Journal of Engineering and Development, Vol. 14, No. 2, June (2010) **ISSN 1813-7822**

Amplitude Control of Single Phase Capacitor Motor

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Abstract

The method is proposed to study the performance of single-phase capacitor motor with amplitude control. This is clearly shown by controlling the speed of the motor with varying the applied voltage to the control voltage while the excitation voltage is constant.

The obtained results show the validity of the method and the accuracy of the equations derived in this work .

الخلاصـــــــة

في هذا البحث تم اقتراح طريقة لإمكانية التحكم في أداء محركات المتسعة أحادية الطور ، حيث من الممكن السيطرة على سرعة هذه المحركات من خلال الجهد المسلط على لفيفة السيطرة مع بقاء جهد الإثارة ثابتا .

إن النتائج التي تم الحصول عليها هي نتائج جيدة تبين صحة هذه الطريقة ودقة المعادلات التي تم استنتاجها في هذا البحث .

1- Introduction:

These types of motors are used for different purpose such as military and medical …ext. The speed of single phase capacitor motor can be controlled by varying the applied voltage to one of its two windings. This is called an amplitude control of the motor. The basic schematic diagram is shown in fig.1, where the capacitor is connected with the main winding (w_1); and the control voltage of the control winding ($V_c = V_3$) is obtained through a regulator (R) from the main supply. Therefore the supply voltage (V_1) and control voltage (V_3) are in phase.

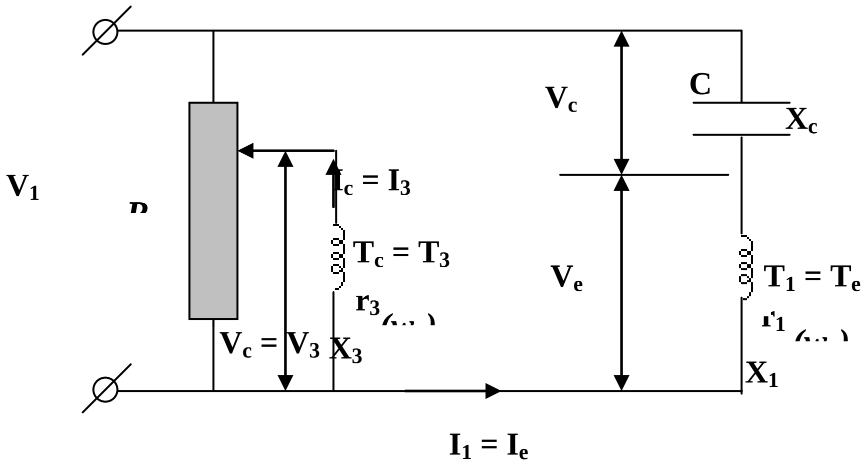

Fig. 1 Basic schematic diagram of single phase capacitor motor

Fig.2 , shows the voltage phaser diagram of such motor, where the values with index (1) related to the main winding and (2) to the squirrel-cage rotor winding and (3) to control winding. [5]

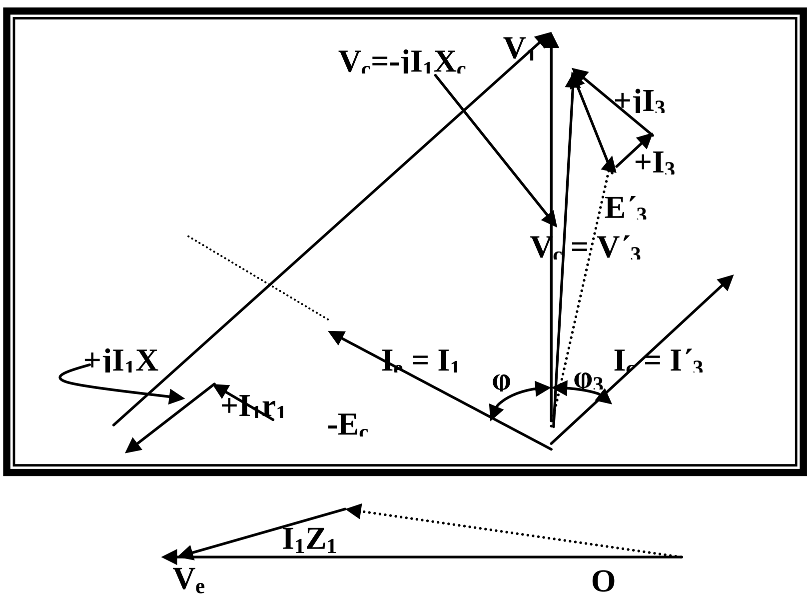

Fig. 2 Voltage phaser diagram of capacitor motor

The parameters of the control and rotor winding are referred to the main (or excitation) winding by using the referring factor which is:

$$K = \frac{K_{w1}T_1}{K_{w3}T_3} \qquad \dots \quad (1)$$

Where K_{w1}, K_{w3} and T_1 , T_3 are the winding factor and the number of turns for the main and control windings respectively; and the control voltage factor is:

$$\alpha = \frac{V_c}{V_1} = \frac{V_3}{V_1} \qquad \dots \quad (2)$$

Also the control voltage referred to that of main winding is:

$$V_3' = KV_3 \qquad \dots \quad (3)$$

Where the effective control factor can be expressed as:

$$\alpha_e = \frac{V_3'}{V_1} = \frac{KV_3}{V_1} = K\alpha \qquad \dots \quad (4)$$

2- The Motor Torque Equations:

At normal operation of this motor, the magnetic field is elliptical, i.e it is a combination of both forward and backward fluxes in the air gap, and therefore, the motor equivalent circuit can be shown as in fig. 3, where (a) for forward field and (b) for backward field.

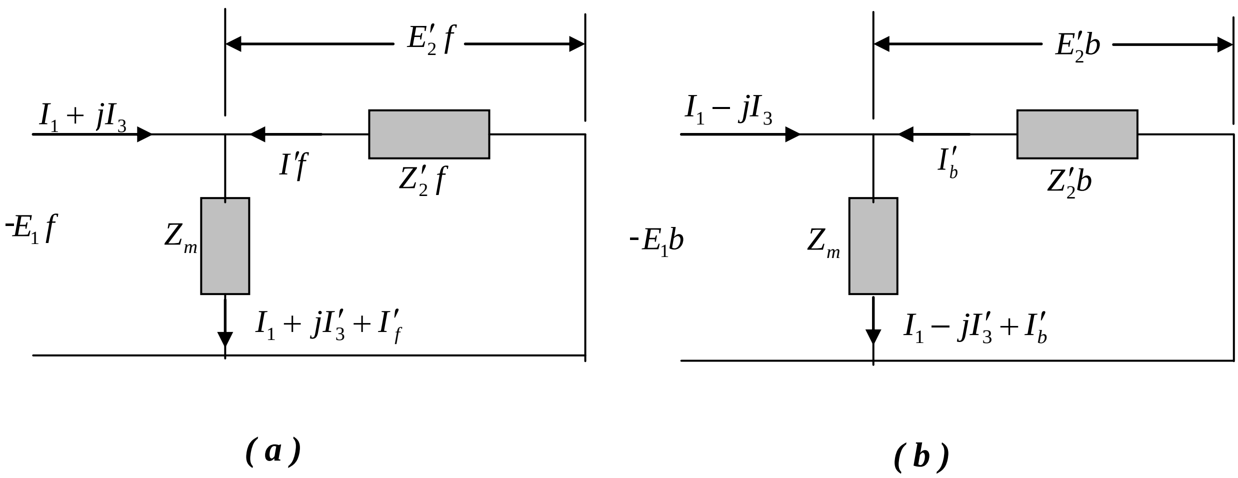

Fig. 3 (a,b) equivalent circuit of forward field and backward field capacitor motor

From figure (a) the main forward current is:

$$I_f = I_1 + jI_3' \qquad \dots \quad (5)$$

And the main backward current is:

$$I_b = I_1 - jI_3' \qquad \dots \quad (6)$$

Where I_1 and I_3' are obtained as in Appendix (A).

The electromagnetic power for forward and backward fields can be expressed as:

$$P_f = m_2 I_f'^2 (\frac{r_2}{s}) \qquad \dots \quad (7)$$

$$P_b = m_2 I_b'^2 (\frac{r_2}{2-s}) \qquad \dots \quad (8)$$

Therefore, the total developed torque in (N – m) is:

$$T = \frac{10^2}{9.81 w_1}(P_f - P_b) = \frac{r_2' \times 10^2}{9.81 w_1}(\frac{I_f'^2}{s} - \frac{I_b'^2}{2-s}) \qquad \dots \quad (9)$$

and when using the equivalent circuit parameters the torque can be expressed as:

$$T = \frac{10^2}{9.81 w_1}\left[(I_1^2 + I_3'^2)(R_f - R_b) + 2I_1 I_3''(R_f + R_b) \times \sin(\varphi_2 - \varphi_1)\right] \qquad \dots \quad (10)$$

Finally the shaft useful torque at rated speed (n_2) is:

$$T_2 = T - T_o \frac{n_2}{n_{2o}} \qquad \dots \quad (11)$$

Where T_o – is the loss torque at no-load speed of (n_{2o}). [2]

3- Motor Behaiver at Starting:

At starting where s = 1 we have that $R_{fs} = R_{bs}$ and therefore, Eq. (10) for the starting condition becomes:

$$T_s = \frac{4 \times 10^2}{9.81 w_1} R_{fs} I_{1s} I_{3s}' \sin(\varphi_{3s} - \varphi_{1s}) \qquad \dots \quad (12)$$

Where I_{1s} and I'_{3s} are obtained as in Appendix (B).

