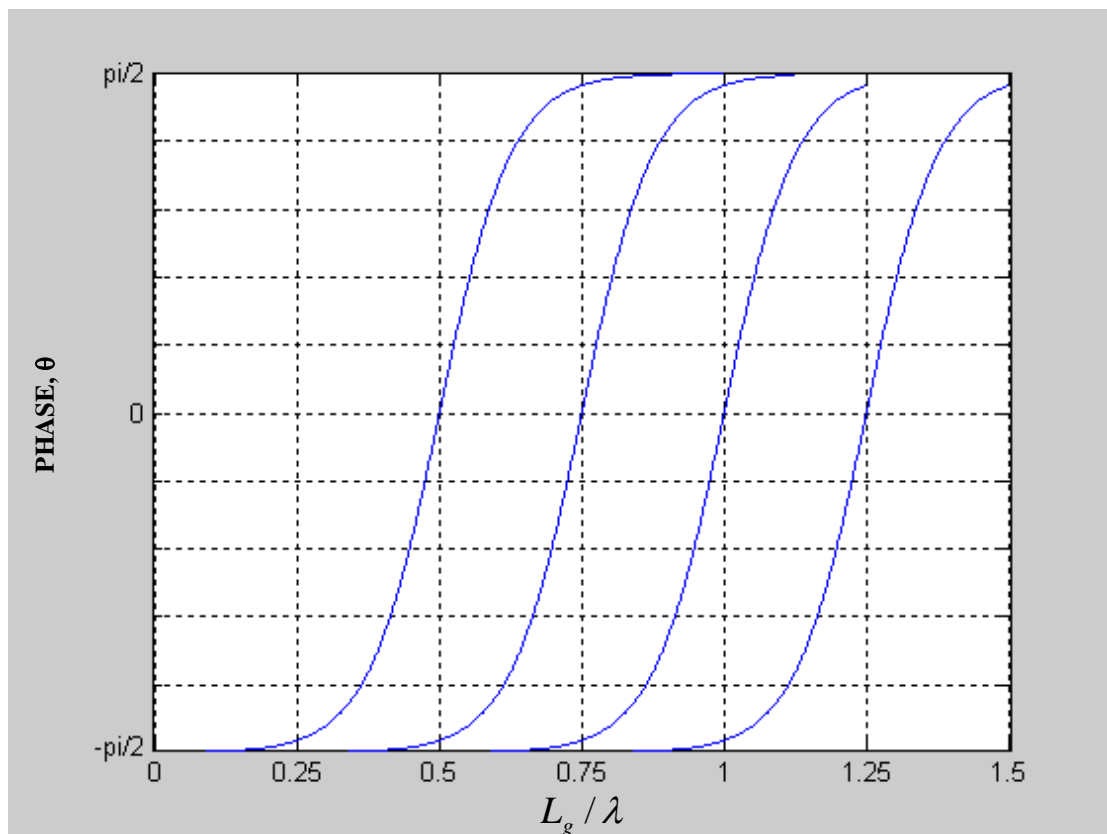**Figure (3-a) Coupling constant C Vs $\mathbf{L_g / \lambda}$, $\lambda = \mathbf{1.3\mu m}$**

Figure (3-b) Dependence of the coupling phase θ on the gap width Lg
for a laser oscillating at the wavelength λ

To illustrate the extent of mode selectivity offered by the coupled-cavity mechanism, we consider solutions of the eigenvalue eq.(14) for a secific C3-type active-active device for

which $r_1 = r_1' = r_2 = r_2' \cong 0.56$. We assume that cavity 2 is biased below threshold such that $\bar{\alpha}_2 = 0$ (in the absence of coupling) and that it does not change significantly with coupling. Eq.(14) is used to obtain $\bar{\alpha}_1$, and the wavelength $\lambda = 2\pi/K_o$ corresponding to various longitudinal various longitudinal modes. **Figure (4)** shows the longitudinal modes and their respective gains for a spesific gap width ($L_g = 1.55\mu m$) and for two sets of cavity lengths L1 and L2 corresponding to long-long and long-short geometries, since the concept of effective reflectivity is approximately valied, $\left|R_{eff}\right|$ versus λ is also shown.