It can be shown that I_{1s} and I_{3s} are independent from each other, and the total starting current is:

$I_s = I_{1s} + I_{3s}$ … (13)

Also the amplitude and phase of I_{1s} is changing with variation of capacitor value, and its phasor drawing a circle as shown in Fig. (4) The circle diameter is also the maximum starting current (I_{1sm}) obtained from the condition ($X_1 - X_c = 0$) which is equal (V_1 / R_1) where $R_1 = r_1 + 2r_{fs}$.

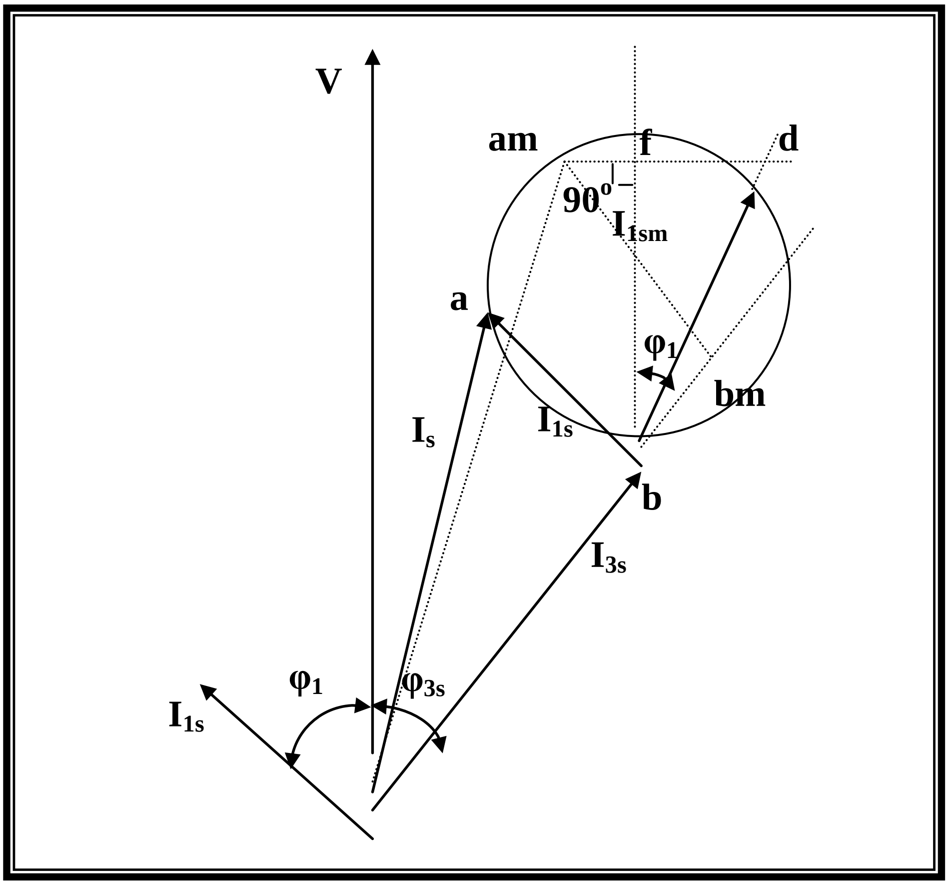

Fig. 4 circle diagram for capacitor motor

The maximum torque is corresponds to (X_c) value which makes (I_{1s} = ab) passing through the circle center; and its value depends on two factors.(i) current value (I_{1sm}) and (ii) its phase shift (φ_{1s}).

From fig.4 the maximum starting torque is obtained at capacitance value of:

$$C_s = \frac{10^6}{2\Pi f_1 X_{cs}} \qquad \ldots \qquad (14)$$

where $$X_{cs} = X_1 \frac{\bar{a}_{md}}{\bar{f}_d}$$

it is evident that as low the stator main winding reactance (X_1) as high the value of the required starting capacitance . [6 , 3]

4- Motor Operating Performance:

4-1 Balanced Operations: [1]

The motor is called balanced if the backward field is vanished or $I'_b = 0$ i.e.

$$I'_b = -(I_1 - I'_3)\frac{Z_b}{Z'_{2b}} = 0 \qquad \ldots \qquad (15)$$

$$I_1 = jI'_3$$

Which means that , then ;

$$R_3 + \alpha K X_o + \alpha K X_1 - \alpha K X_c + R_o = 0$$

$$X_3 - \alpha K R_o - \alpha K (R_1 + r_c) + X_o = 0$$

and the control factor at circular field (α_c) is:

$$\alpha_c = \frac{X_3 + X_o}{K(R_1 + R_o + r_c)}$$

The required capacitive reactance to insure circular field is:

$$X_{cc} = \frac{(R_1 + R_o + r_c)(R_3 + R_o) + (X_1 + X_o)(X_3 + X_o)}{X_3 + X_o}$$

and For balanced operation at starting (s = 1) and for $r_c = 0$

$$\alpha_{cs} = \frac{X_3}{KR_1}$$

$$X_{cs} = \frac{R_1 R_3 + X_1 X_3}{X_3}$$

4-2 Load Performance: [7]

Using q as the relative rotor speed (i.e = w_1 / w_2)

Then the forward slip $S_f = S = 1 - q$ and the backward slip $S_b = 2 - S = 1 + q$.

Then the torque equation in (N – m) from eq. 9 is:

$$T = \frac{r_2' \times 10^2}{9.81 w_1}\left(\frac{I_f'^2}{1-q} - \frac{I_b'^2}{1+q}\right) \qquad \dots \quad (16)$$

To simplify this expression assume that: $r_1 = r_3' = 0, X_1 = X_3' = 0, X_2' = 0$

$r_m = 0$, $r_c = 0$ and considering that $\zeta = \dfrac{r_2'}{X_m}$ and $\beta = X_c / 2X_m$

from Appendix (C) the electromagnetic torque becomes in (N – m) as:

$$T_{em} = \frac{V_1^2 \beta \alpha_e \cdot 10^2}{9.81 w_1 X_m \left[\zeta^2 (1-\beta)^2 + \beta^2\right]}\left\{1 - q^2 - q\left[\frac{\alpha_e \beta (\zeta^2 + q^2 + 1)}{2\zeta} + \frac{\zeta(1+\alpha_e^2)}{2\beta\alpha_e} - \frac{\alpha_e(\zeta^2 + \beta)}{\zeta}\right]\right\} \qquad \dots \quad (17)$$

At starting (q = 0) and the equation simplified to:

$$T_{em} = \frac{V_1^2 \beta \alpha_e \cdot 10^2}{9.81 w_1 X_m \left[\zeta^2 (1-\beta)^2 + \beta^2\right]} \qquad \dots. \quad (18)$$

The ratio of electromagnetic torque to the starting torque, therefore, is:

$$m = \frac{T_{em}}{T_s} = \left\{1 - q^2 - q\left[\frac{\alpha_e \beta (\zeta^2 + q^2 + 1)}{2\zeta} + \frac{\zeta(1+\alpha_e^2)}{2\beta\alpha_e} - \frac{\alpha_e(\zeta^2 + \beta)}{\zeta}\right]\right\} \qquad \dots \quad (19)$$

At balanced operation when $q = 0$, $\alpha_e = \zeta$, $\beta = 1$, the ratio of starting torque to that at balanced operation is:

$$m_s = \frac{T_s}{T_{sc}} = \frac{\beta \alpha_e}{\zeta\left[\zeta^2 (1-\beta)^2 + \beta^2\right]} \qquad \text{p.u.}$$

The shaft useful torque in p.u. can be expressed as in Eq. 11 by:

$$m_{ss} = m - m_o \frac{q}{q_o} \qquad \ldots \qquad (20)$$

Where m_o – is the p.u. loss torque at no – load relative speed of q_o.

4-3 Motor Mechanical Power: [8]

The mechanical useful power (P_2) can be expressed in watts as:

$P_2 = q\, P_{em}$

And in p.u. as:

$$P_2 = \frac{P_2}{P_{em}} = q\left\{1 - q^2 - q\left[\frac{\alpha_e \beta(\zeta^2 + q^2 + 1)}{2\zeta} + \frac{\zeta(1+\alpha_e^2)}{2\beta\alpha_e} - \frac{\alpha_e(\zeta^2 + \beta)}{\zeta}\right]\right\} \qquad \ldots \qquad (21)$$

4-4 The Performance Curves: [4]

To check the validity of the obtained performance equations for the proposed motor, there different motor parameters were taken, as given in table (1).

For reference the motors are called A, B and C. The performance curves required to compare the motor quality are:

a- $m_s = f(\beta)$ for different ζ values , when $\zeta = \dfrac{r_2'}{X_m} = 0.5$ and $\beta = \dfrac{X_c}{2X_m}$.

b- $m = f(q)$ for different α_e values

c- $P_2 = f(q)$ for different α_e values

The results obtained from these relationships are given in Figs (5 – 7).

These result are obtain by using computer program (Visual Basic) , input the data as show in table 1 for three type of motor (a,b,c) and the result are shown in fig. 5,6,7 respectively .

Table 1: The description of the motors

Symbol	Motor type A	Motor type B	Motor type C
P_2	190	170	20
n	1405	1400	1450
I	1.4	2.24	0.403
I_1	1.2	0.99	0.424
I_3	1.25	0.86	0.212
Cos φ	0.995	0.985	0.7
η	66 %	67 %	48 %
T_1	800	806	1430 & 838
K_{w1}	0.846	0.837	0.904
r_1	16.6	21	71
r_2'	20	23	73
X_m	200	240	528
X_1	13	15	42
X_2'	8	10.5	35
r_m	13	16	20
X_c	235	368	2090
K	1	0.85	0.595

The motor type **A**:

SCREEN (2) DESIGE DATA

For the motor type A

When A= 0.109		When A= 0.5		When A= 1		When A= 2		When A= 3	
β	ms	β	ms	β	ms	β	ms	β	ms
0	0.000	0	0.000	0	0.000	0	0.000	0	0.000
0.1	55.249	0.1	0.941	0.1	0.122	0.1	0.015	0.1	0.005
0.2	43.103	0.2	2.000	0.2	0.294	0.2	0.038	0.2	0.011
0.3	31.612	0.3	2.824	0.3	0.517	0.3	0.073	0.3	0.022
0.4	24.450	0.4	3.200	0.4	0.769	0.4	0.125	0.4	0.039
0.5	19.802	0.5	3.200	0.5	1.000	0.5	0.200	0.5	0.067
0.6	16.593	0.6	3.000	0.6	1.154	0.6	0.300	0.6	0.111
0.62	16.069	0.62	2.949	0.62	1.172	0.62	0.322	0.62	0.123
0.64	15.576	0.64	2.896	0.64	1.187	0.64	0.345	0.64	0.135
0.656	16.938	0.656	3.031	0.656	1.140	0.656	0.286	0.656	0.104
0.66	15.111	0.66	2.842	0.66	1.197	0.66	0.367	0.66	0.149
0.68	14.673	0.68	2.787	0.68	1.204	0.68	0.390	0.68	0.164
0.7	14.260	0.7	2.732	0.7	1.207	0.7	0.412	0.7	0.179
0.8	12.492	0.8	2.462	0.8	1.176	0.8	0.500	0.8	0.267
0.9	11.110	0.9	2.215	0.9	1.098	0.9	0.529	0.9	0.333
1	10.000	1	2.000	1	1.000	1	0.500	1	0.333
1.1	9.090	1.1	1.814	1.1	0.902	1.1	0.440	1.1	0.282
1.2	8.331	1.2	1.655	1.2	0.811	1.2	0.375	1.2	0.222
1.3	7.688	1.3	1.518	1.3	0.730	1.3	0.317	1.3	0.173
1.4	7.137	1.4	1.400	1.4	0.660	1.4	0.269	1.4	0.137

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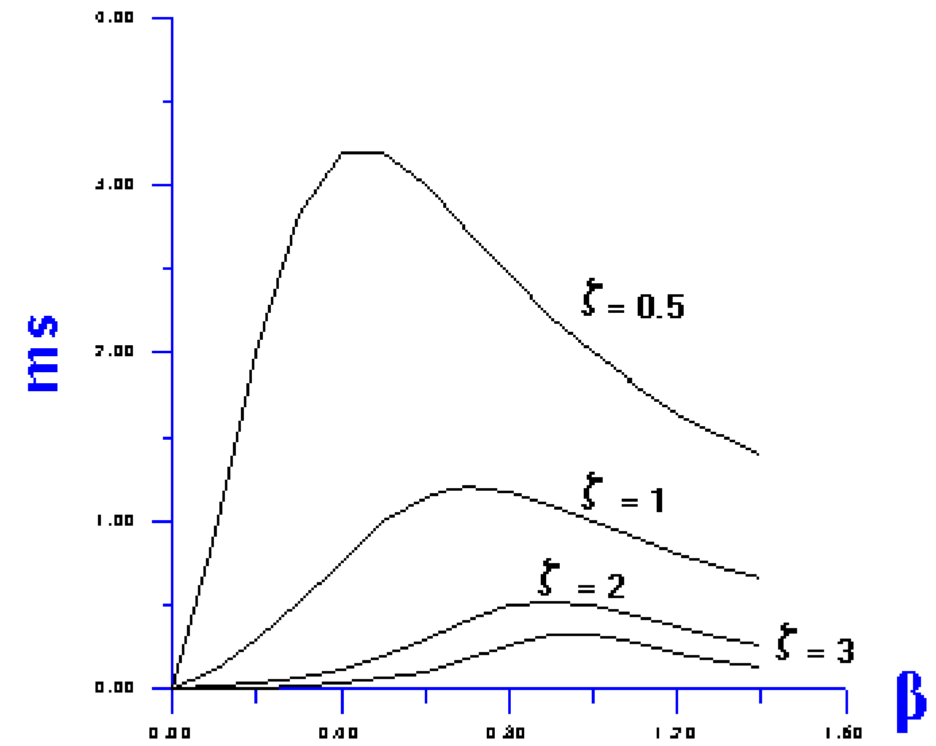

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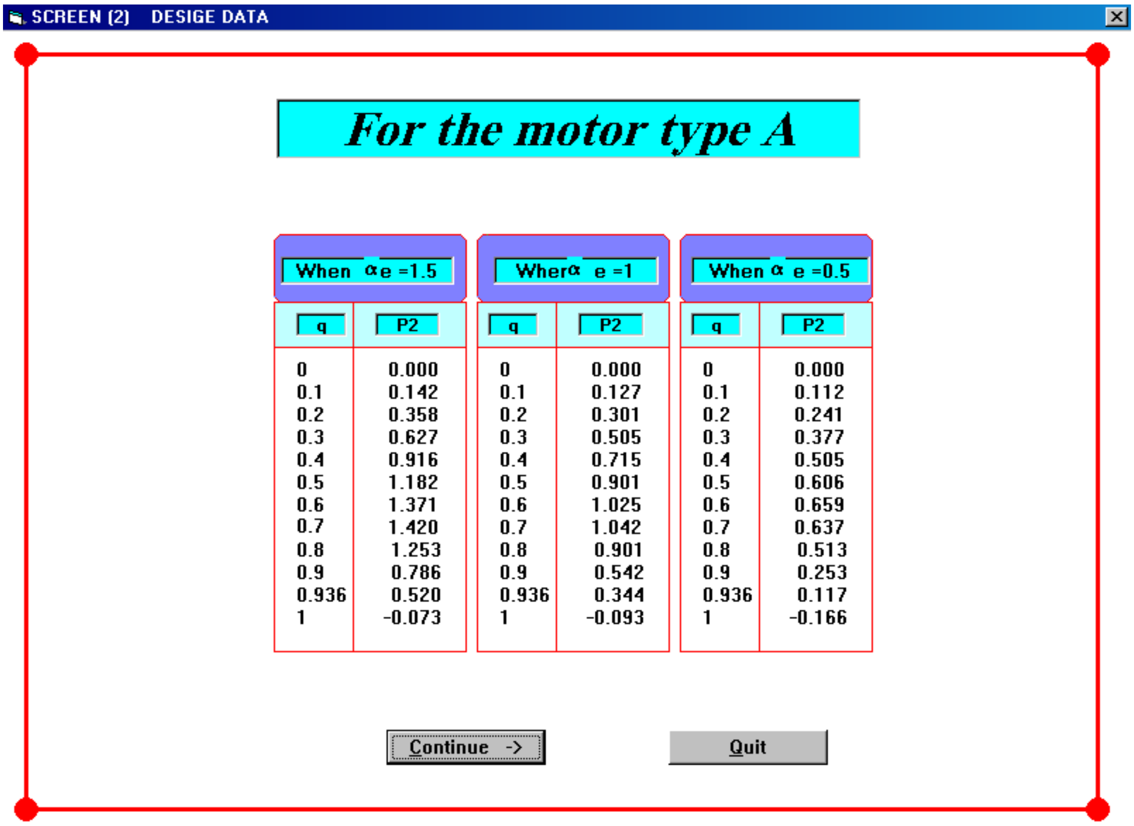
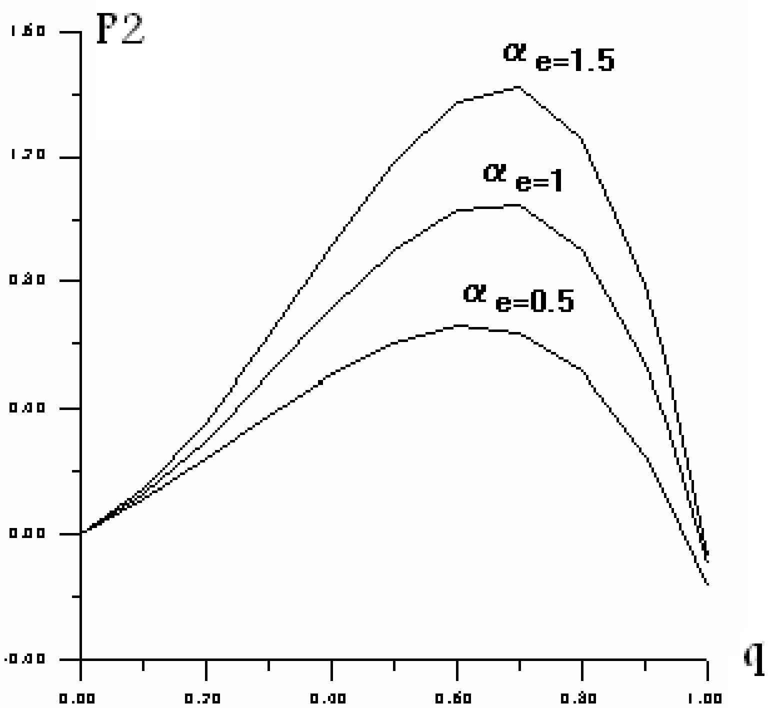