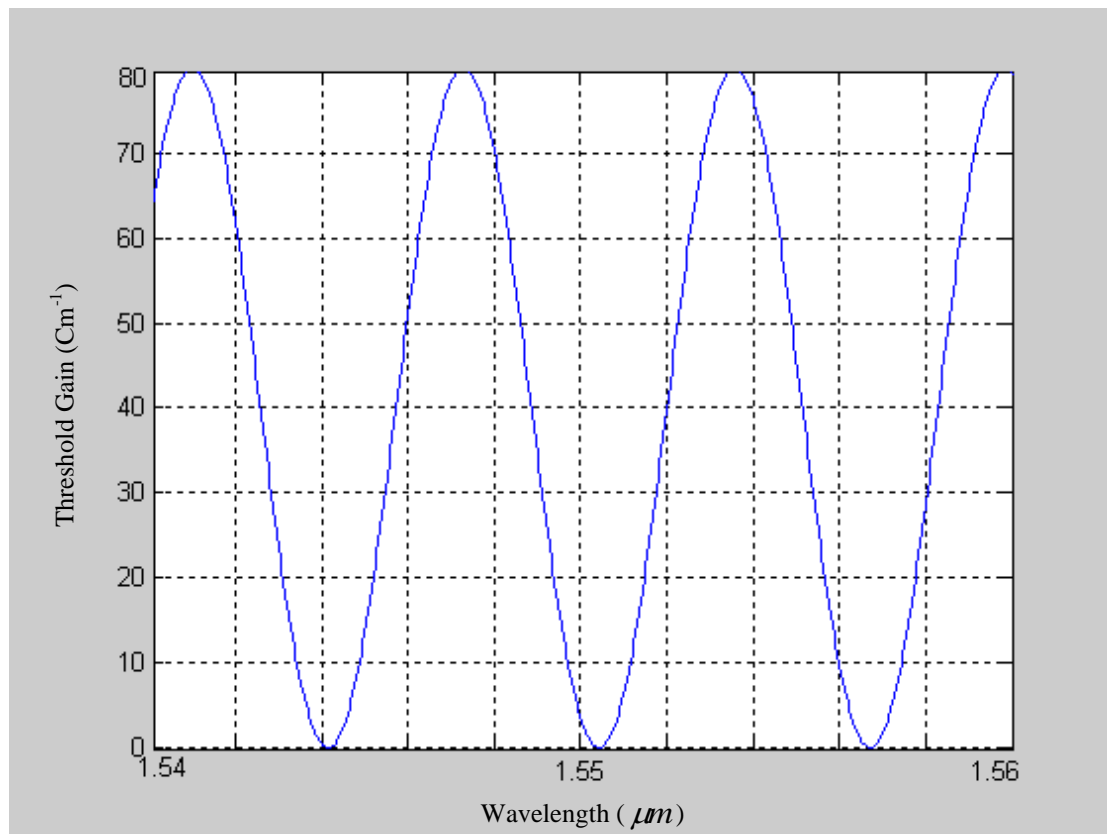$L_1 = 240 \; \mu m$
$L_2 = 210 \; \mu m$
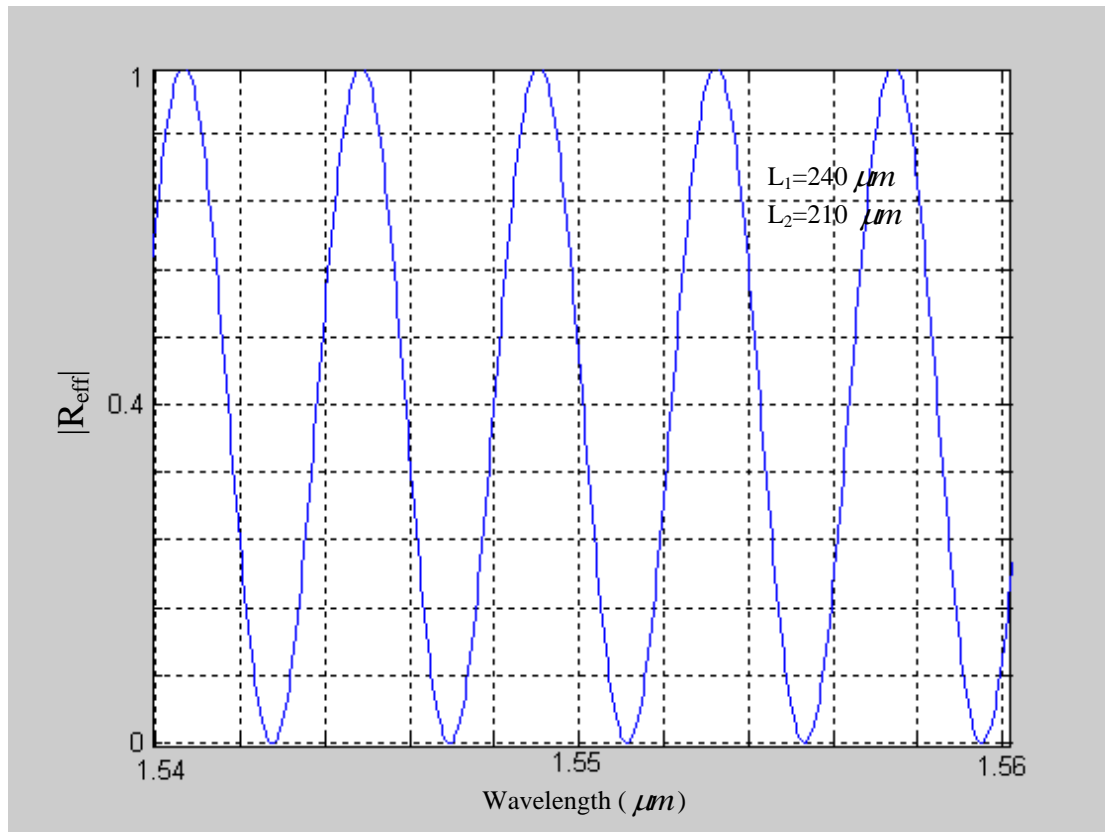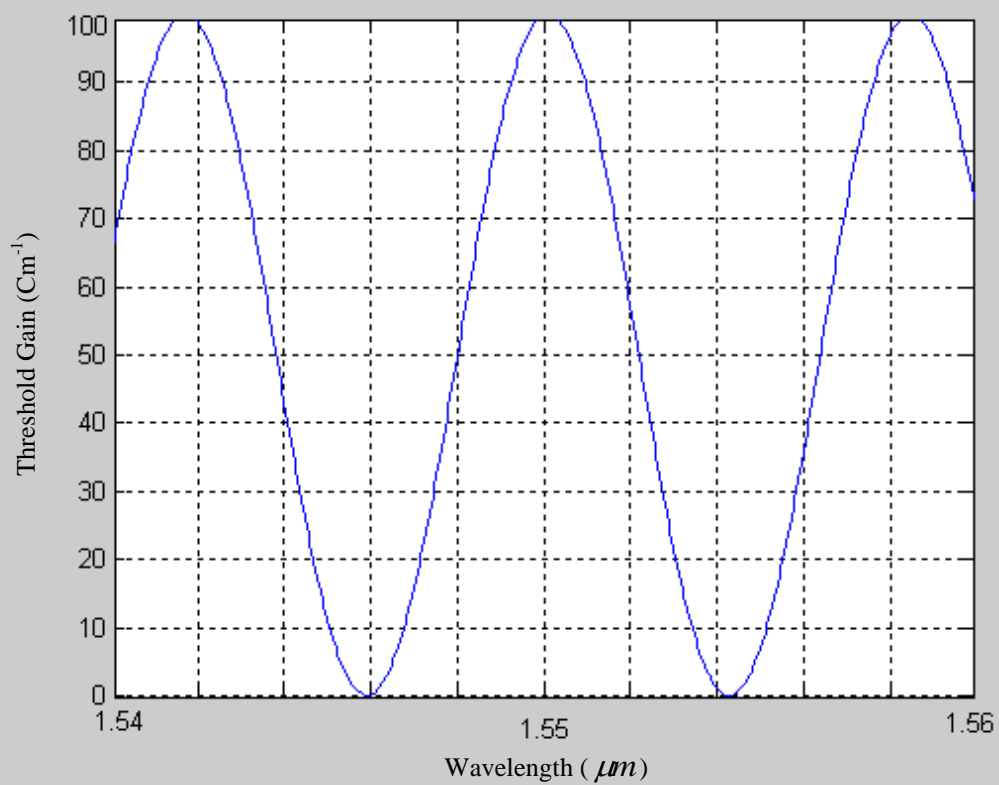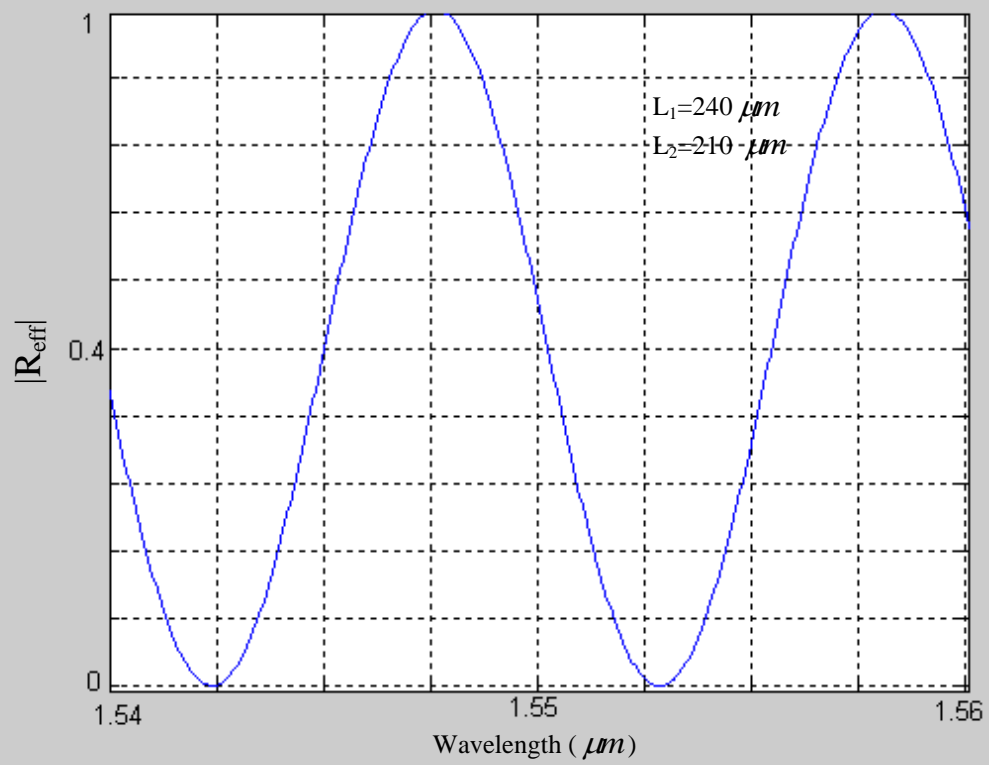
Figure (4) mode discrimination in coupled cavity laser employing long-short and long-long geometries, in each case the upper two figures shows the wavelength variation of the effective laser-facet reflectivity while the lower two figures shows longitudinal modes

4. Conclusion

We are studied and analysed the effects of coupling constant, longitudinal modes, and threshold gain on coupled-cavity semiconductors which the coupling constant is depended on coupling phase, effective gap loss, and the air gap of width. The other side the longitudinal modes and threshold are depended on the effective reflectivity and the lengths of cavities where this lengths and the effective reflectivity will limit the type of modes (i.e. long-long and long-short geometries).

5. References

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