When αe =1.5		When αe =1		When αe =0.5	
q	P2	q	P2	q	P2
0	0.000	0	0.000	0	0.000
0.1	0.142	0.1	0.127	0.1	0.112
0.2	0.358	0.2	0.301	0.2	0.241
0.3	0.627	0.3	0.505	0.3	0.377
0.4	0.916	0.4	0.715	0.4	0.505
0.5	1.182	0.5	0.901	0.5	0.606
0.6	1.371	0.6	1.025	0.6	0.659
0.7	1.420	0.7	1.042	0.7	0.637
0.8	1.253	0.8	0.901	0.8	0.513
0.9	0.786	0.9	0.542	0.9	0.253
0.936	0.520	0.936	0.344	0.936	0.117
1	-0.073	1	-0.093	1	-0.166



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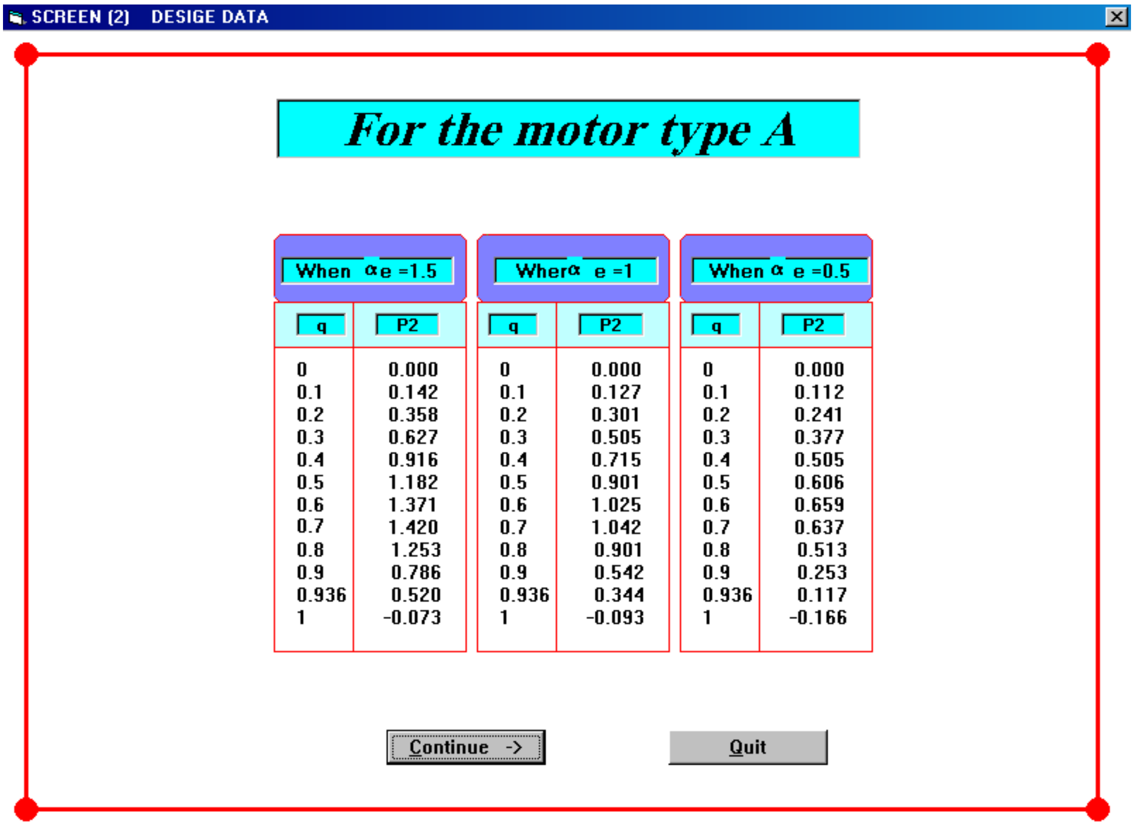
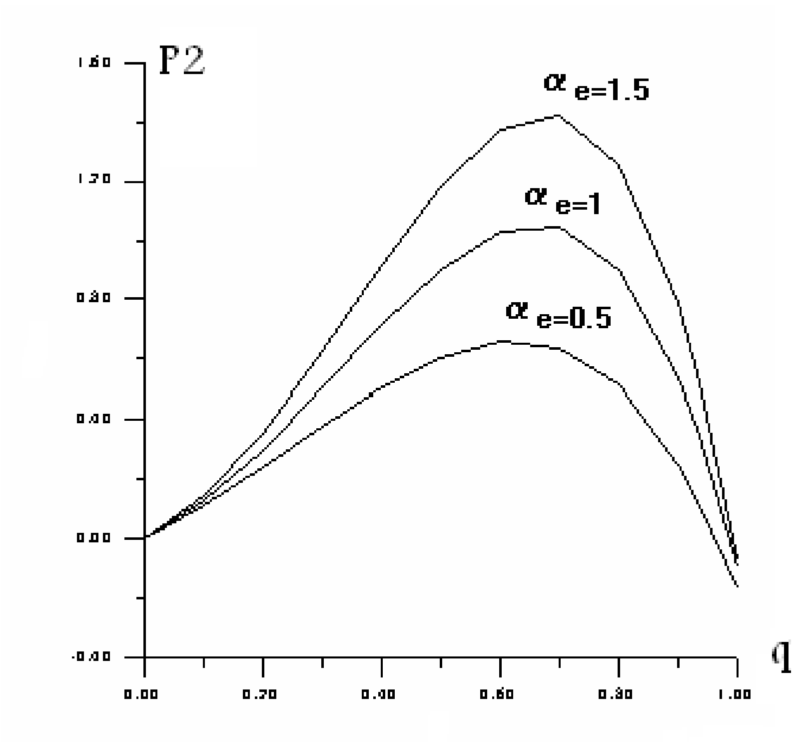

SCREEN (2) DESIGE DATA

For the motor type A

When α_e =1.5		When α_e =1		When α_e =0.5	
q	P2	q	P2	q	P2
0	0.000	0	0.000	0	0.000
0.1	0.142	0.1	0.127	0.1	0.112
0.2	0.358	0.2	0.301	0.2	0.241
0.3	0.627	0.3	0.505	0.3	0.377
0.4	0.916	0.4	0.715	0.4	0.505
0.5	1.182	0.5	0.901	0.5	0.606
0.6	1.371	0.6	1.025	0.6	0.659
0.7	1.420	0.7	1.042	0.7	0.637
0.8	1.253	0.8	0.901	0.8	0.513
0.9	0.786	0.9	0.542	0.9	0.253
0.936	0.520	0.936	0.344	0.936	0.117
1	-0.073	1	-0.093	1	-0.166

Continue -> Quit



- c -

Fig. 5 (a,b,c) : a- $m_s = f(\beta)$ for different ζ values , b- m = f (q) for different α_e values c- $P_2 = f(q)$ for different α_e values

The motor type B: - a -

SCREEN (2) DESIGE DATA

For the motor type B

When A= 0.103		When A= 0.5		When A= 1		When A= 2		When A= 3	
β	ms	β	ms	β	ms	β	ms	β	ms
0	0.000	0	0.000	0	0.000	0	0.000	0	0.000
0.1	50.860	0.1	0.800	0.1	0.104	0.1	0.013	0.1	0.004
0.2	38.666	0.2	1.700	0.2	0.250	0.2	0.033	0.2	0.010
0.3	28.157	0.3	2.400	0.3	0.440	0.3	0.062	0.3	0.019
0.4	21.725	0.4	2.720	0.4	0.654	0.4	0.106	0.4	0.033
0.5	17.578	0.5	2.720	0.5	0.850	0.5	0.170	0.5	0.057
0.6	14.723	0.6	2.550	0.6	0.981	0.6	0.255	0.6	0.094
0.62	14.257	0.62	2.507	0.62	0.997	0.62	0.274	0.62	0.104
0.64	13.819	0.64	2.462	0.64	1.009	0.64	0.293	0.64	0.115
0.66	13.406	0.66	2.416	0.66	1.018	0.66	0.312	0.66	0.127
0.68	13.017	0.68	2.369	0.68	1.023	0.68	0.331	0.68	0.139
0.7	12.649	0.7	2.322	0.7	1.026	0.7	0.350	0.7	0.153
0.8	11.081	0.8	2.092	0.8	1.000	0.8	0.425	0.8	0.227
0.822	11.559	0.822	2.167	0.822	1.015	0.822	0.404	0.822	0.202
0.9	9.854	0.9	1.883	0.9	0.933	0.9	0.450	0.9	0.283
1	8.870	1	1.700	1	0.850	1	0.425	1	0.283
1.1	8.063	1.1	1.542	1.1	0.766	1.1	0.374	1.1	0.240
1.2	7.389	1.2	1.407	1.2	0.689	1.2	0.319	1.2	0.189
1.3	6.819	1.3	1.291	1.3	0.621	1.3	0.270	1.3	0.147
1.4	6.331	1.4	1.190	1.4	0.561	1.4	0.229	1.4	0.117

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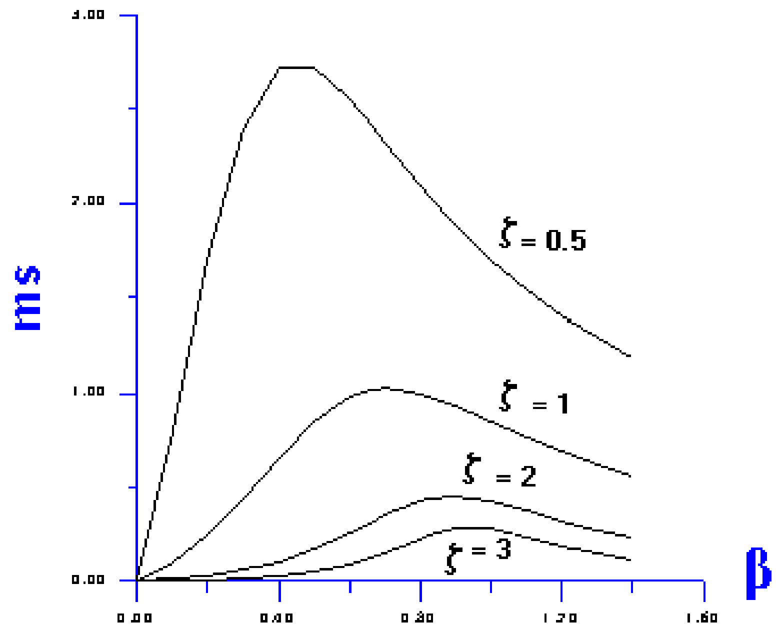


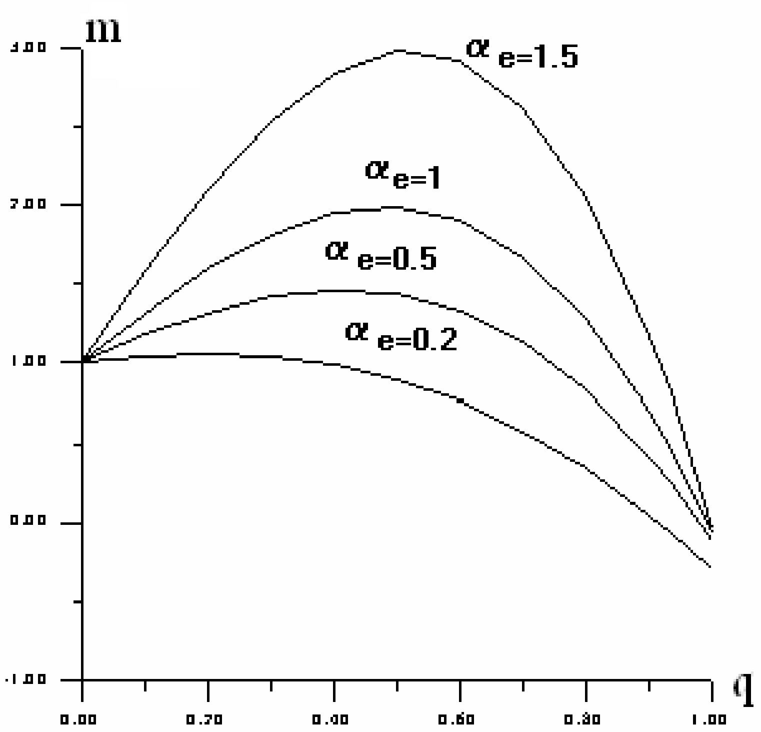
SCREEN (2) DESIGE DATA

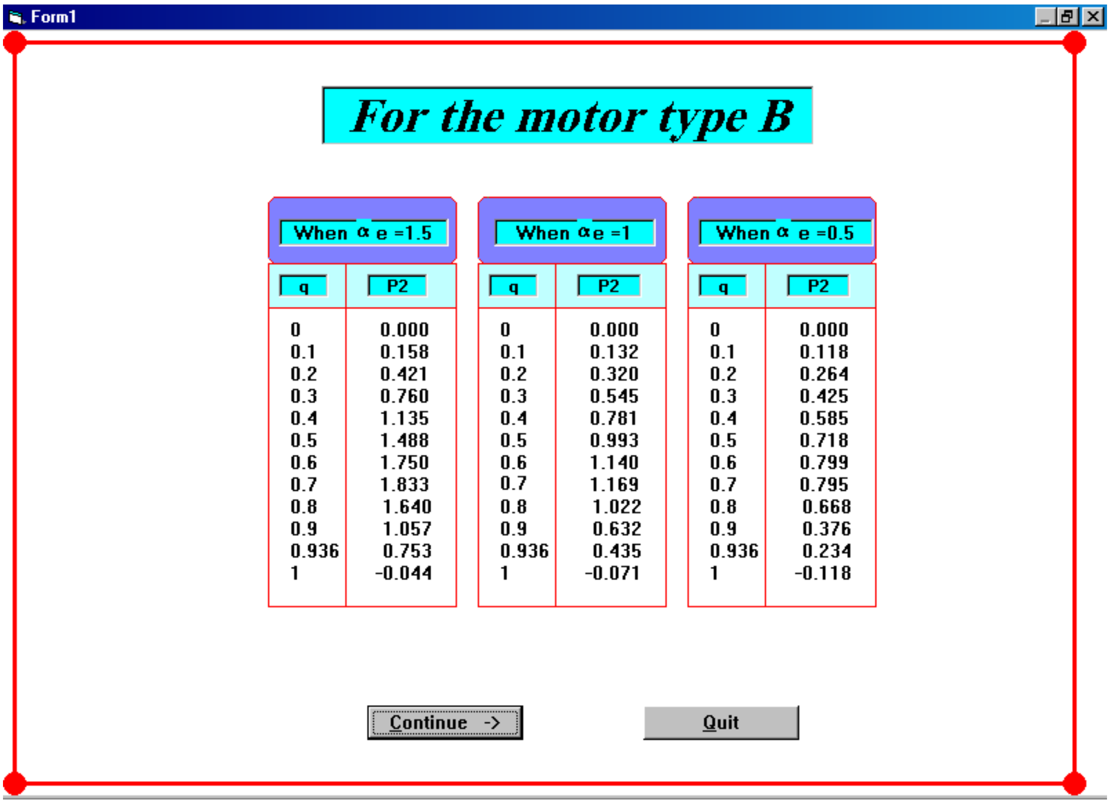
For the motor type B

When α e =1.5		When αe =1		When α e =0.5		When α e =0.2	
q	m	q	m	q	m	q	m
0	1.000	0	1.000	0	1.000	0	1.000
0.1	1.579	0.1	1.319	0.1	1.175	0.1	1.038
0.2	2.103	0.2	1.598	0.2	1.319	0.2	1.051
0.3	2.534	0.3	1.815	0.3	1.418	0.3	1.034
0.4	2.837	0.4	1.952	0.4	1.461	0.4	0.984
0.5	2.977	0.5	1.987	0.5	1.437	0.5	0.893
0.6	2.916	0.6	1.900	0.6	1.332	0.6	0.759
0.7	2.619	0.7	1.670	0.7	1.135	0.7	0.576
0.8	2.051	0.8	1.278	0.8	0.835	0.8	0.340
0.9	1.174	0.9	0.703	0.9	0.418	0.9	0.045
0.936	0.807	0.936	0.467	0.936	0.251	0.936	-0.067
1	-0.044	1	-0.071	1	-0.118	1	-0.292

Continue -> Quit

- b -





Form1

For the motor type B

When α e =1.5		When α e =1		When α e =0.5	
q	P2	q	P2	q	P2
0	0.000	0	0.000	0	0.000
0.1	0.158	0.1	0.132	0.1	0.118
0.2	0.421	0.2	0.320	0.2	0.264
0.3	0.760	0.3	0.545	0.3	0.425
0.4	1.135	0.4	0.781	0.4	0.585
0.5	1.488	0.5	0.993	0.5	0.718
0.6	1.750	0.6	1.140	0.6	0.799
0.7	1.833	0.7	1.169	0.7	0.795
0.8	1.640	0.8	1.022	0.8	0.668
0.9	1.057	0.9	0.632	0.9	0.376
0.936	0.753	0.936	0.435	0.936	0.234
1	-0.044	1	-0.071	1	-0.118

Continue -> Quit

- c -

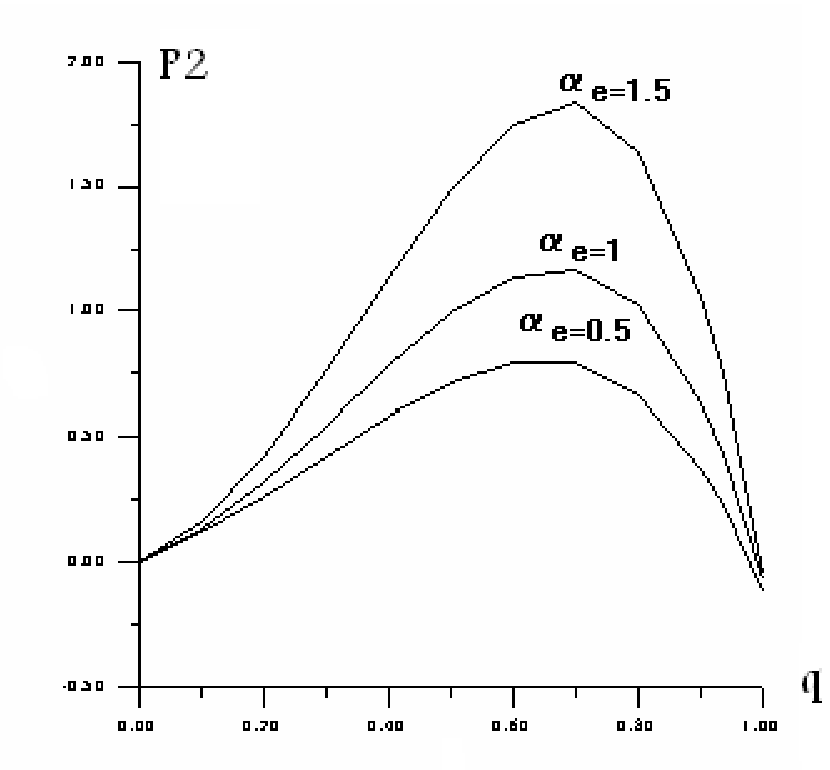


Fig. 6 (a,b,c) : a- $m_s = f (\beta)$ for different ζ values , b- $m = f (q)$ for different α_e values

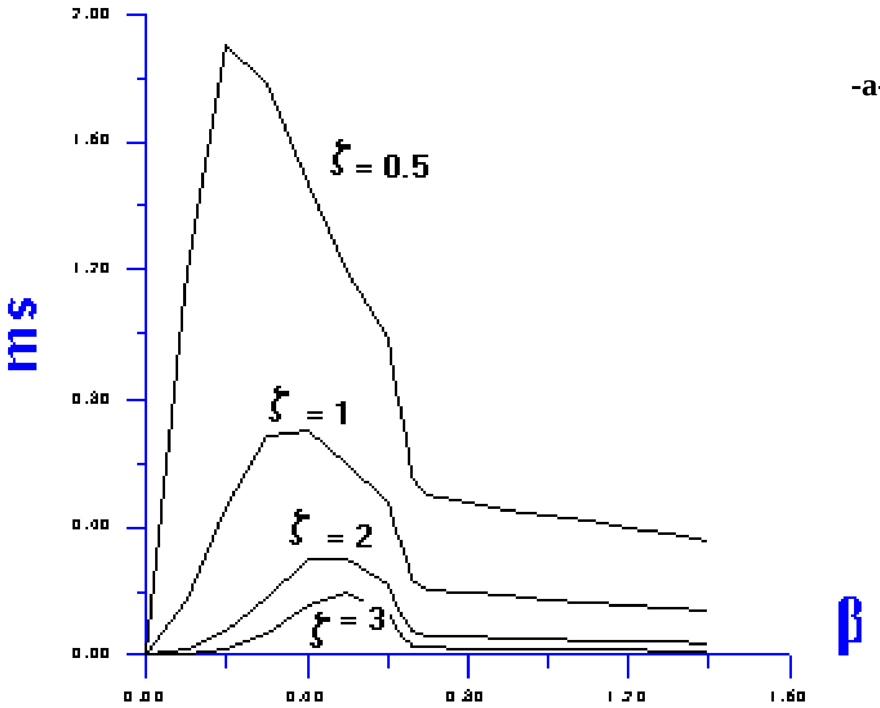
The motor type **C**:

SCREEN (2) DESIGE DATA

For the motor type C

When A= 0.142		When A= 0.5		When A= 1		When A= 2		When A= 3	
β	ms	β	ms	β	ms	β	ms	β	ms
0	0.000	0	0.000	0	0.000	0	0.000	0	0.000
0.1	16.478	0.1	1.190	0.1	0.175	0.1	0.023	0.1	0.007
0.2	10.315	0.2	1.904	0.2	0.458	0.2	0.074	0.2	0.023
0.3	7.112	0.3	1.785	0.3	0.687	0.3	0.179	0.3	0.066
0.4	5.373	0.4	1.465	0.4	0.700	0.4	0.298	0.4	0.159
0.5	4.304	0.5	1.190	0.5	0.595	0.5	0.298	0.5	0.198
0.6	3.584	0.6	0.985	0.6	0.482	0.6	0.223	0.6	0.132
0.62	3.069	0.62	0.833	0.62	0.393	0.62	0.160	0.62	0.082
0.64	2.683	0.64	0.718	0.64	0.326	0.64	0.119	0.64	0.055
0.656	2.382	0.656	0.630	0.656	0.276	0.656	0.092	0.656	0.040
0.66	2.164	0.66	0.567	0.66	0.242	0.66	0.076	0.66	0.031
0.68	2.039	0.68	0.530	0.68	0.222	0.68	0.068	0.68	0.027
0.7	1.945	0.7	0.503	0.7	0.208	0.7	0.062	0.7	0.025
0.8	1.860	0.8	0.479	0.8	0.196	0.8	0.057	0.8	0.022
0.9	1.782	0.9	0.457	0.9	0.185	0.9	0.053	0.9	0.020
1	1.710	1	0.437	1	0.175	1	0.049	1	0.019
1.1	1.643	1.1	0.418	1.1	0.166	1.1	0.046	1.1	0.017
1.2	1.582	1.2	0.401	1.2	0.158	1.2	0.043	1.2	0.016
1.3	1.525	1.3	0.385	1.3	0.150	1.3	0.040	1.3	0.015
1.4	1.422	1.4	0.357	1.4	0.137	1.4	0.036	1.4	0.013

Continue -> Quit



-a-

SCREEN (2) DESIGE DATA

For the motor type C

When αe =1.5	
q	m
0	1.000
0.1	2.046
0.2	3.007
0.3	3.819
0.4	4.418
0.5	4.739
0.6	4.719
0.7	4.291
0.8	3.393
0.9	1.960
0.936	4.101
1	-0.053

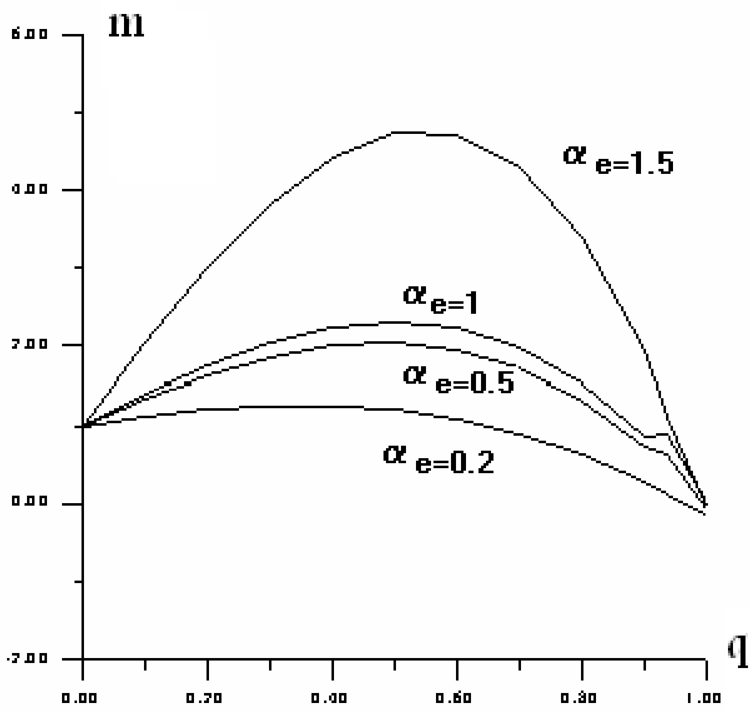
When α e =1	
q	m
0	1.000
0.1	1.404
0.2	1.762
0.3	2.049
0.4	2.239
0.5	2.308
0.6	2.228
0.7	1.975
0.8	1.524
0.9	0.847
0.936	1.896
1	0.032

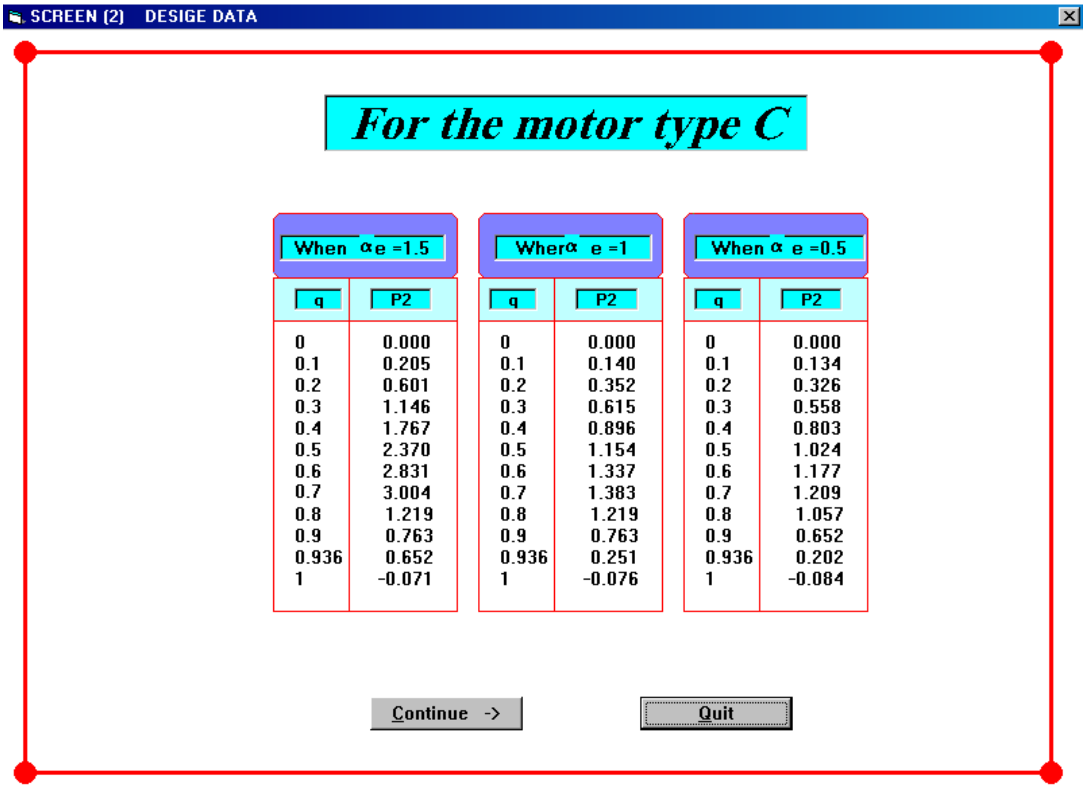
When α e =0.5	
q	m
0	1.000
0.1	1.336
0.2	1.630
0.3	1.861
0.4	2.008
0.5	2.049
0.6	1.962
0.7	1.727
0.8	1.321
0.9	0.724
0.936	1.636
1	-0.063

When α e =0.2	
q	m
0	1.000
0.1	1.114
0.2	1.199
0.3	1.246
0.4	1.248
0.5	1.196
0.6	1.081
0.7	0.894
0.8	0.627
0.9	0.272
0.936	0.831
1	-0.132

Continue ->   Quit

- b -





When αe =1.5 q	When αe =1.5 P2	When α e =1 q	When α e =1 P2	When α e =0.5 q	When α e =0.5 P2
0	0.000	0	0.000	0	0.000
0.1	0.205	0.1	0.140	0.1	0.134
0.2	0.601	0.2	0.352	0.2	0.326
0.3	1.146	0.3	0.615	0.3	0.558
0.4	1.767	0.4	0.896	0.4	0.803
0.5	2.370	0.5	1.154	0.5	1.024
0.6	2.831	0.6	1.337	0.6	1.177
0.7	3.004	0.7	1.383	0.7	1.209
0.8	1.219	0.8	1.219	0.8	1.057
0.9	0.763	0.9	0.763	0.9	0.652
0.936	0.652	0.936	0.251	0.936	0.202
1	-0.071	1	-0.076	1	-0.084

- c -

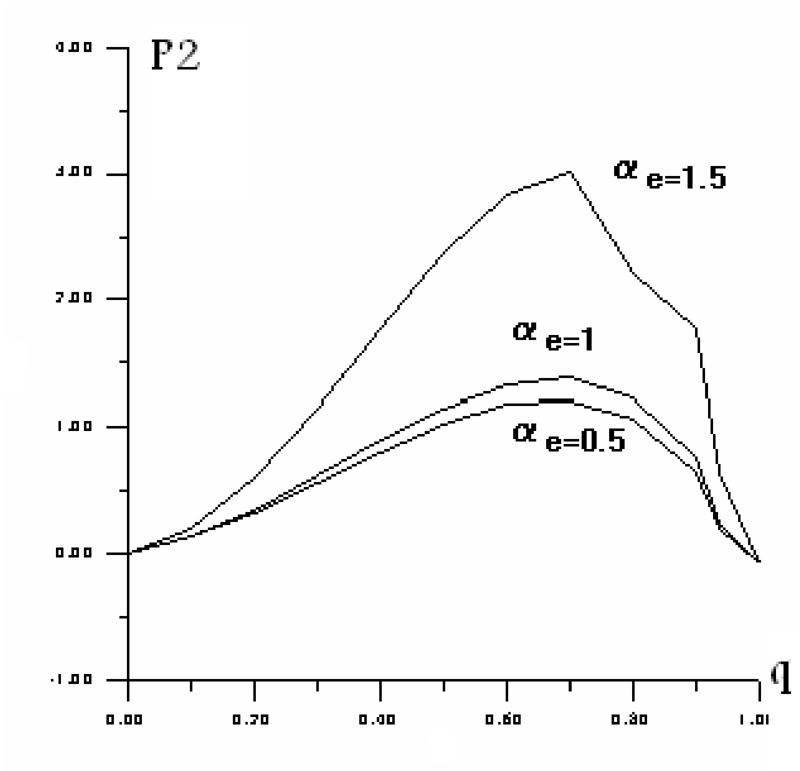


Fig. 7 (a,b,c) : a- m_s = f (β) for different ζ values , b- m = f (q) for different $α_e$ values

Appendix (A)

From Fig.3 ; the magnetizing impedance referred to the main winding is:

$Z_m = r_m + j\, X_m$

where:

$$r_m = P_{Fe} / 2I_m^2 \qquad [\Omega] \qquad \ldots \text{ (A - 1)}$$

and the forward and backward induced emf is:

$$\left.\begin{aligned} E_{1f} &= -(I_1 + jI_3')Z_f \\ E_{1b} &= -(I_1 + jI_3')Z_b \end{aligned}\quad \text{and} \quad \begin{aligned} E_{2f} &= I_f' Z_{2f}' \\ E_{2b} &= I_f' Z_{2b}' \end{aligned}\right\} \qquad \ldots \text{ (A - 2)}$$

Also the forward and backward rotor impedance is expressed by:

$$\left.\begin{aligned} Z_{2f}' &= \frac{r_2'}{s} + jX_2' \\ Z_{2b}' &= \frac{r_2'}{2-s} + jX_2' \end{aligned}\right\} \qquad \ldots \text{ (A - 3)}$$

Where:

$s = (w_1 - w_2) / w_1$ and $w_1 = 2\pi f_1 / P$, w_2- rotor angular velocity or $s = (n_1 - n) / n_1$

The total forward and backward parameters are:

$$\left.\begin{aligned} Z_f &= Z_m \frac{Z_{2f}'}{Z_m + Z_{2f}'} = R_f + jX_f & I_f' &= -(I_1 + jI_3')\frac{Z_f}{Z_{2f}'} \\ Z_b &= Z_m \frac{Z_{2b}'}{Z_m + Z_{2b}'} = R_b + jX_b & I_b' &= -(I_1 + jI_3')\frac{Z_b}{Z_{2b}'} \end{aligned}\right\} \qquad \ldots \text{ (A - 4)}$$

$$\left.\begin{aligned} R_f &= \frac{\frac{r_2'}{s}(r_m^2 + X_m^2) + r_m(\frac{r_2'^2}{s^2} + X_2'^2)}{(\frac{r_2'}{s} + r_m)^2 + (X_2' + X_m)^2} , & R_b &= \frac{\frac{r_2'}{2-s}(r_m^2 + X_m^2) + r_m(\frac{r_2'^2}{(2-s)^2} + X_2'^2)}{(\frac{r_2'}{(2-s)} + r_m)^2 + (X_2' + X_m)^2} \\ X_f &= \frac{X_2'(r_m^2 + X_m^2) + X_m(\frac{r_2'^2}{s^2} + X_2'^2)}{(\frac{r_2'}{s} + r_m)^2 + (X_2' + X_m)^2} , & X_b &= \frac{X_2'(r_m^2 + X_m^2) + X_m(\frac{r_2'^2}{(2-s)^2} + X_2'^2)}{(\frac{r_2'}{(2-s)} + r_m)^2 + (X_2' + X_m)^2} \end{aligned}\right\} \qquad \ldots \text{ (A - 5)}$$

The total emf in the main winding:

$E_1 = E_f + E_{1b} = - I_1 (Z_f + Z_b) – j I'_3 (Z_f – Z_b)$

And the total emf in the control winding:

$$E'_3 = E'_{3f} + E'_{3b}$$

Where

$$E'_{3f} = -jE_{1f} = +j(I_1 + jI'_3)Z_f$$

$$E'_{3b} = -jE_{1b} = +j(I_1 + jI'_3)Z_b \qquad \dots \text{ (A - 6)}$$

Or

$$E'_3 = jI_1(Z_f - Z_b) - I'_3(Z_f + Z_b) \qquad \dots \text{ (A - 7)}$$

From Kirchhoff equation

$$\left.\begin{aligned} V_1 + E_1 &= I_1(Z_1 + Z_c) \\ V'_3 + E'_3 &= I'_3 Z'_3 \end{aligned}\right\} \qquad \dots \text{ (A - 8)}$$

Using equations (A-6) and (A-7) in equation (A-8) we have:

$$\left.\begin{aligned} V_1 &= I_1\left[(R_1 + r_c) + j(X_1 - X_c)\right] + jI'_3(R_o + jX_o) \\ V'_3 &= -jI_1(R_o + JX_o) + I'_3(R_3 + jX_3) \end{aligned}\right\} \qquad \dots \text{ (A - 9)}$$

Where

$$R_o = R_f - R_b \qquad X_o = X_f - X_b$$
$$R_1 = r_1 + R_f + R_b \qquad X_1 = x_1 + X_f + X_b$$
$$R_3 = r_3 + R_f + R_b \qquad X_3 = x'_3 + X_f + X_b$$

The winding currents from eq. (A-8) are

$$\left.\begin{aligned} I_1 &= \frac{V_1(R_3 - jX_3) - jV_3'(R_o + jX_o)}{[(R_1 + r_c) + j(X_1 - X_c)](R_3 + jX_3) - (R_o + jX_o)^2} \\ I_3' &= \frac{V_3'[(R_1 + r_c) + j(X_1 - X_c)] + jV_1(R_o + jX_o)}{[(R_1 + r_c) + j(X_1 - X_c)](R_3 + jX_3) - (R_o + jX_o)^2} \end{aligned}\right\} \quad \ldots \; (A\text{-}10)$$

Appendix (B)

The starting current is:

$$I_{1s} = \frac{V_1}{\sqrt{R_1^2 + (X_1 - X_c)^2}} \quad , \quad \tan\varphi_{1s} = \frac{X_1 - X_c}{R_1} \quad \ldots \; (B\text{-}1)$$

and

$$I_{3s}' = \frac{V_3'}{\sqrt{R_3^2 + X_3^2}} \quad , \quad \tan\varphi_{3s} = \frac{X_3}{R_3} \quad \ldots \; (B\text{-}2)$$

$$R_1 = r_1 + 2R_{fs} \qquad R_3 = r_3' + 2R_{fs}$$

$$X_1 = x_1 + 2X_{fs} \qquad X_3 = x_3 + 2X_{fs}$$

Then the forward resistance and reactance at starting are:

$$\left.\begin{aligned} R_{fs} &= \frac{r_2' Z_m^2 + r_m Z_2'^2}{(r_2' + r_m)^2 + (X_2' + X_m)^2} \\ X_{fs} &= \frac{X_2' X_m^2 + X_m Z_2'^2}{(r_2' + r_m)^2 + (X_2' + X_m)^2} \end{aligned}\right\} \quad \ldots \; (B\text{-}3)$$

Appendix (C)

From assumptions given in (4-2) we have that:

$$\left.\begin{aligned} R_f &= \frac{\zeta X_m(1-q)}{\zeta^2+(1-q)^2} \qquad X_f = \frac{\zeta^2 X_m}{\zeta^2+(1-q)^2} \\ R_b &= \frac{\zeta X_o(1+q)}{\zeta^2+(1+q)^2} \qquad X_b = \frac{\zeta^2 X_m}{\zeta^2+(1+q)^2} \end{aligned}\right\} \quad \dots \ (\text{C - 1})$$

$$Z'_{2f} = \frac{\zeta X_m}{1-q} \qquad Z'_{2b} = \frac{\zeta X_m}{1+q}$$

$$\left.\begin{aligned} R_o &= \frac{2q\zeta X_m(1-q^2-\zeta^2)}{\left[\zeta^2+(1-q)^2\right]\left[\zeta^2+(1+q)^2\right]} \\ X_o &= \frac{4q\zeta^2 X_m}{\left[\zeta^2+(1-q)^2\right]\left[\zeta^2+(1+q)^2\right]} \end{aligned}\right\} \quad \dots \ (\text{C - 2})$$

$$\left.\begin{aligned} R_1 = R_3 &= \frac{2\zeta X_m(1-q^2+\zeta^2)}{\left[\zeta^2+(1-q)^2\right]\left[\zeta^2+(1+q)^2\right]} \\ X_1 = X_3 &= \frac{2\zeta X_m(1+q^2+\zeta^2)}{\left[\zeta^2+(1-q)^2\right]\left[\zeta^2+(1+q)^2\right]} \end{aligned}\right\} \quad \dots \ (\text{C - 3})$$

The electromagnetic power developed is then:

$$P_{em} = \frac{V_1^2\beta\alpha_e}{X_m\left[\zeta^2(1-\beta)^2+\beta^2\right]}\left\{1-q^2-q\left[\frac{\alpha_e\beta(\zeta^2+q^2+1)}{2\zeta}+\frac{\zeta(1+\alpha_e^2)}{2\beta\alpha_e}-\frac{\alpha_e(\zeta^2+\beta)}{\zeta}\right]\right\}$$

$$[W] \qquad \dots \ (\text{C - 4})$$

5- Conclusion:

The validity of the equations derived in this paper is checked by the accuracy of the curves obtained as in Figs (5 – 7) . The curves consistent with the theory of the amplitude control capacitor motor. It is clearly shown that the output power and torque are directly proportional with the effective control factor. While the torque at circular field is inversely proportional with the ratio of rotor resistance to reactance (ζ).

No.	Description	Symbol	Unit
1.	Main (excitation) winding	T_1	---
2.	Auxiliary (control) winding	T_3	---
3.	Frequency	f	Hz
4.	Rated voltage	V	V
5.	No. of phases	m	---
6.	Pole pairs	P	---
7.	Capacitance voltage	Vc	V
8.	Capacitance reactance	Xc	Ω
9.	Referring factor	K	---
10.	Control factor	α_c	---
11.	Starting control factor	α_{cs}	---
12.	Effective control factor	α_e	---
13.	Forward current	If	A
14.	Backward current	Ib	A
15.	Electromagnetic power	Pf	W
16.	Developed torque	T	N – m
17.	Starting torque	Ts	N – m
18.	Starting current	Is	A
19.	Electromagnetic torque	Tem	N – m
20.	Shaft useful torque	m_{ss}	---
21.	Mechanical useful power	P_2	W
22.	Power factor	cosφ	---

6- References:

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